Chiral extensions of regular toroids

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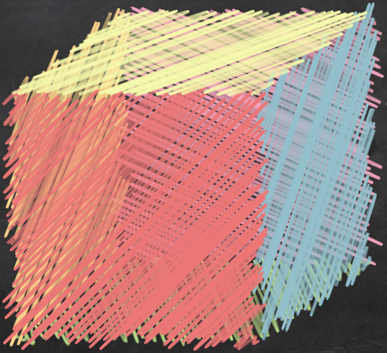
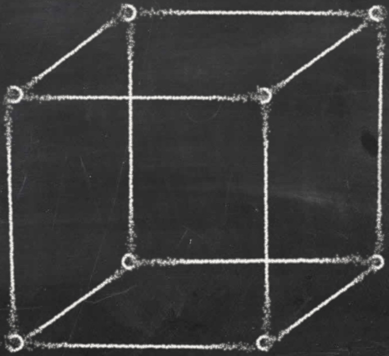
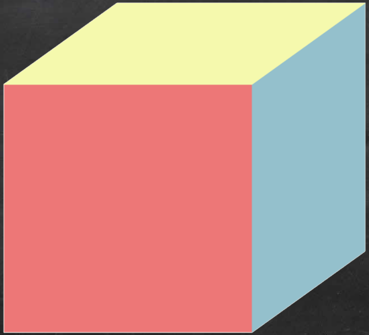
joint work with Daniel Pellicer and Micael Toledo

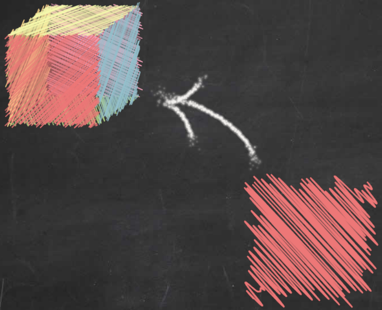
Centro de Ciencias Matemáticas - UNAM

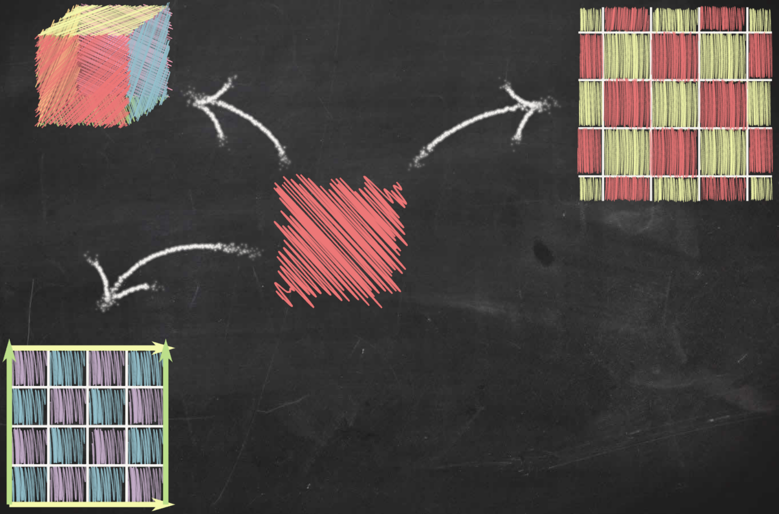
8th PhD Summer School in Discrete Mathematics
Rogla, Slovenia July 2018

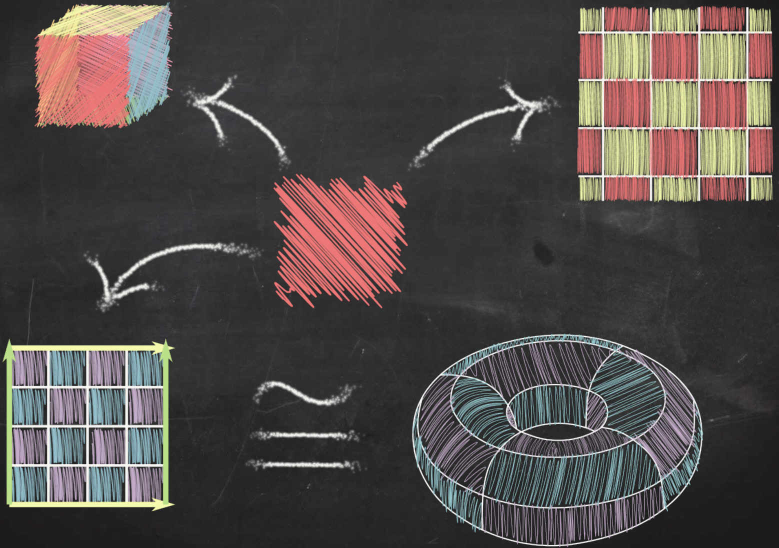


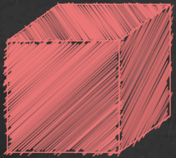












Recursive Construction

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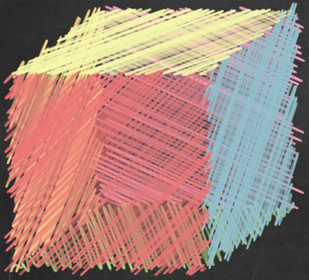
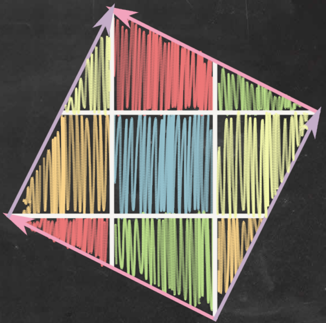
Strong Connectivity Conditions

Recursive Construction

Strong Connectivity Conditions

Abstract Polytopes





Regular polytopes

Regular polytopes

- Platonic solids

Regular polytopes

- Platonic solids
- Convex

Regular polytopes

- Platonic solids
- Convex
- Tillings

Regular polytopes

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Chiral polytopes

- 70's: Chiral maps and twisted honeycombs

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- 2018: There are plenty but they are huge

Given an abstract n-polytope K,

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does there exist a chiral $(n+1)$-polytope $\mathcal{P}$ with all its facets isomorphic to $\mathcal{K}$?

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Determined by its group:

Chiral polytopes

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- Generators and relations

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Regular o chiral facets, regular subfacets

$\mathcal{P}$

chiral

K

chiral

w/reg facets

P

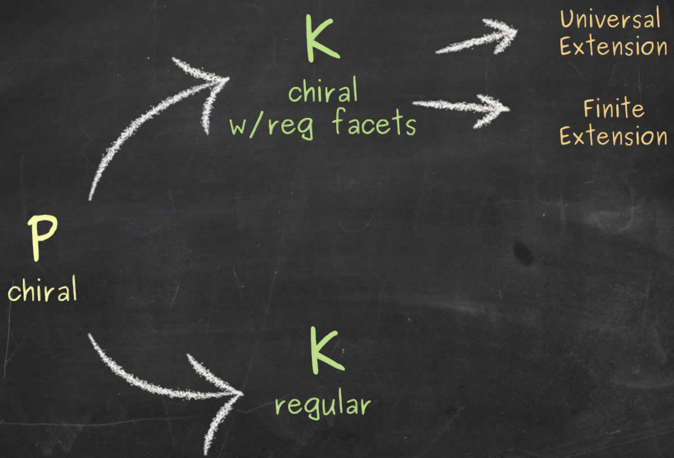
chiral

P
chiral

K
chiral
w/reg facets

Universal
Extension


K
chiral
w/reg facets
Universal
Extension
Finite
Extension
P
chiral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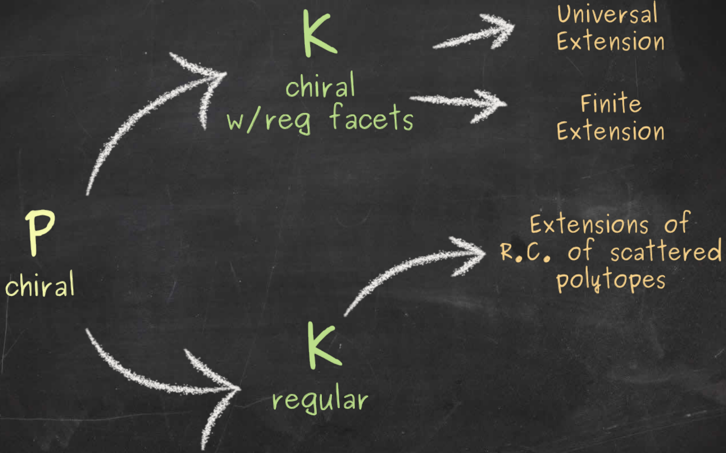
P
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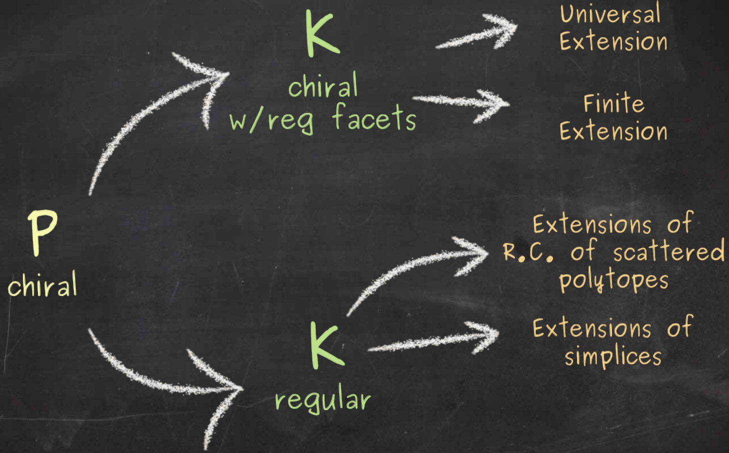
Extensions of
R.C. of scattered
polytopes


P
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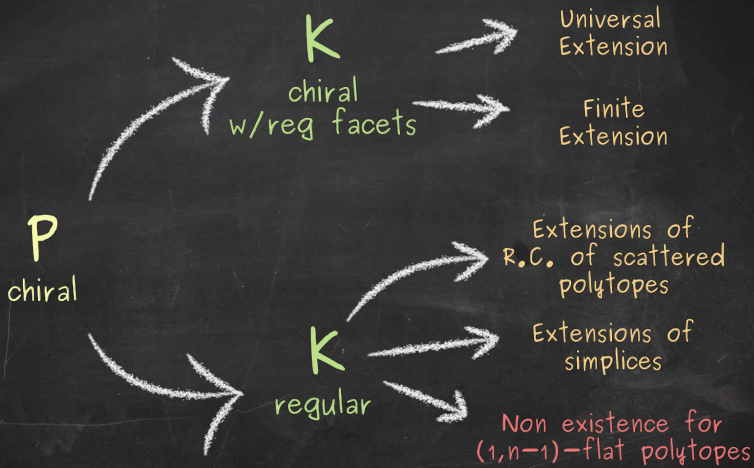
Extensions of
simplices


P
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Extensions of
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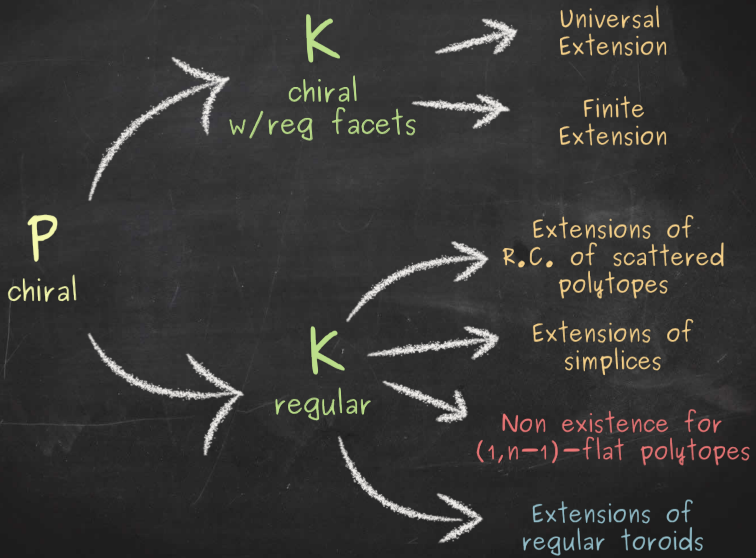
Non existence for
(1,n−1)−flat polytopes


K
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Non existence for
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Extensions of
regular toroids

Theorem (Pellicer, Potocnik, Toledo, 2018)

For every $n \geqslant 3$ and every $I \subsetneq \{0, \ldots, n-1\}$ there exists an n-maniplex of type 2_I.

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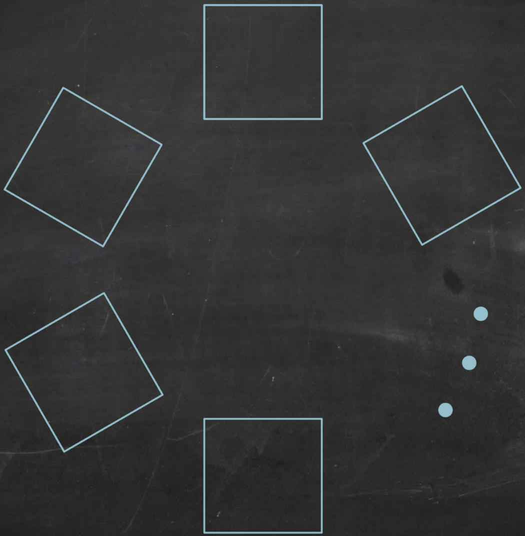
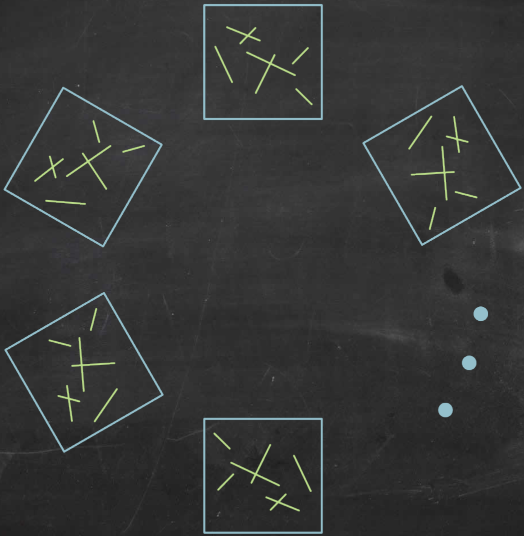
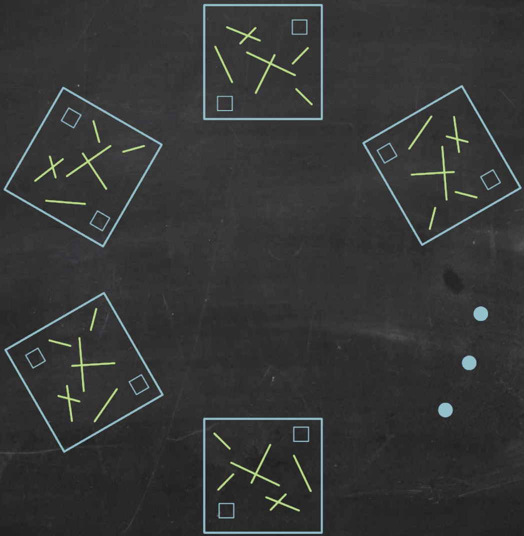
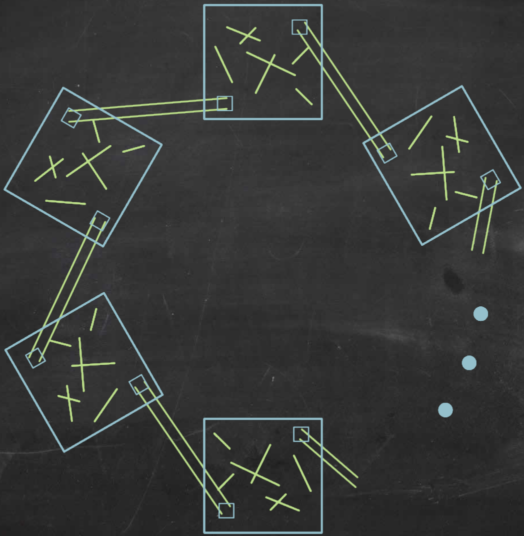
Theorem (M., Pellicer, Toledo)
For every $n \geqslant 3$ and every $a \geqslant 2n+1$ there exists a chiral extension of the regular toroid $\{4, 3, \dots, 3, 4\}_{(a,0,\dots,0)}$









Thank you!