Voltage operations on maniplexes, maps and polytopes

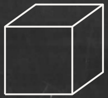
Antonio Montero

Joint work with Isabel Hubard and Elías Mochan

University of Ljubljana

November 2022

The idea



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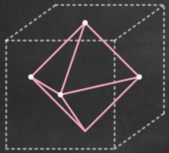

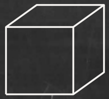

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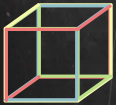
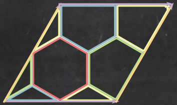
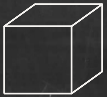
 $\cong$  

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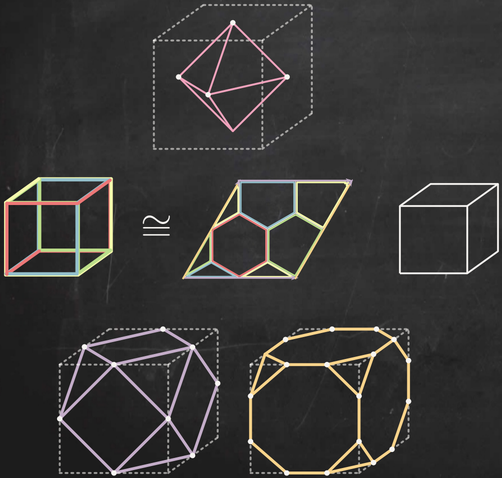

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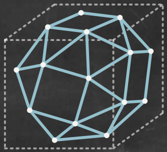

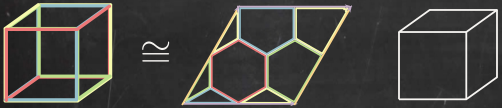

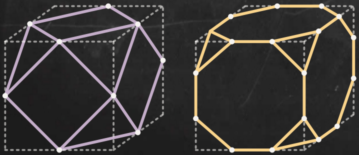

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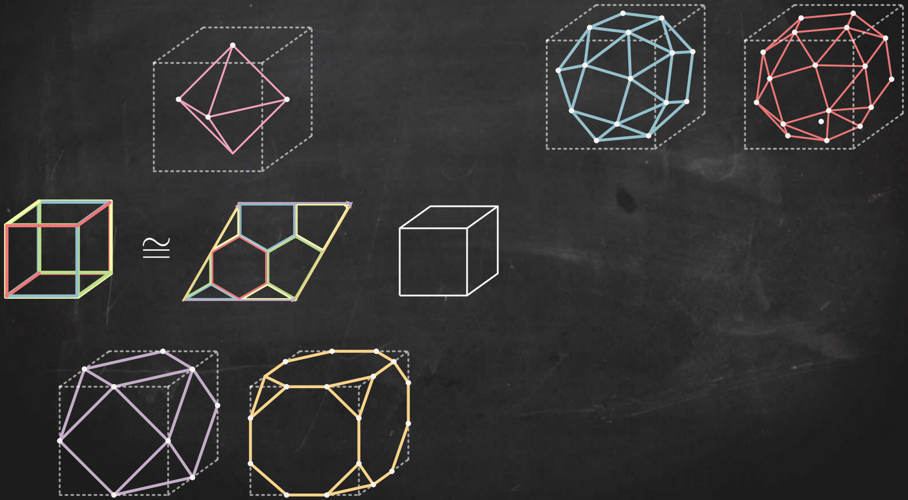

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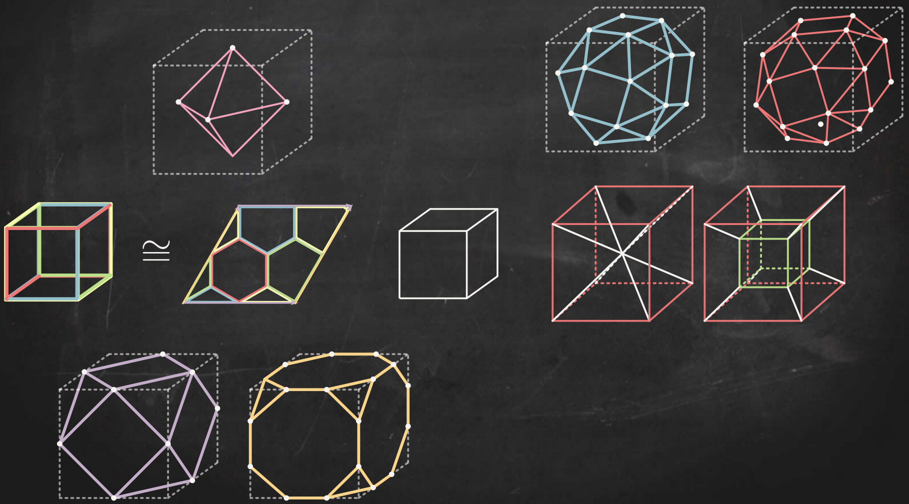

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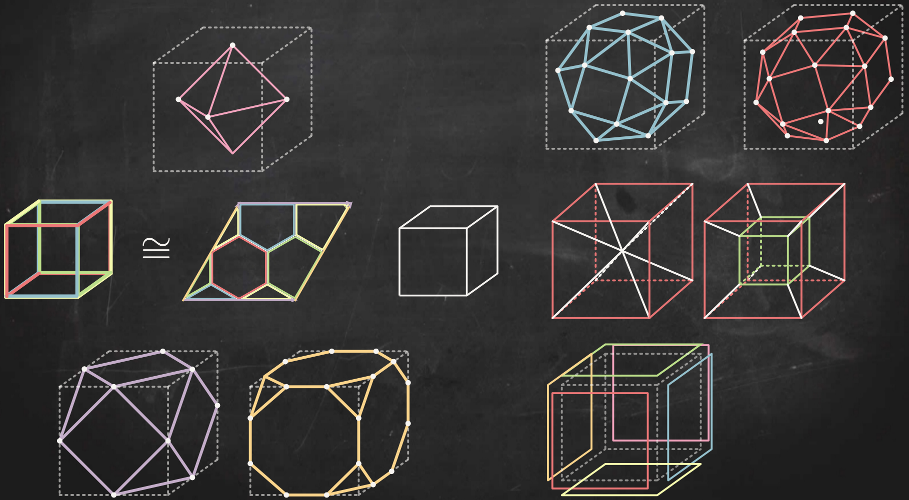

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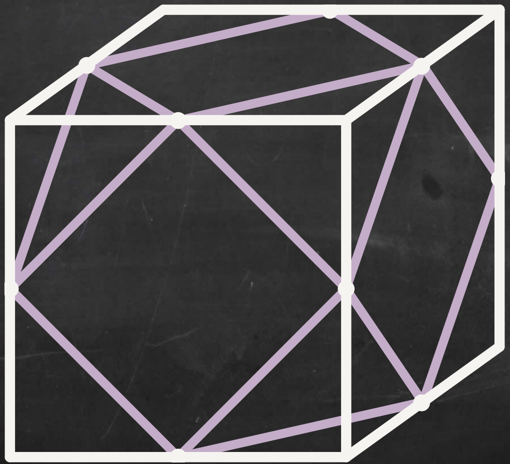

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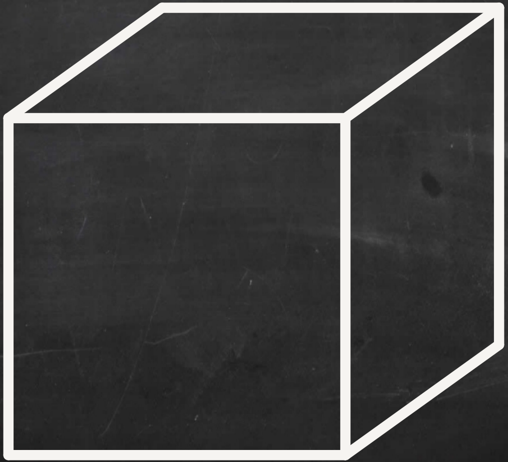

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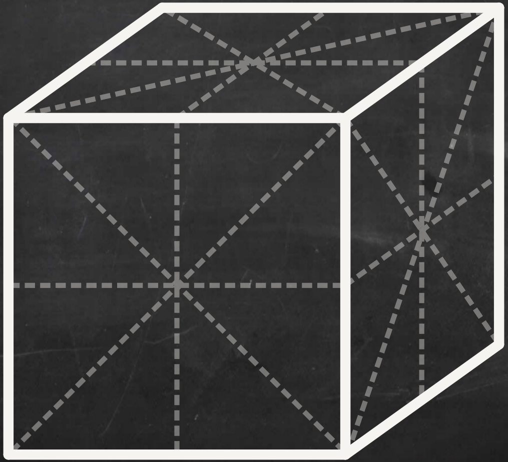

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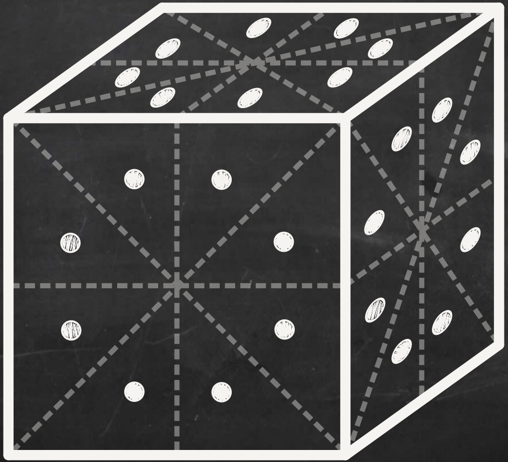

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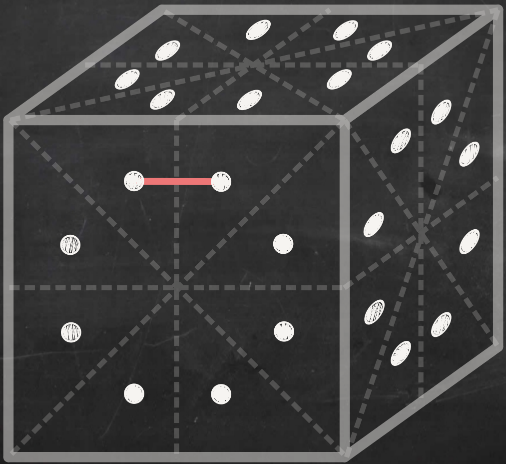

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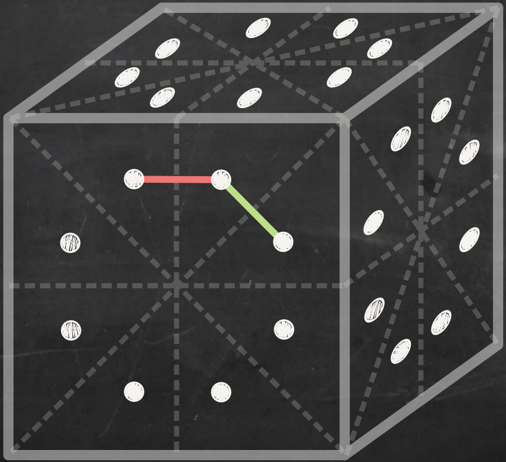

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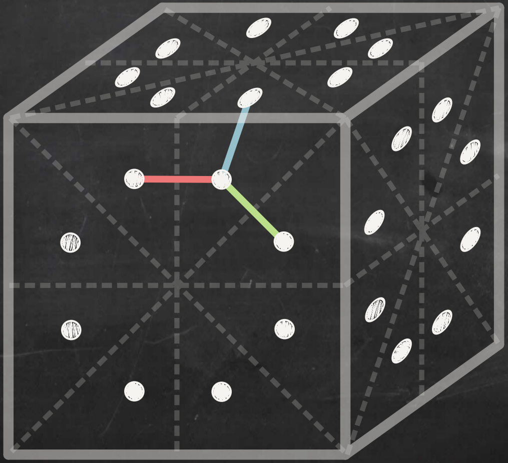

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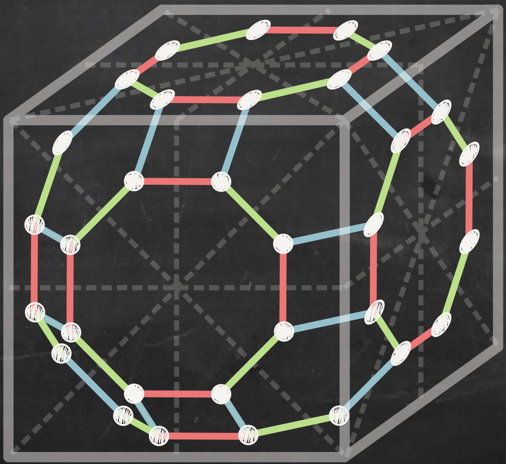
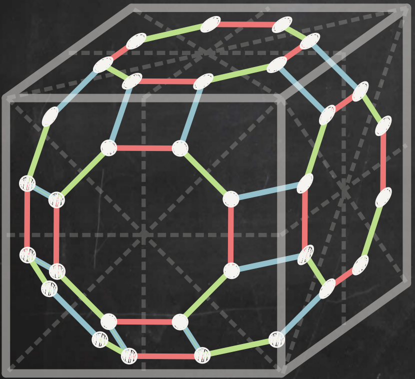


Flag-graphs of polytopes



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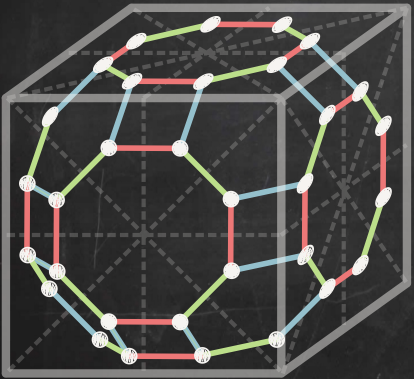

* Connected simple graph.

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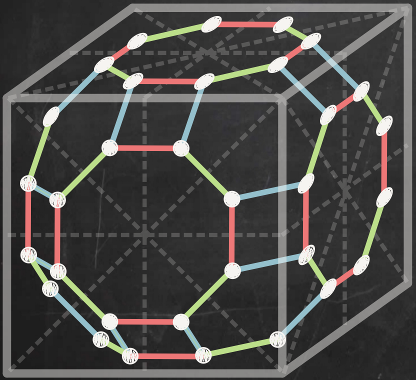

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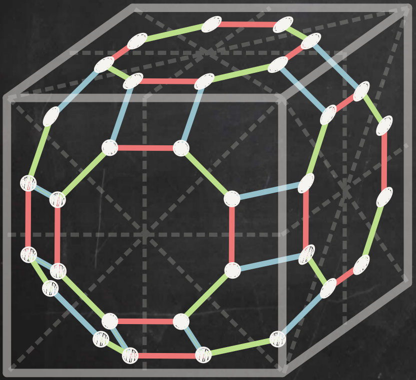

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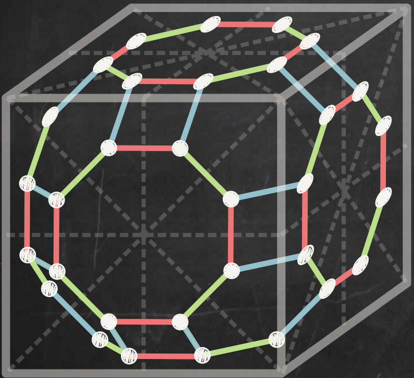

- Connected simple graph.
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- If $|i-j|>1$, then the (i,j)-factors are alternating squares

Maniplexes

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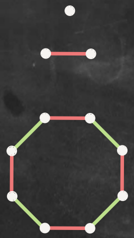

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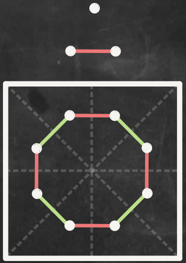

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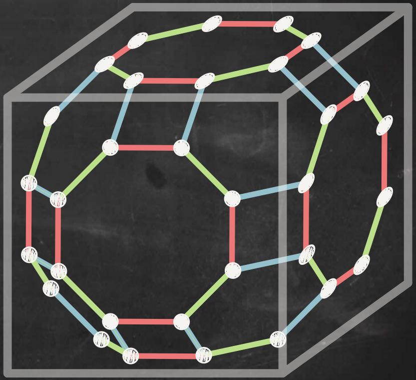

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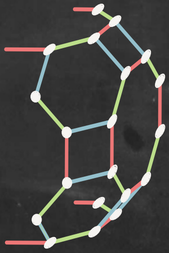

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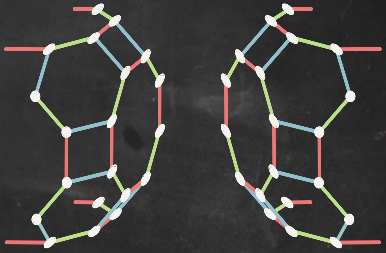

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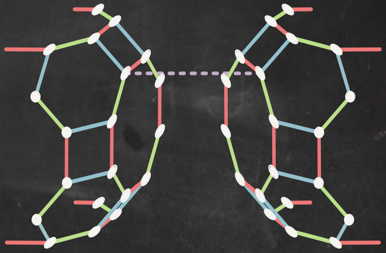

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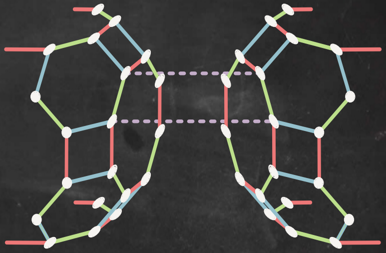

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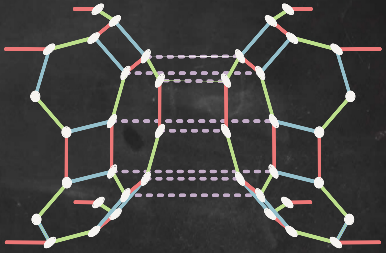

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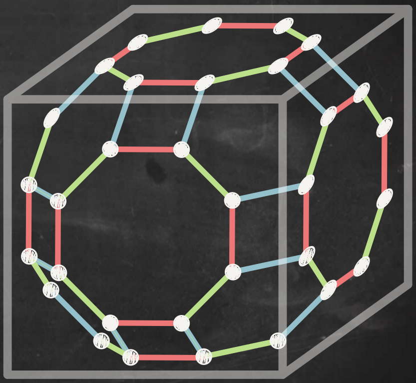

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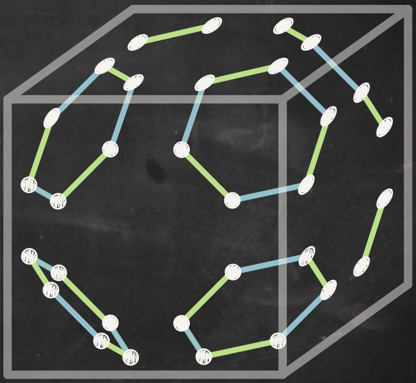

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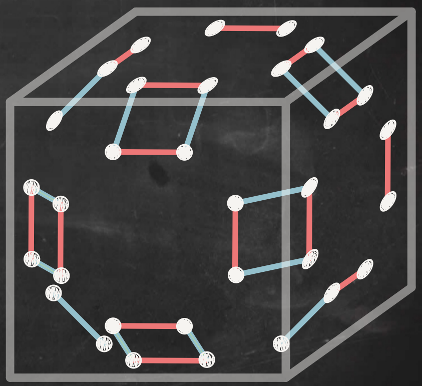

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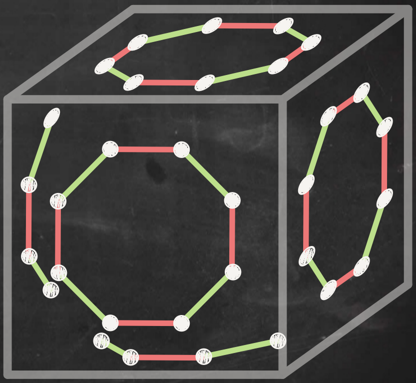

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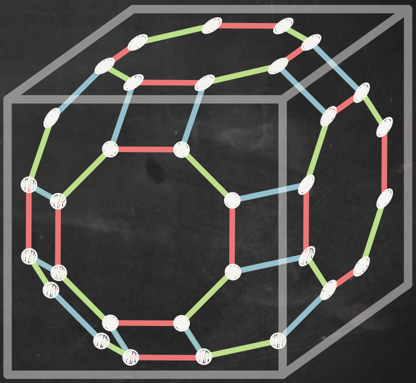

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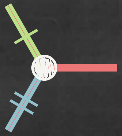

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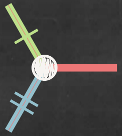

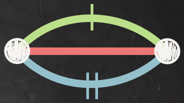

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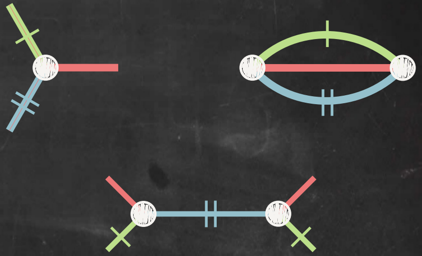

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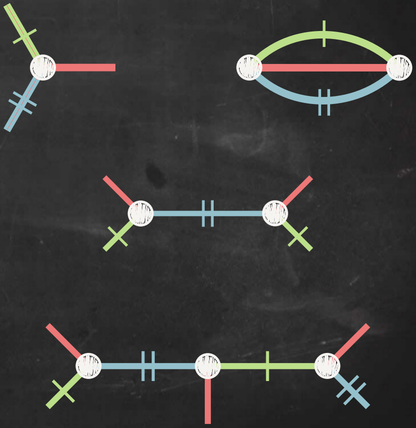

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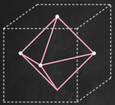
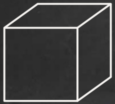
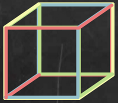
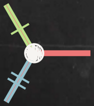
* If $\Gamma \leqslant \mathrm{Aut}(\mathcal{M})$, then the symmetry type graph of $\mathcal{M}$ (with respect to Γ) is the connected premaniplex $\mathcal{M}/\Gamma$.

Symmetry type graph









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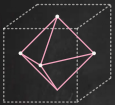
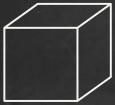
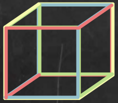
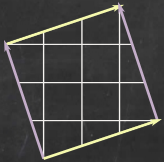
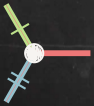

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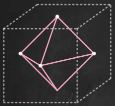
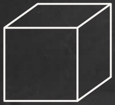
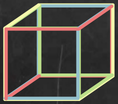
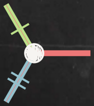
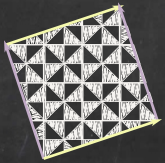


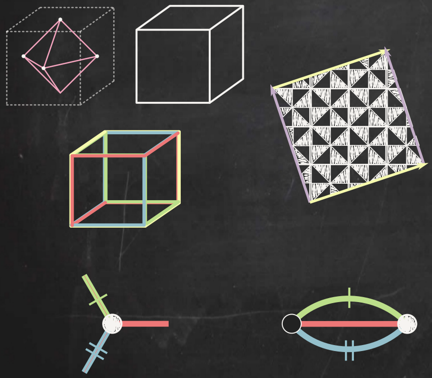








Symmetry type graph



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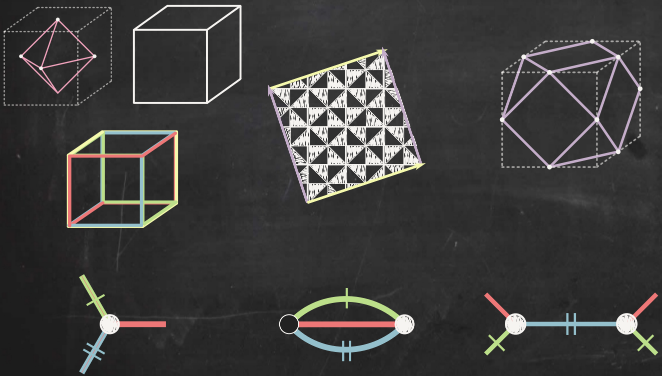

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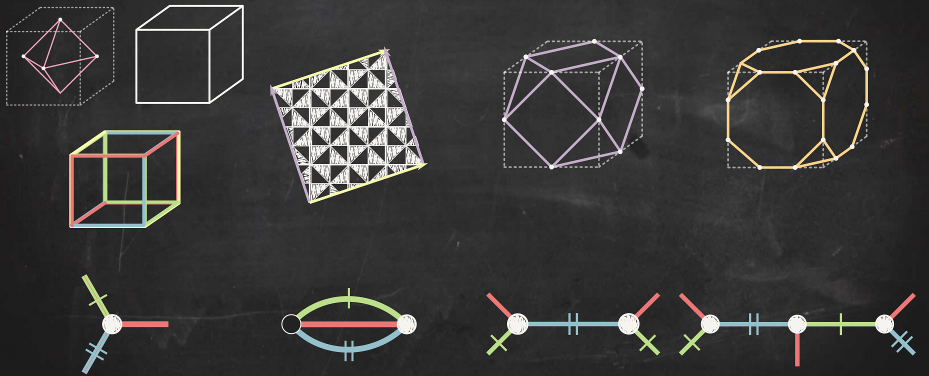

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A good action

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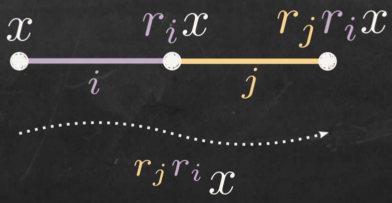

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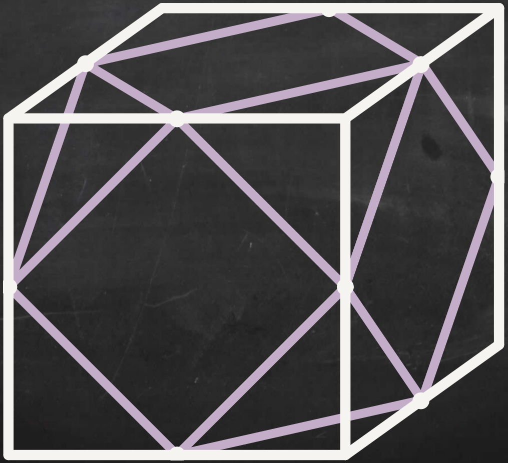
Voltage operations



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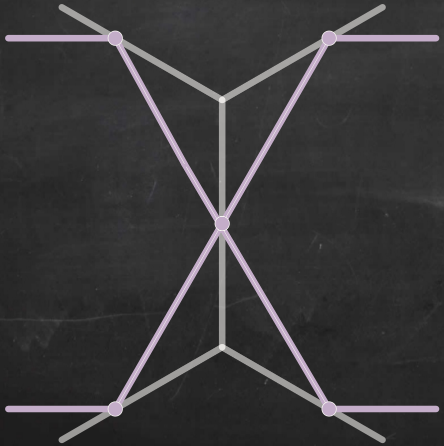

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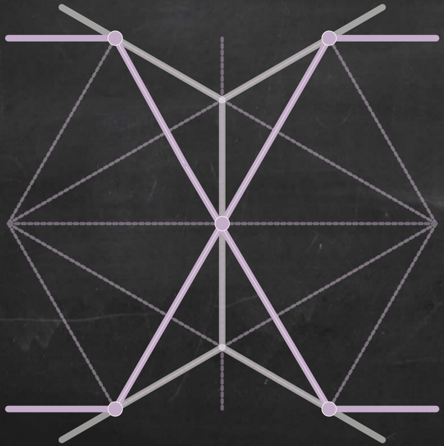

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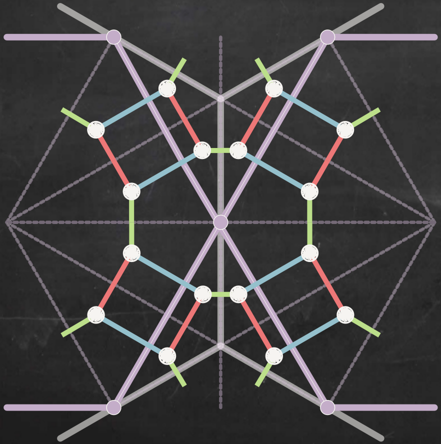

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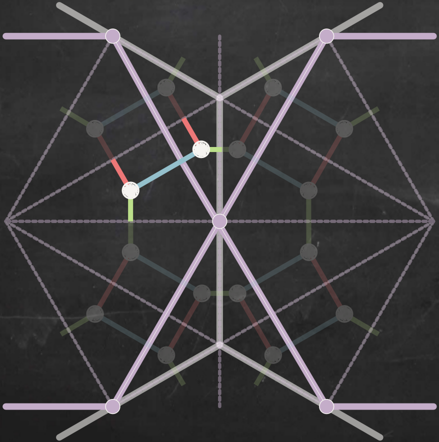

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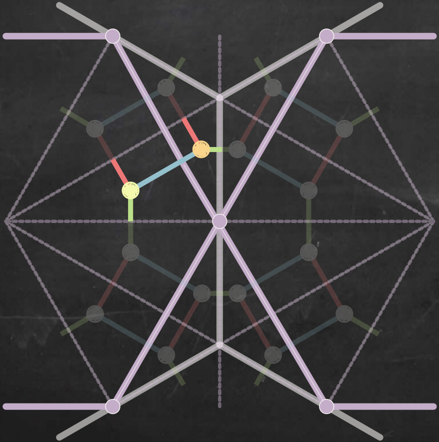

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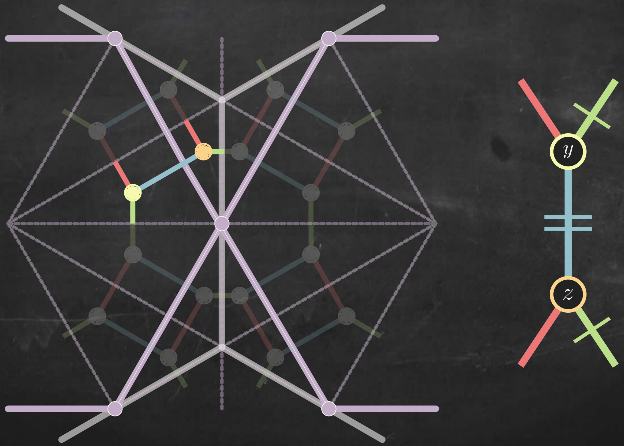

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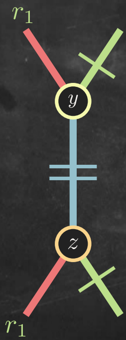

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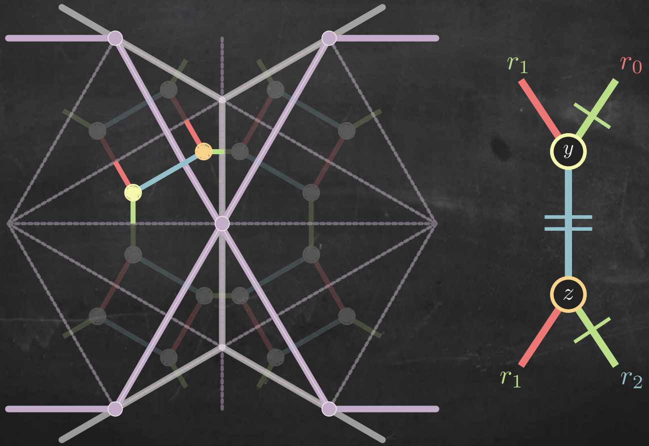

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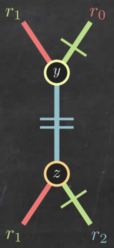
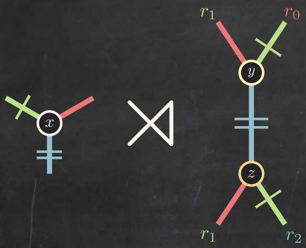


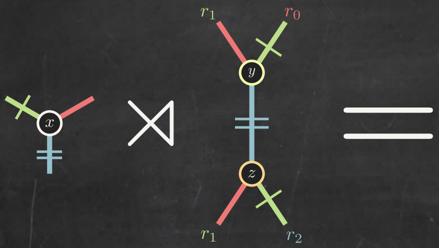
$$(\mathcal{Y}, \eta)$$

Voltage operations



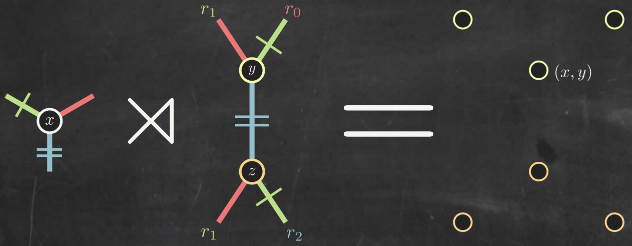
$\mathcal{X}$ $(\mathcal{Y}, \eta)$

Voltage operations



$\mathcal{X}$ $\quad$ $(\mathcal{Y}, \eta)$ $\quad$ $\mathcal{X} \rtimes_\eta \mathcal{Y}$

Voltage operations



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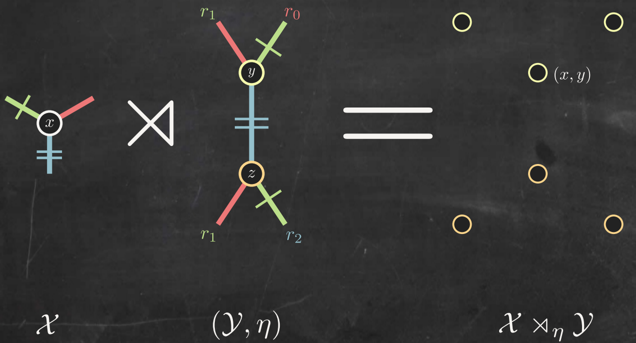
$$V(\mathcal{X} \rtimes_\eta \mathcal{Y}) = V(\mathcal{X}) \times V(\mathcal{Y})$$

Voltage operations



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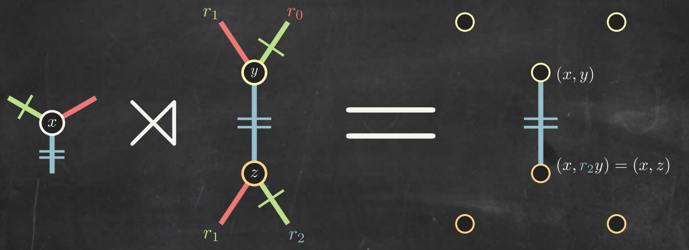
Voltage operations



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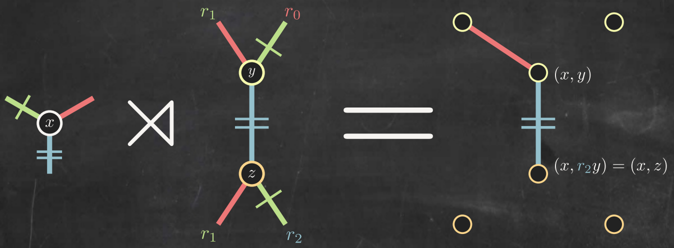

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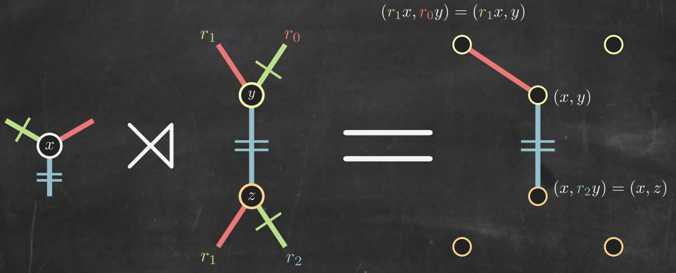

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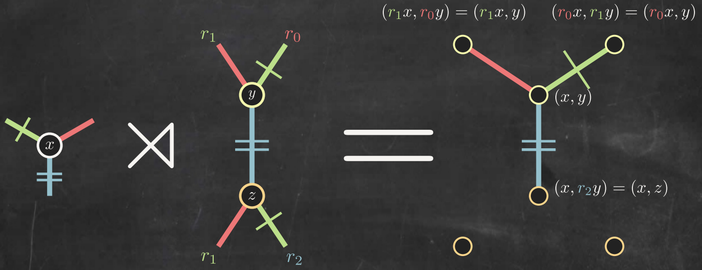

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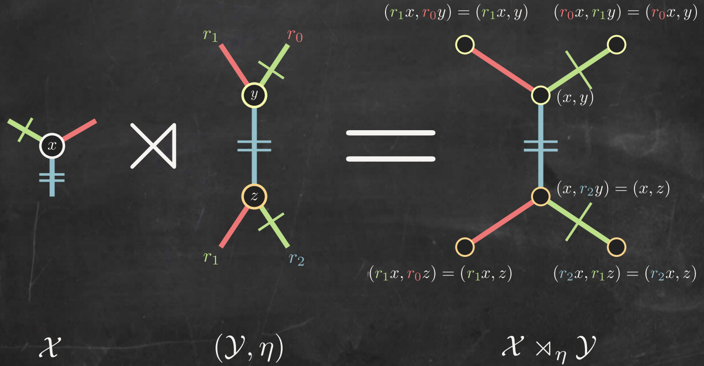

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Voltage operations

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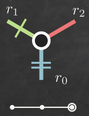
Voltage operations



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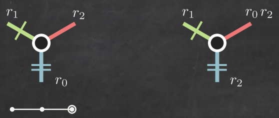

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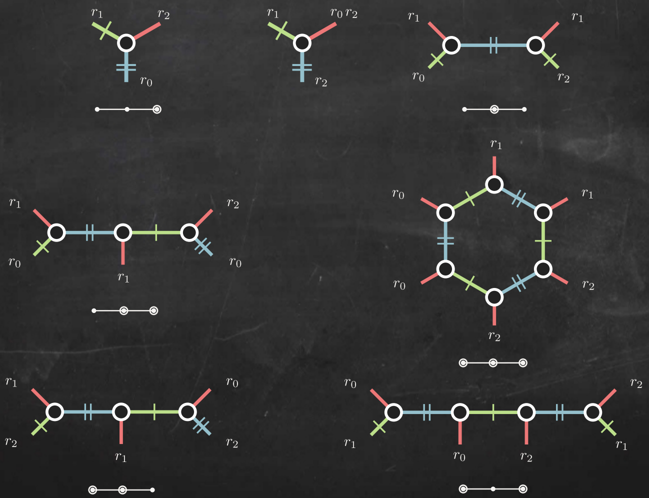
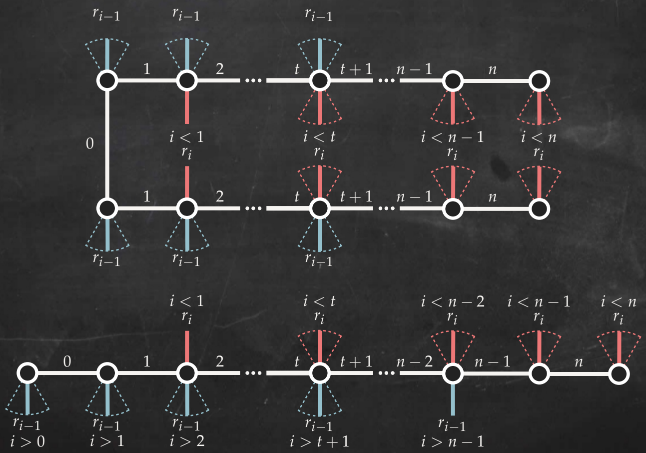


Voltage operations
Prisms and pyramids



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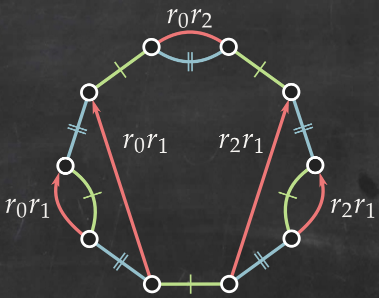

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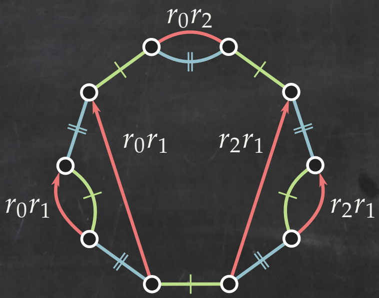

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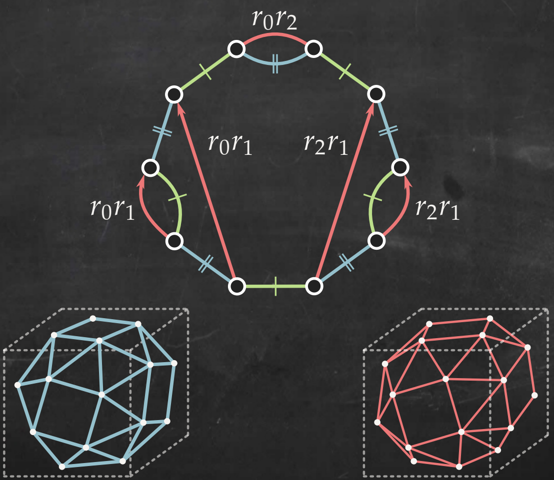

Parallel product (mix)

If $\mathcal{Y}$ is a premaniplex, we denote by $\mu_{\mathcal{Y}}$ the voltage assignment that gives the voltage r_i to each dart of colour i.

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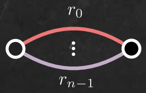
$\mathcal{X} \rtimes$  $= MOC(\mathcal{X})$

Voltage operations

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If $(\mathcal{Y}_1, \eta_1)$ is an (n, m)-voltage operator, and $(\mathcal{Y}_2, \eta_2)$ is a (m, ℓ)-voltage operator, then there exists a voltage $\theta : \mathcal{Y}_1 \rtimes_{\eta_2} \mathcal{Y}_2 \to W_n$ such that

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Corollary

If $\mathcal{M}$ is a non-orientable maniplex, then $\mathrm{Snub}(\mathcal{M})$ is isomorphic to a connected component of $\mathrm{Snub}(MOC(\mathcal{M}))$.

Voltage operations
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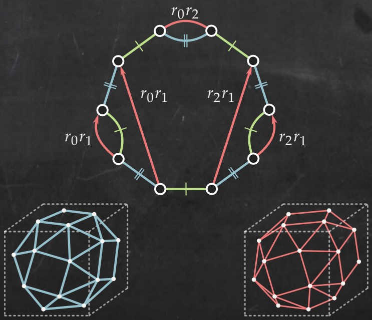

Voltage operations

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Voltage operations

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If $\mathcal{Z} = \mathcal{X}/\Gamma$ for $\Gamma \leqslant \mathrm{Aut}(\mathcal{X})$, then $\mathcal{Z} \rtimes \mathcal{Y} = (\mathcal{X} \rtimes \mathcal{Y})/\Gamma$.

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Assume that $\mathcal{O}$ is a mapping that assigns a premaniplex $\mathcal{O}(\mathcal{X})$ to each premaniplex $\mathcal{X}$, then $\mathcal{O}$ is a voltage operation if and only if

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Voltage operations

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Assume that $\mathcal{O}$ is a mapping that assigns a premaniplex $\mathcal{O}(\mathcal{X})$ to each premaniplex $\mathcal{X}$, then $\mathcal{O}$ is a voltage operation if and only if

* $\mathrm{Aut}(\mathcal{U})$ acts by automorphisms on $\mathcal{O}(\mathcal{U})$ and
* for every $\Gamma \leqslant \mathrm{Aut}(\mathcal{U})$,

$$\mathcal{O}(\mathcal{U}/\Gamma) \cong \mathcal{O}(\mathcal{U})/\Gamma.$$

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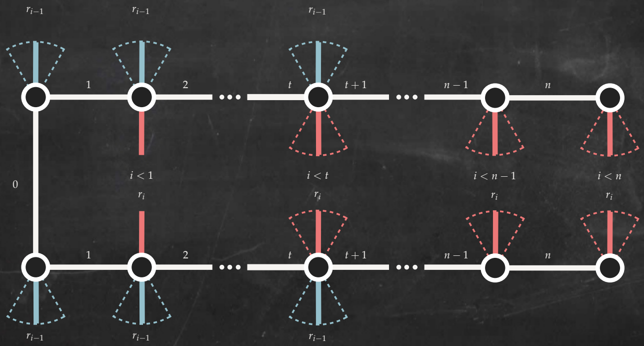

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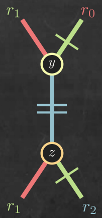

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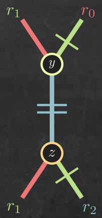

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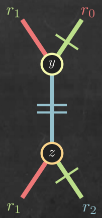

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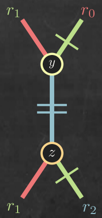

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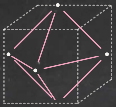

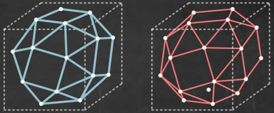

Thank you!