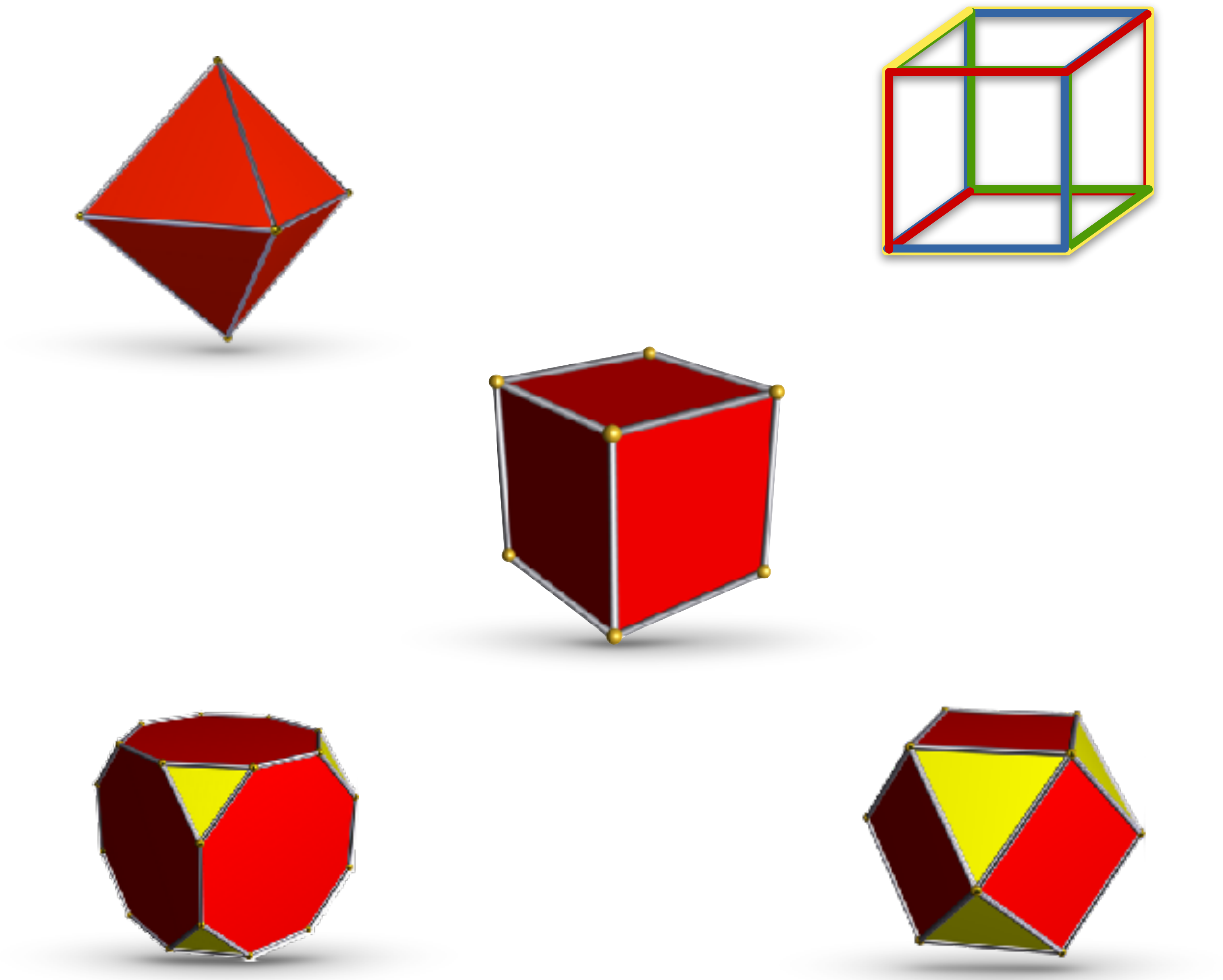
Operaciones en poliedros, politopos y similares.

Antonio Montero

Universidad de Ljubljana

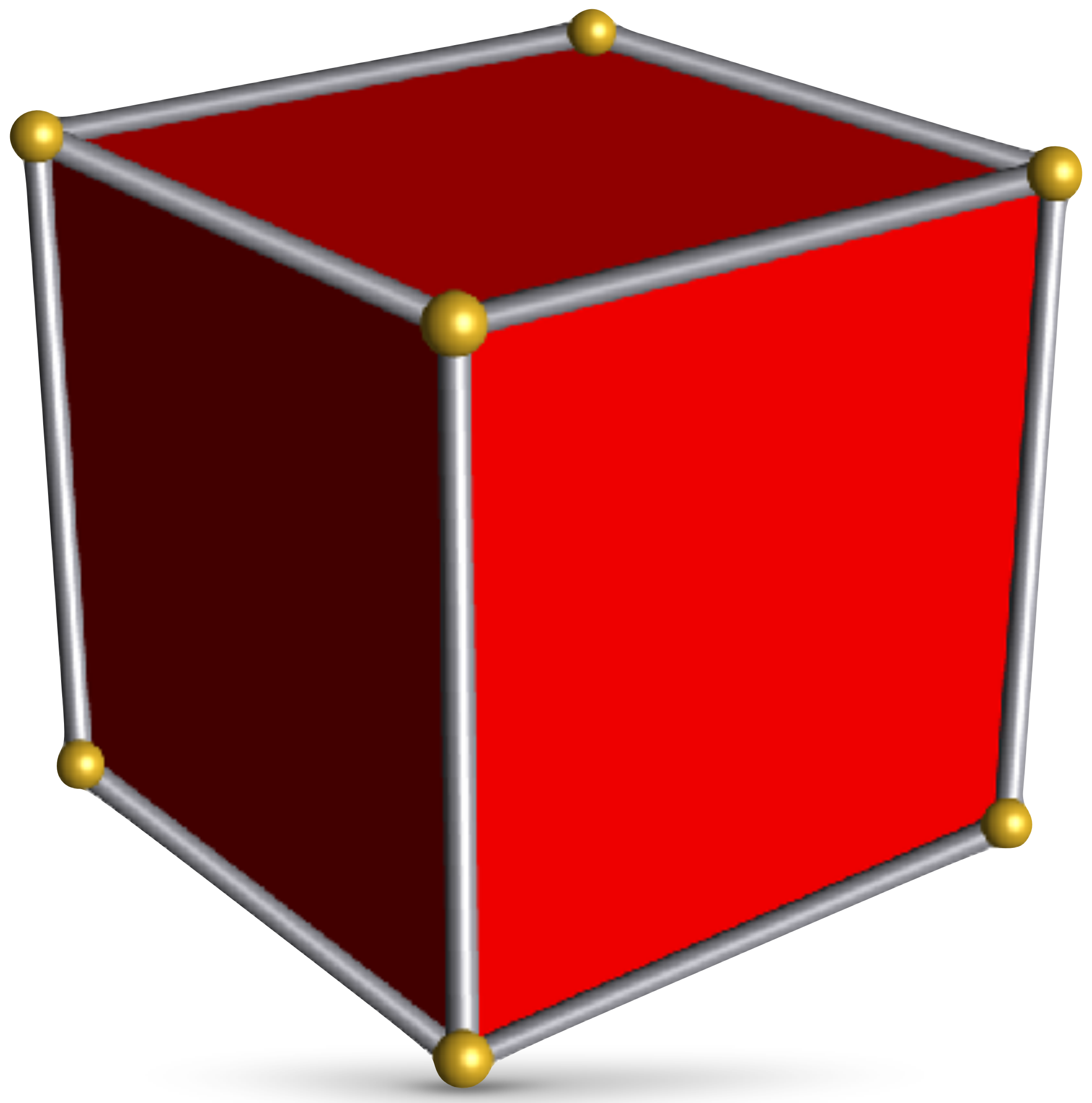
Basado en trabajo conjunto con Isabel Hubard y Elías Mochán

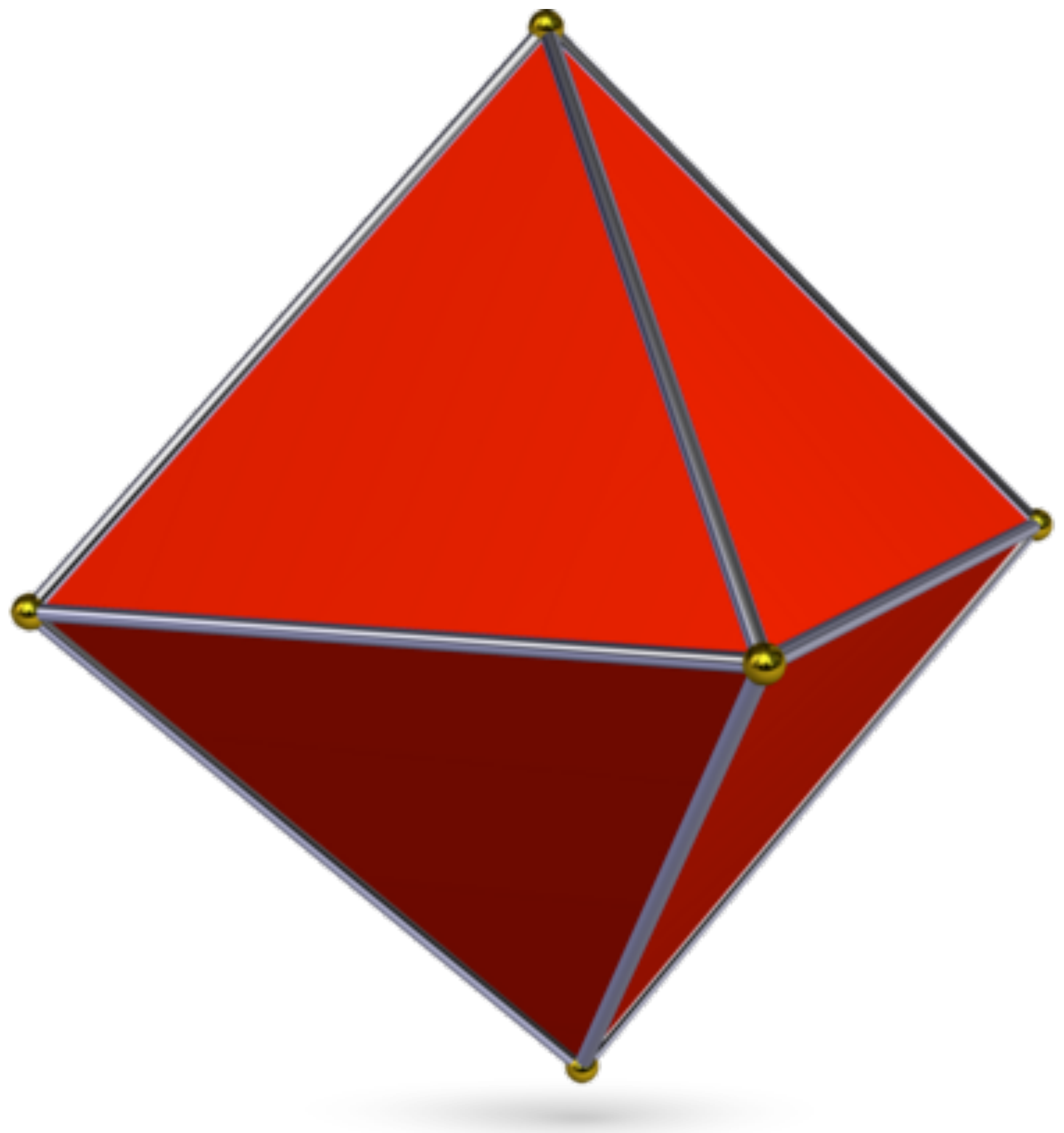


Seminario (preguntón) de Matemáticas Discretas

Septiembre 2026, Instituto de Matemáticas, UNAM Campus Juriquilla.

Operaciones





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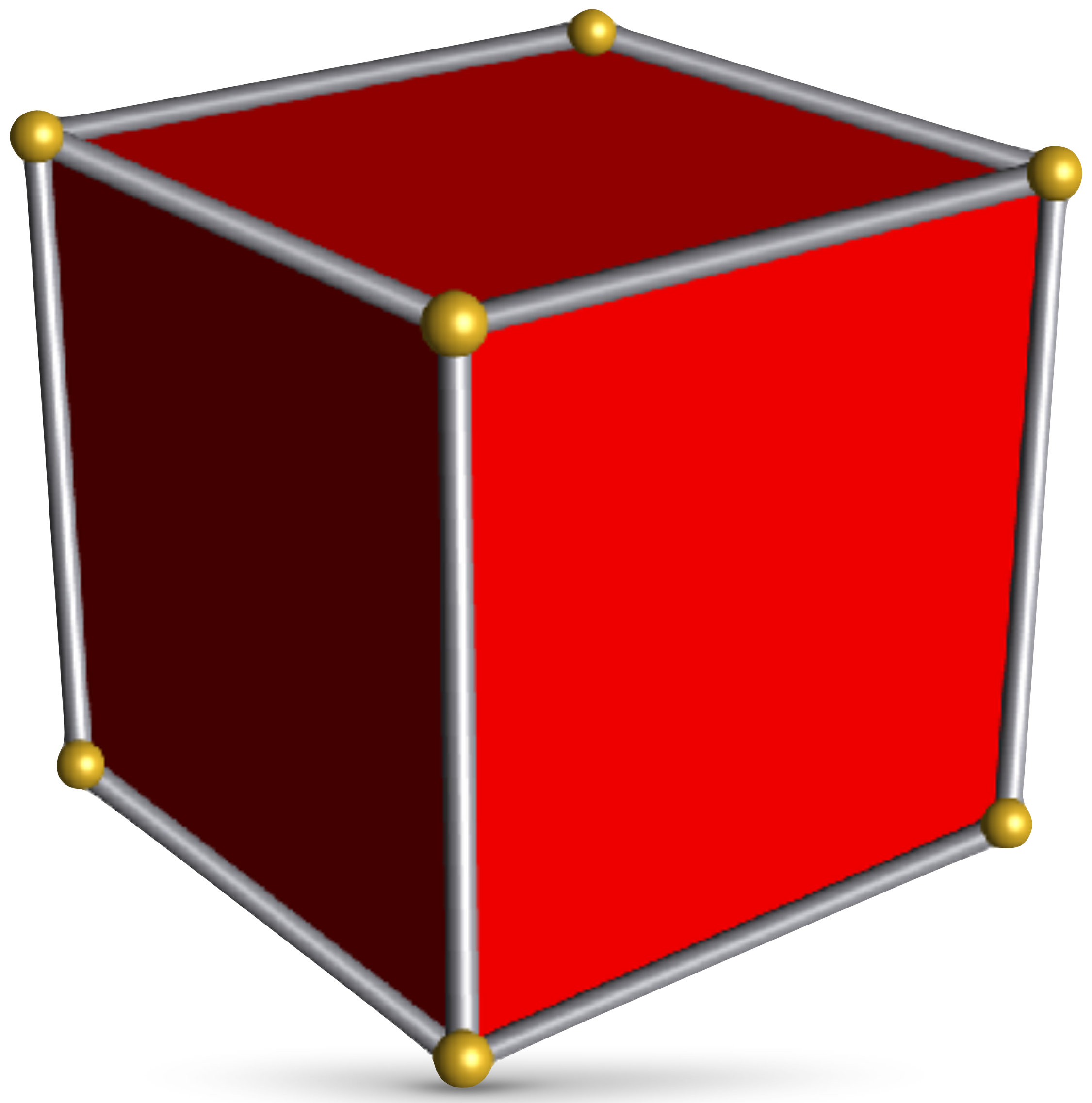

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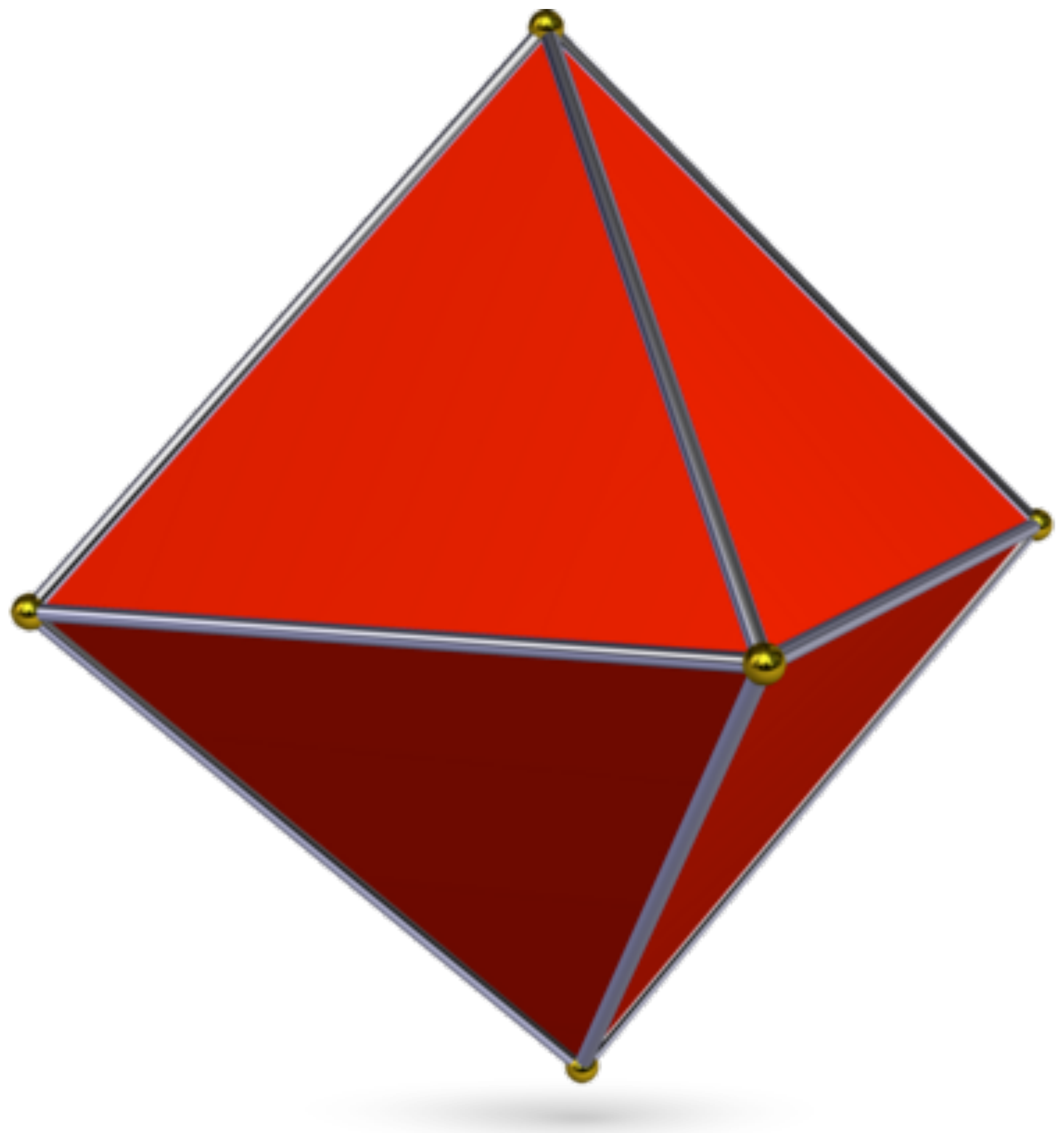

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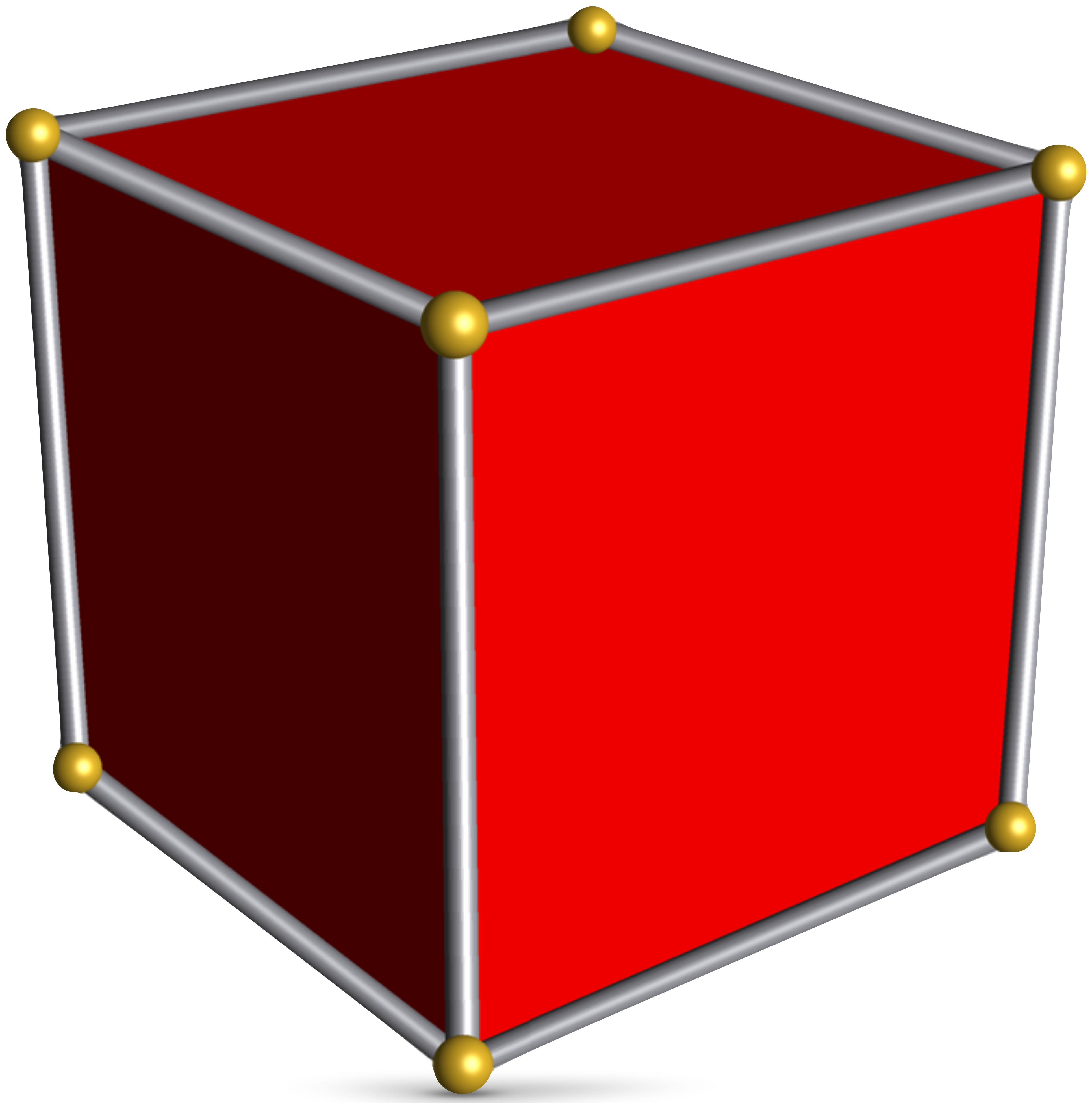

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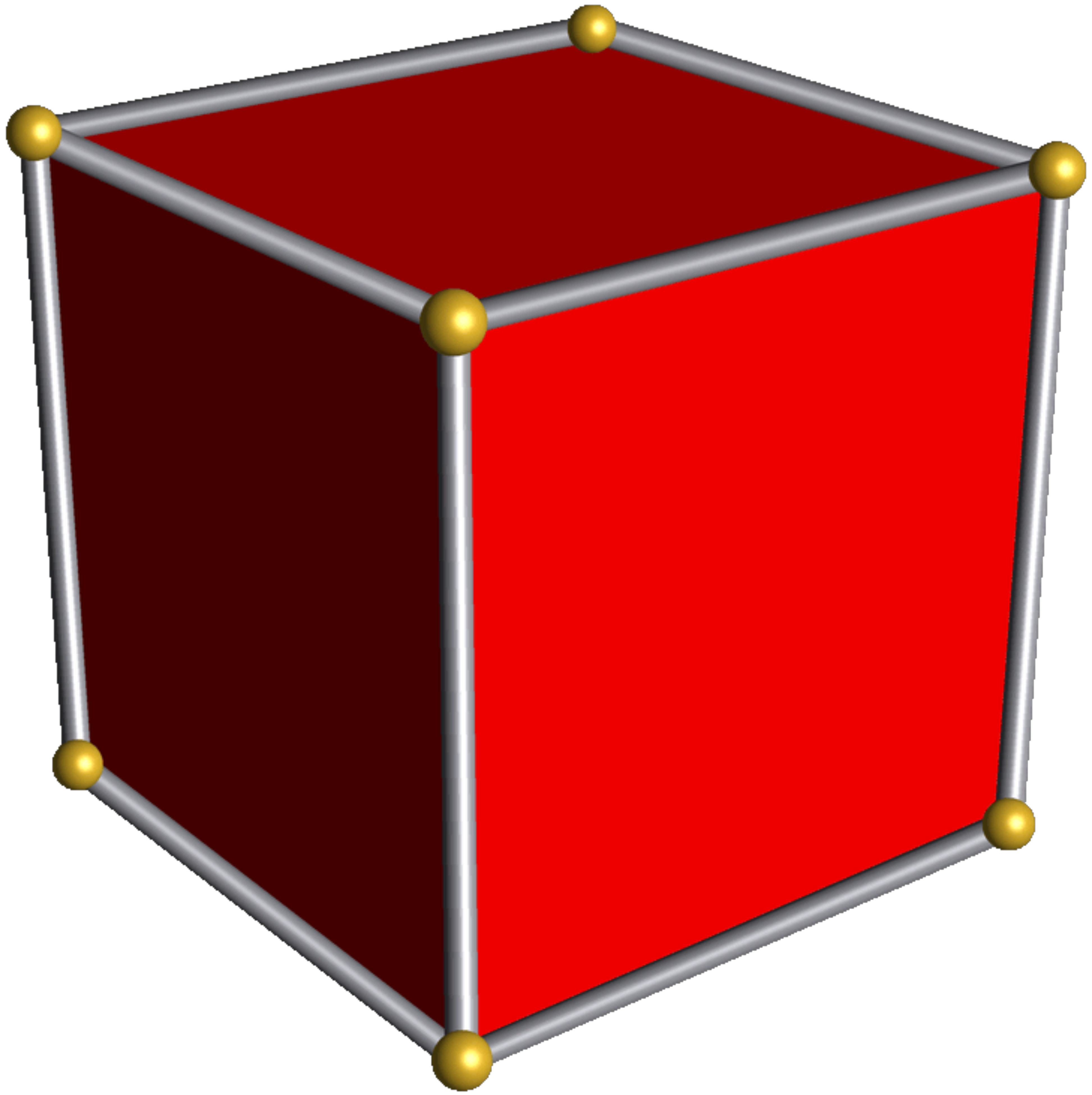

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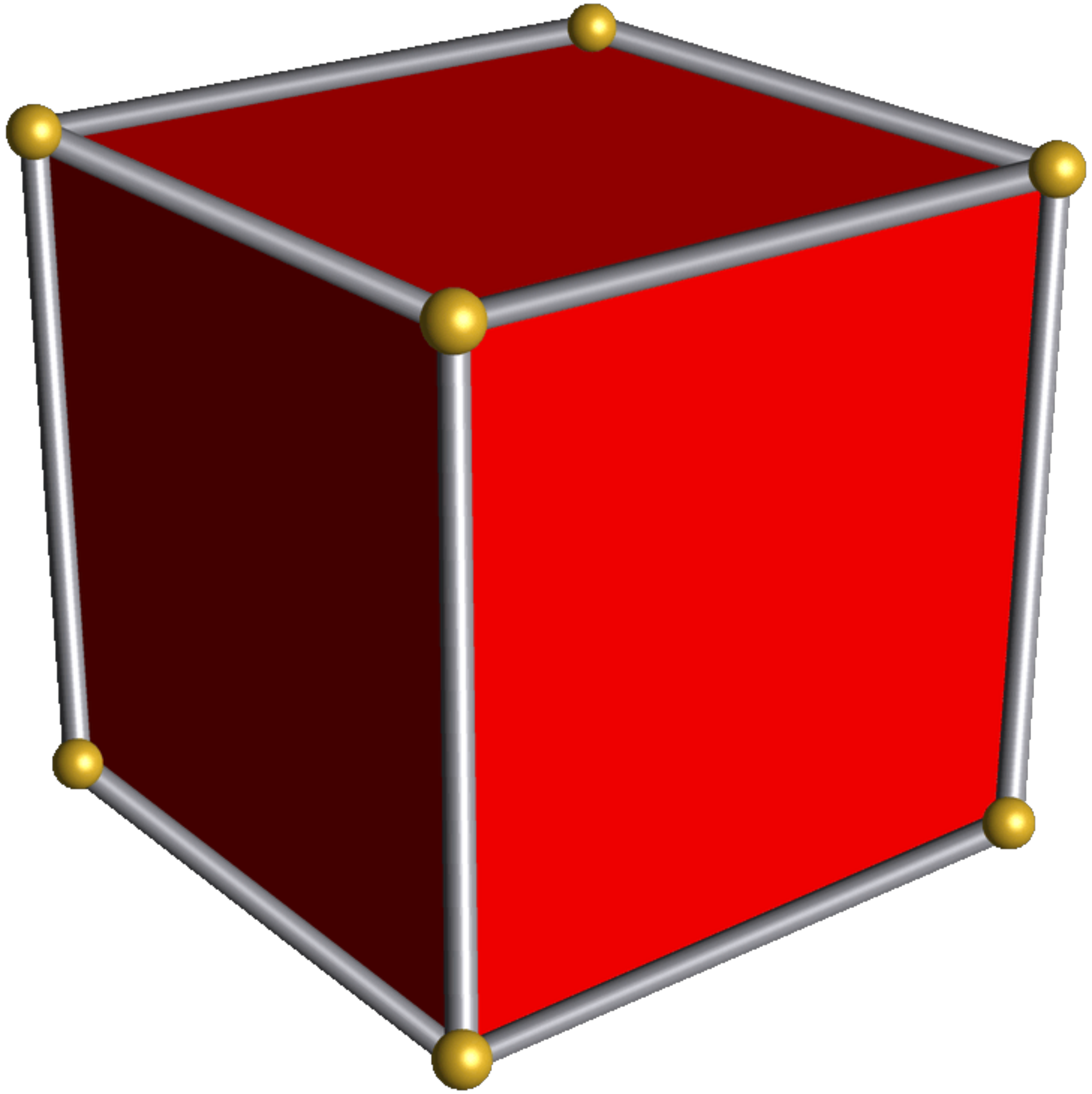

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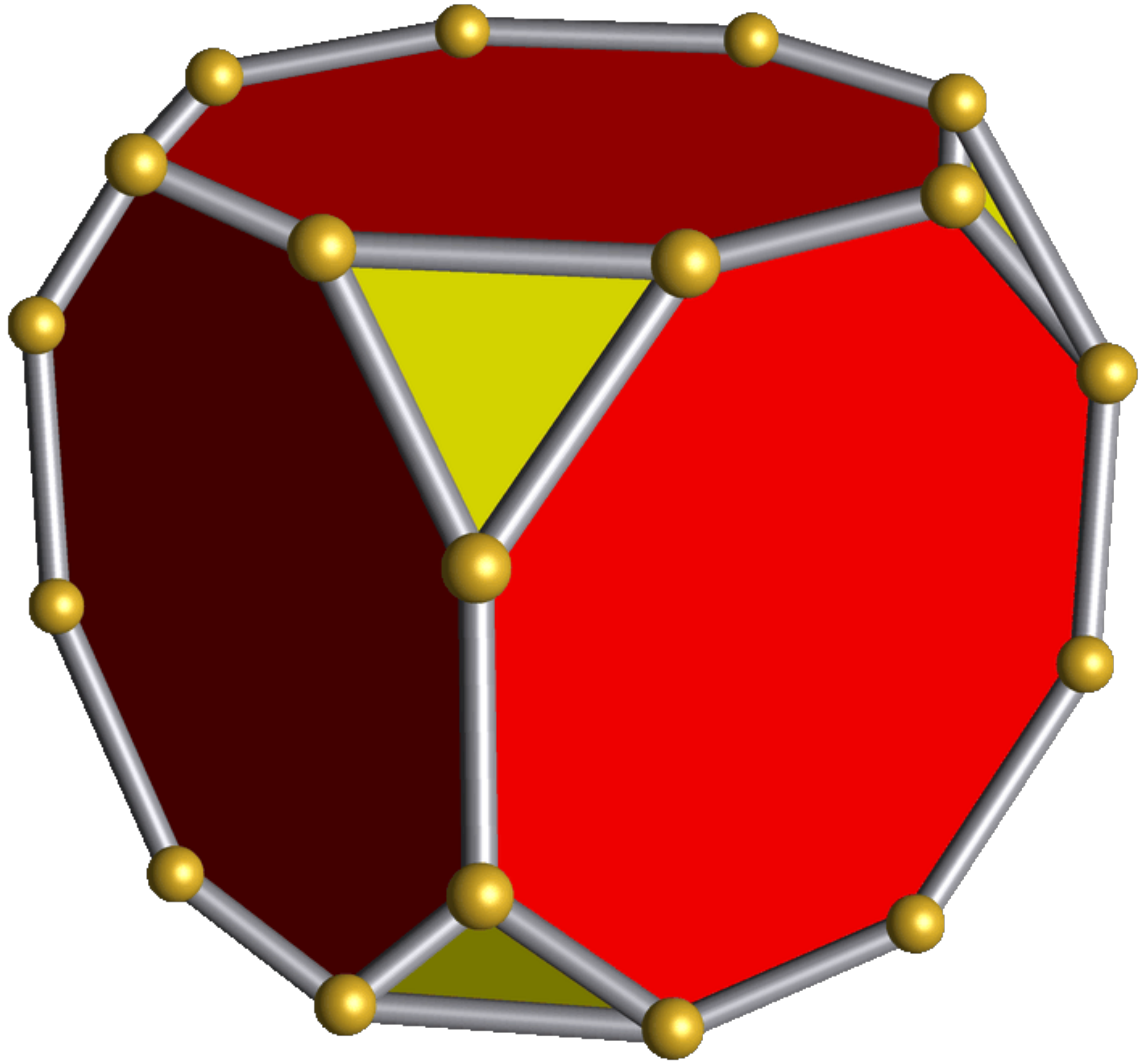

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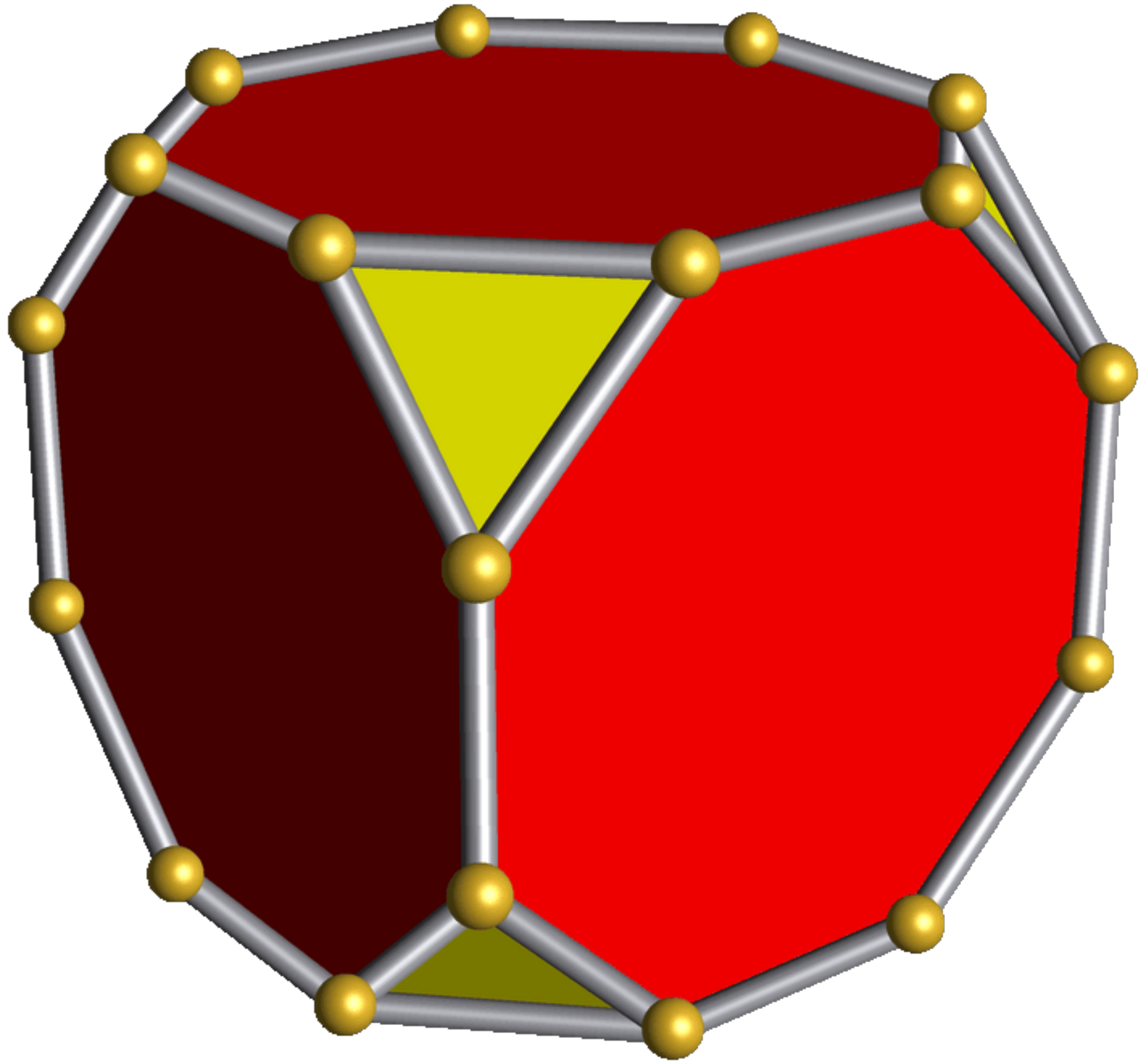

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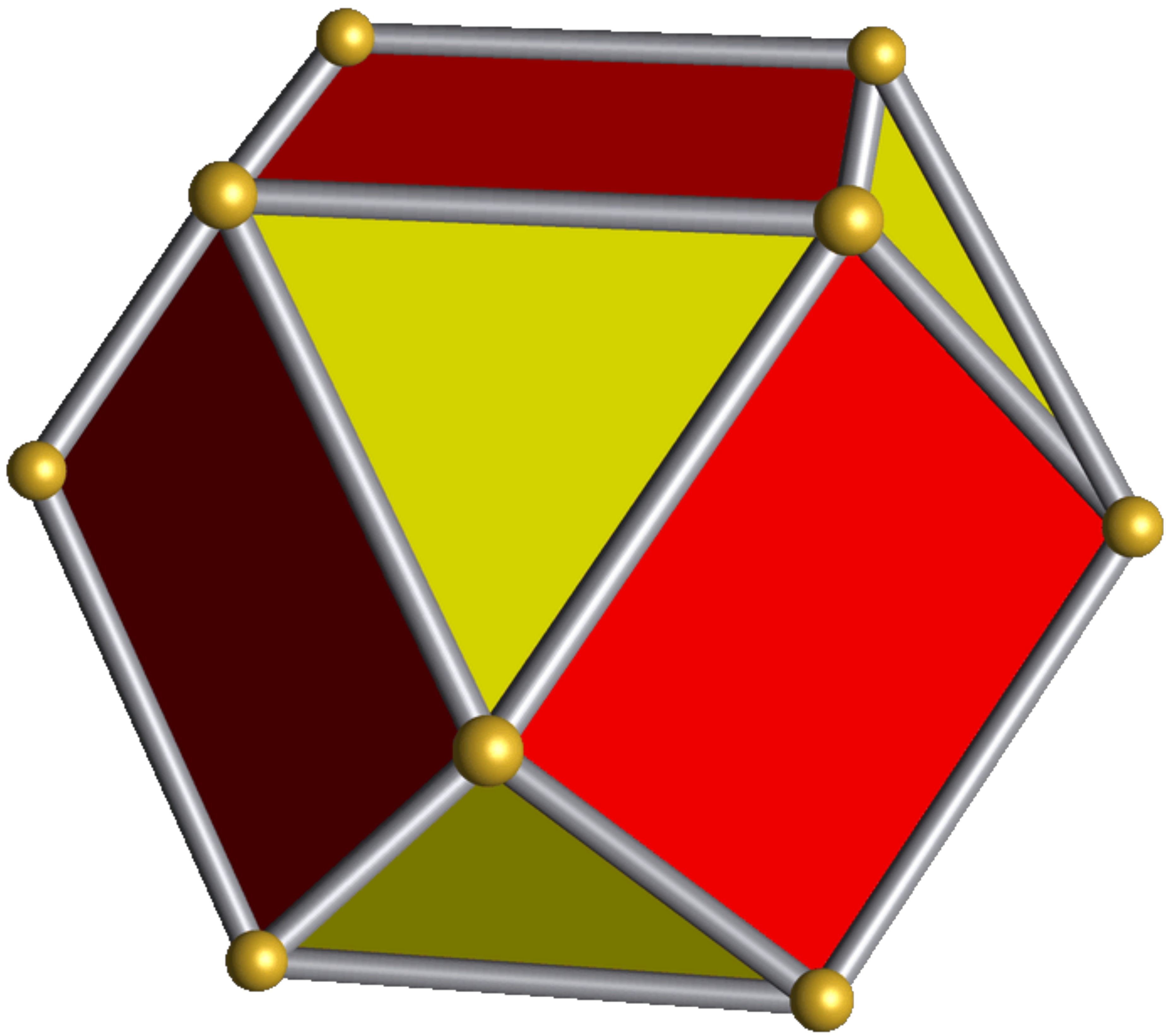

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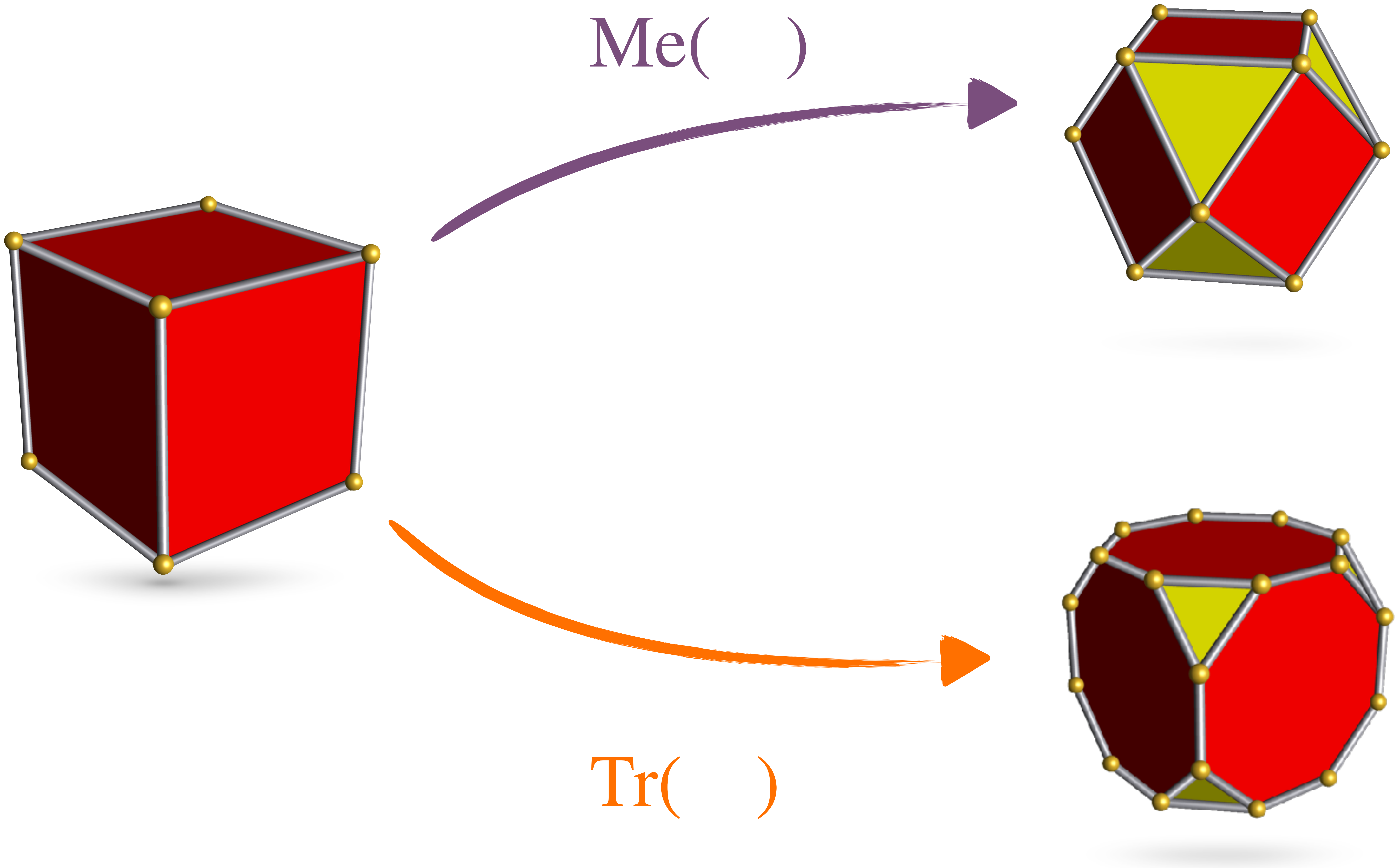

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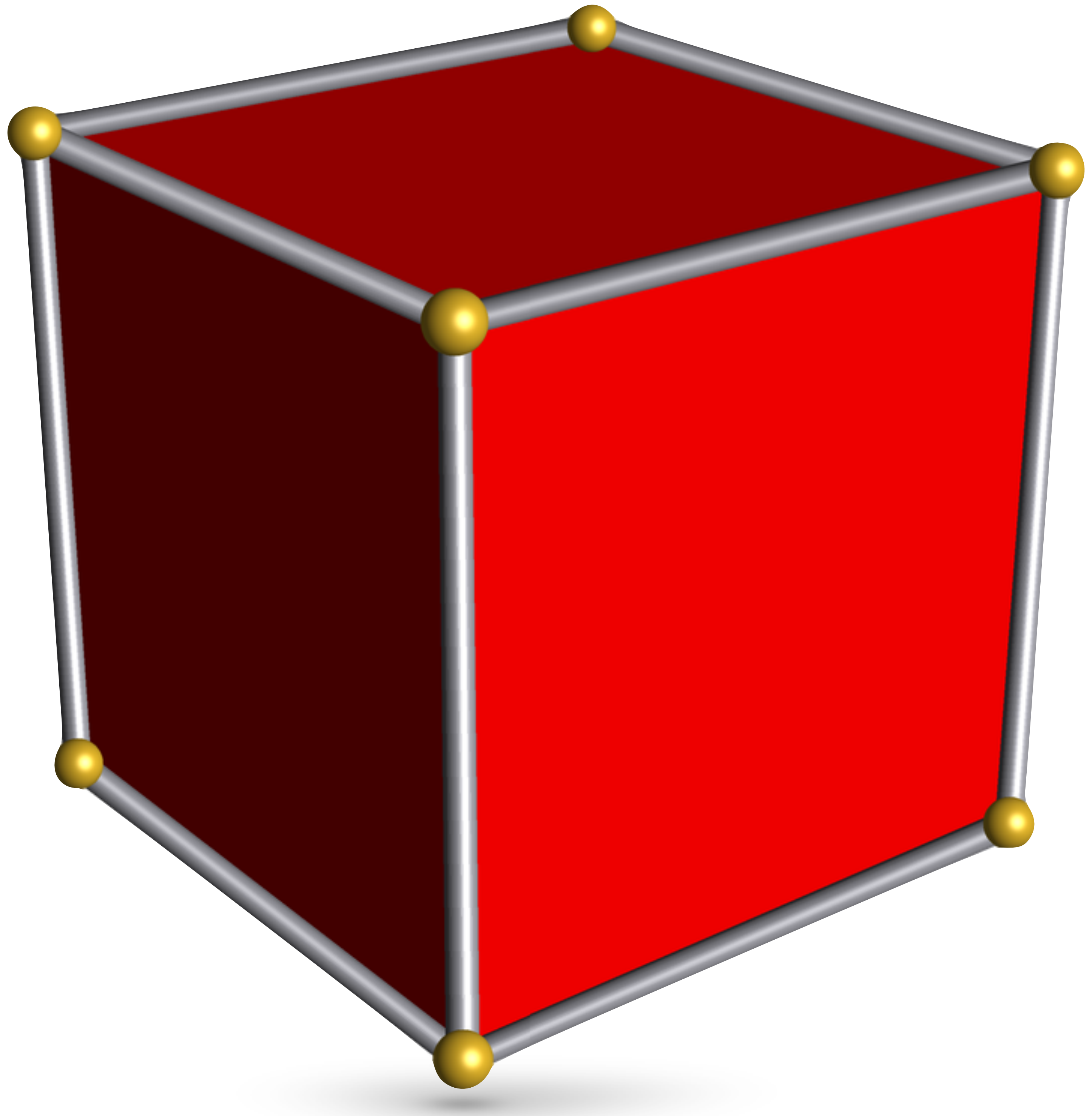

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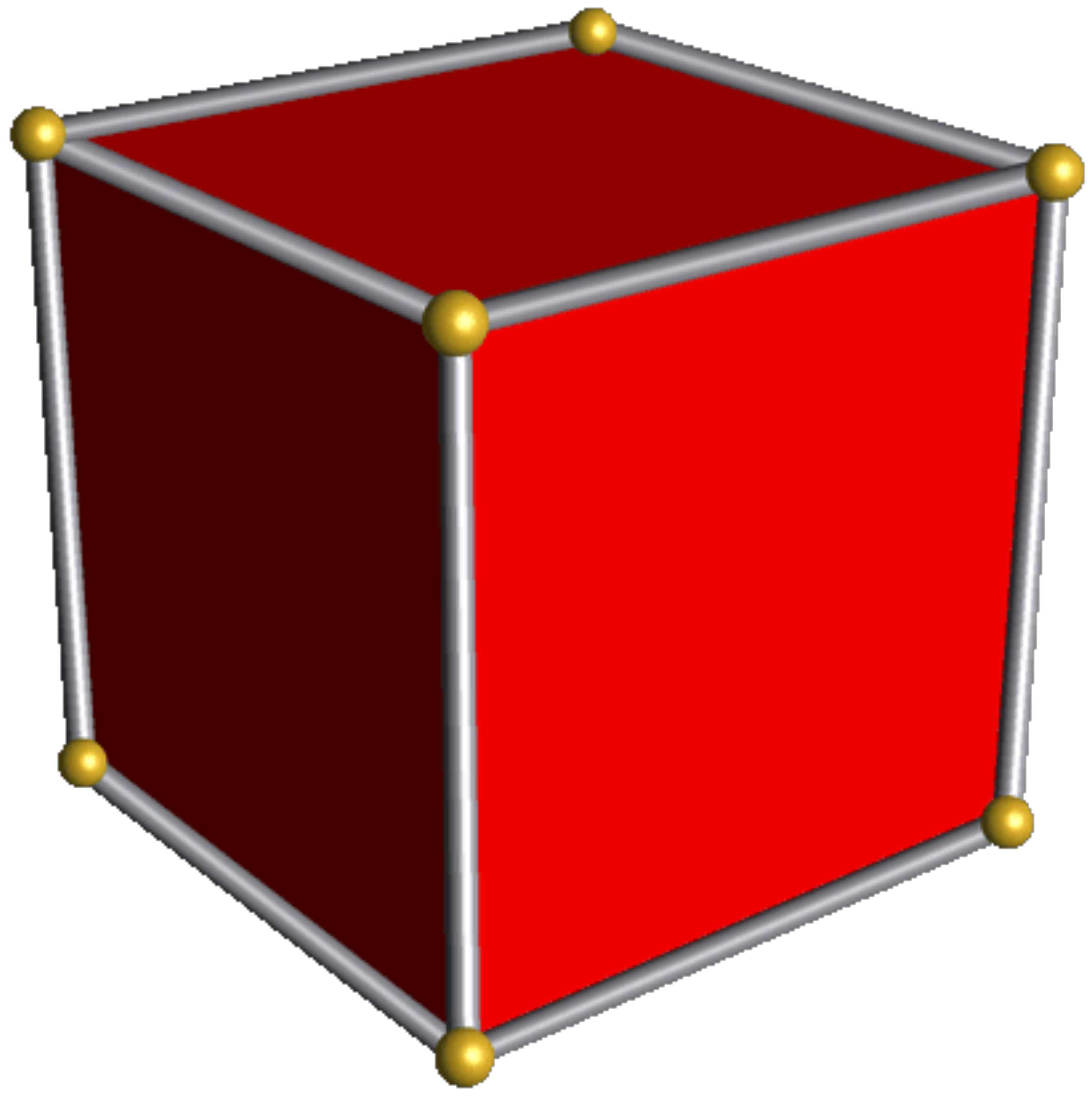

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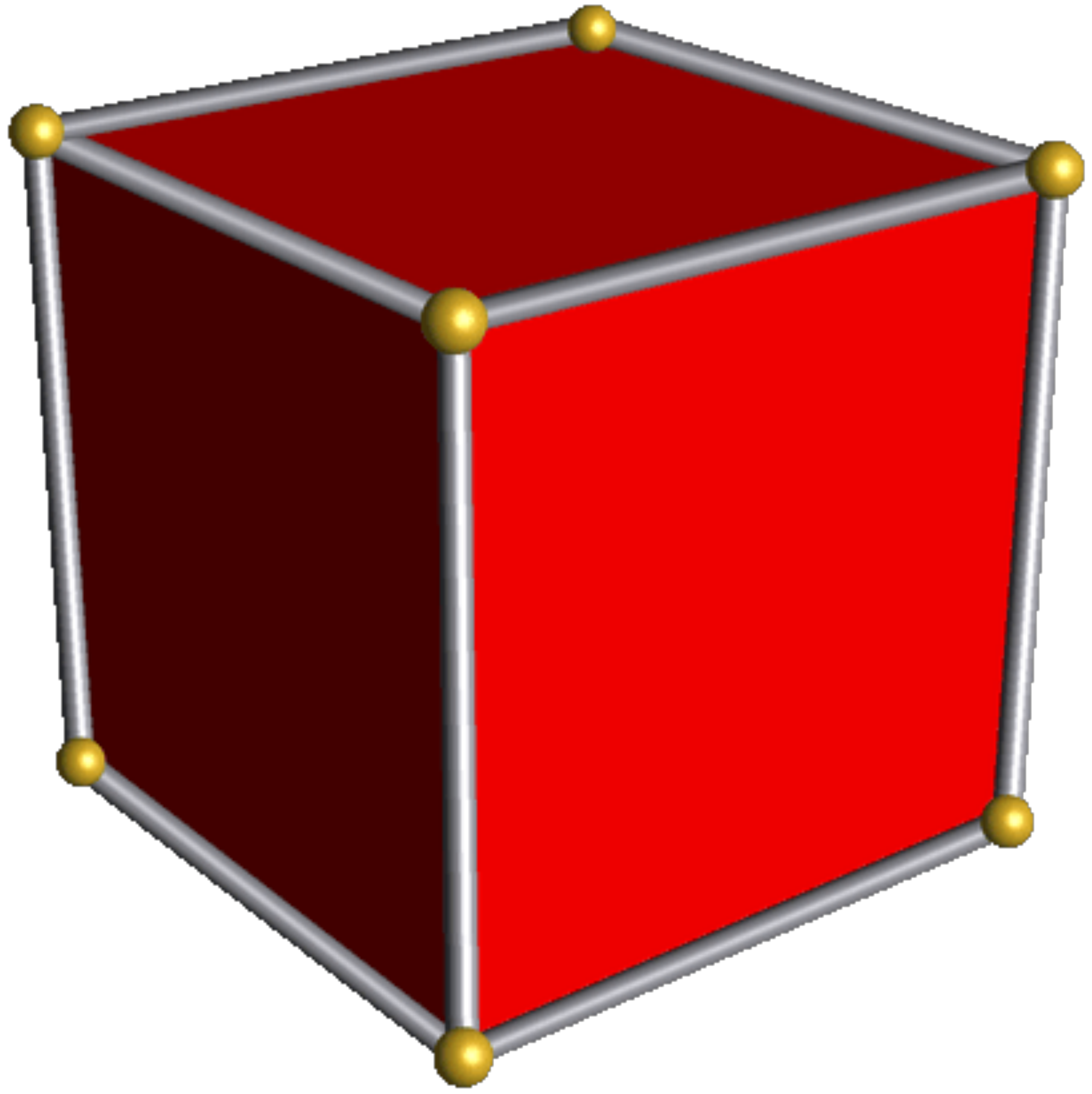

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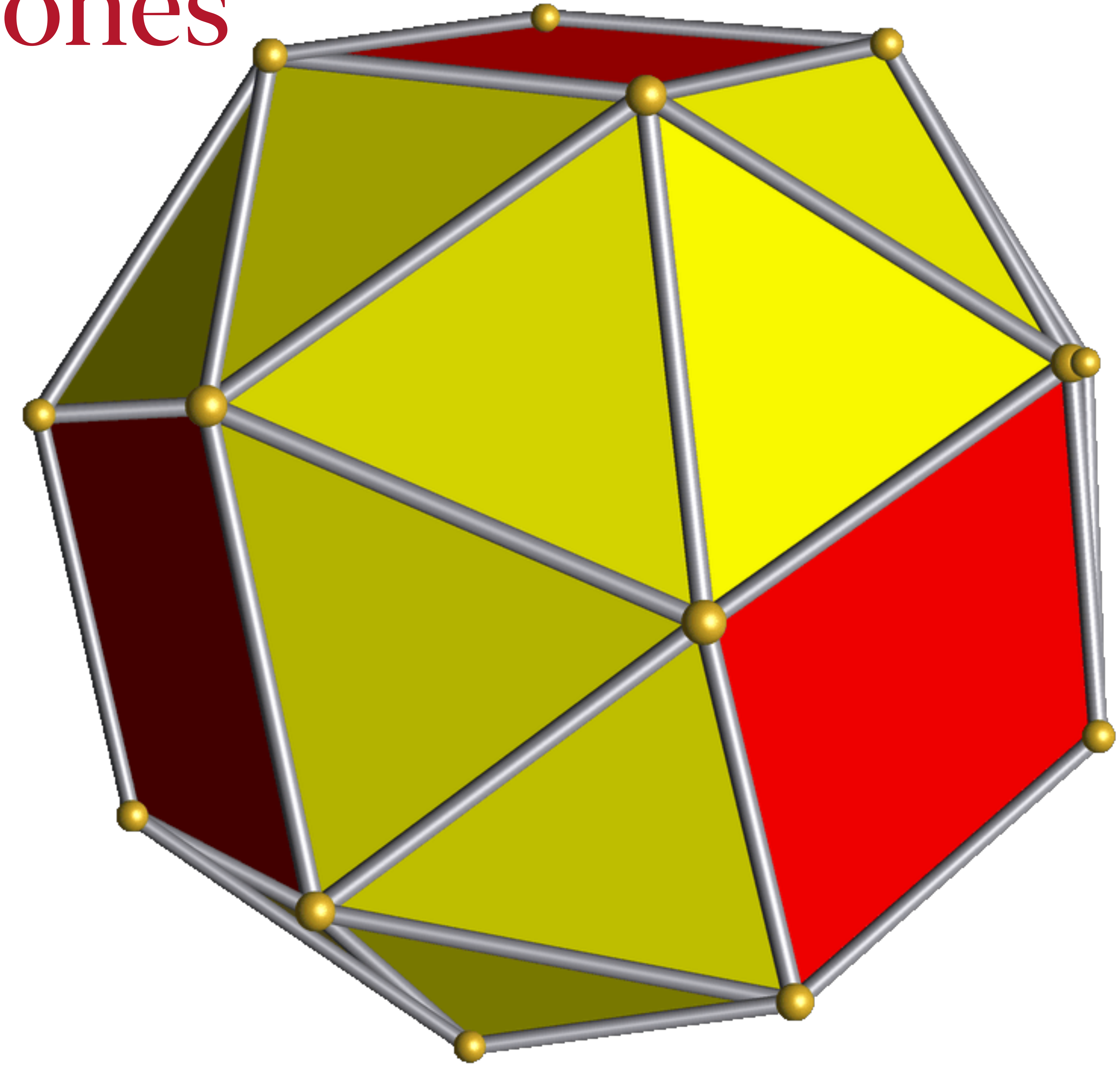

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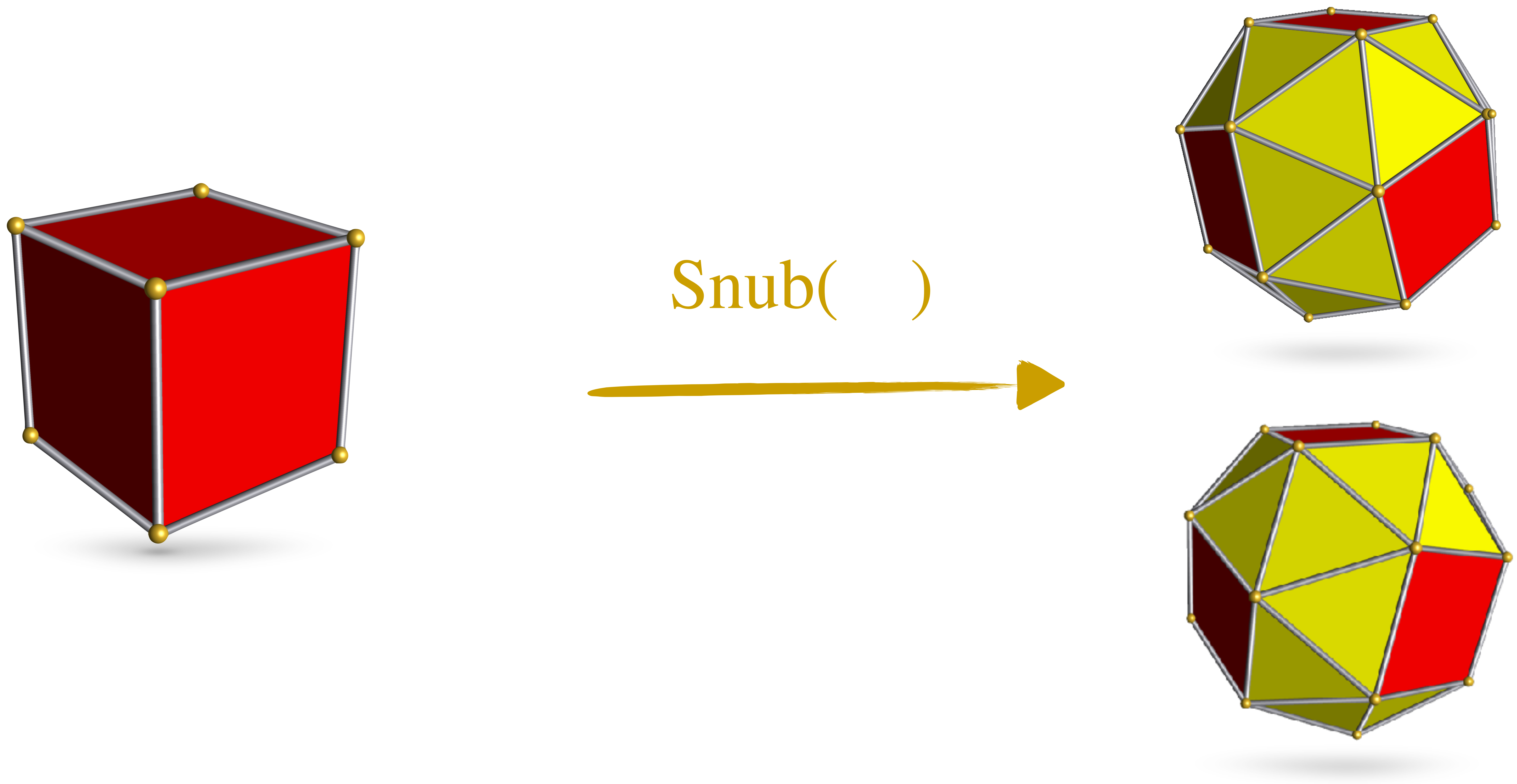

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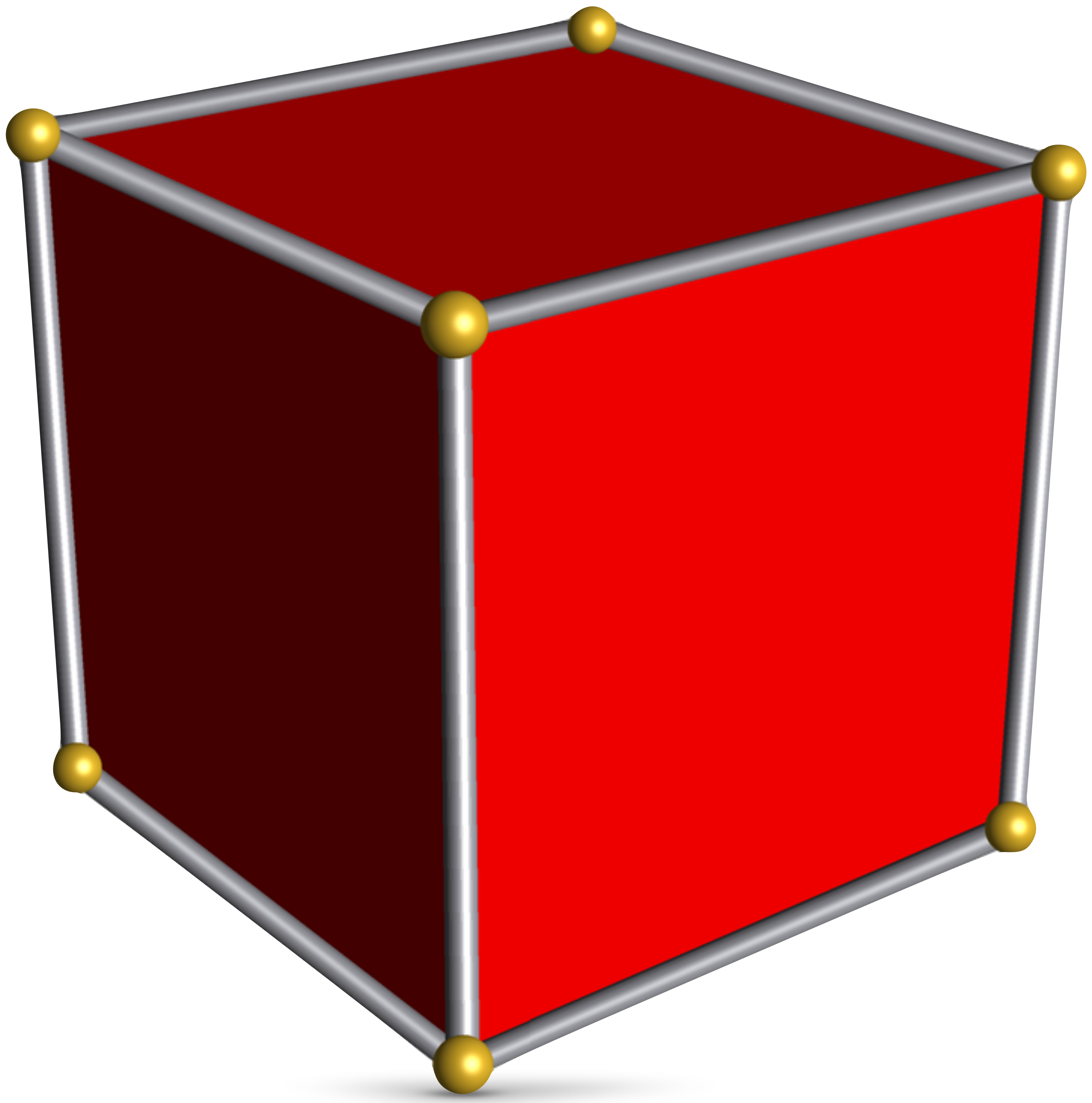

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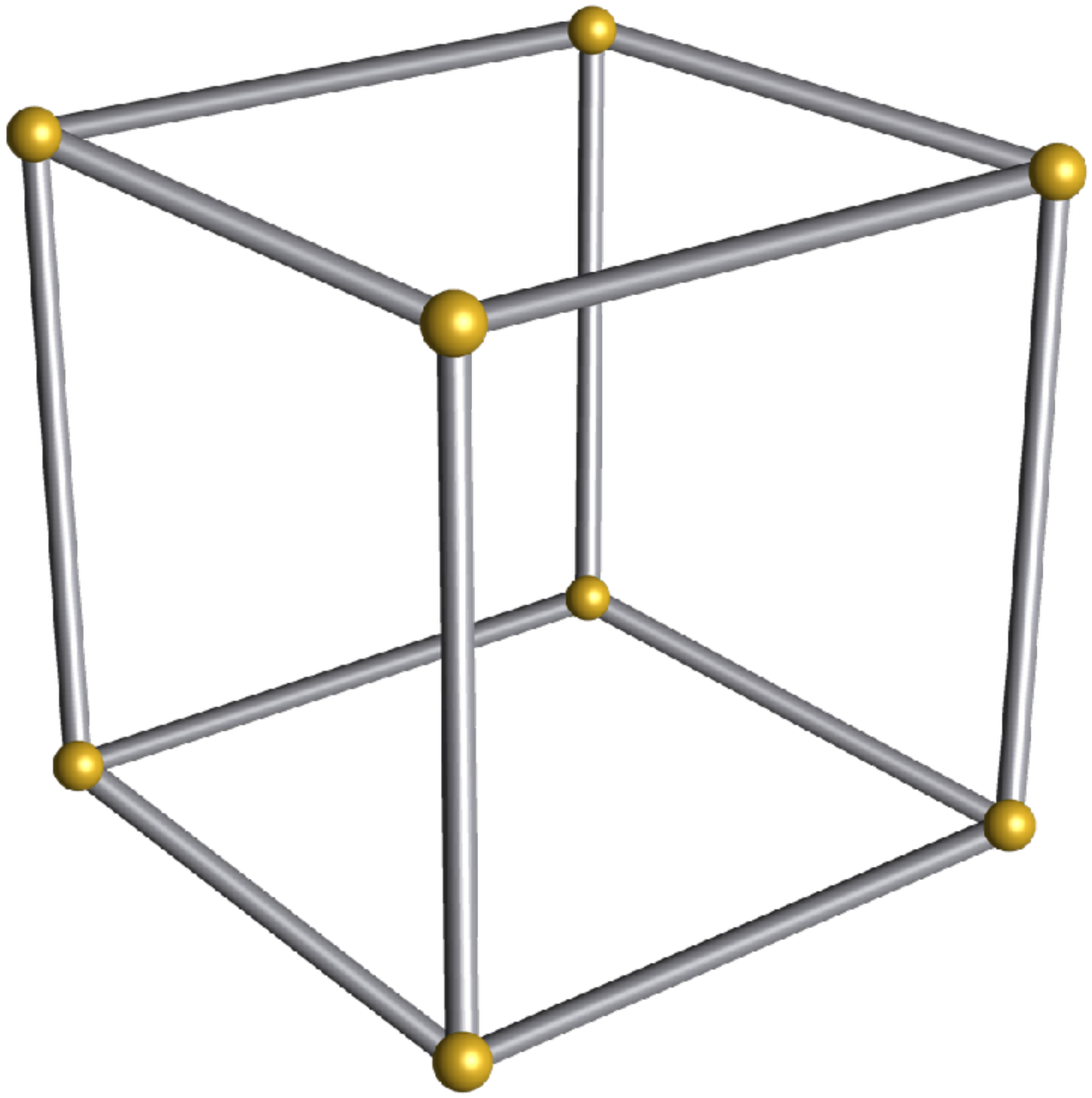

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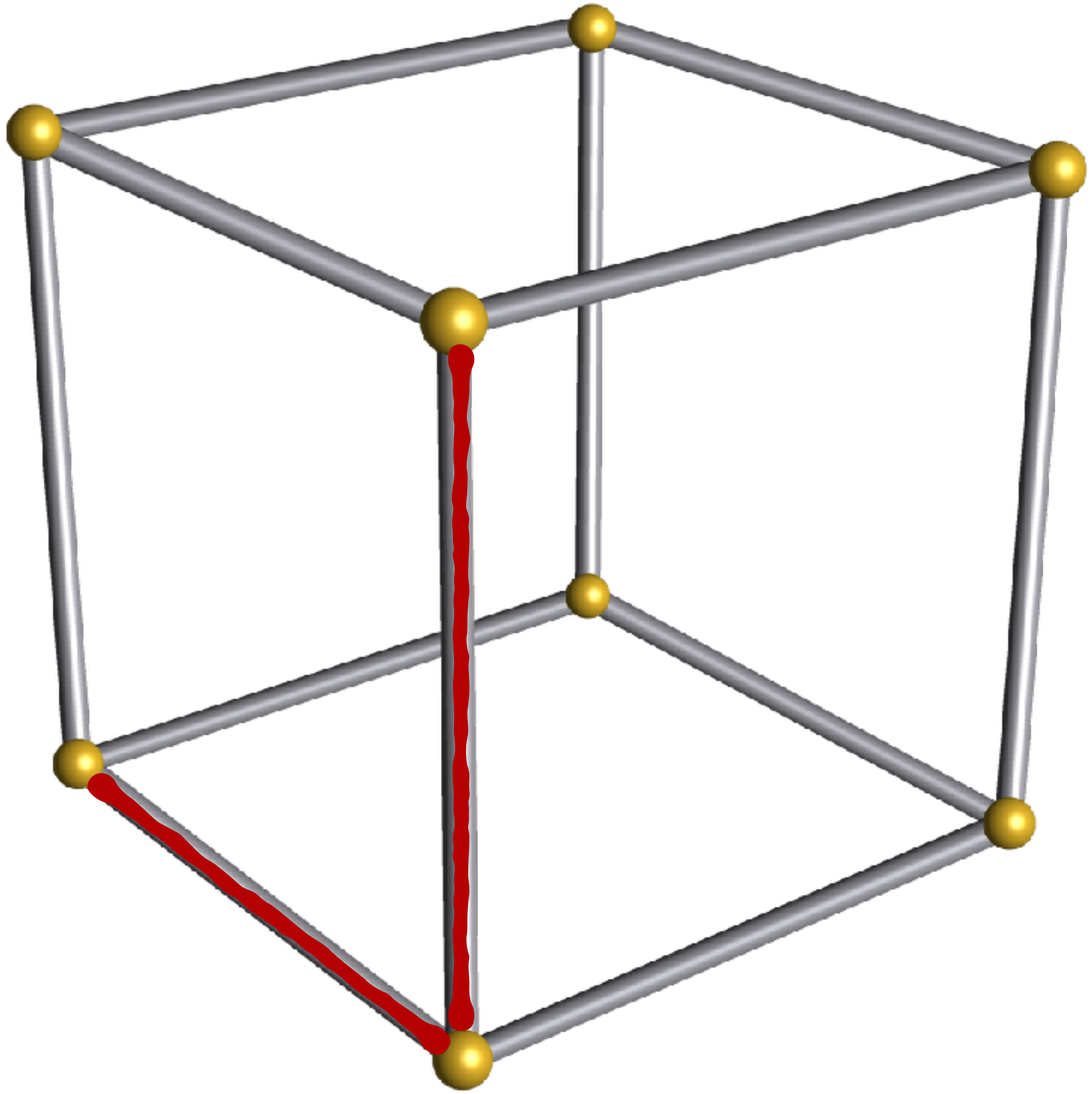

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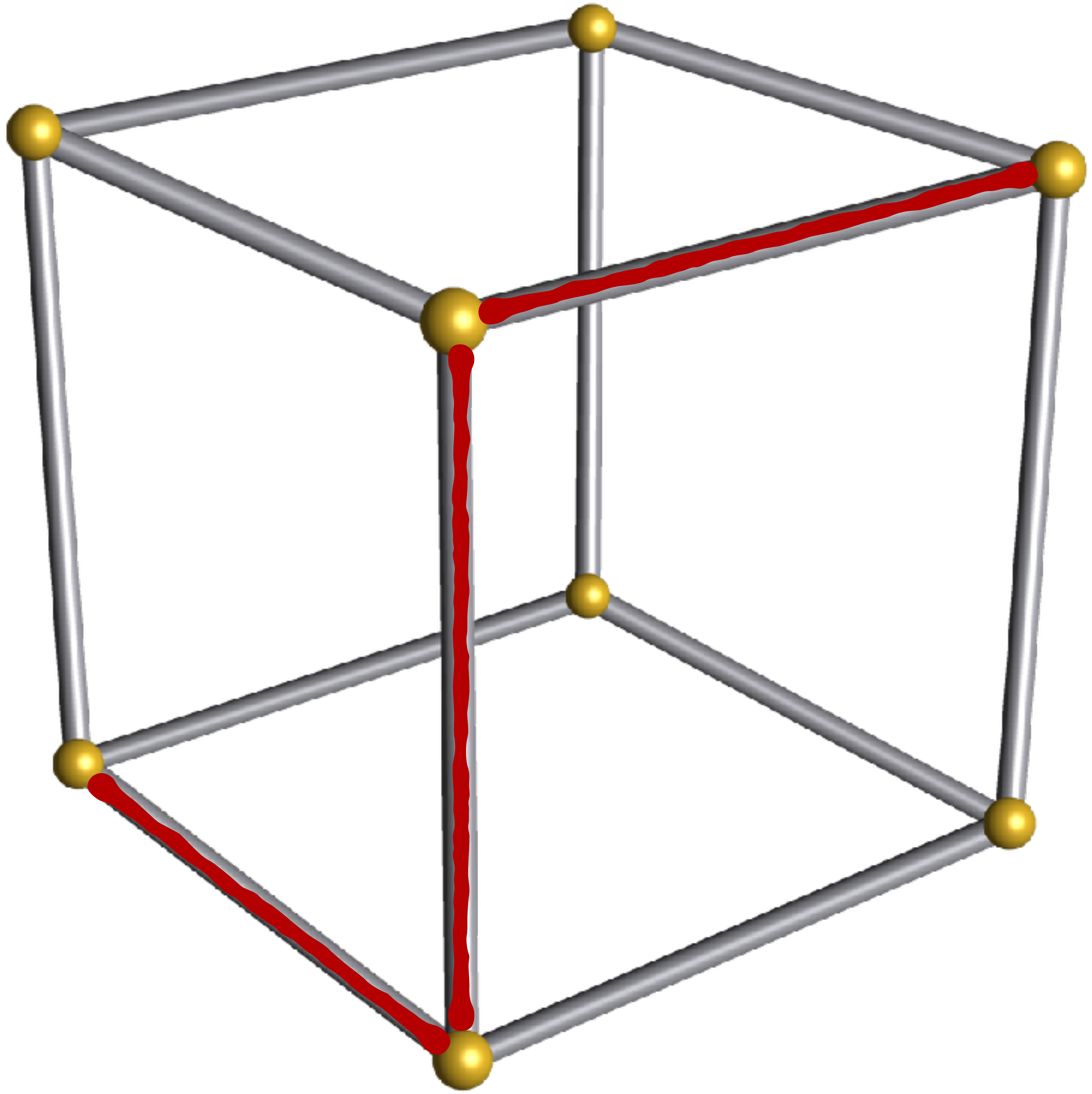

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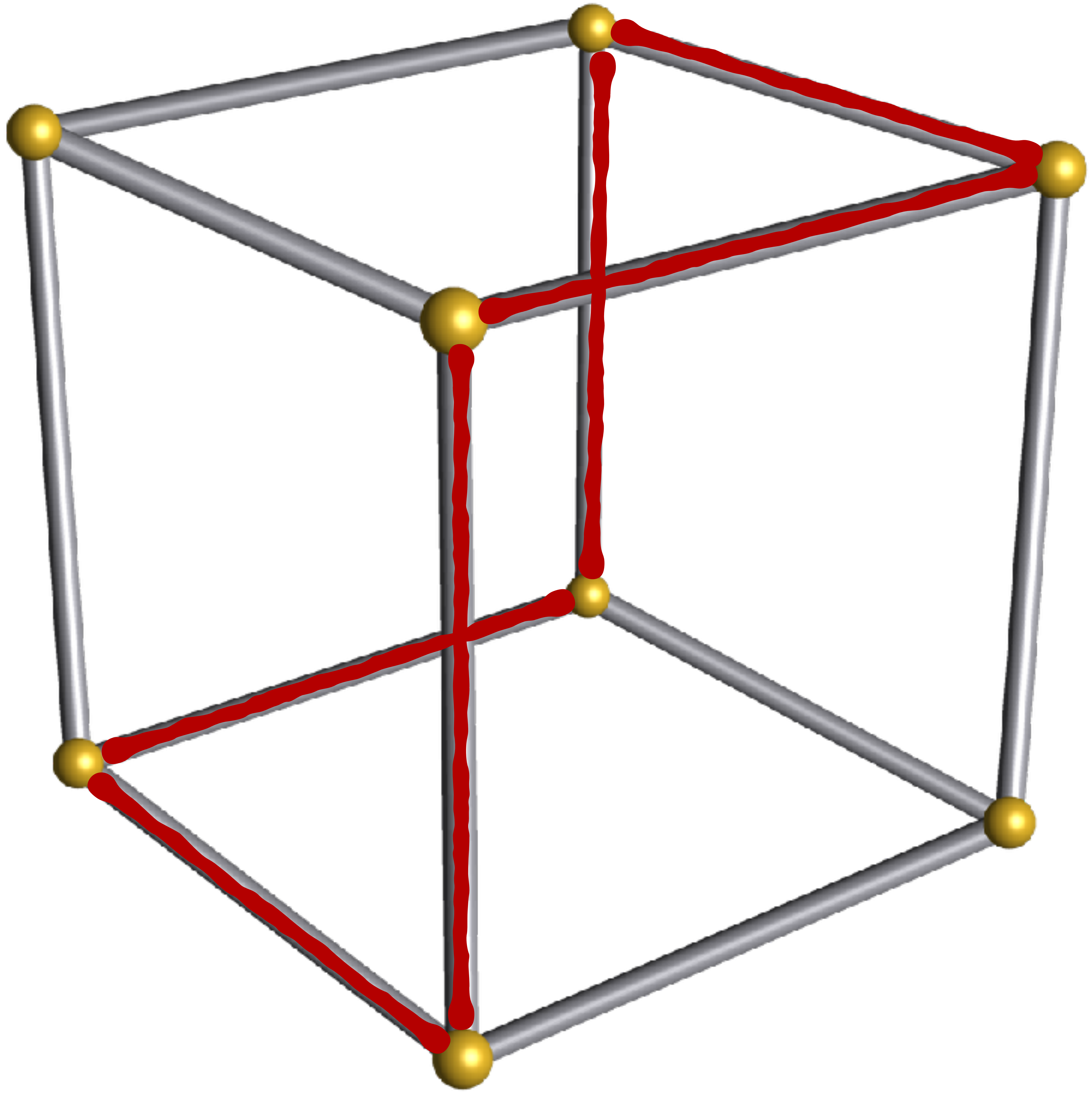

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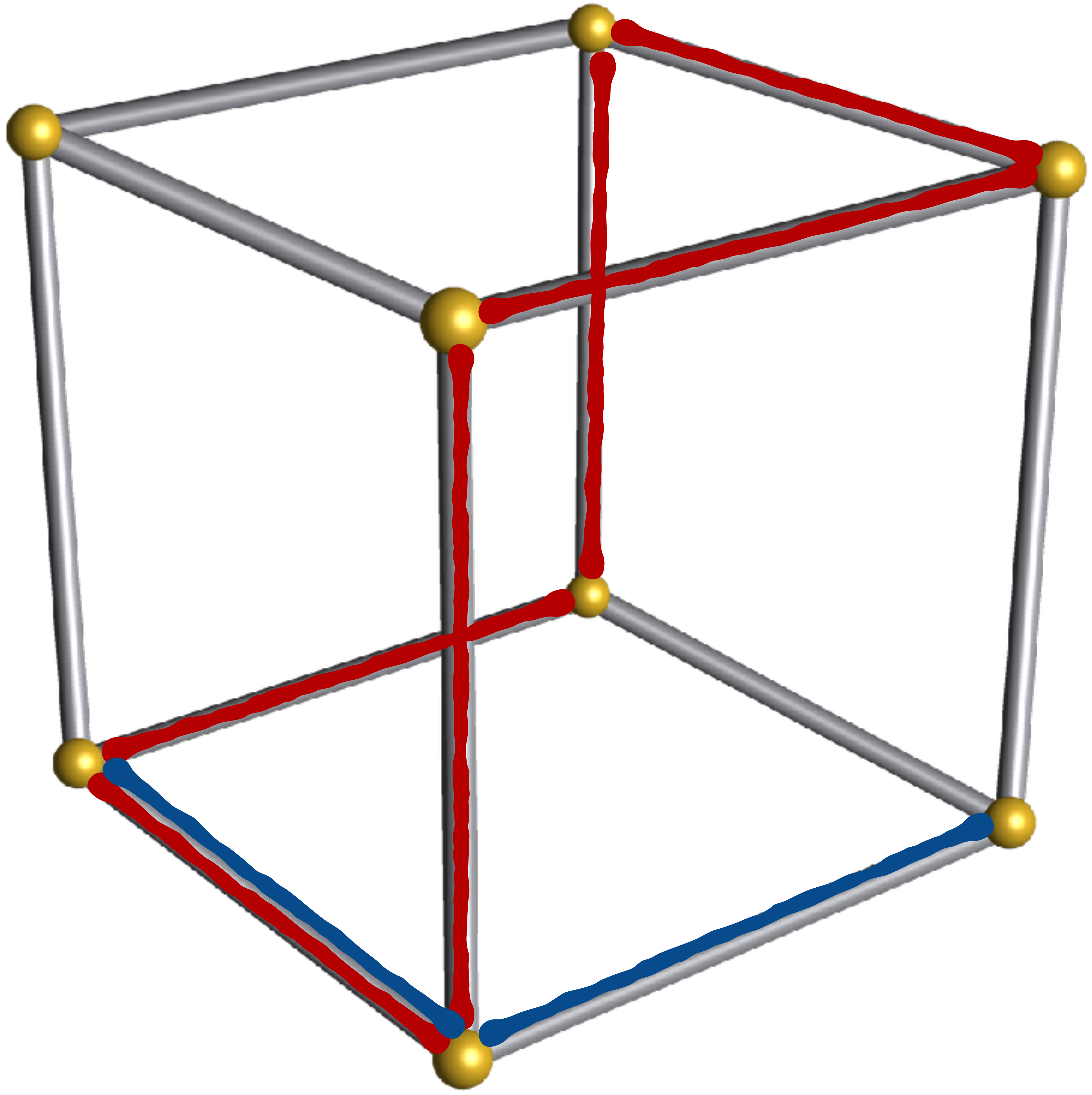

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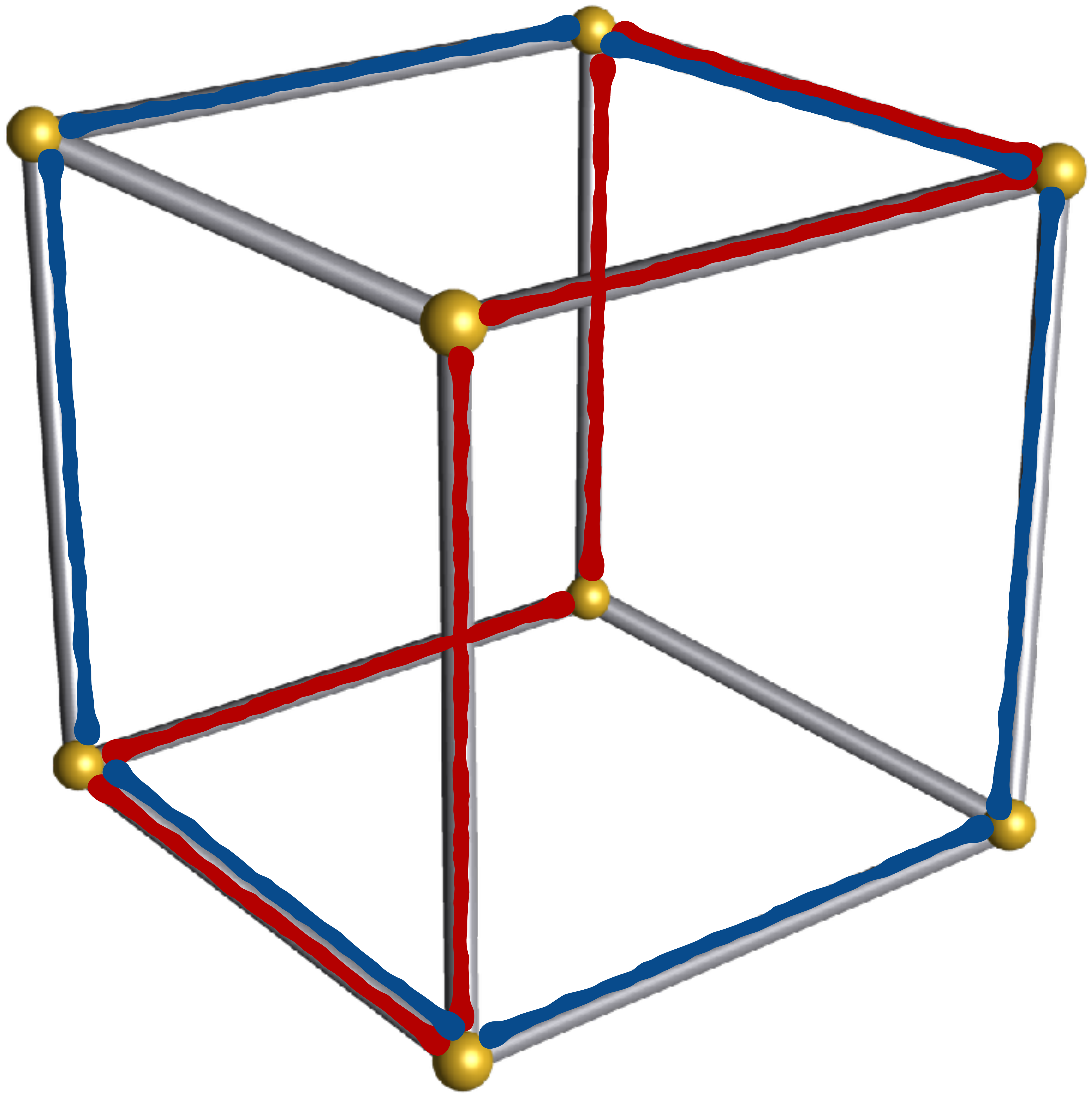

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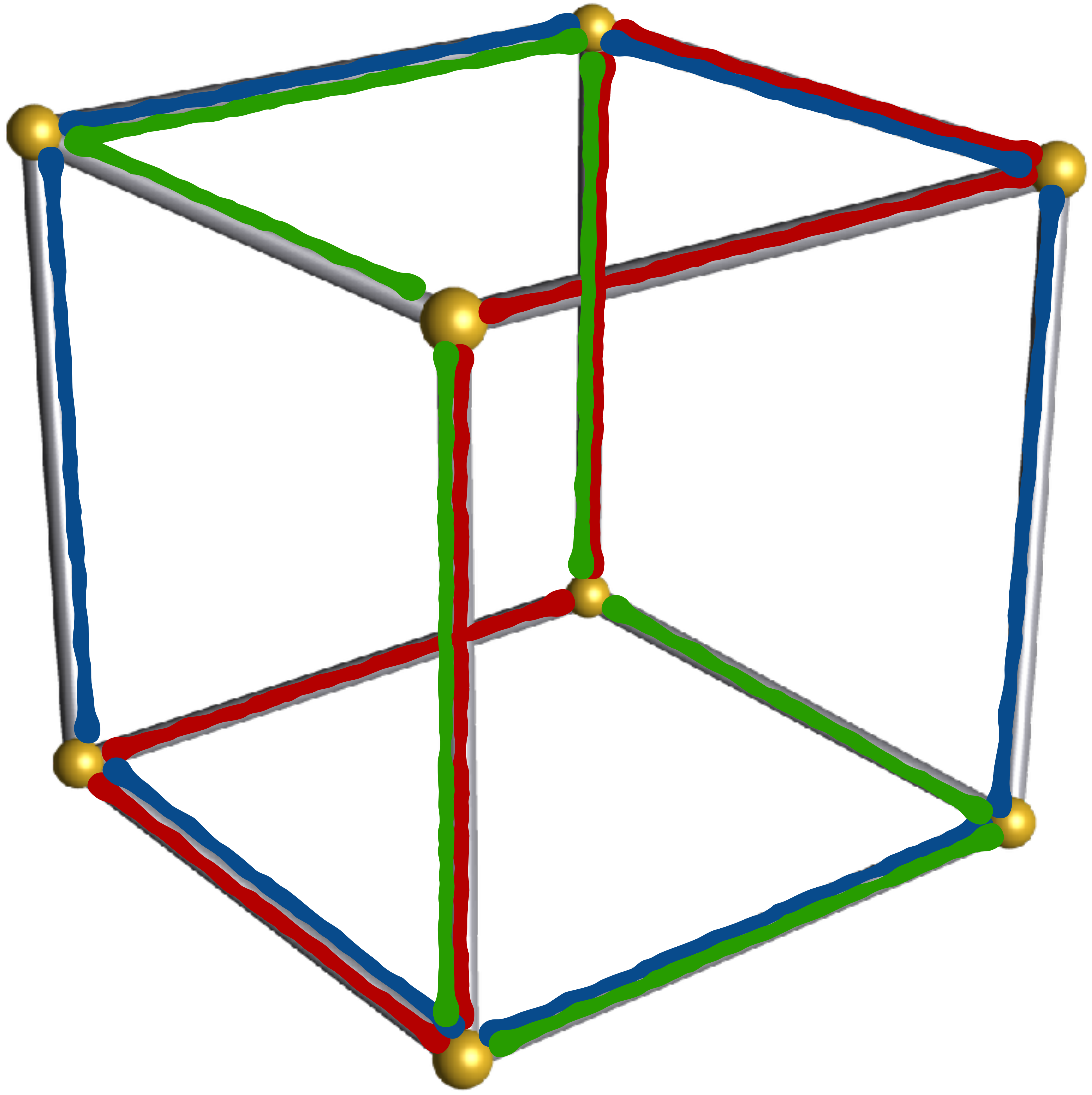

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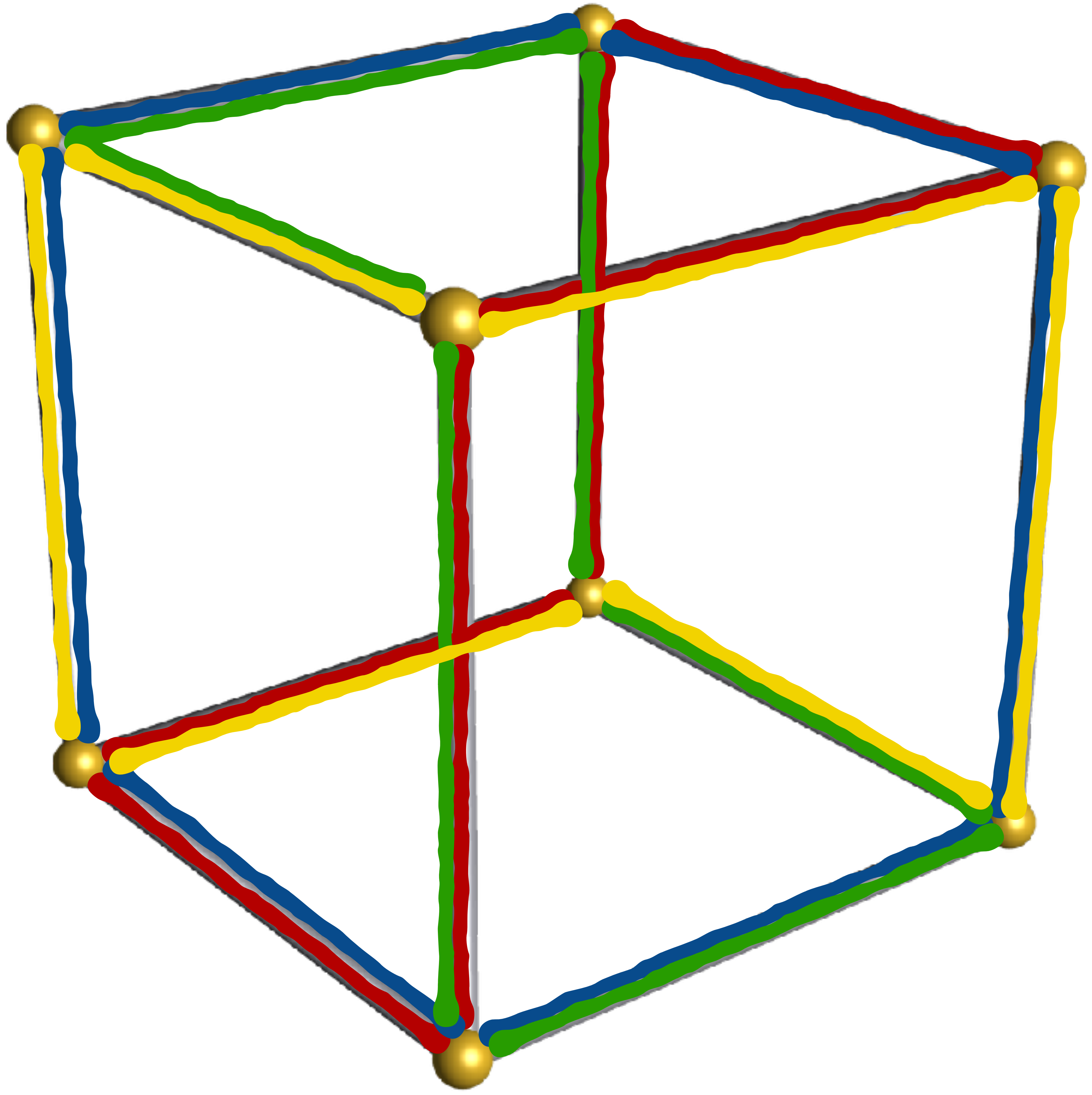

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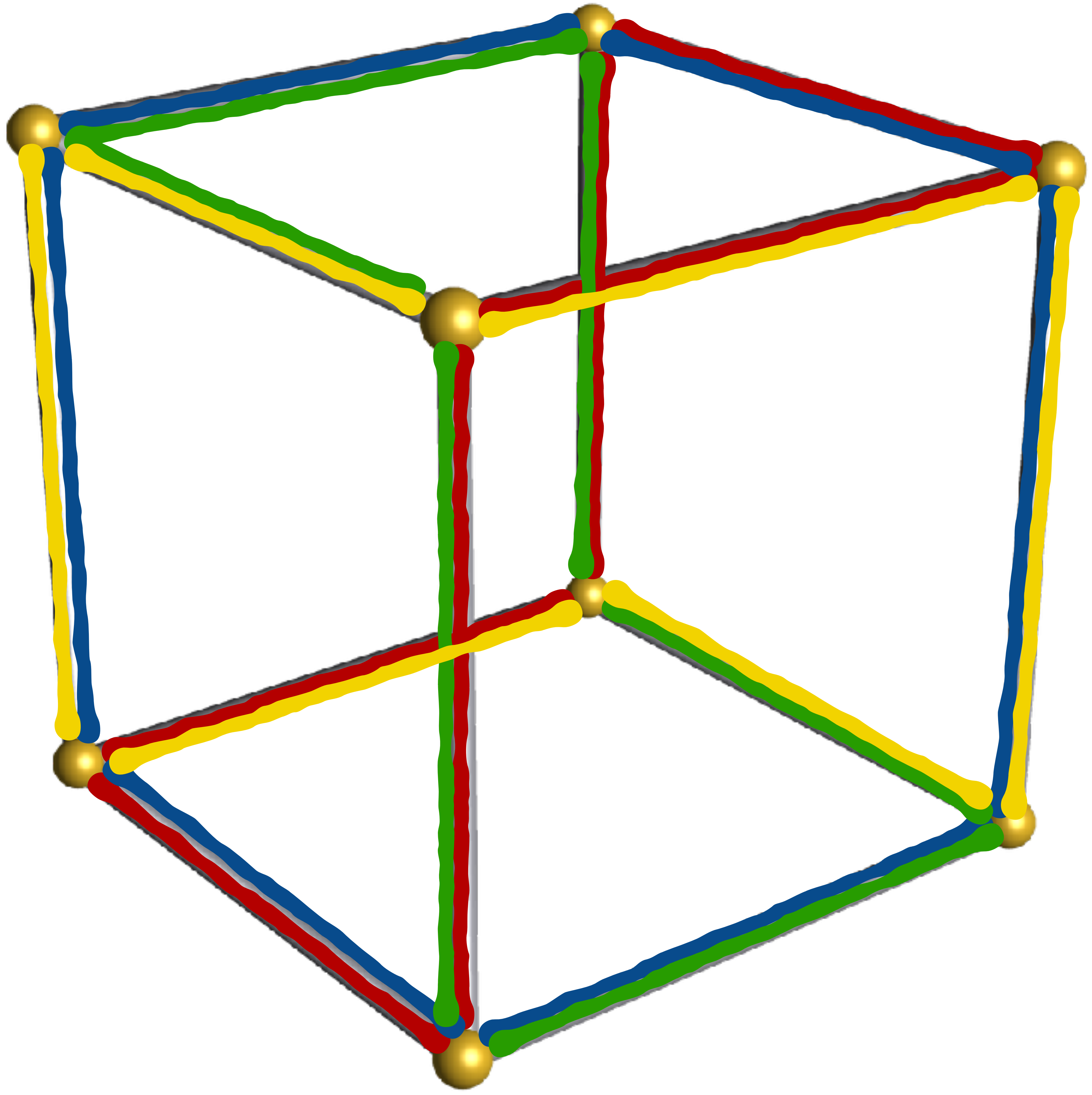

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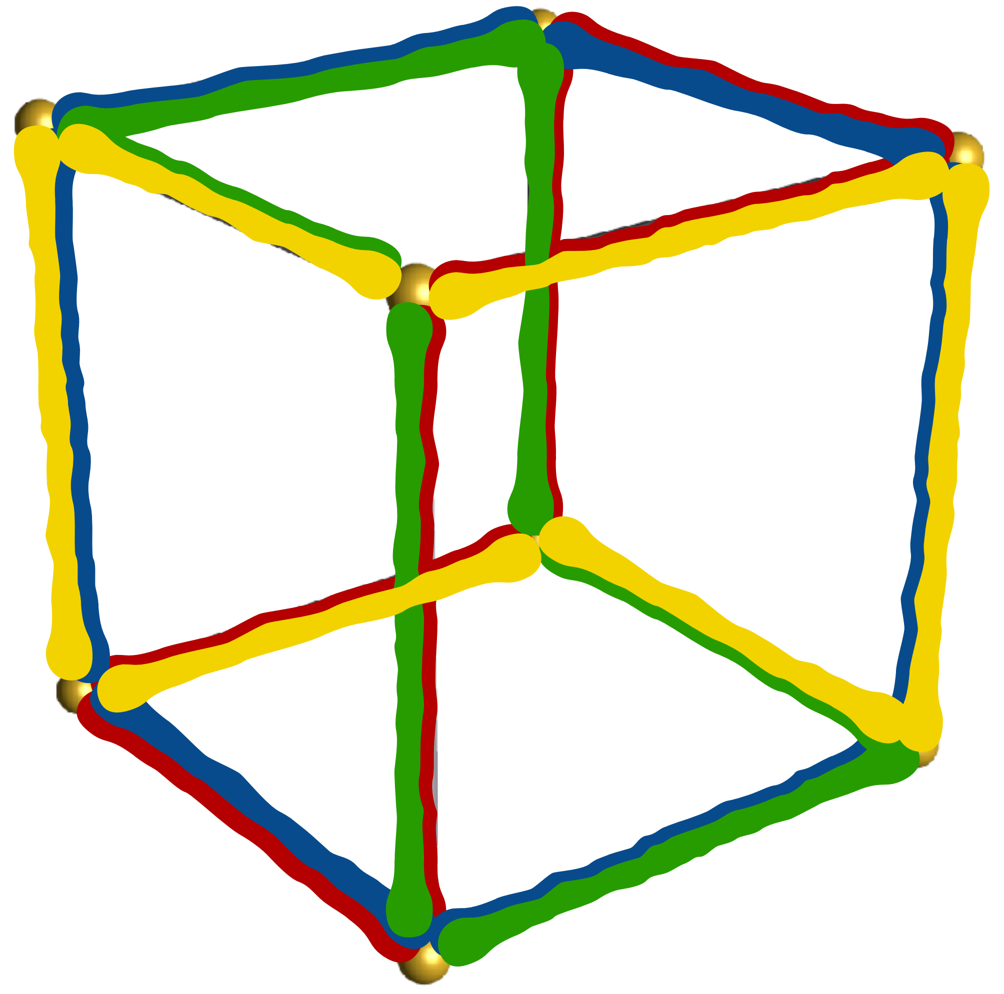

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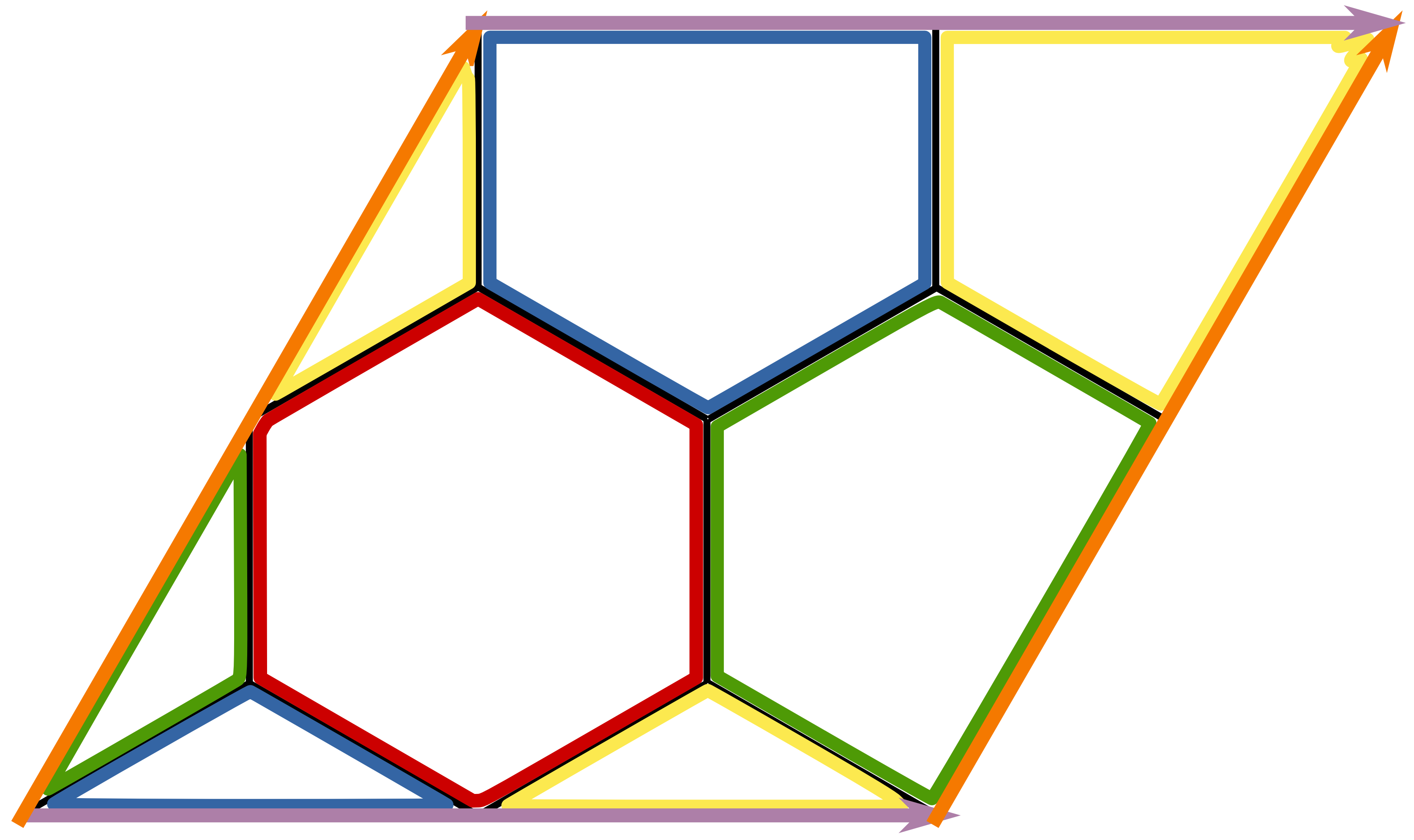

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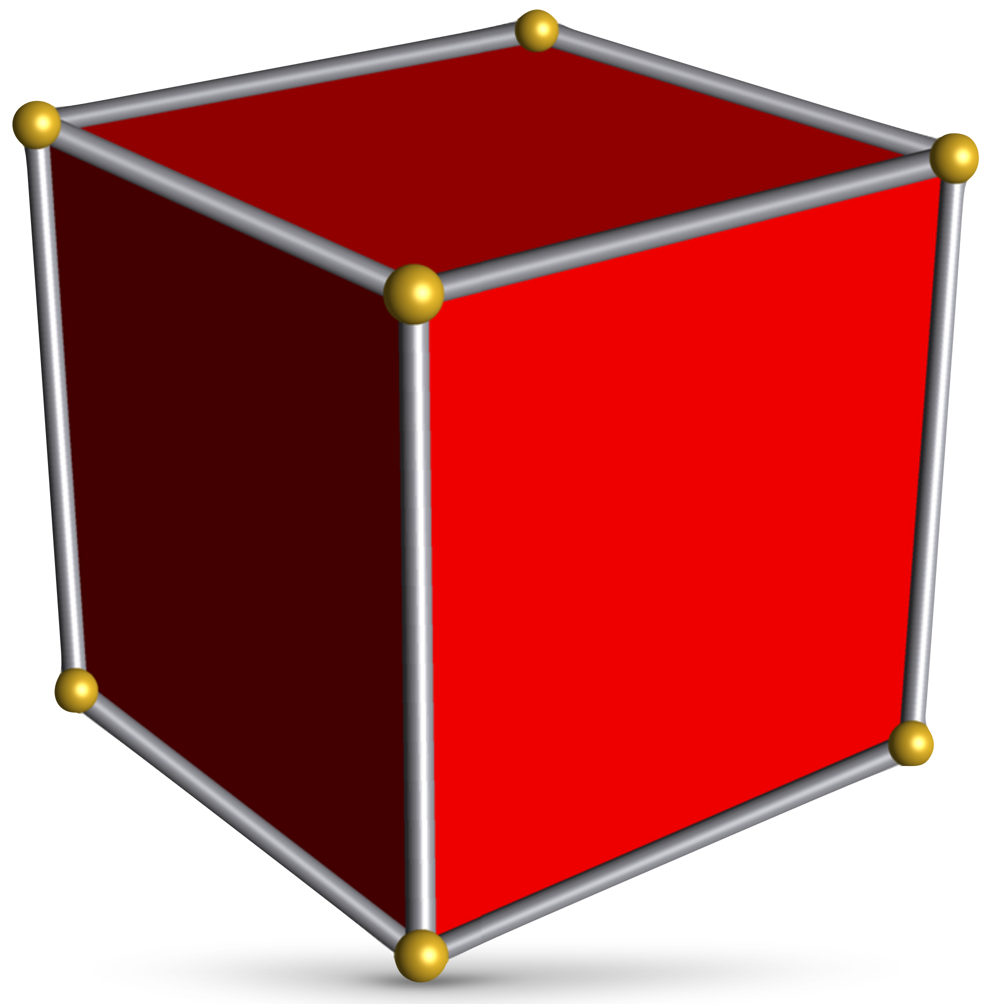

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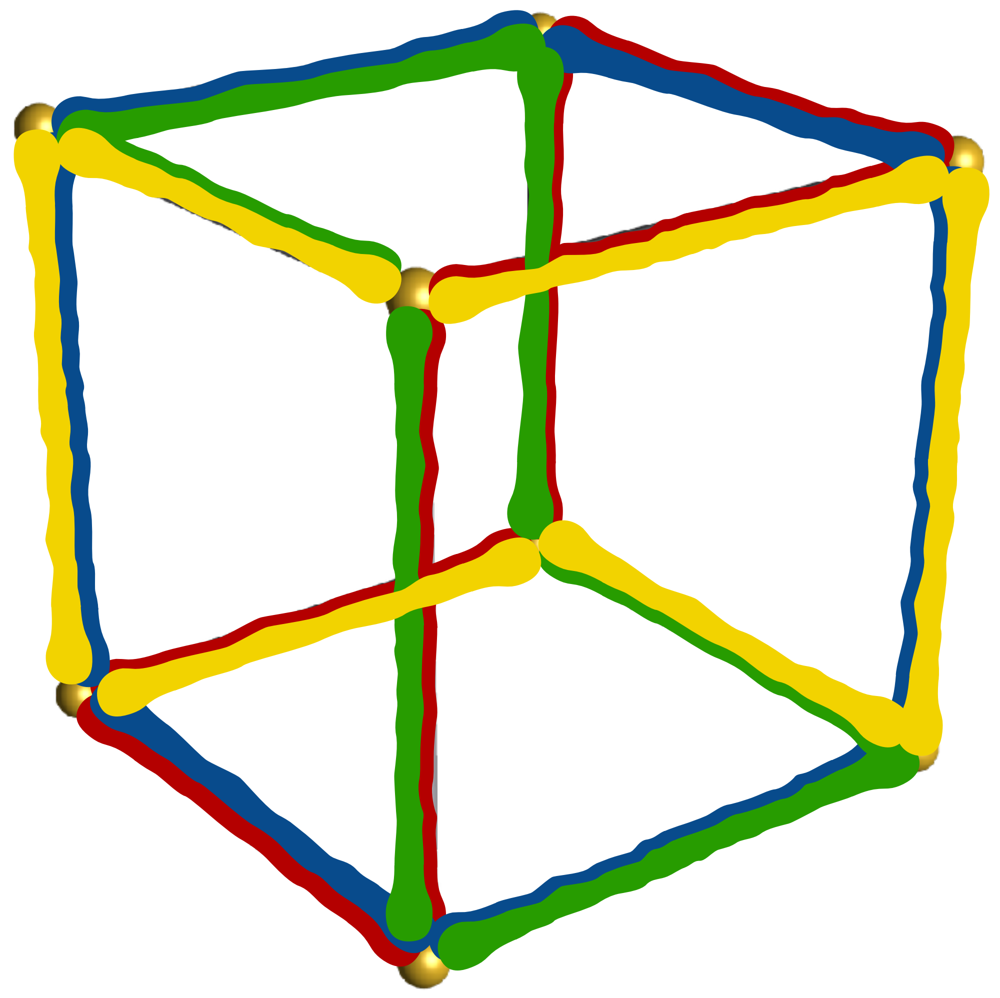

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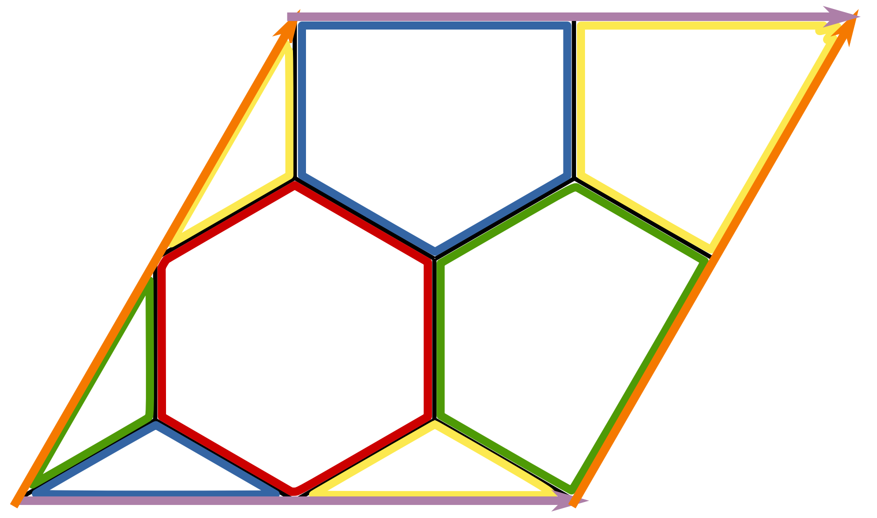

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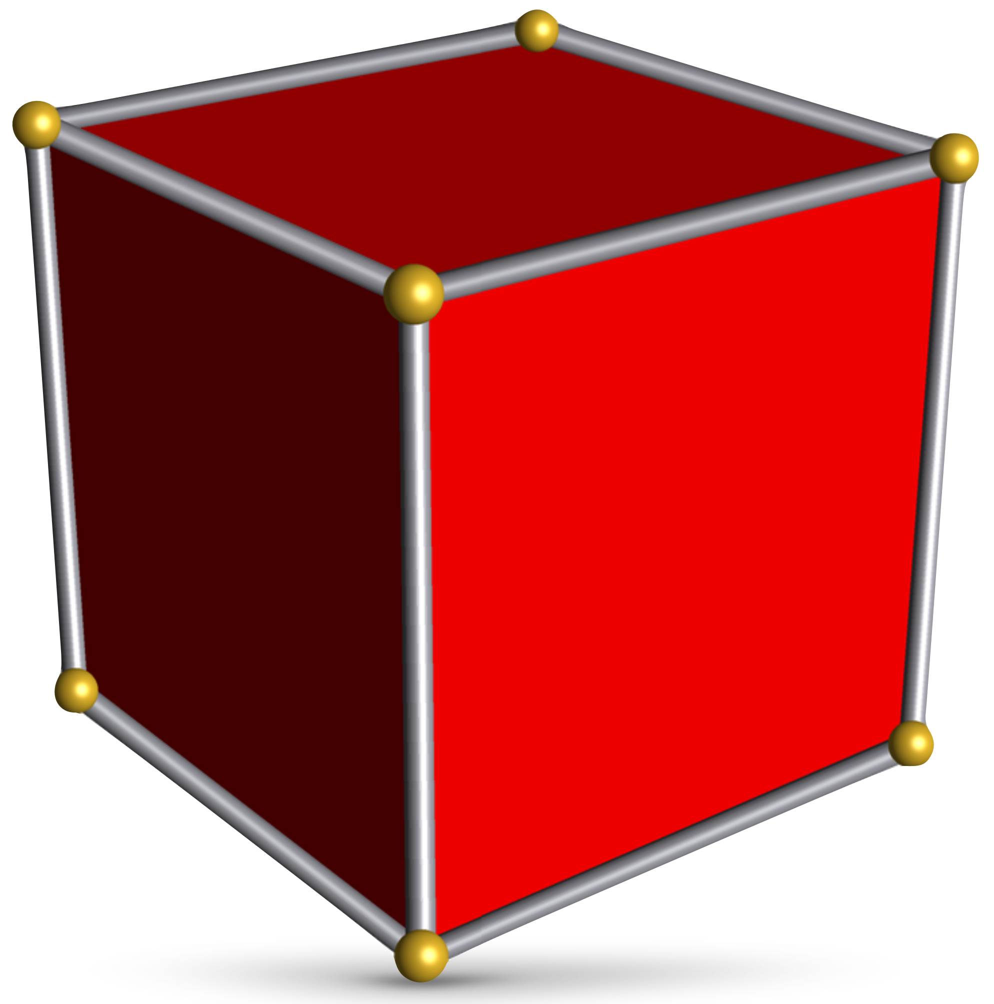

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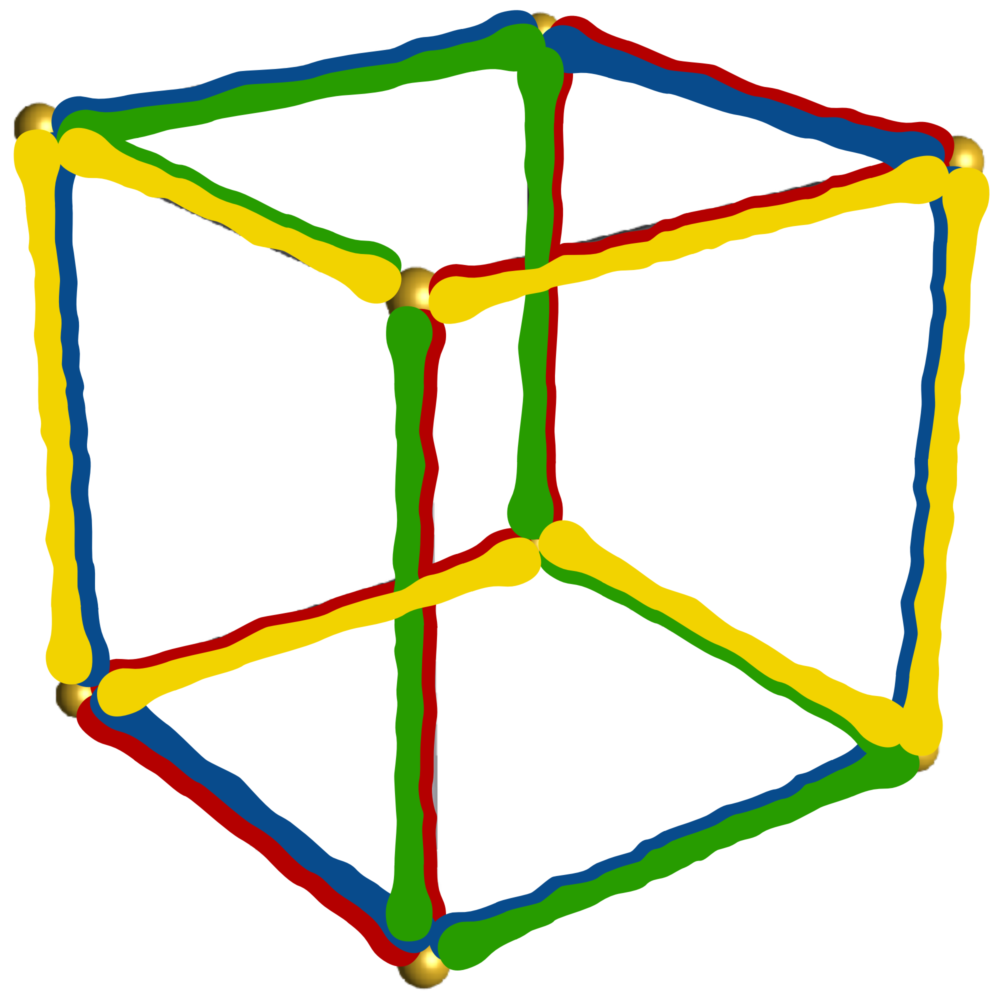

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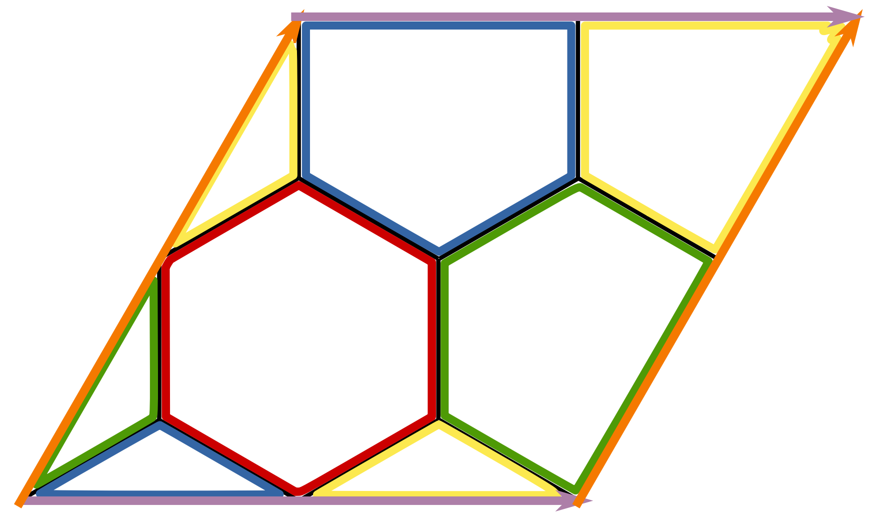

Operaciones

Operaciones

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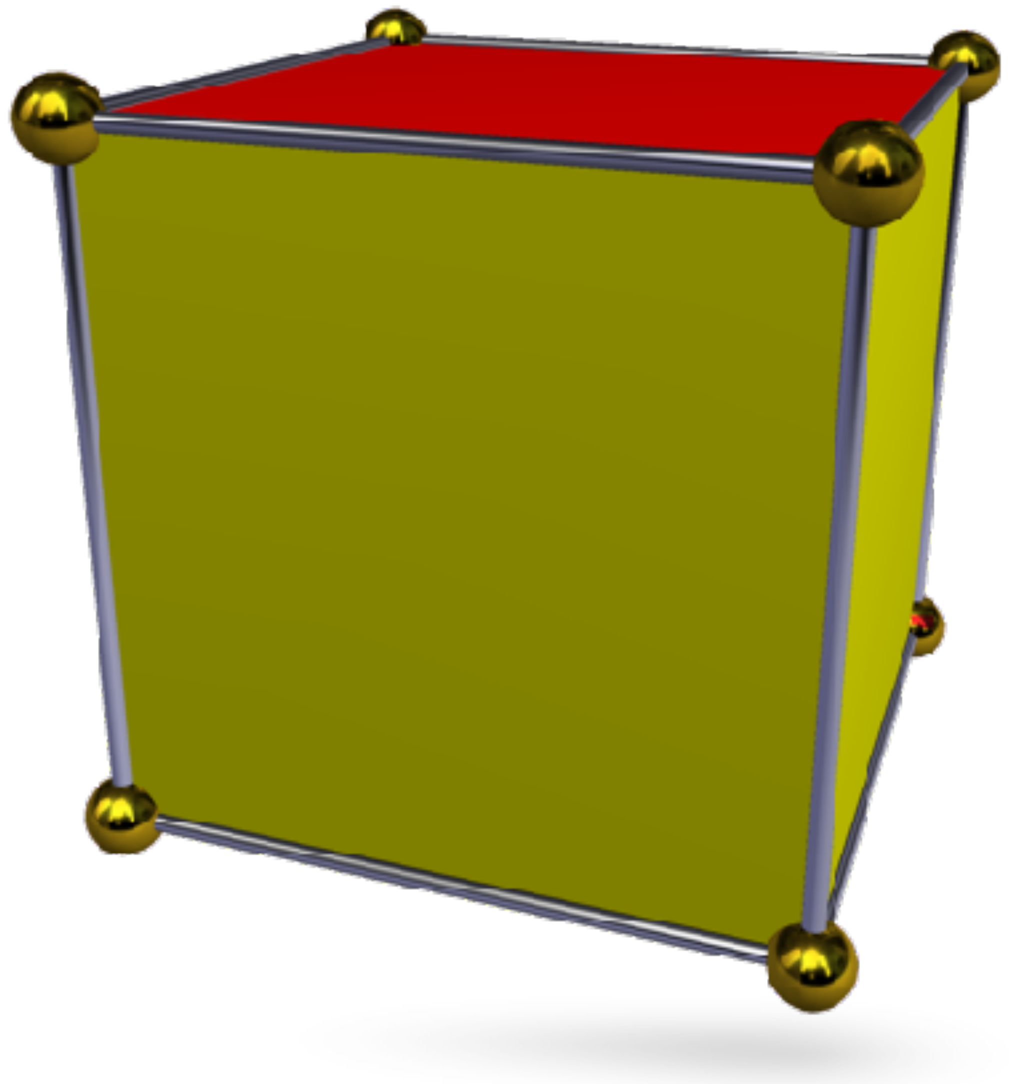

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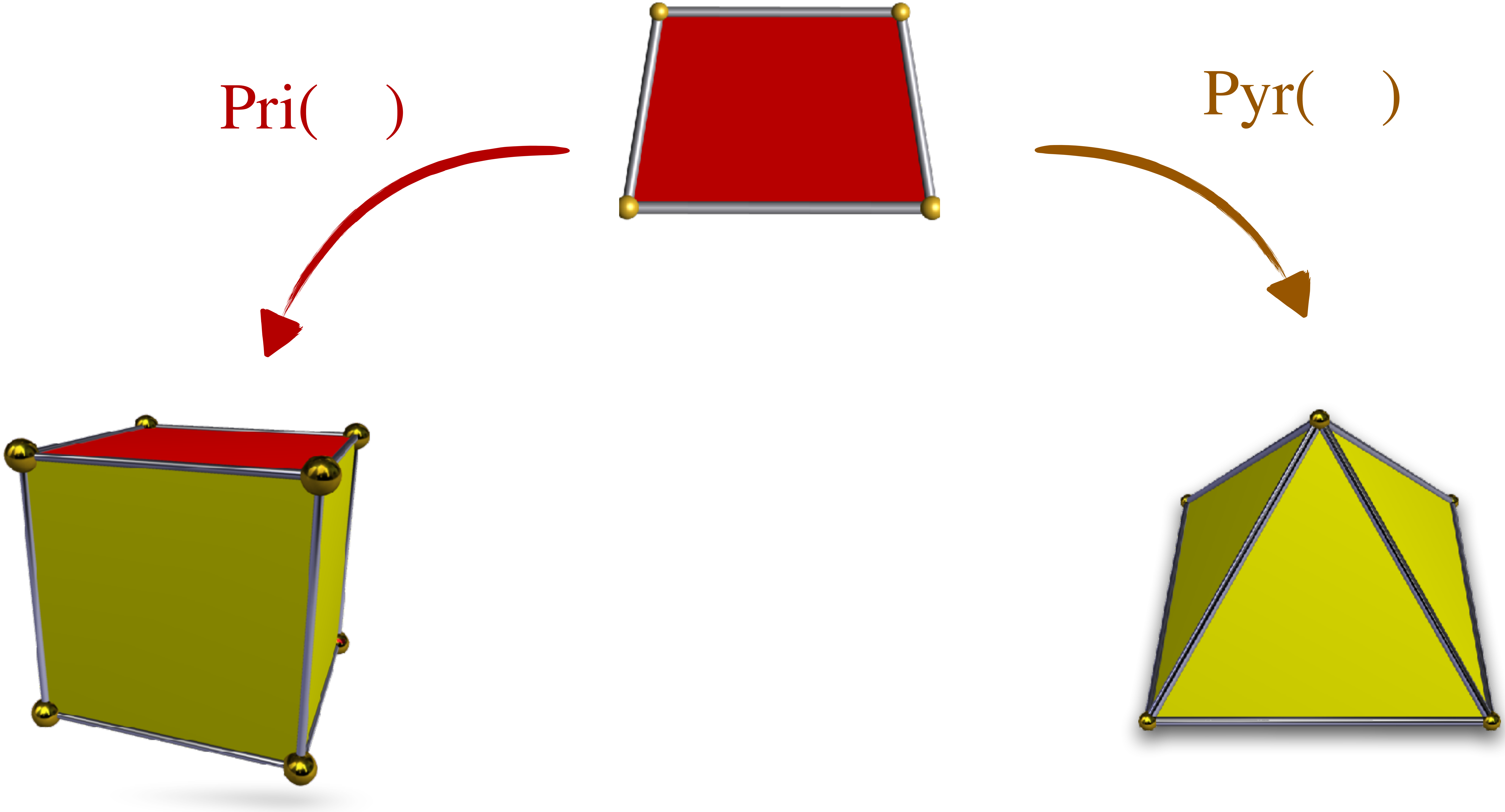

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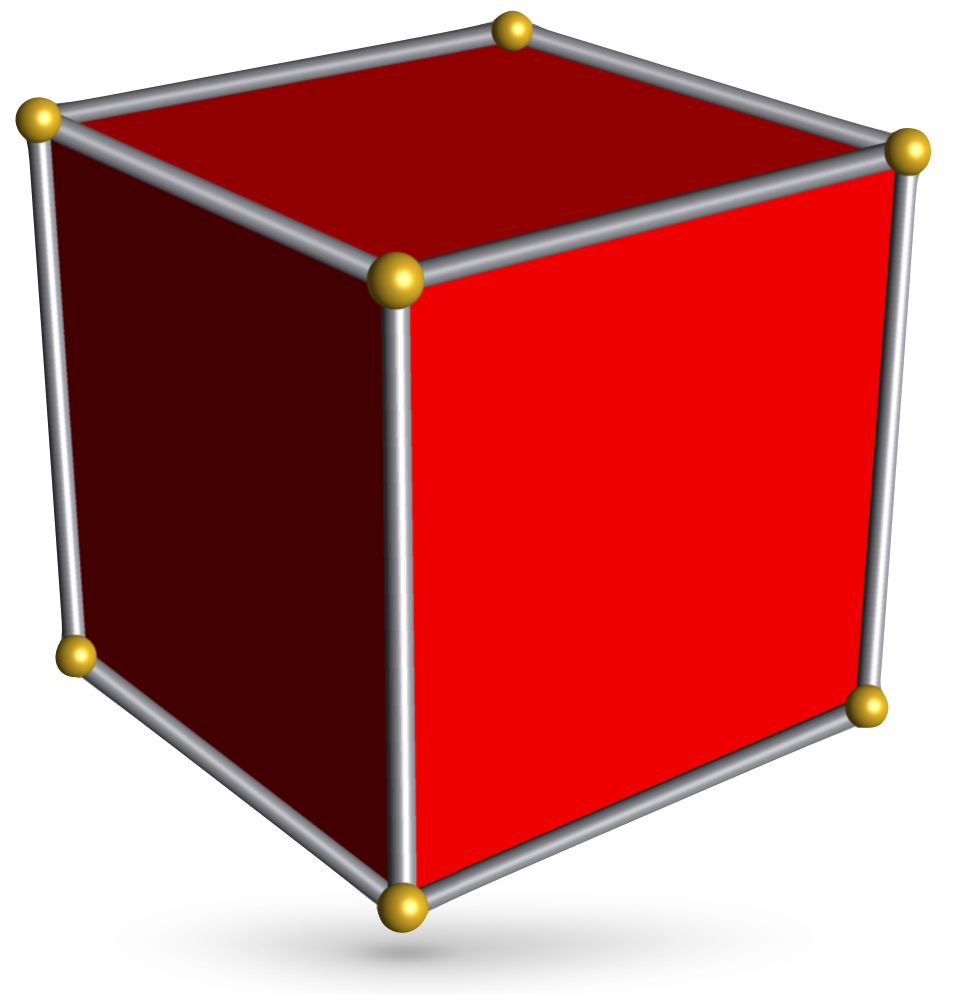

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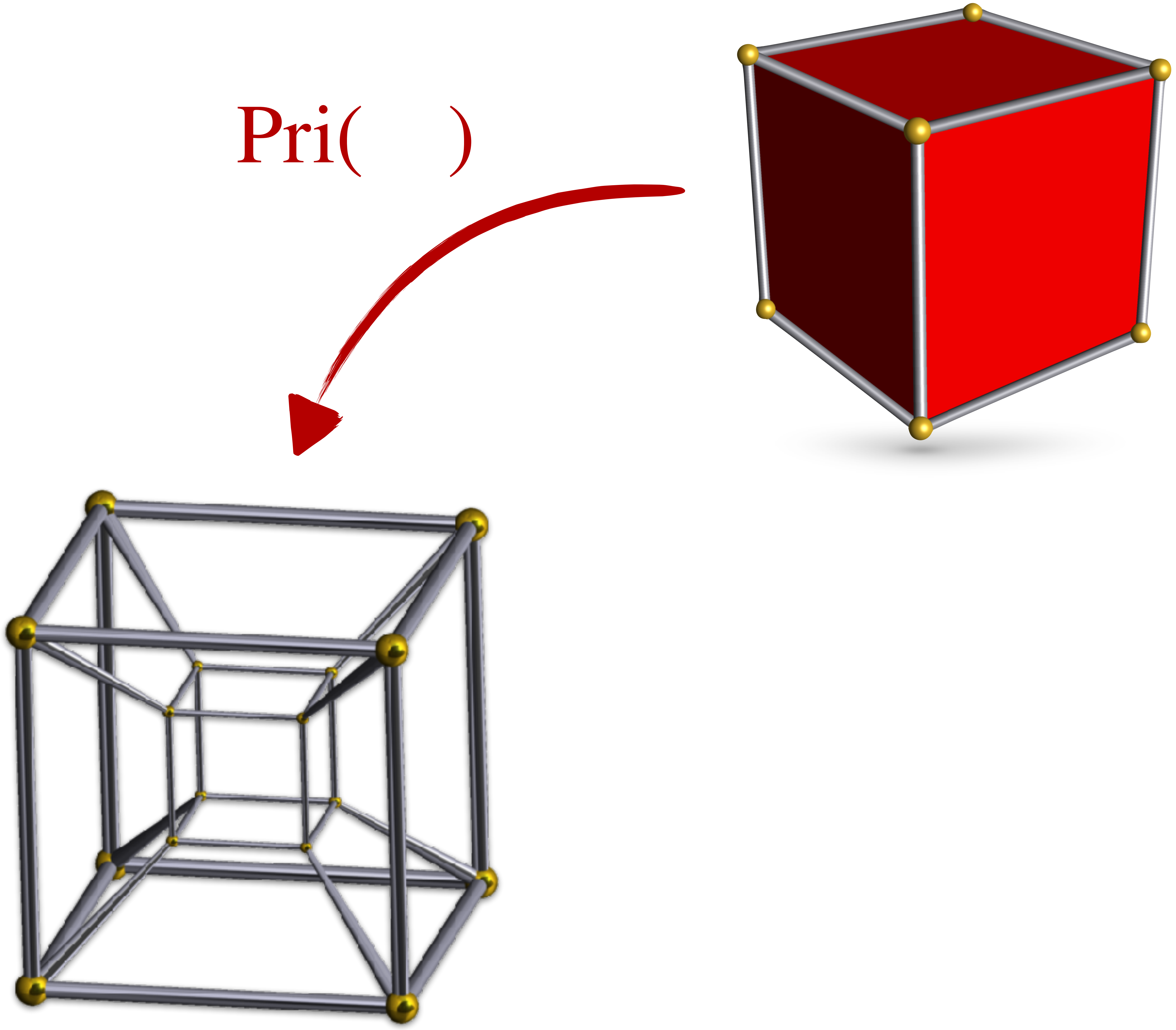

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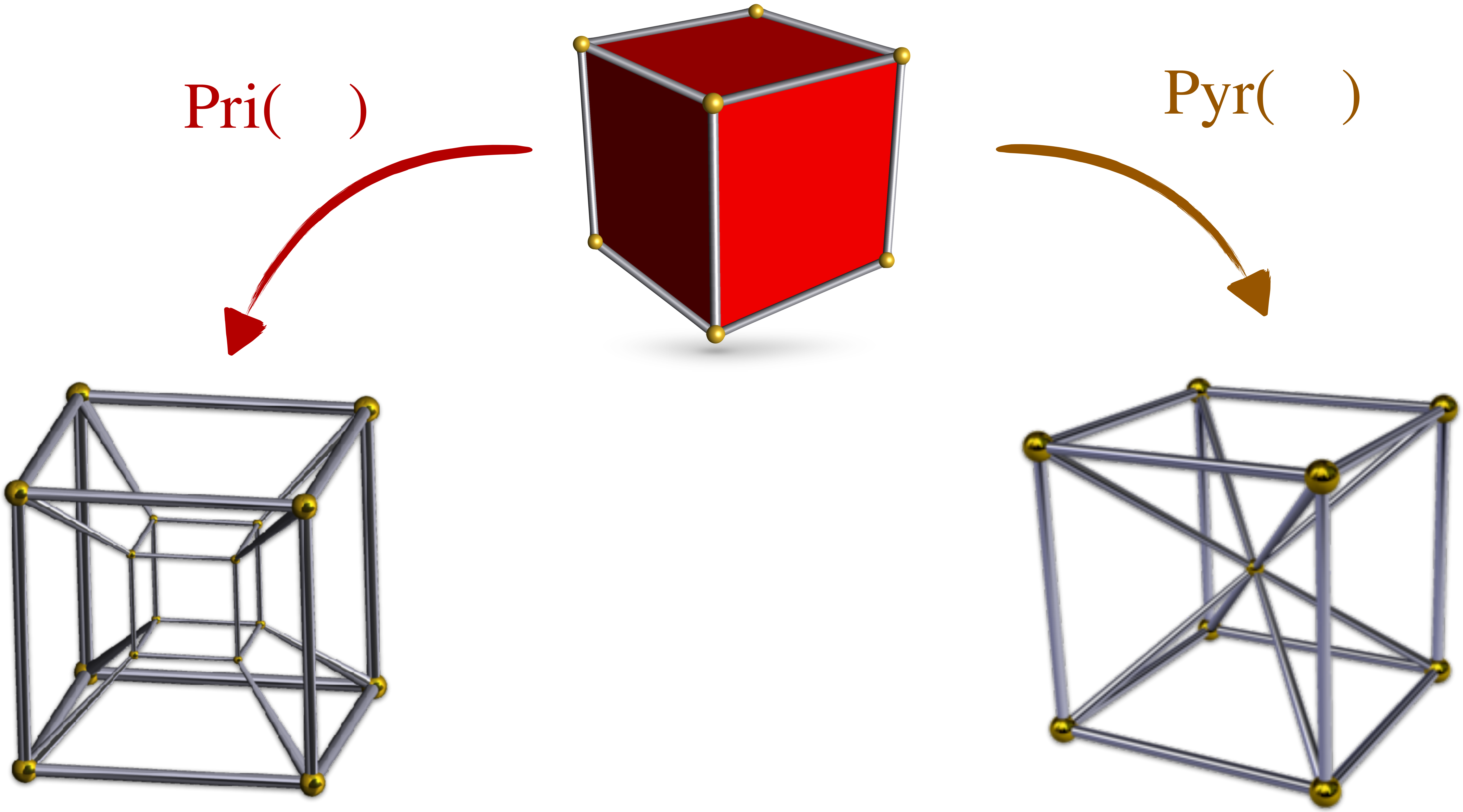

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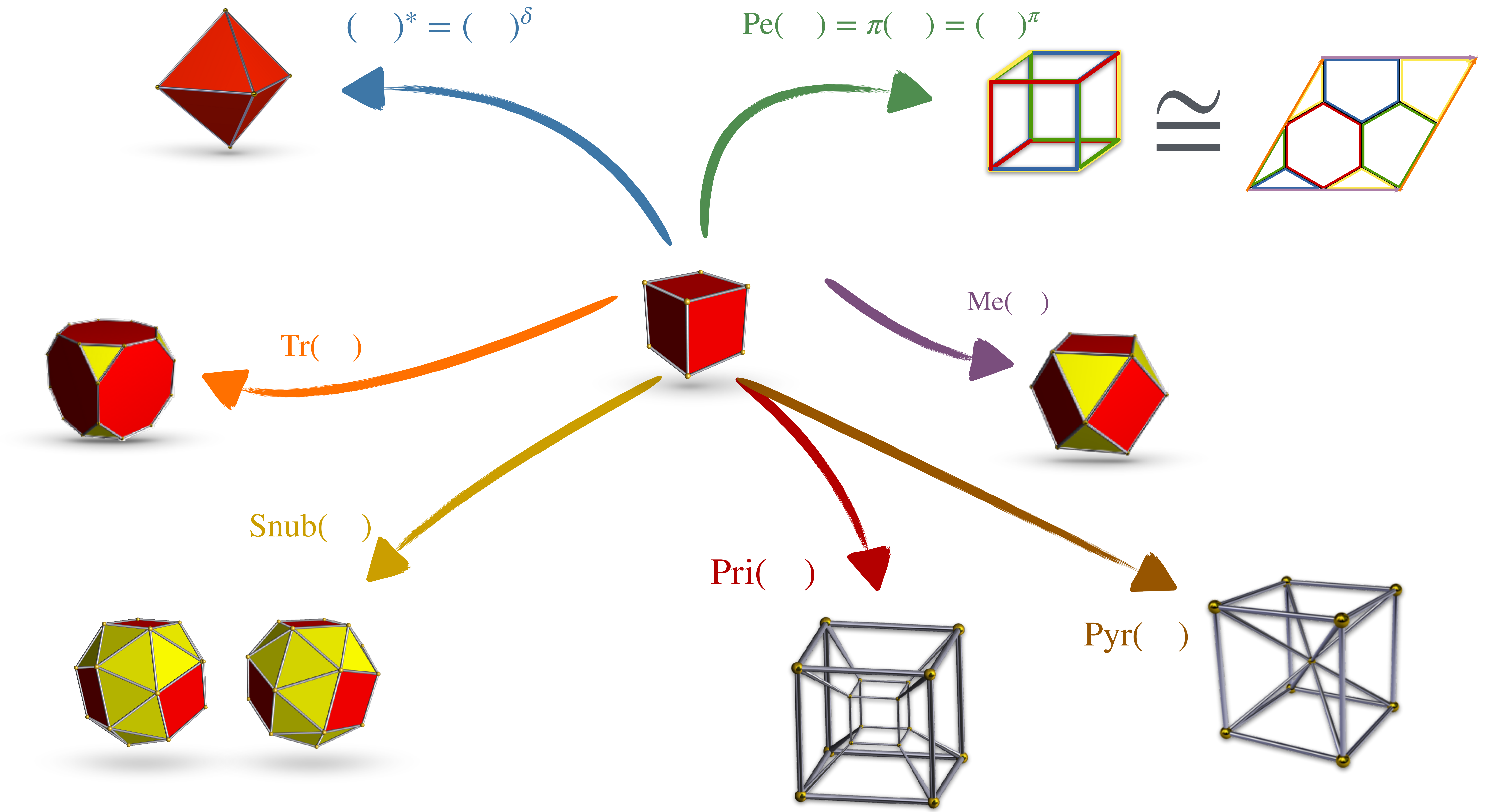

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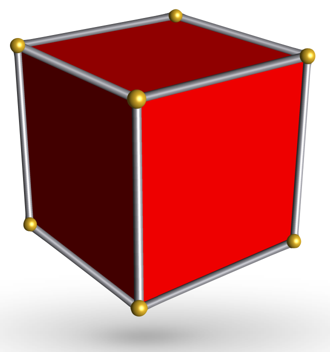
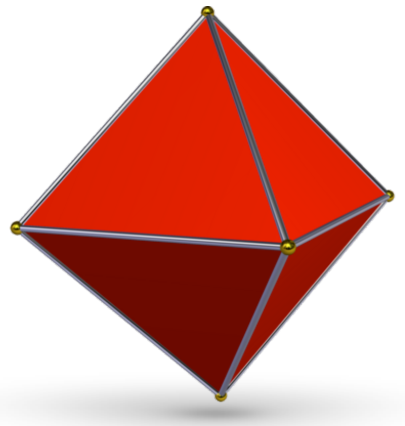
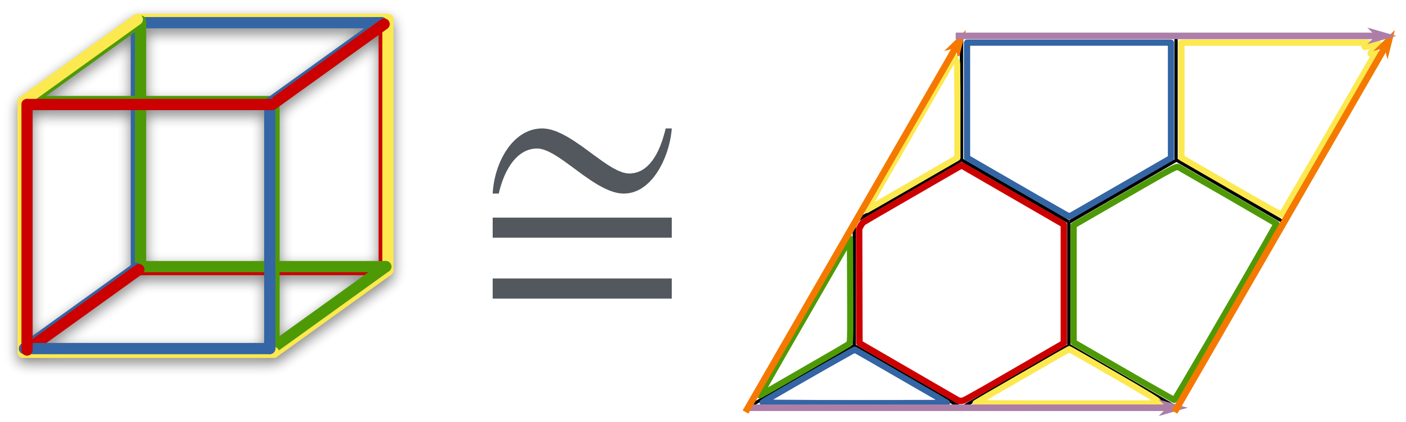




$(\quad)^* = (\quad)^\delta$



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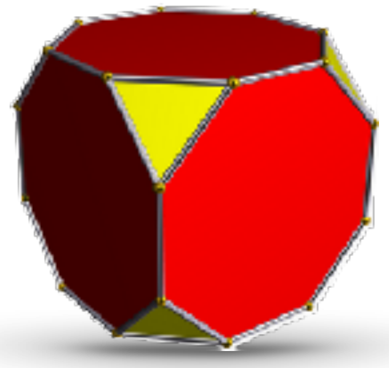
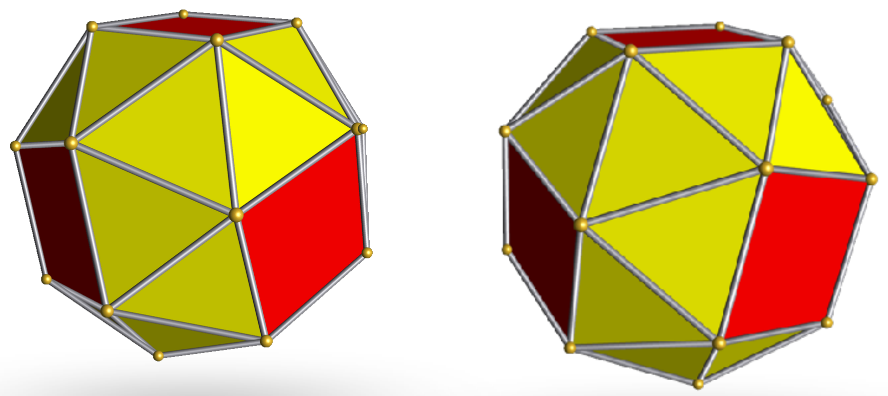


Tr(   )

Me(   )



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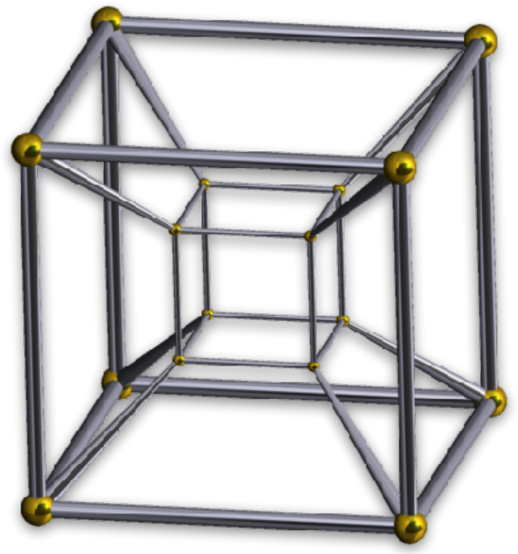

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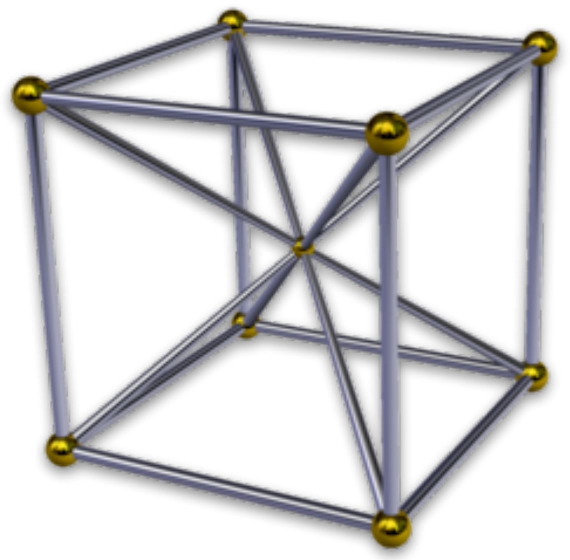

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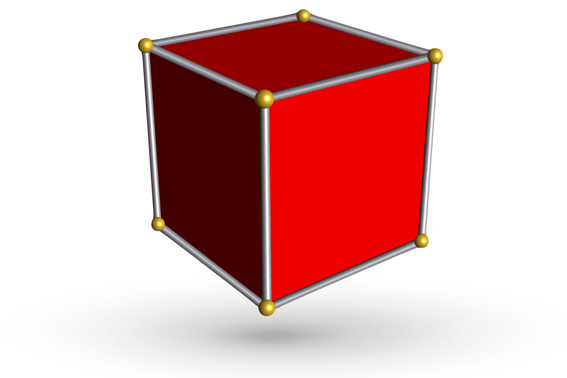
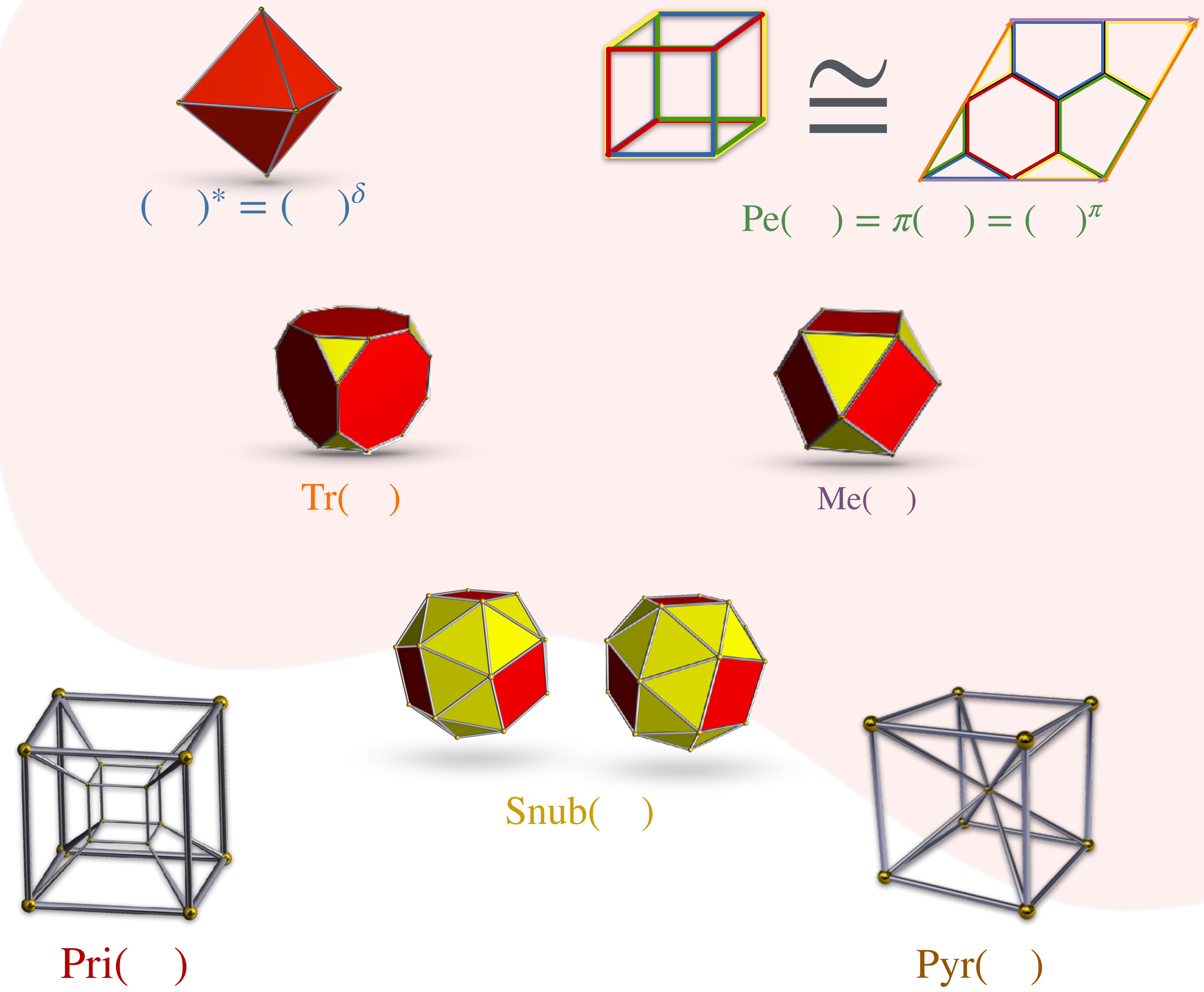


- Grupo de automorfismos



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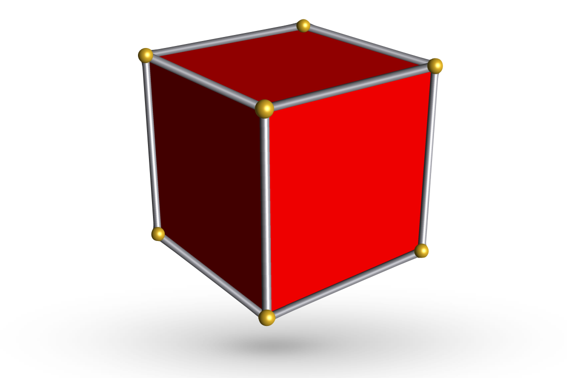
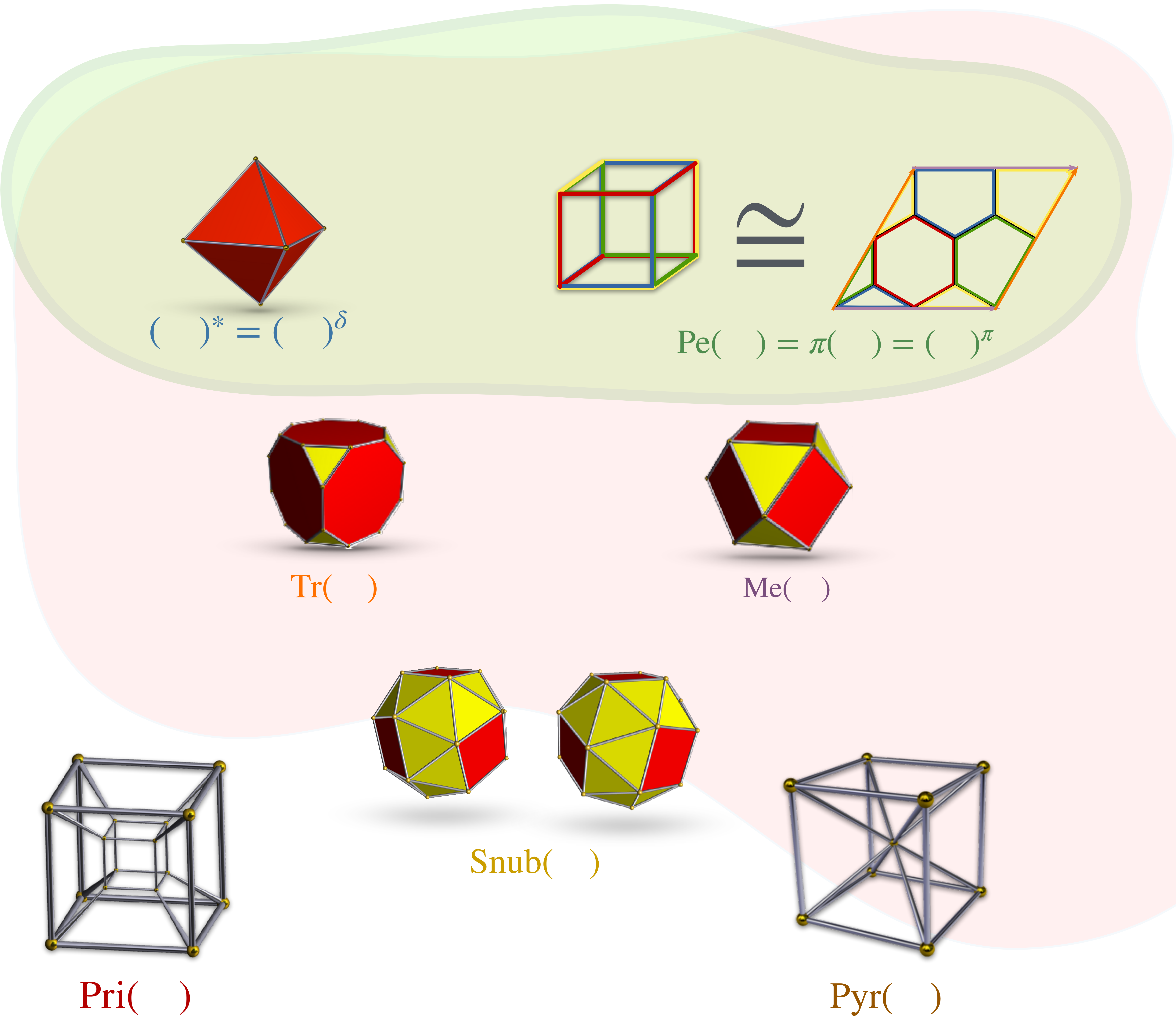
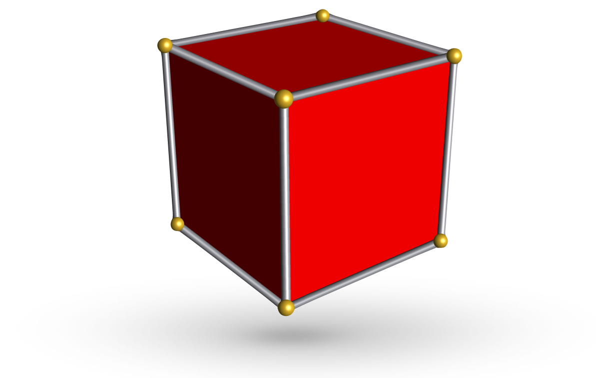
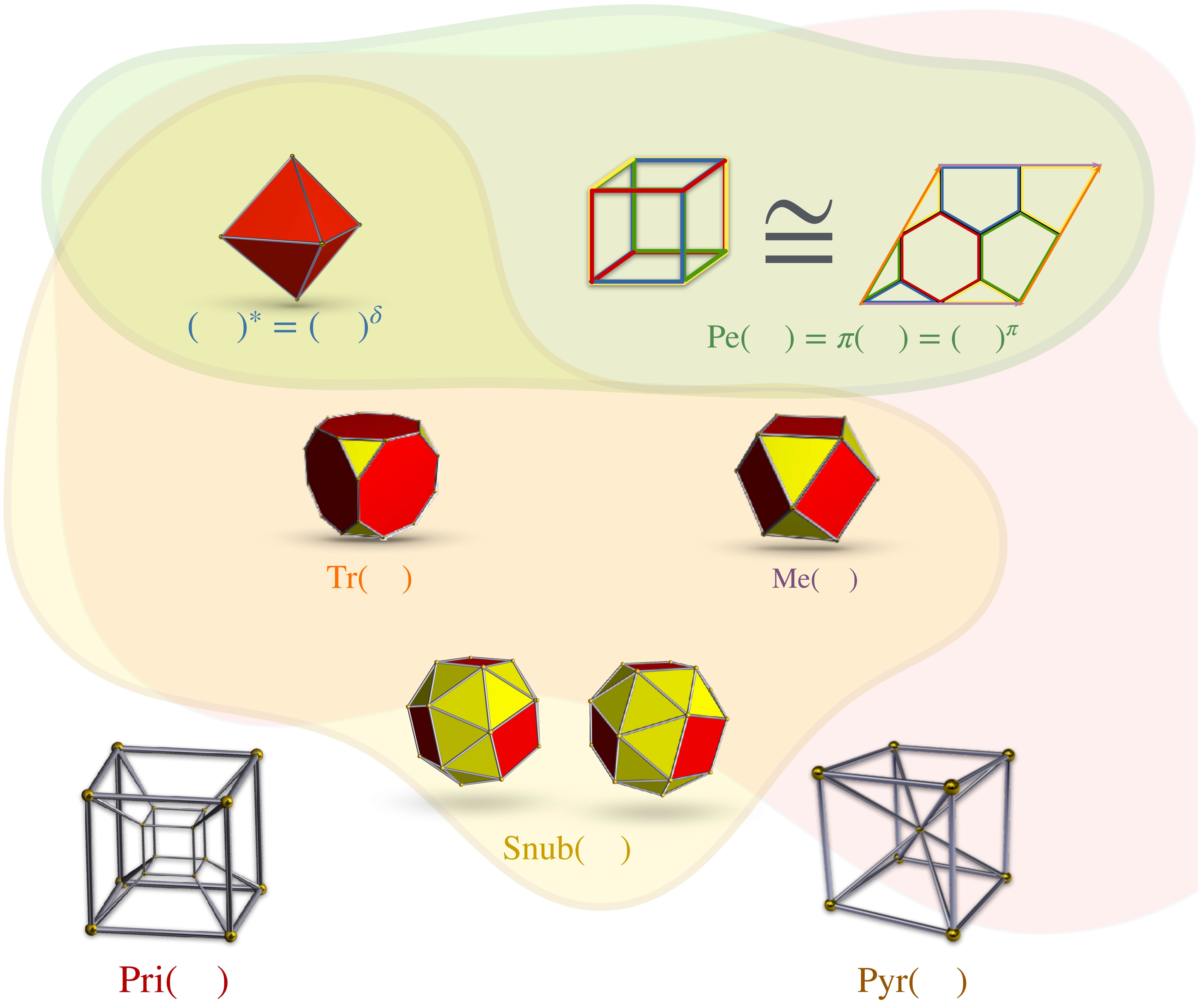




- Grupo de automorfismos
- Tamaño

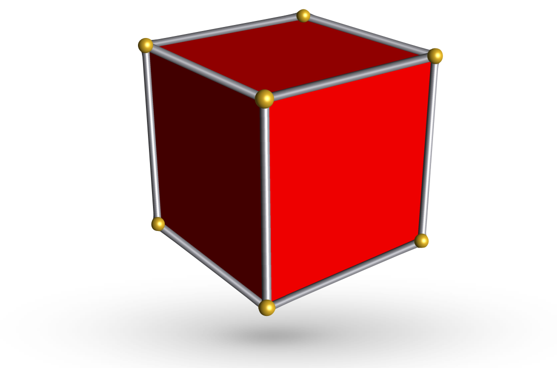
Operaciones



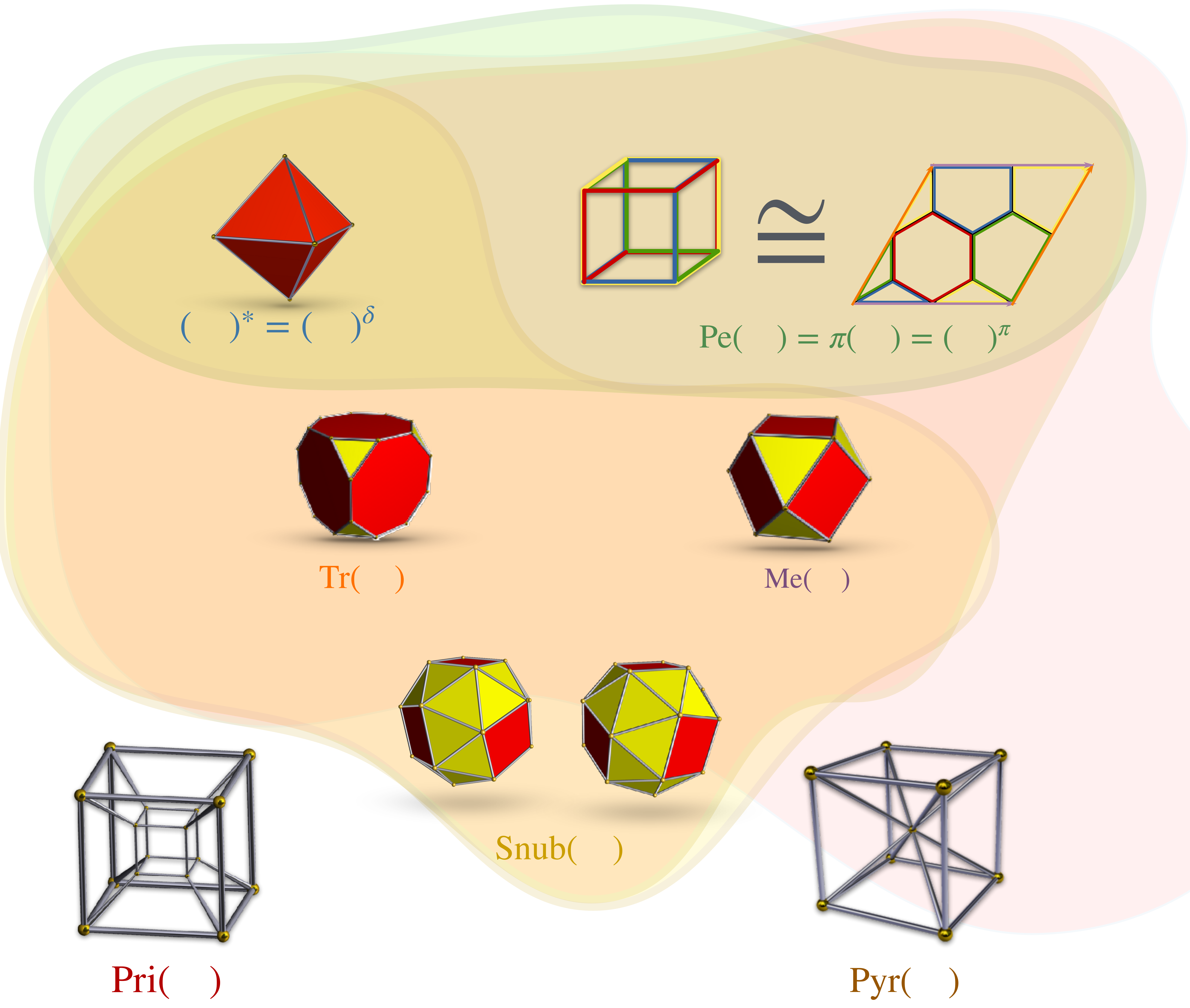


- Grupo de automorfismos
- Tamaño
- Superficie

Operaciones



- Grupo de automorfismos
- Tamaño
- Superficie
- Rango (dimension)



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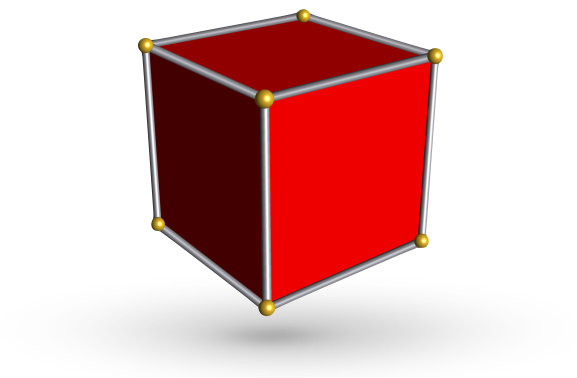
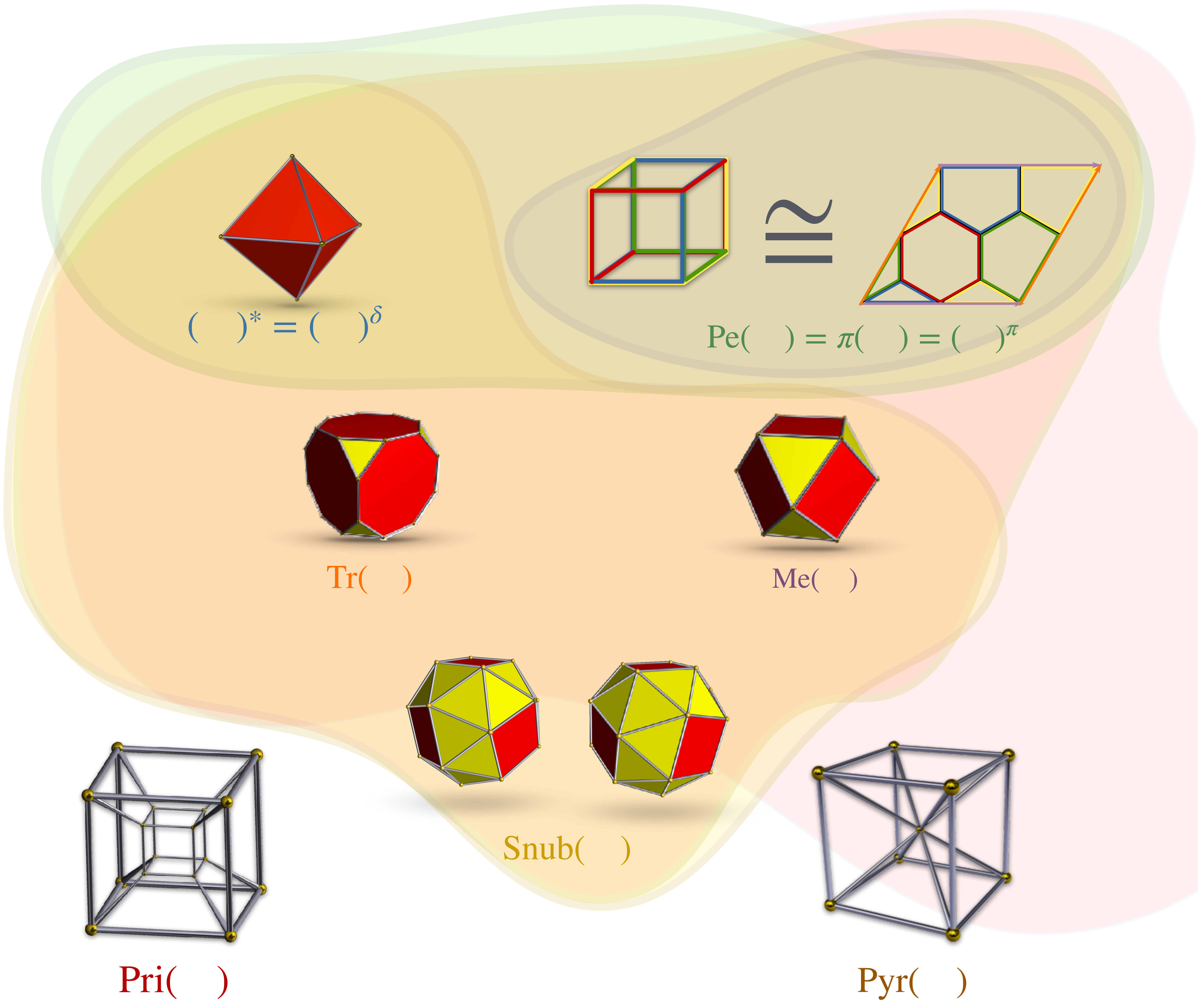


- Grupo de automorfismos
- Tamaño
- Superficie
- Rango (dimension)
- 1-esqueleto



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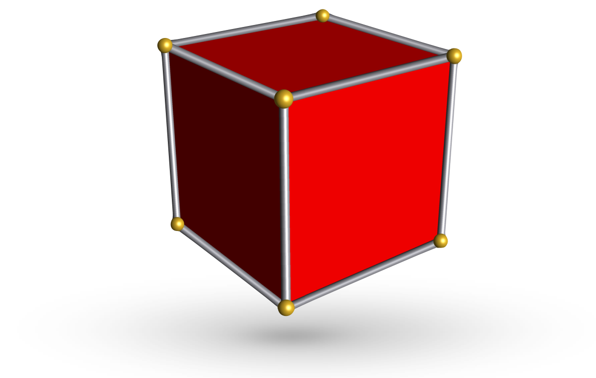

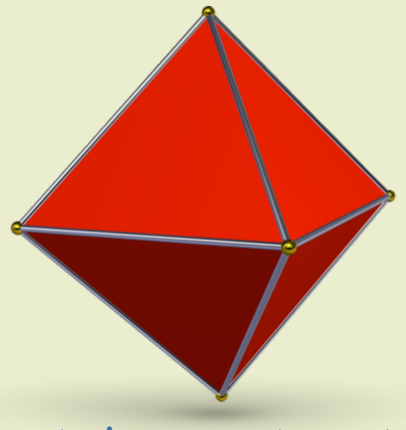

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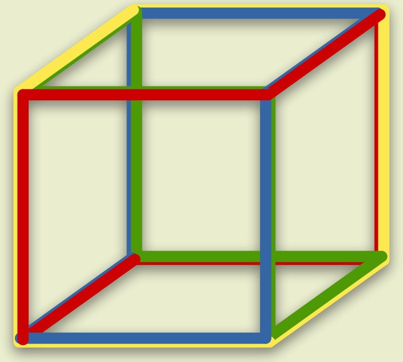

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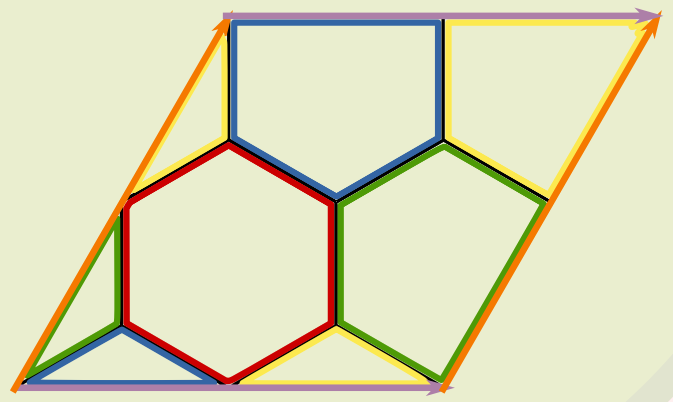
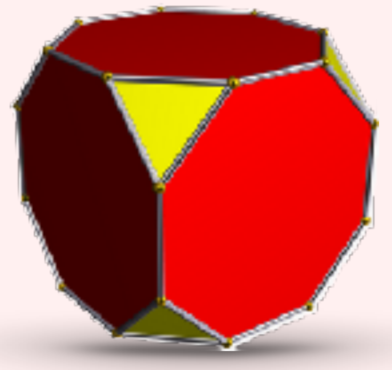


$\mathrm{Pe}(\quad) = \pi(\quad) = (\quad)^{\pi}$



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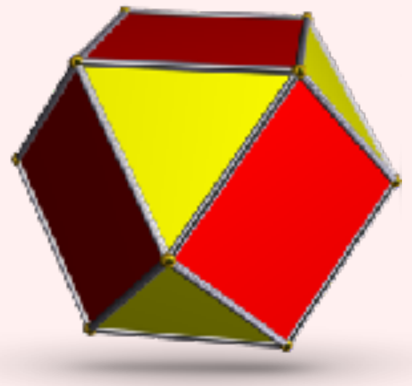

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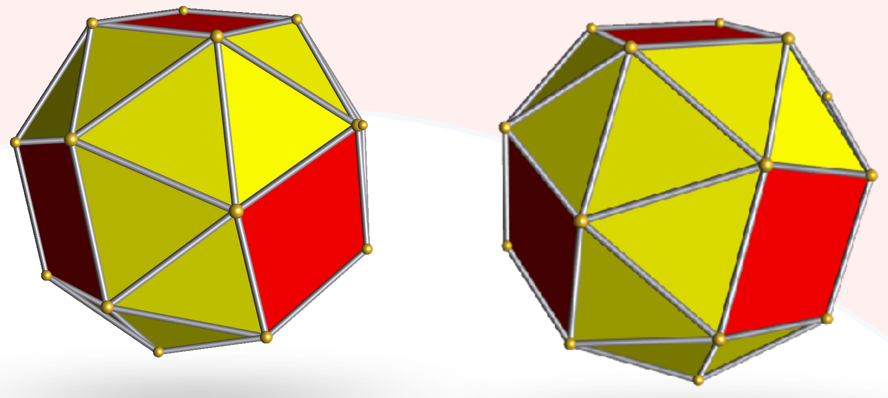

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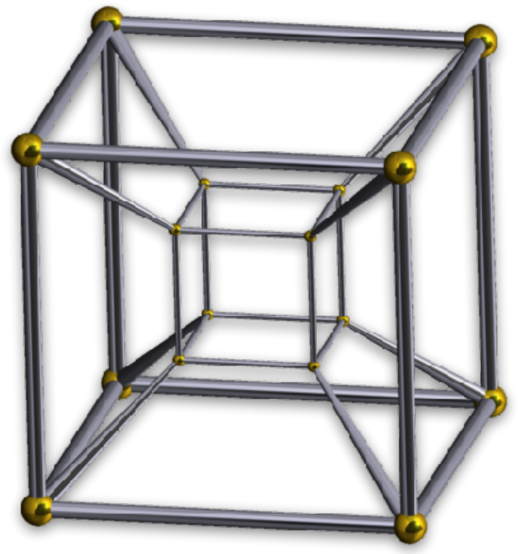

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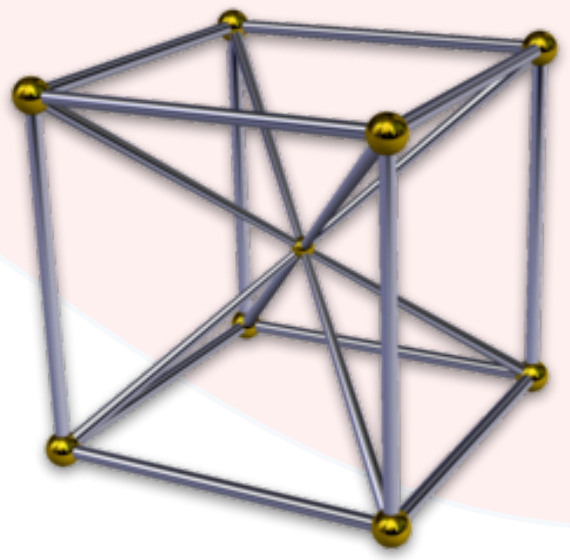
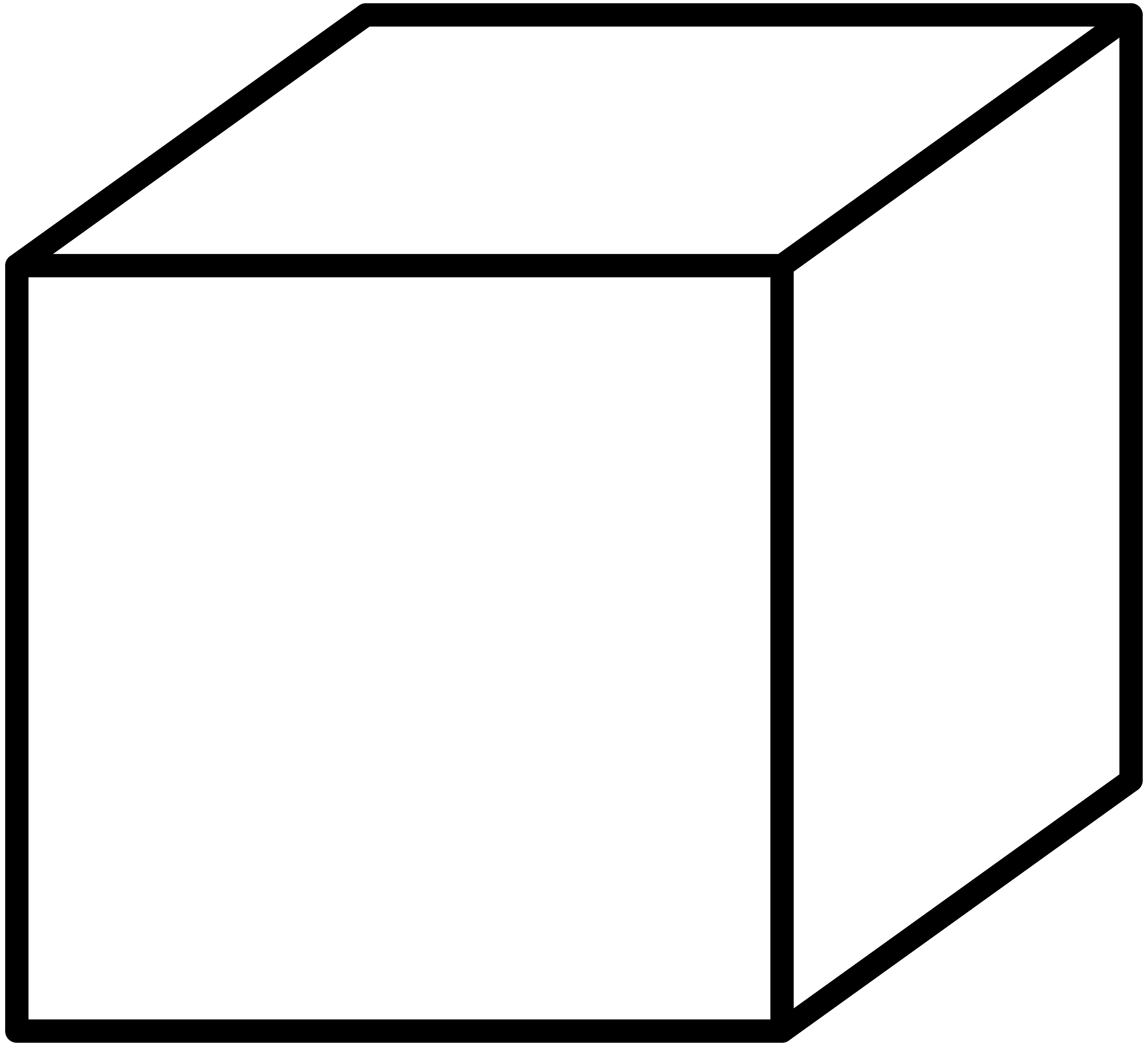


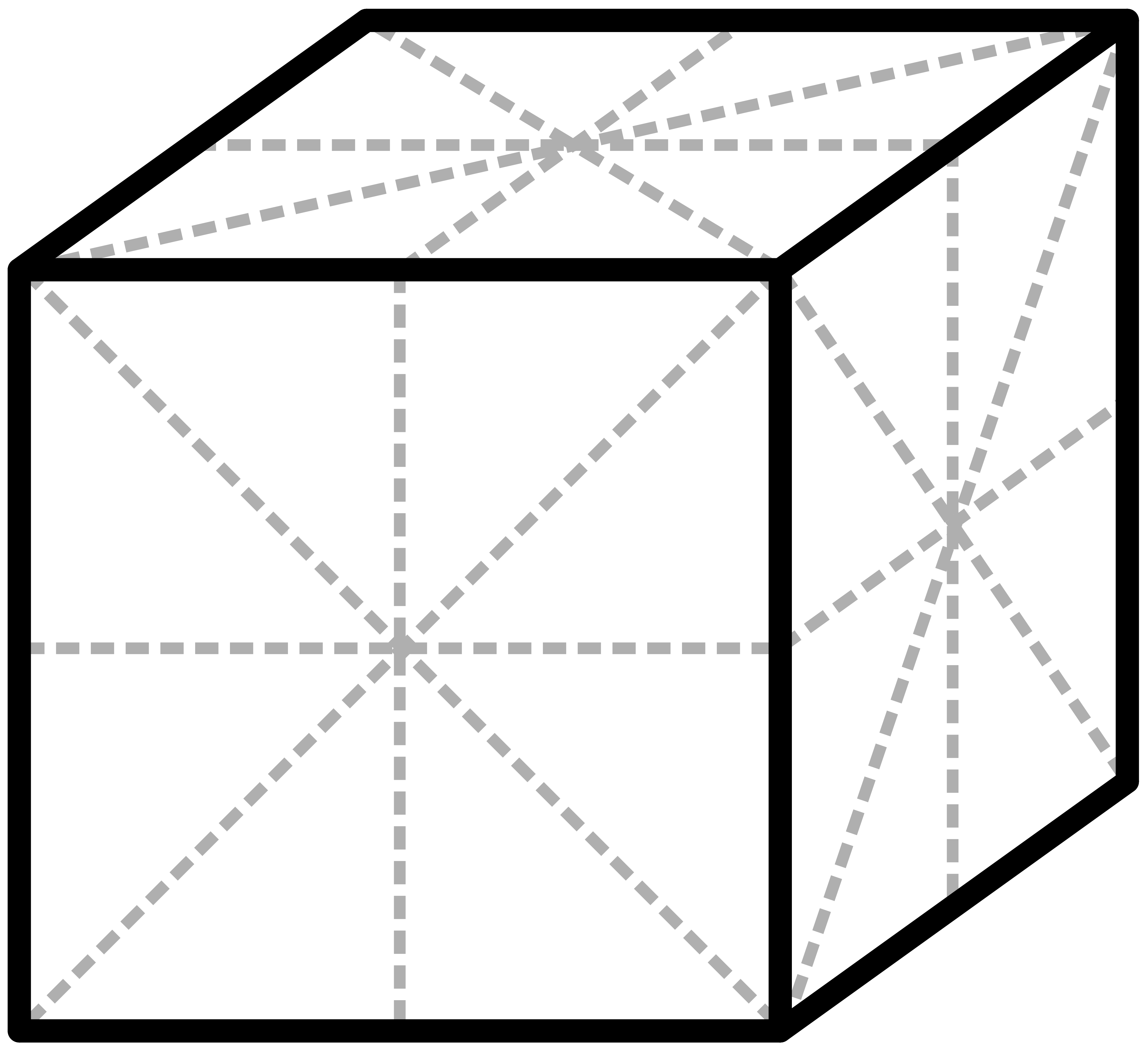
Pyr( )

- Grupo de automorfismos
- Tamaño

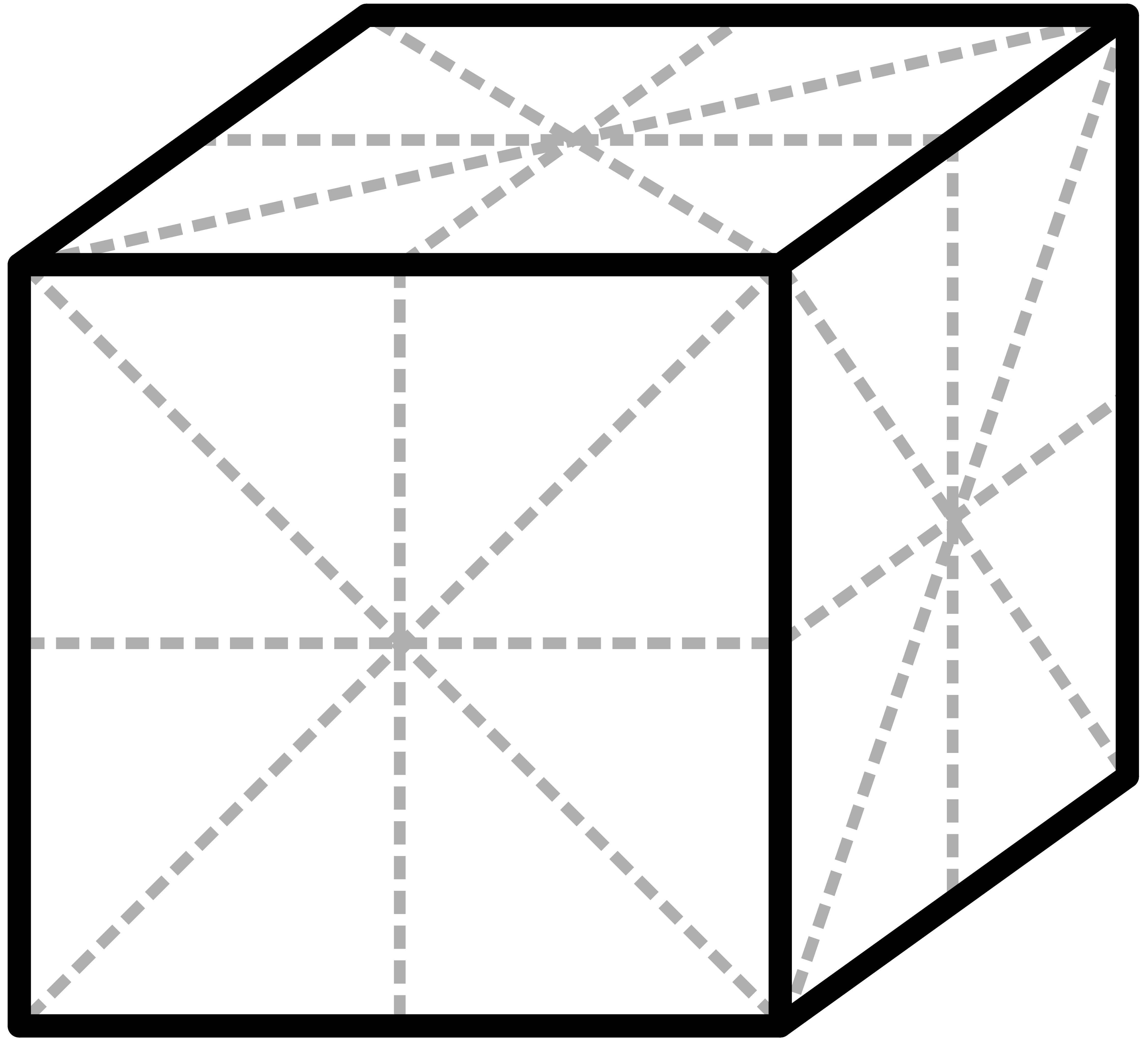
Politopos y maniplexes



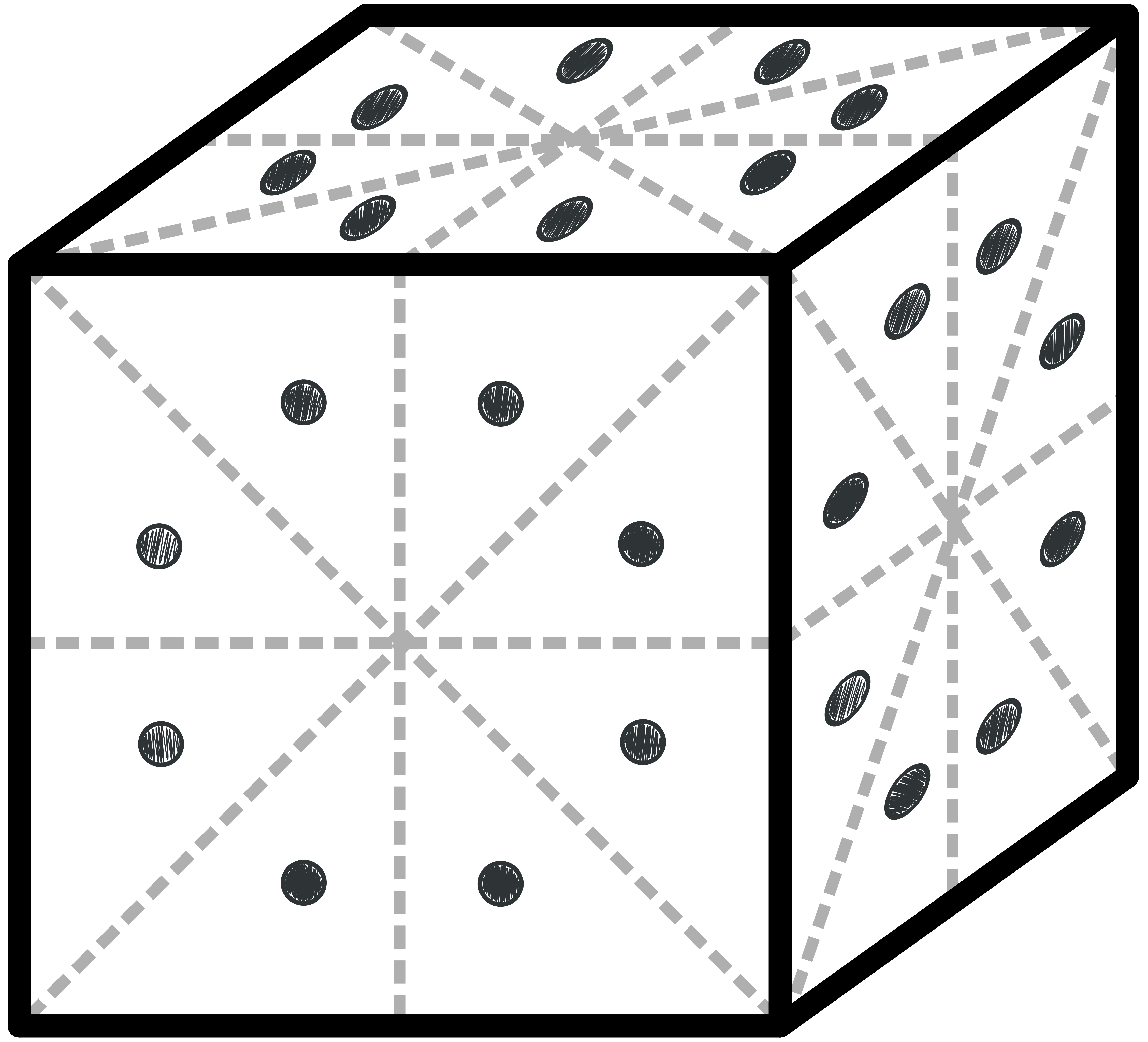
Politopos y maniplexes



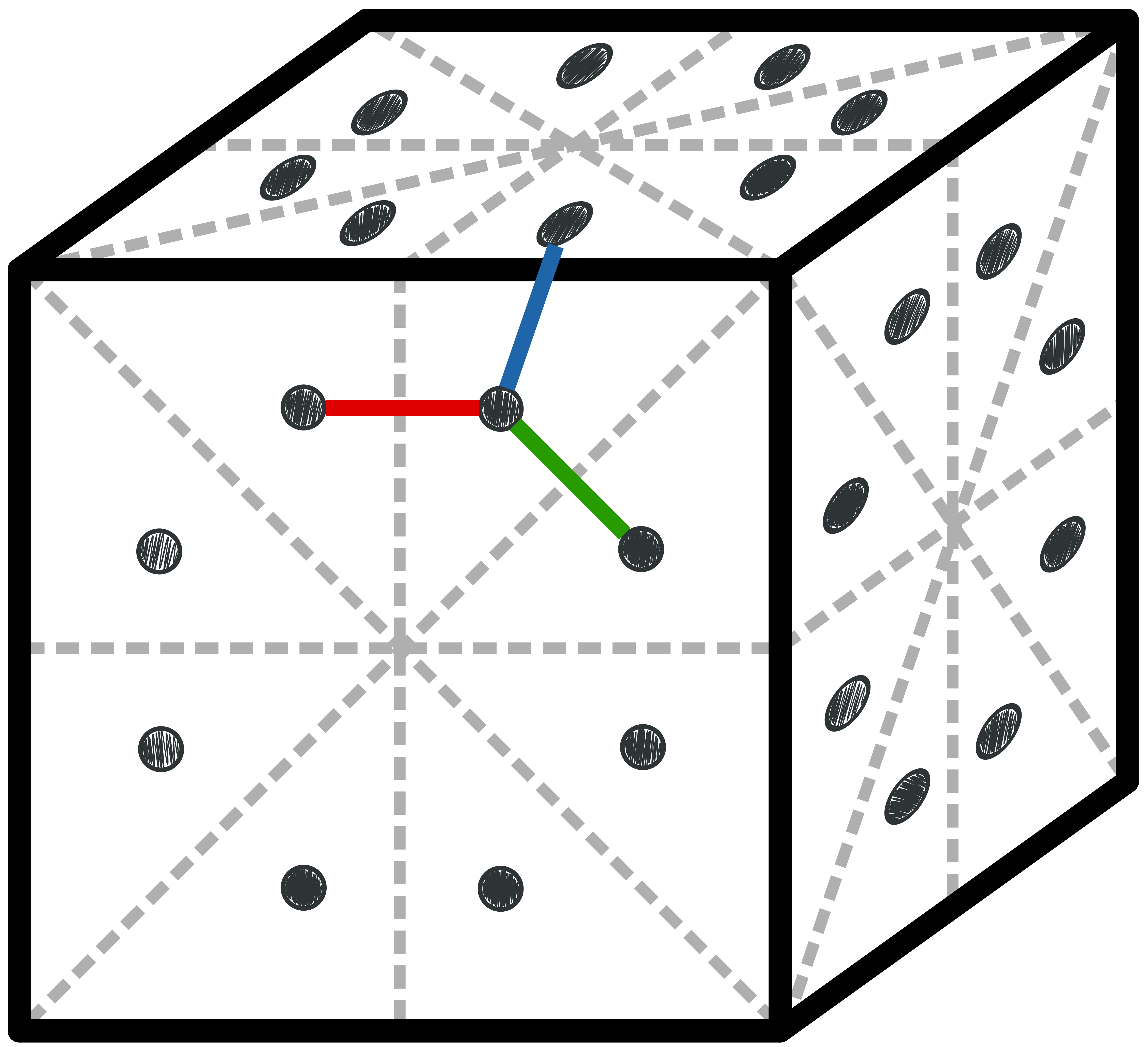
Politopos y maniplexes



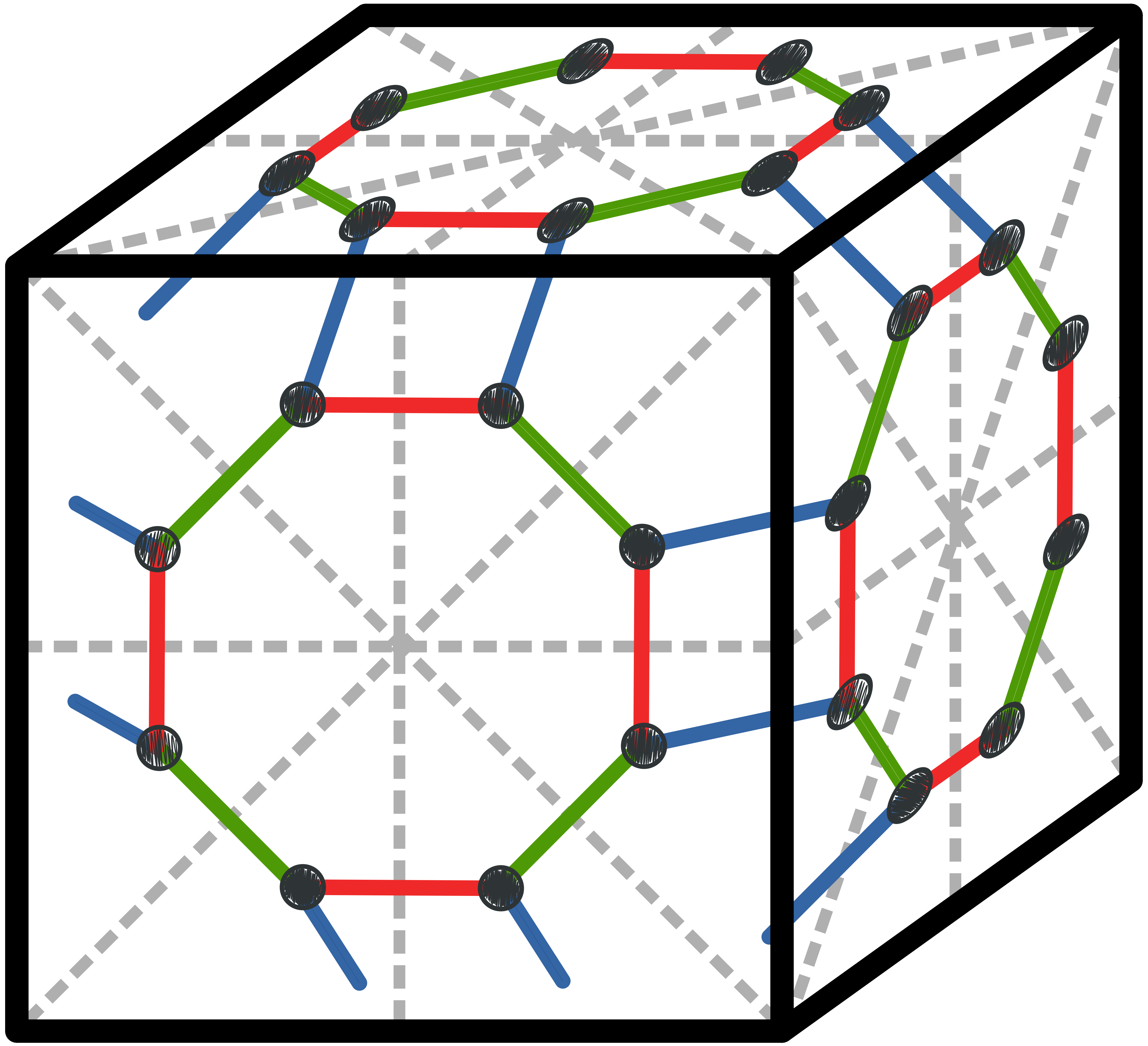
Politopos y maniplexes



Politopos y maniplexes

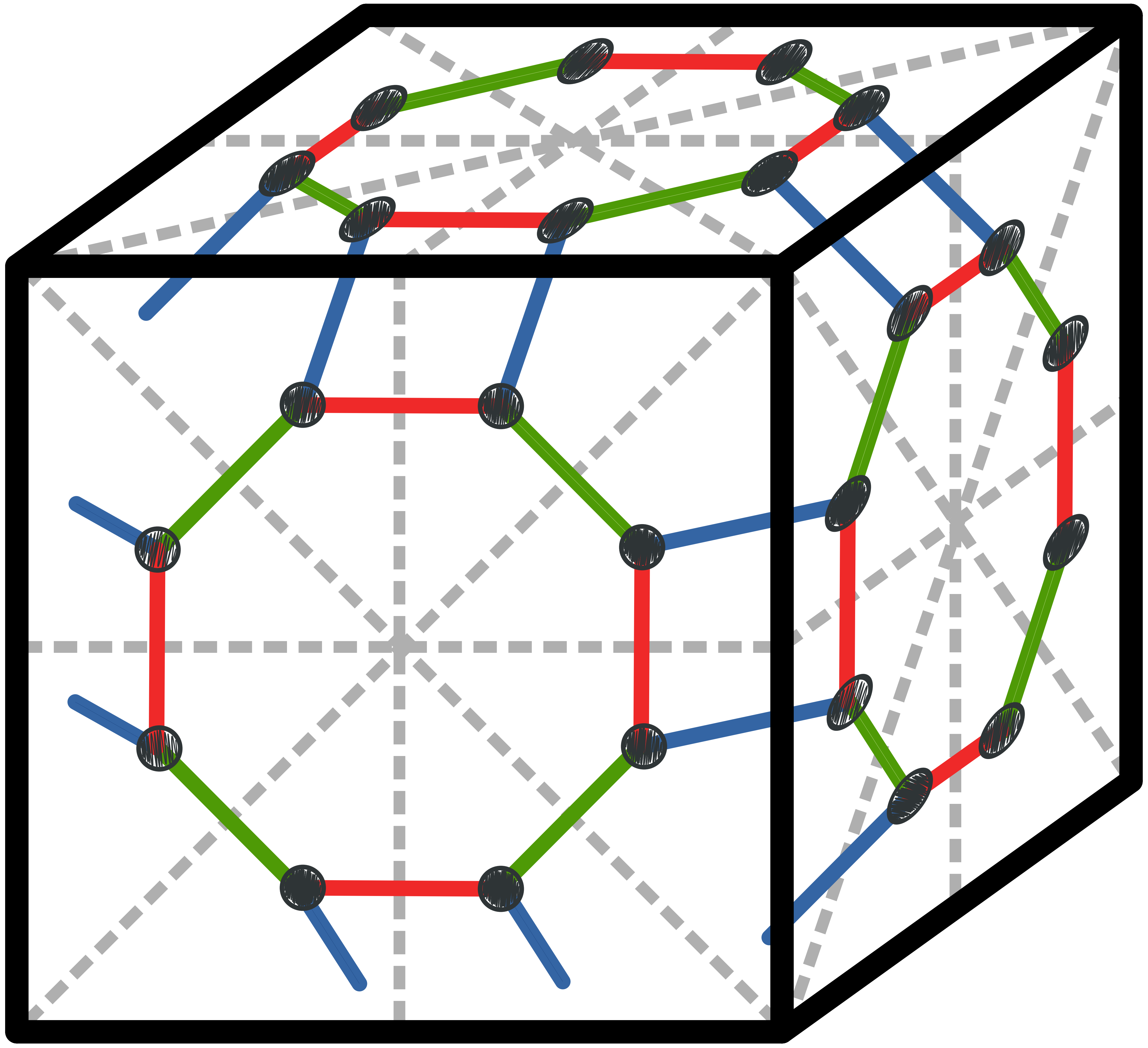


Politopos y maniplexes



Politopos y maniplexes

La gráfica de banderas $\mathrm{Fl}(\mathscr{M})$ de un mapa $\mathscr{M}$:



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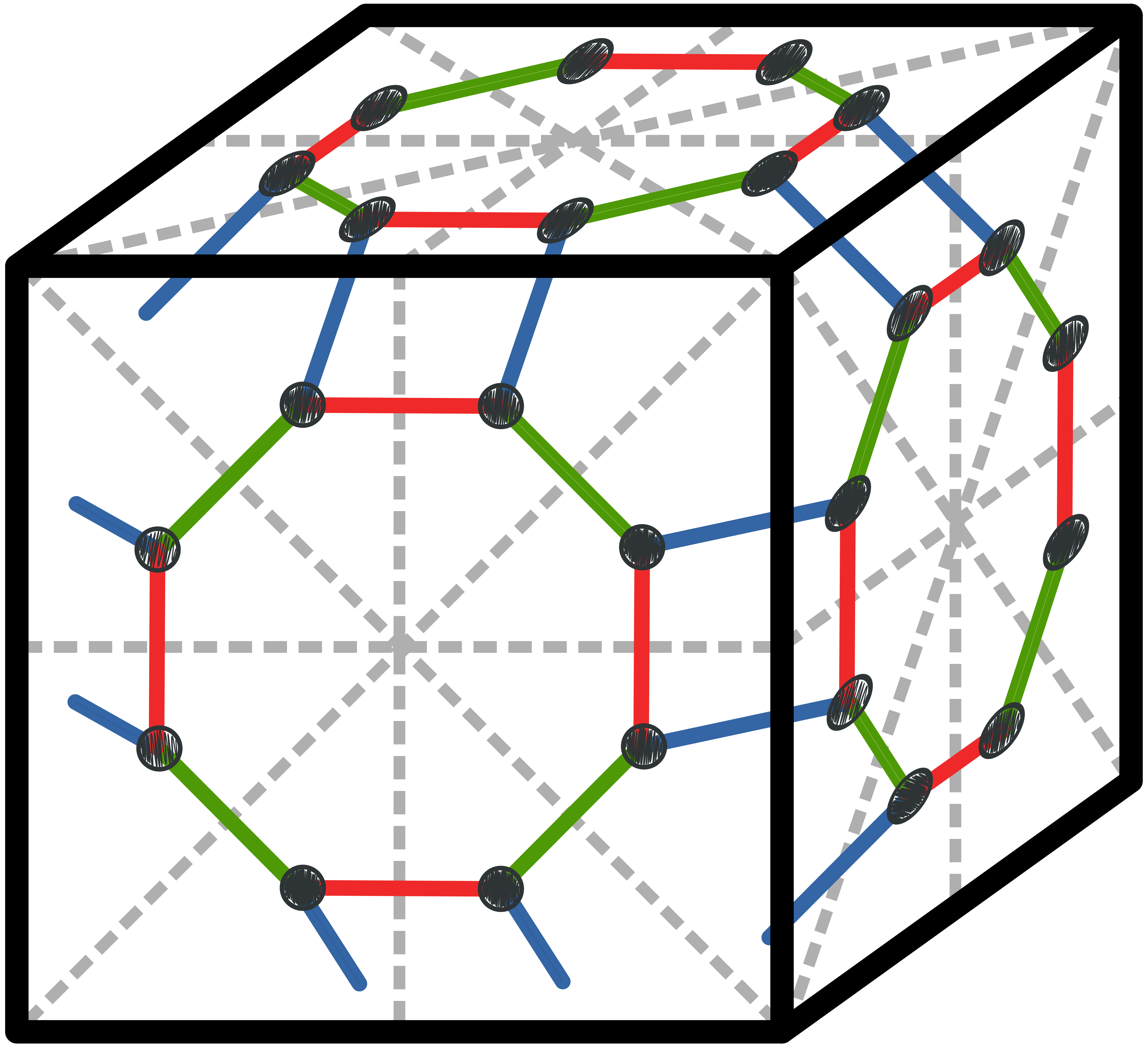

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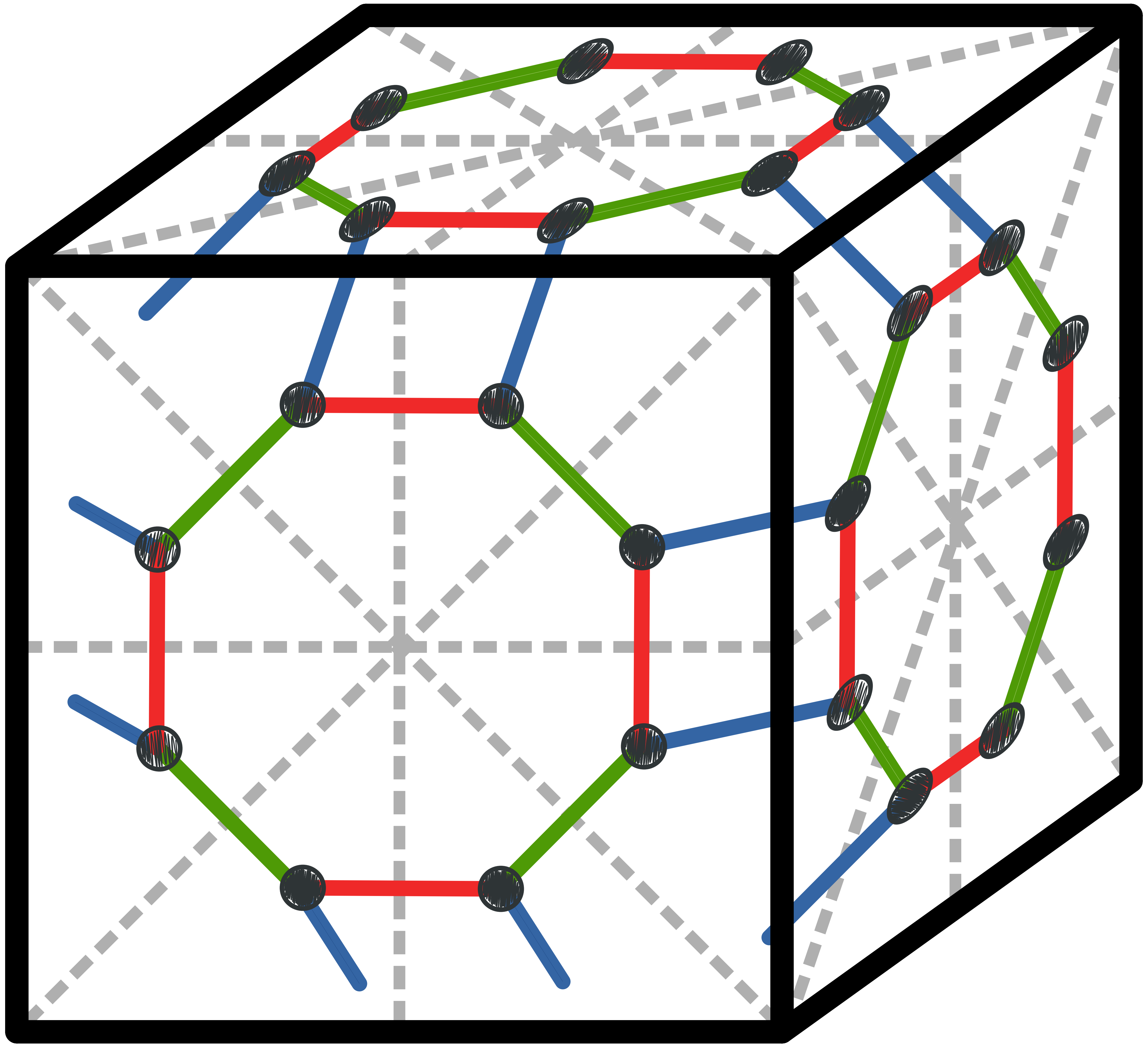
$\mathrm{Fl}(\mathscr{M})$ contiene toda la información combinatoria de $\mathscr{M}$



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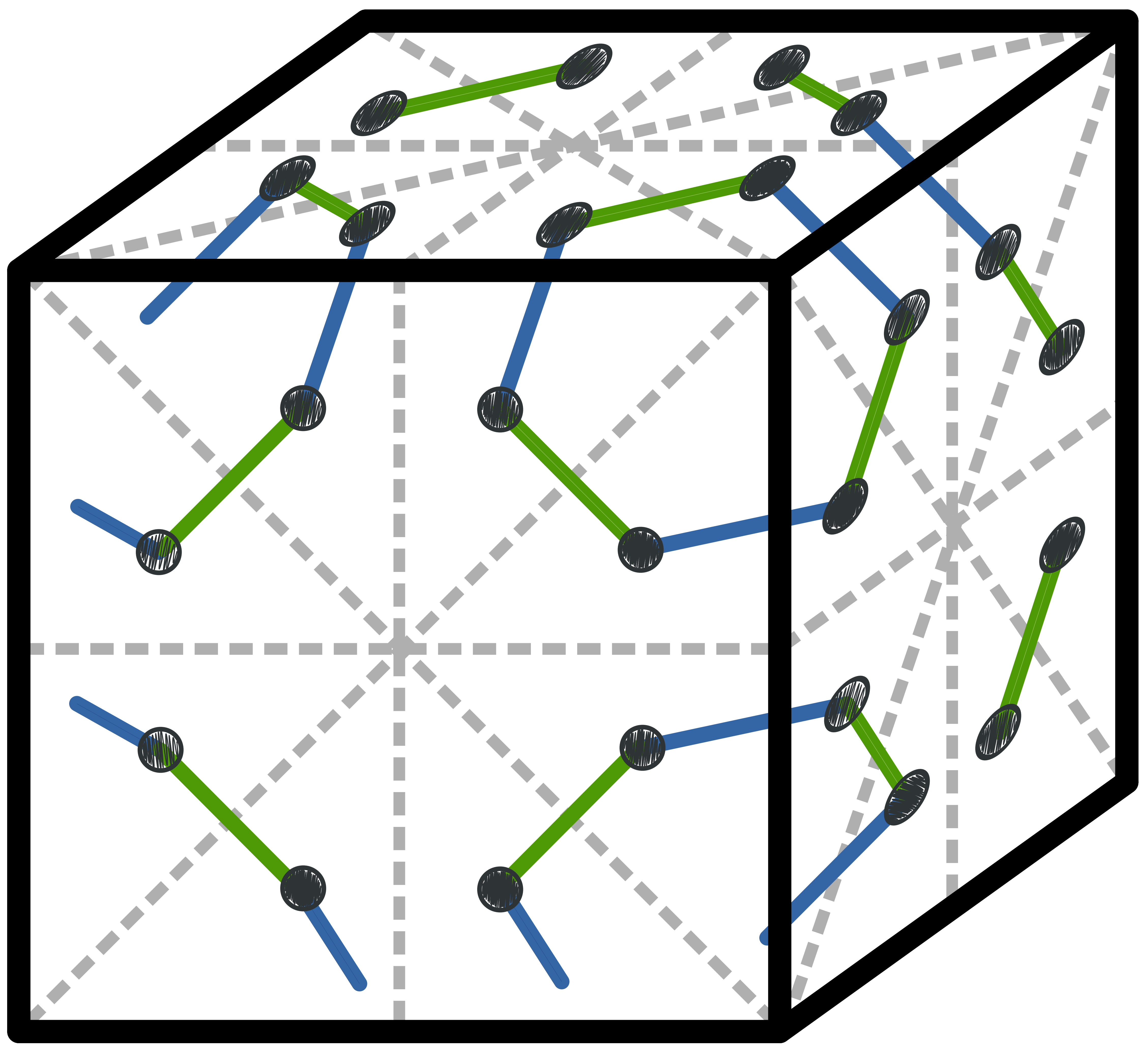

Politopos y maniplexes

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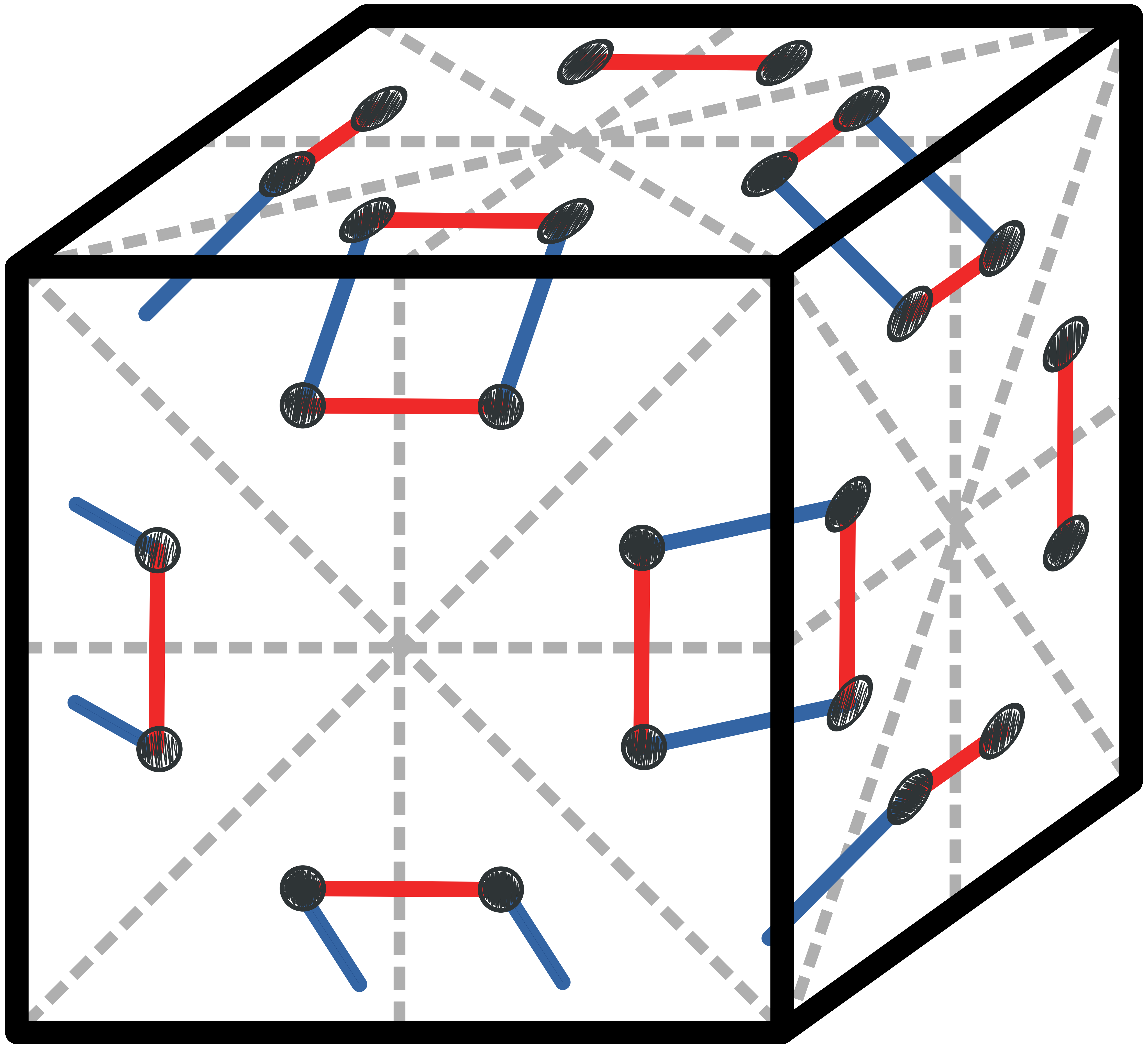

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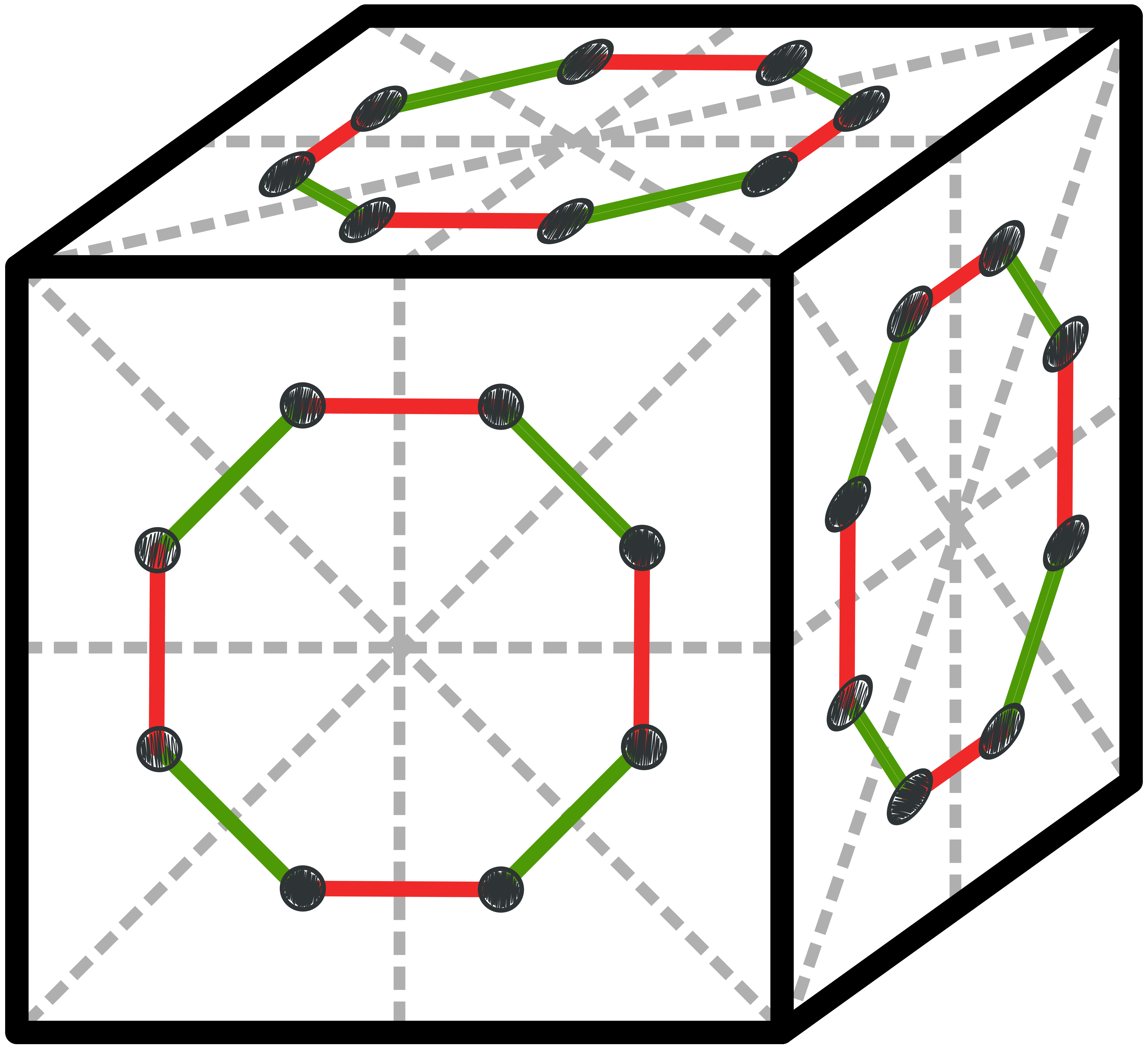

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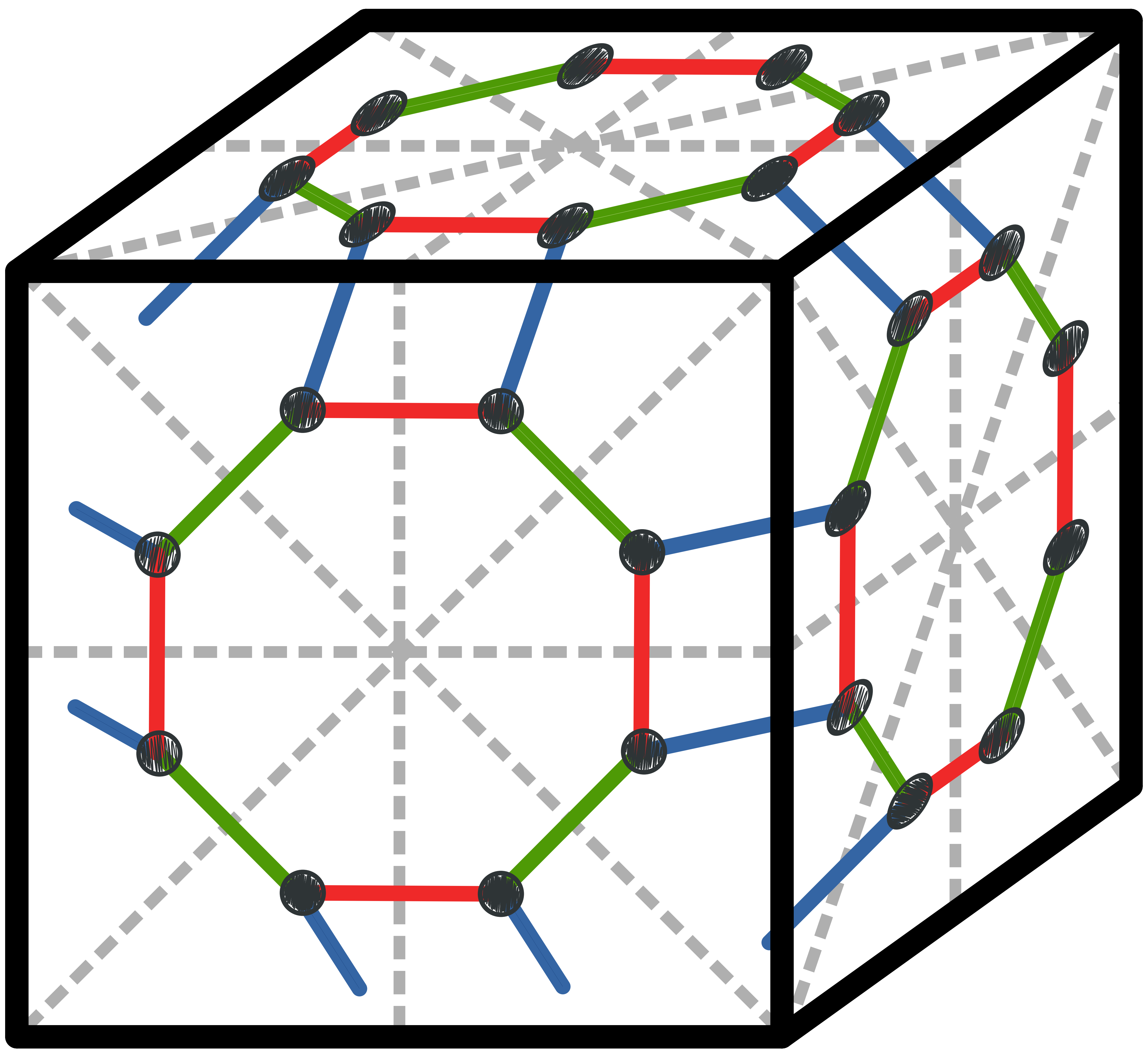

Politopos y maniplexes

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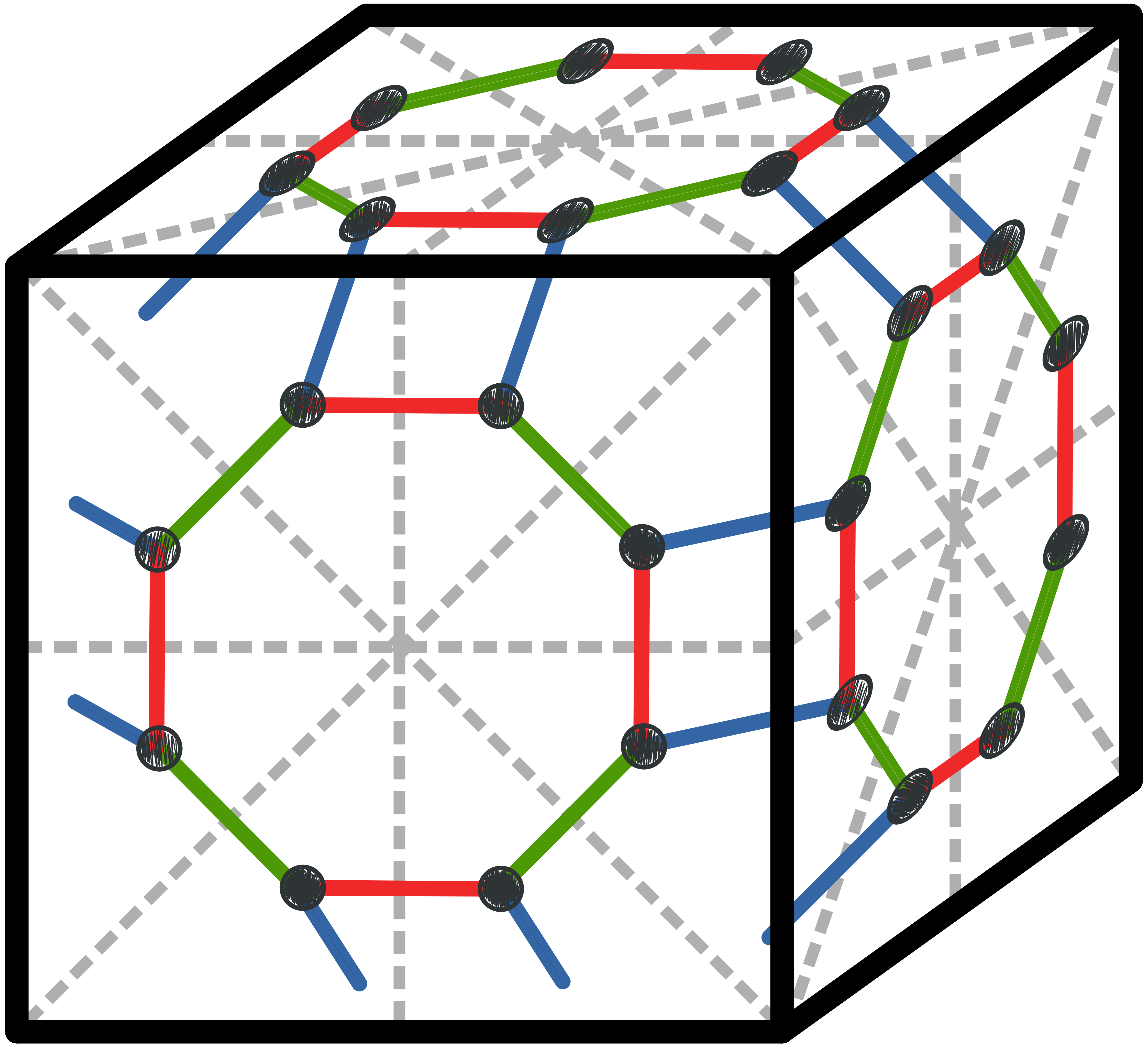

Politopos y maniplexes

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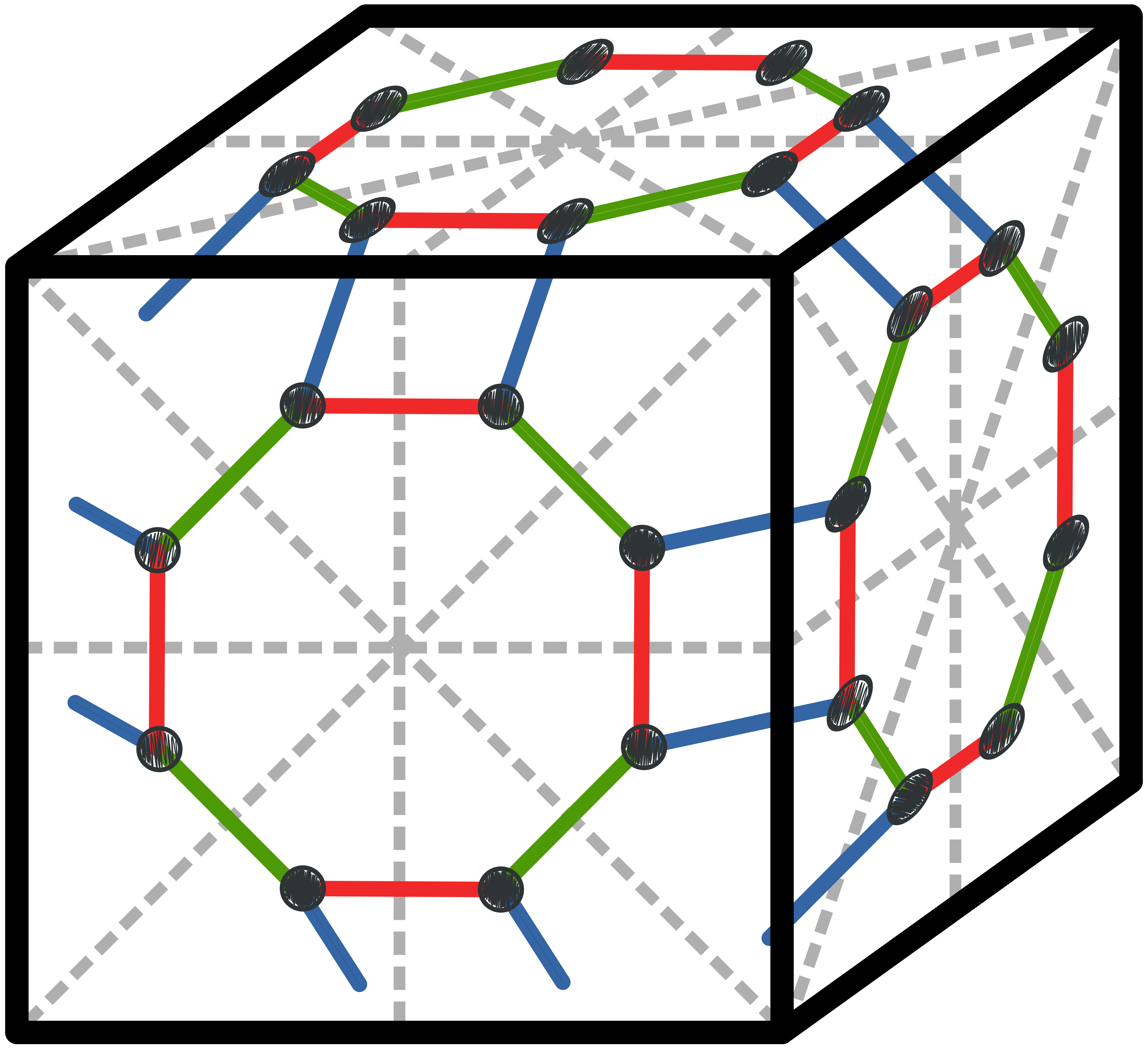

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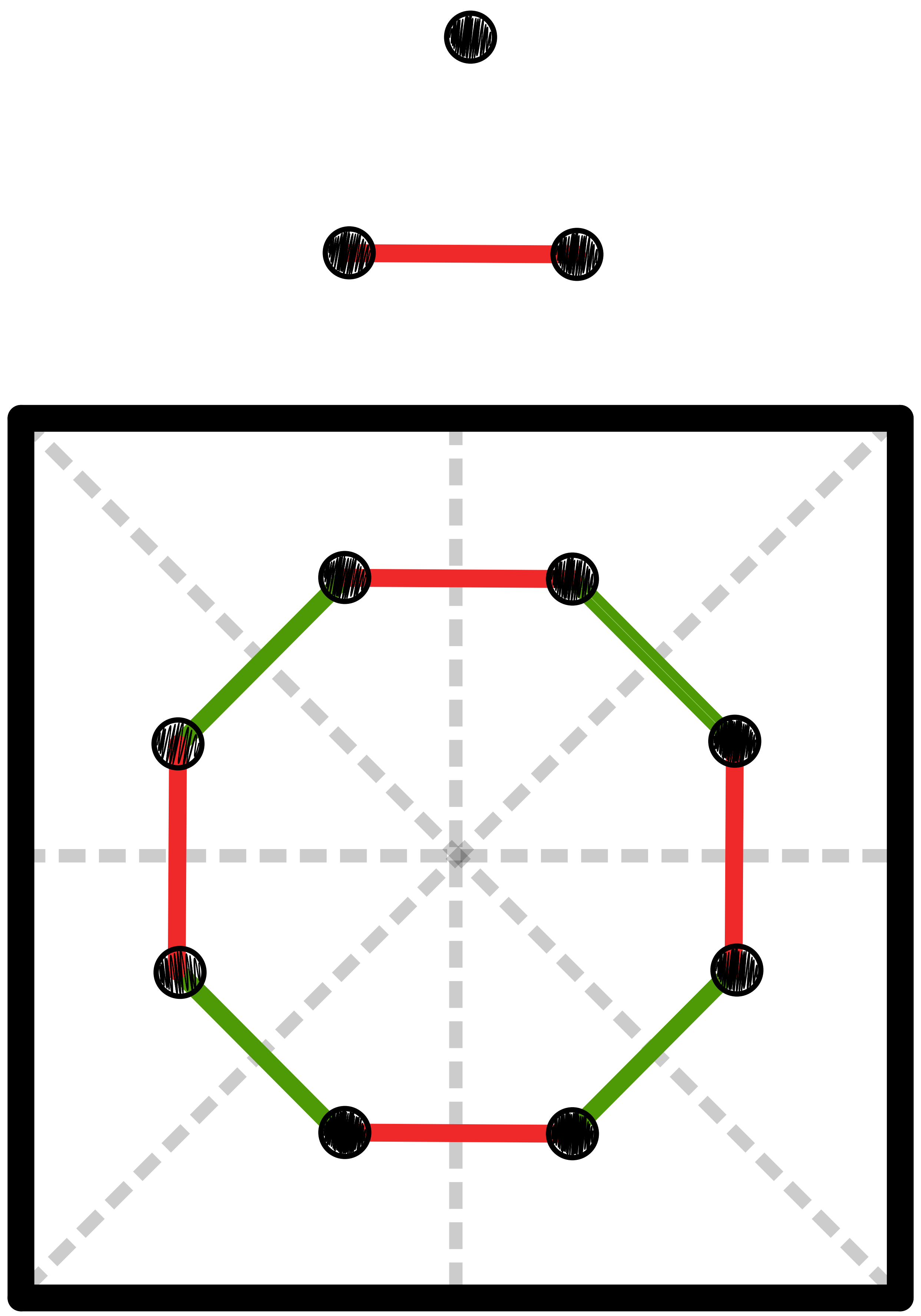

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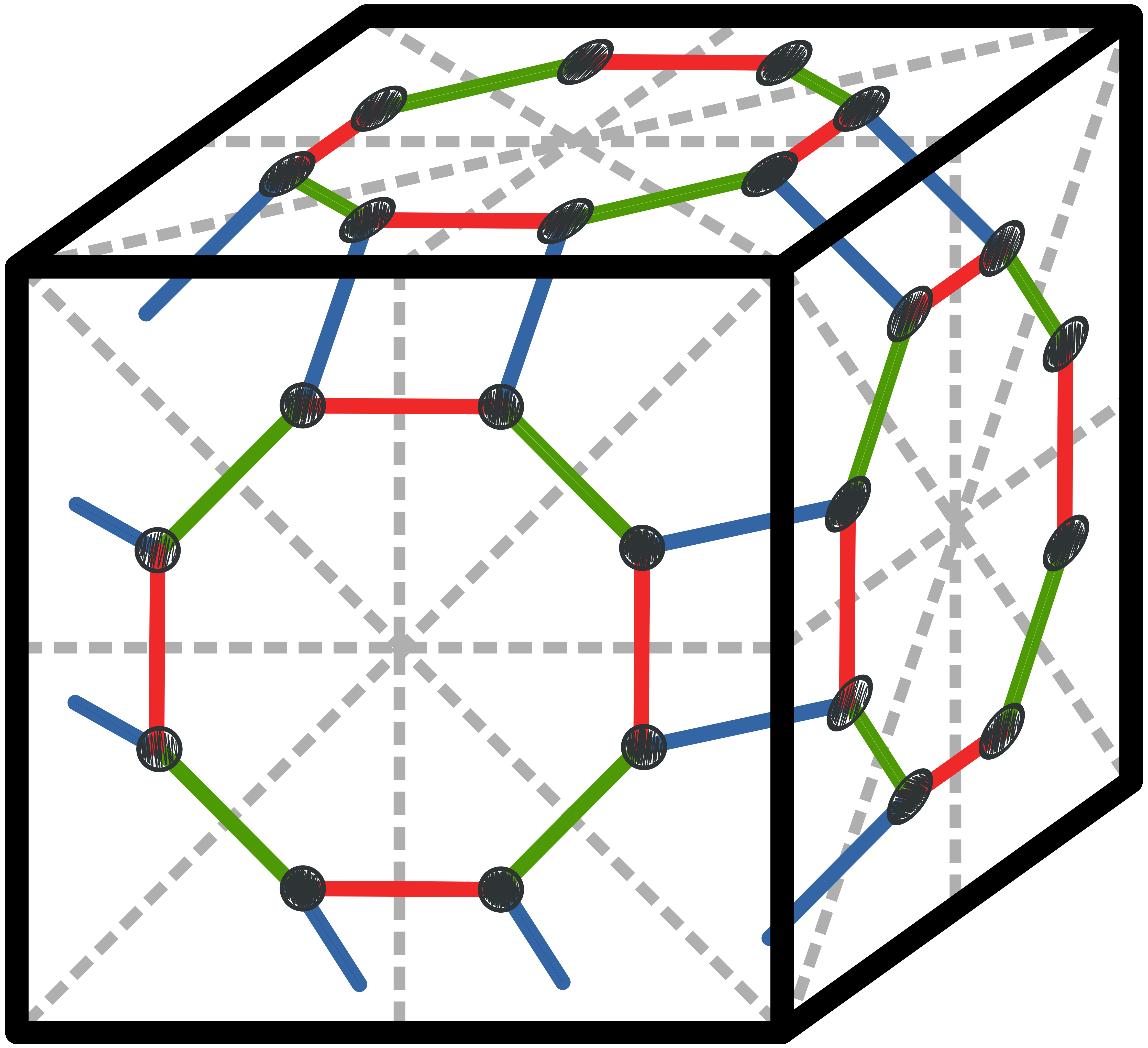

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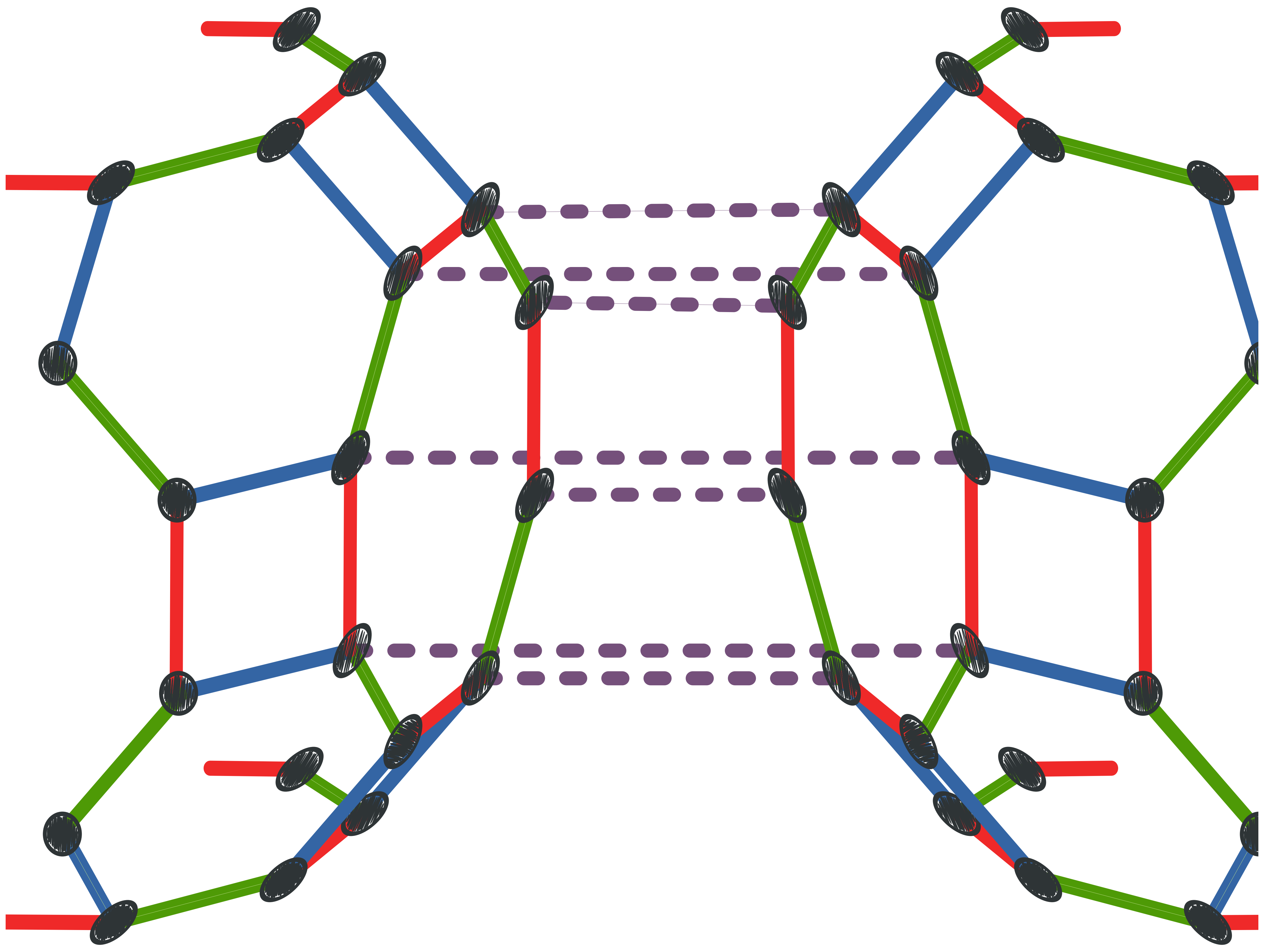

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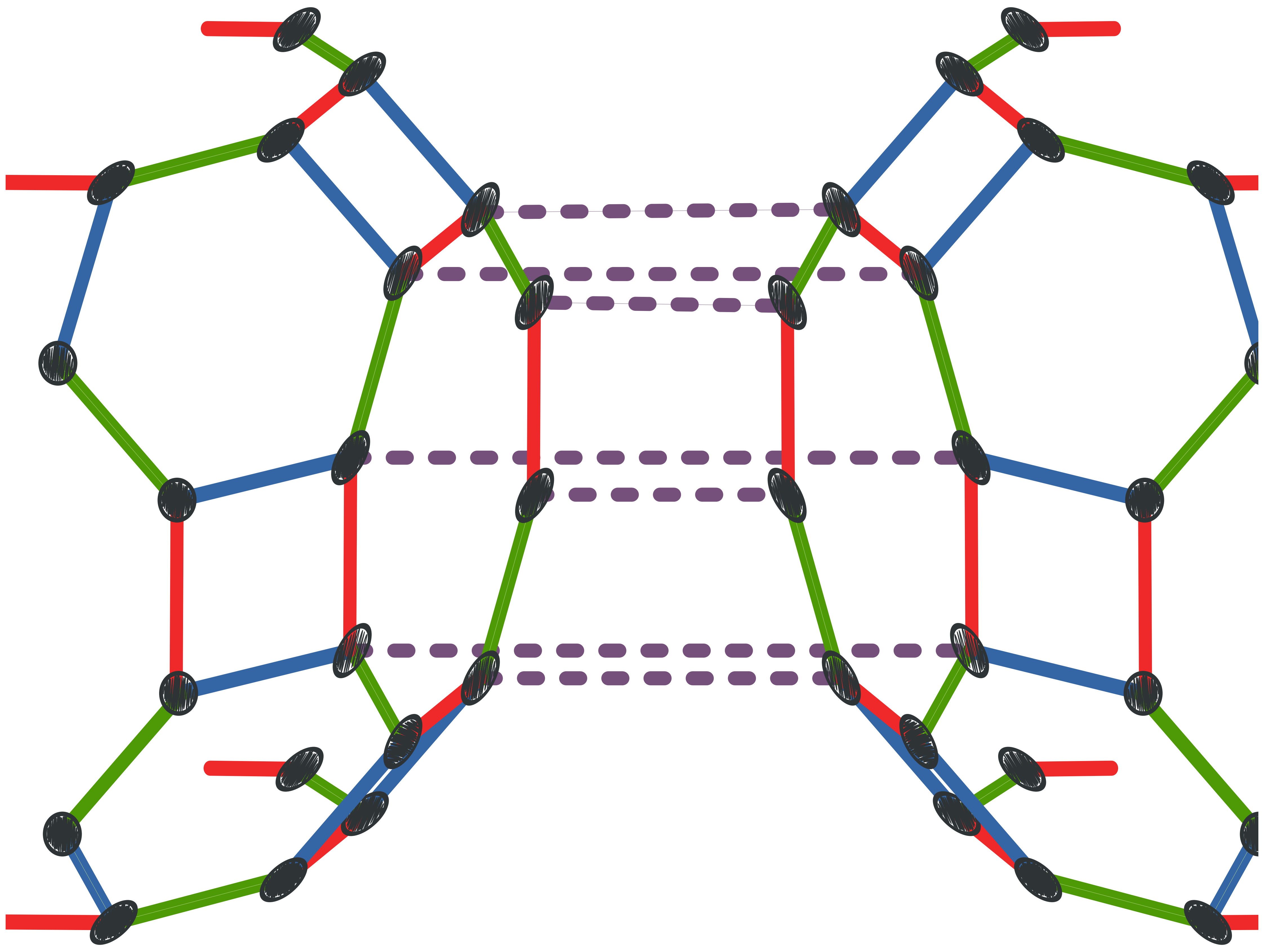
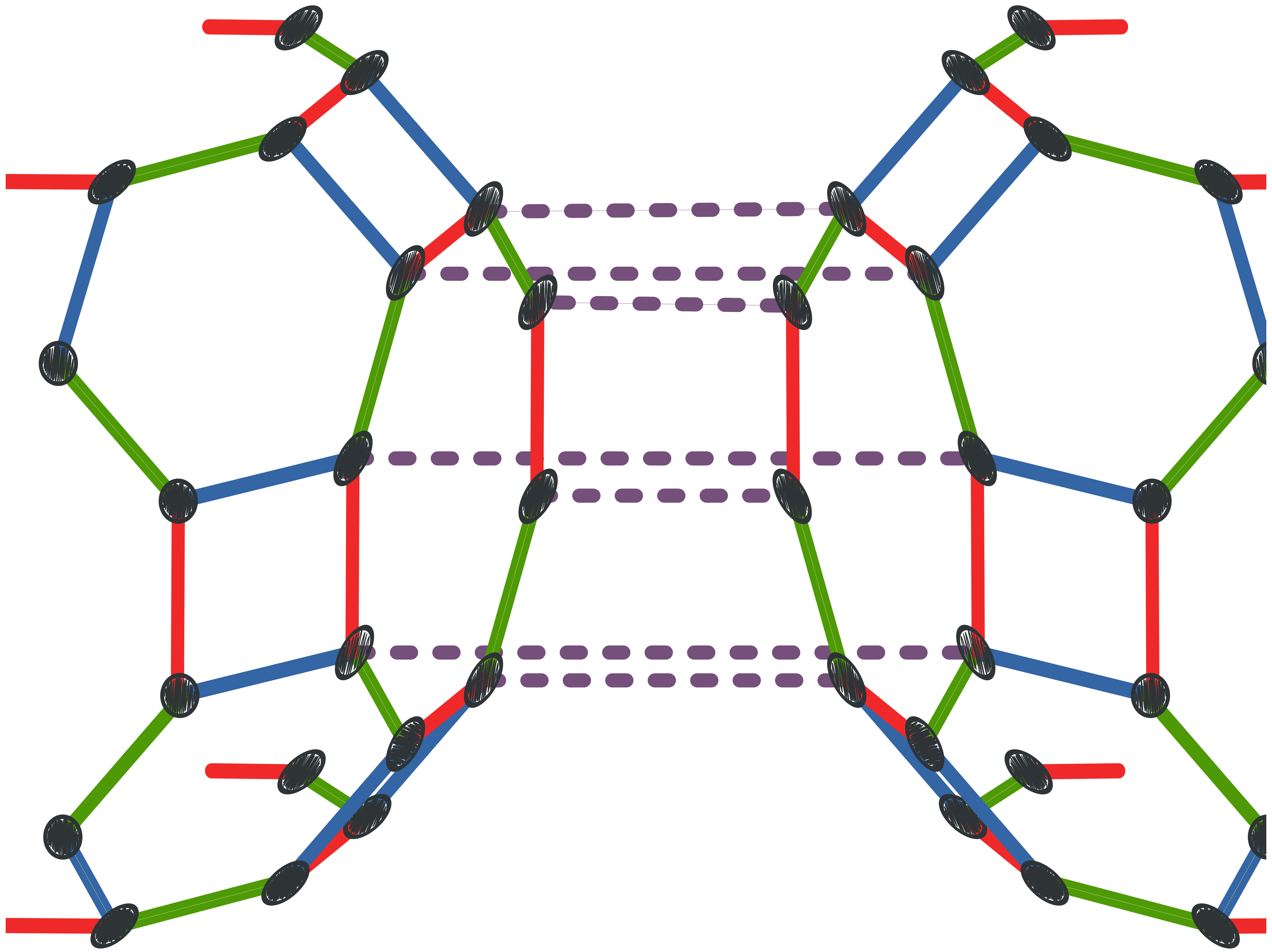
Hubard, Garza-Vargas (2017):
Los politopos abstractos son maniplexes que cumplen la PIC



Politopos y maniplexes



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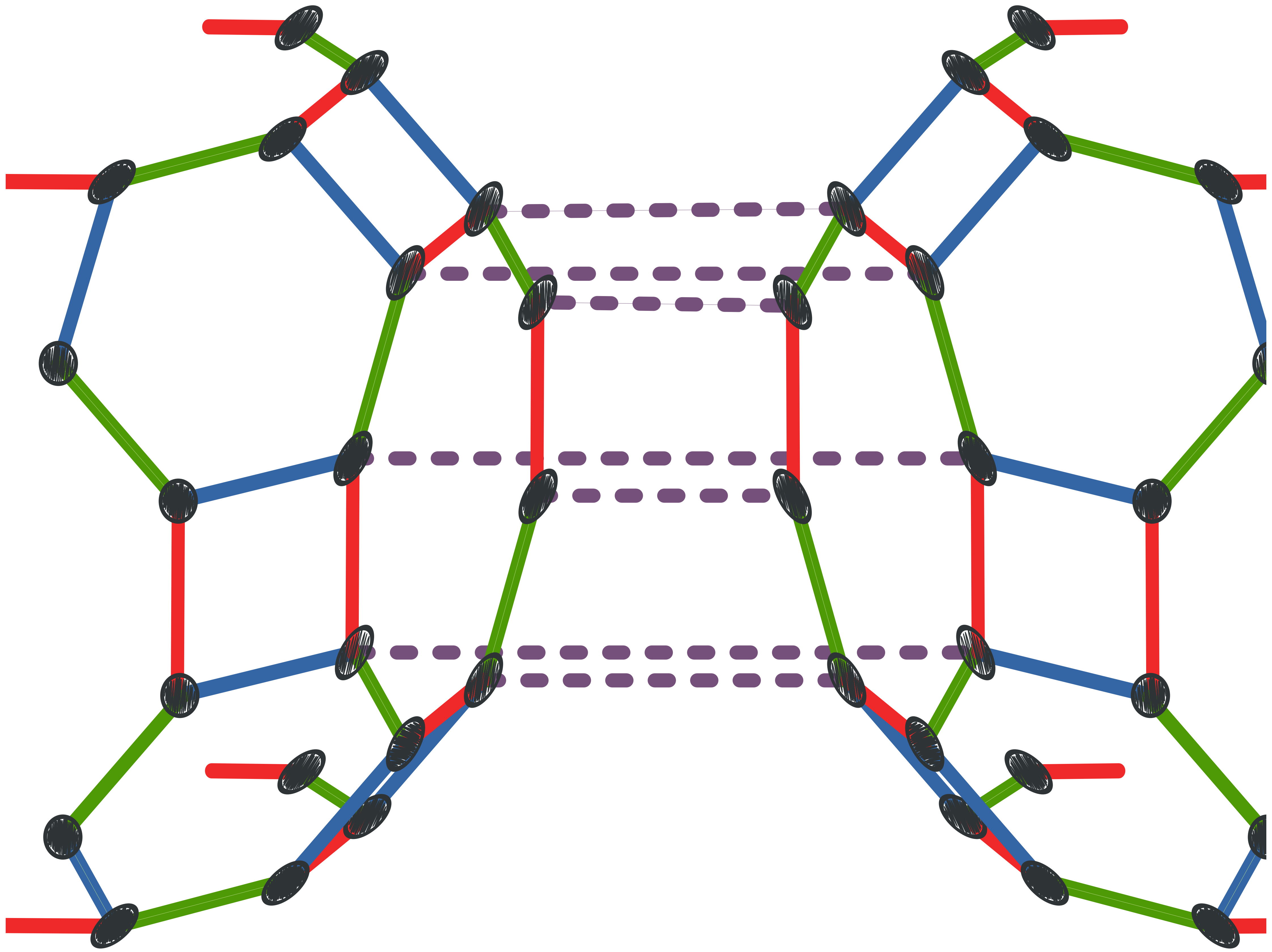

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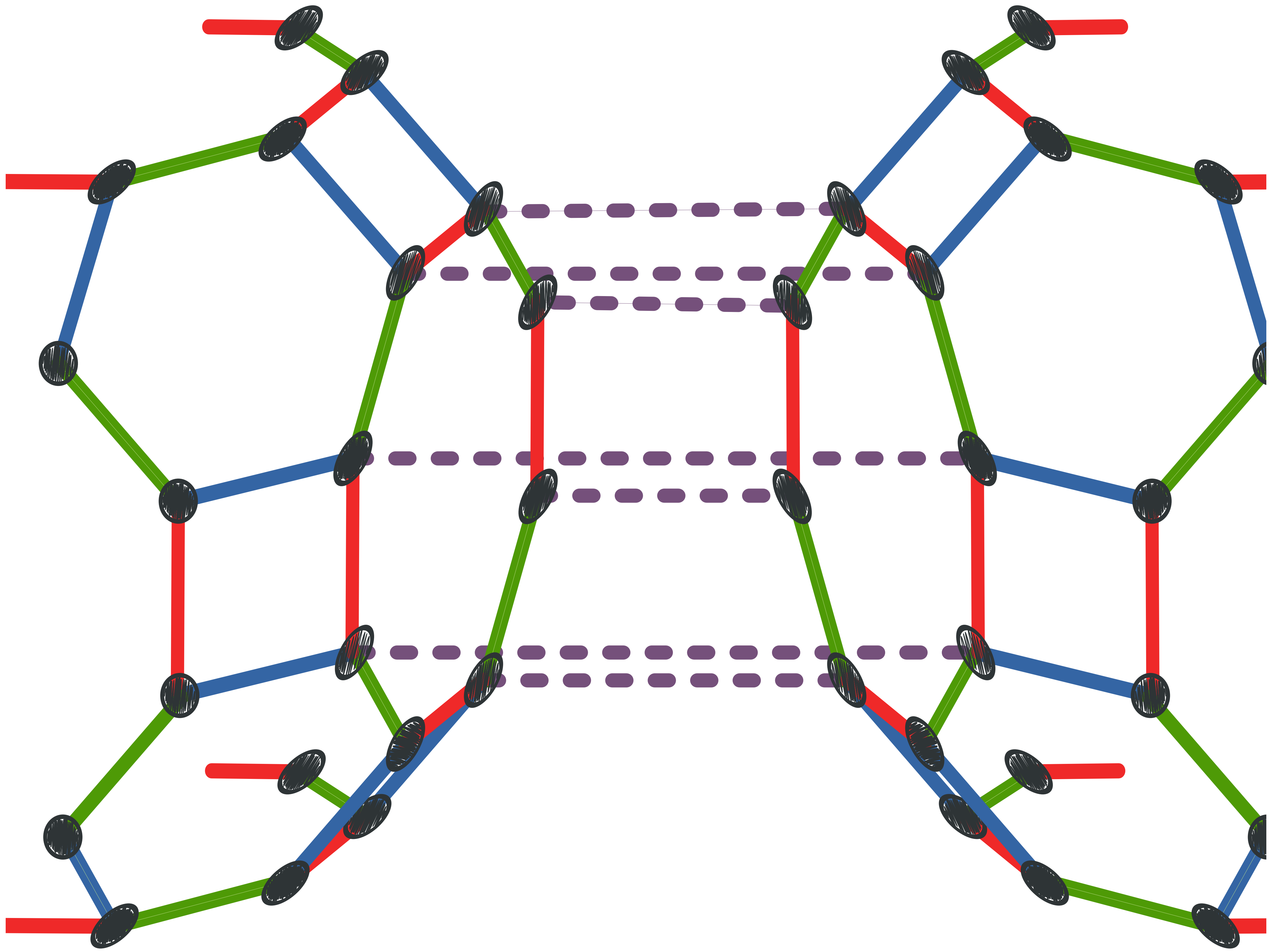
- El grupo $\mathrm{Aut}(\mathscr{M})$ actúa libremente en las banderas.

Politopos y maniplexes



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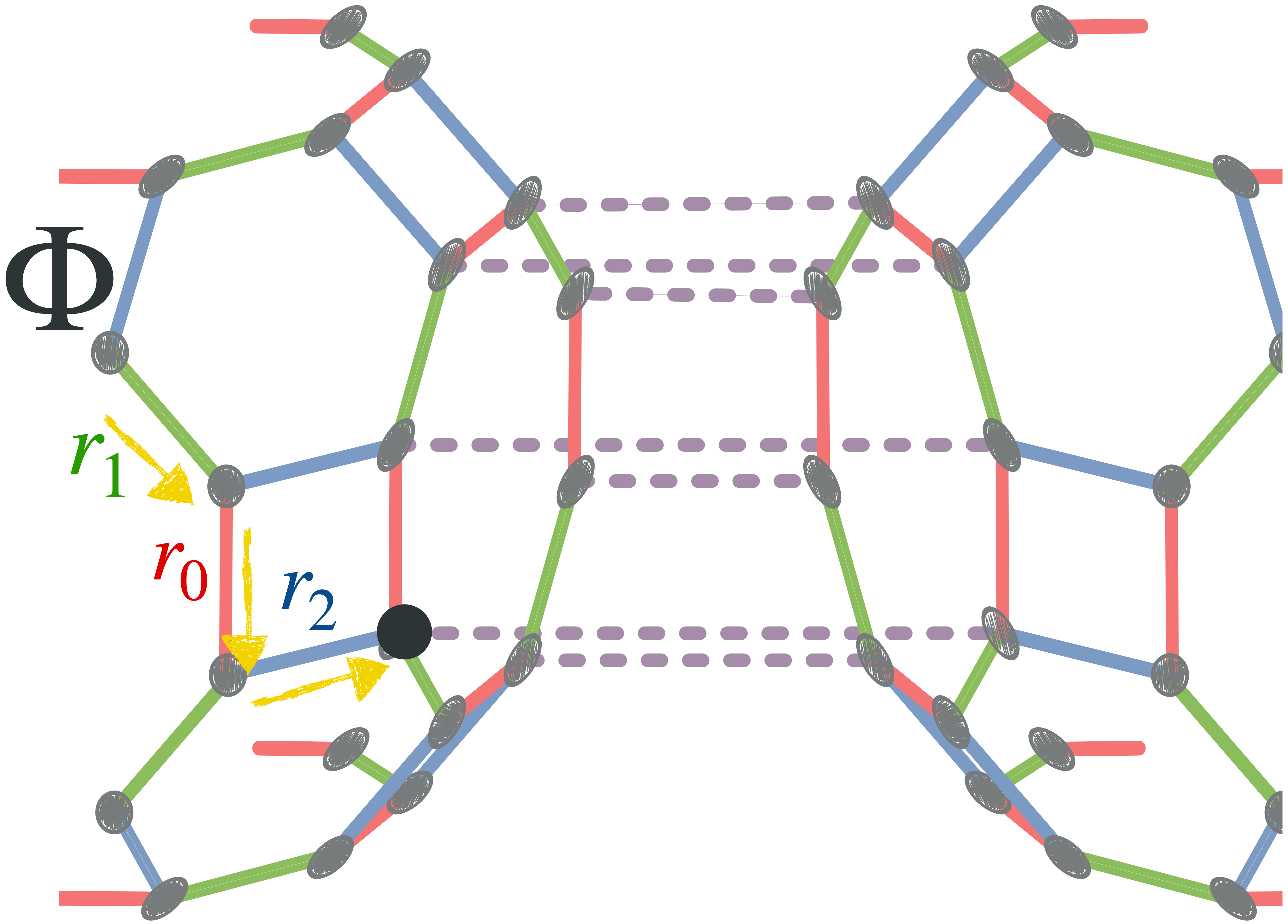
- El grupo

$$W_n = \langle r_0, \ldots, r_{n-1} \mid (r_i)^2 = (r_i r_j)^2 = 1 \rangle$$

actúa en $\mathscr{M}$ por conexiones.

Politopos y maniplexes



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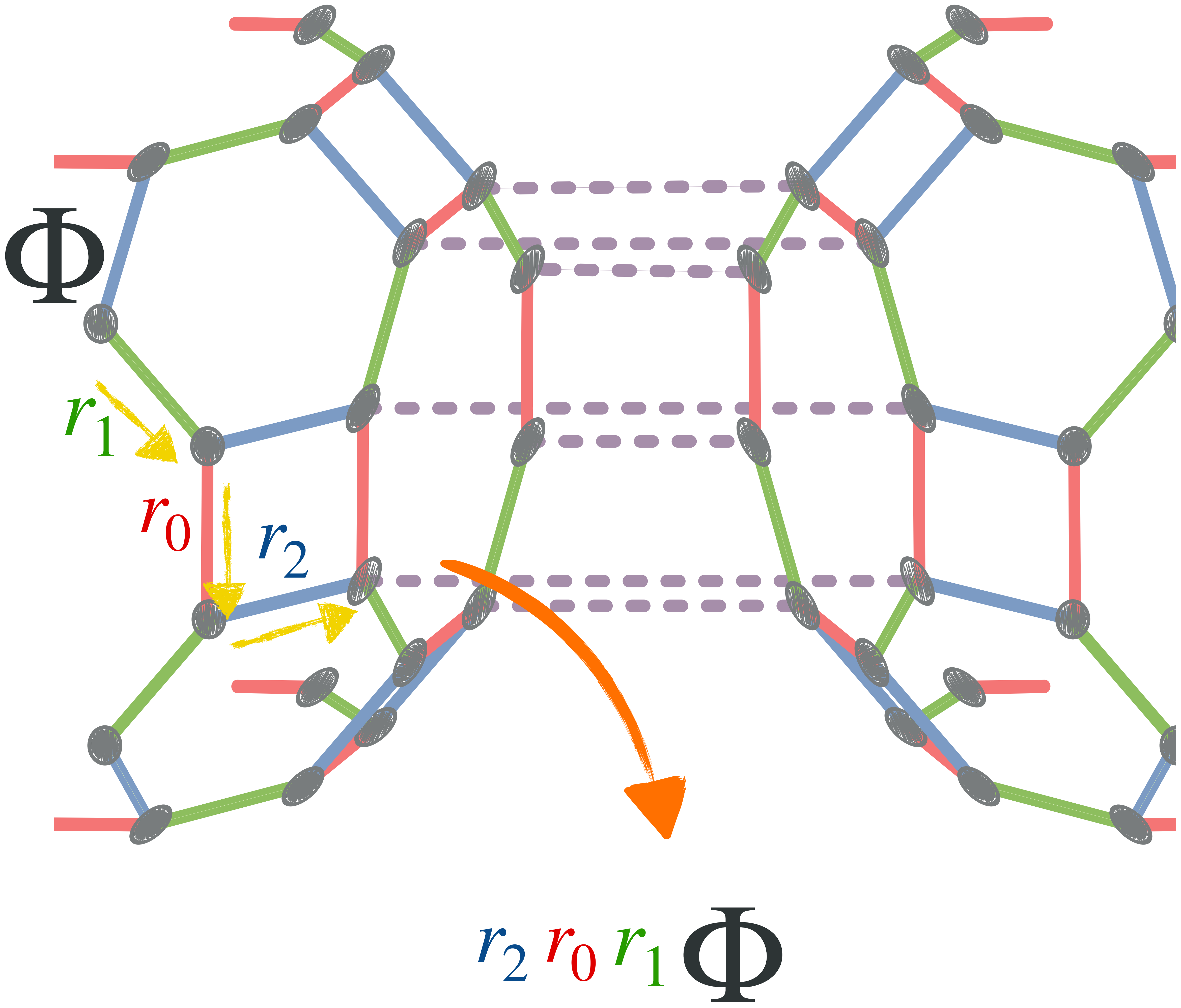

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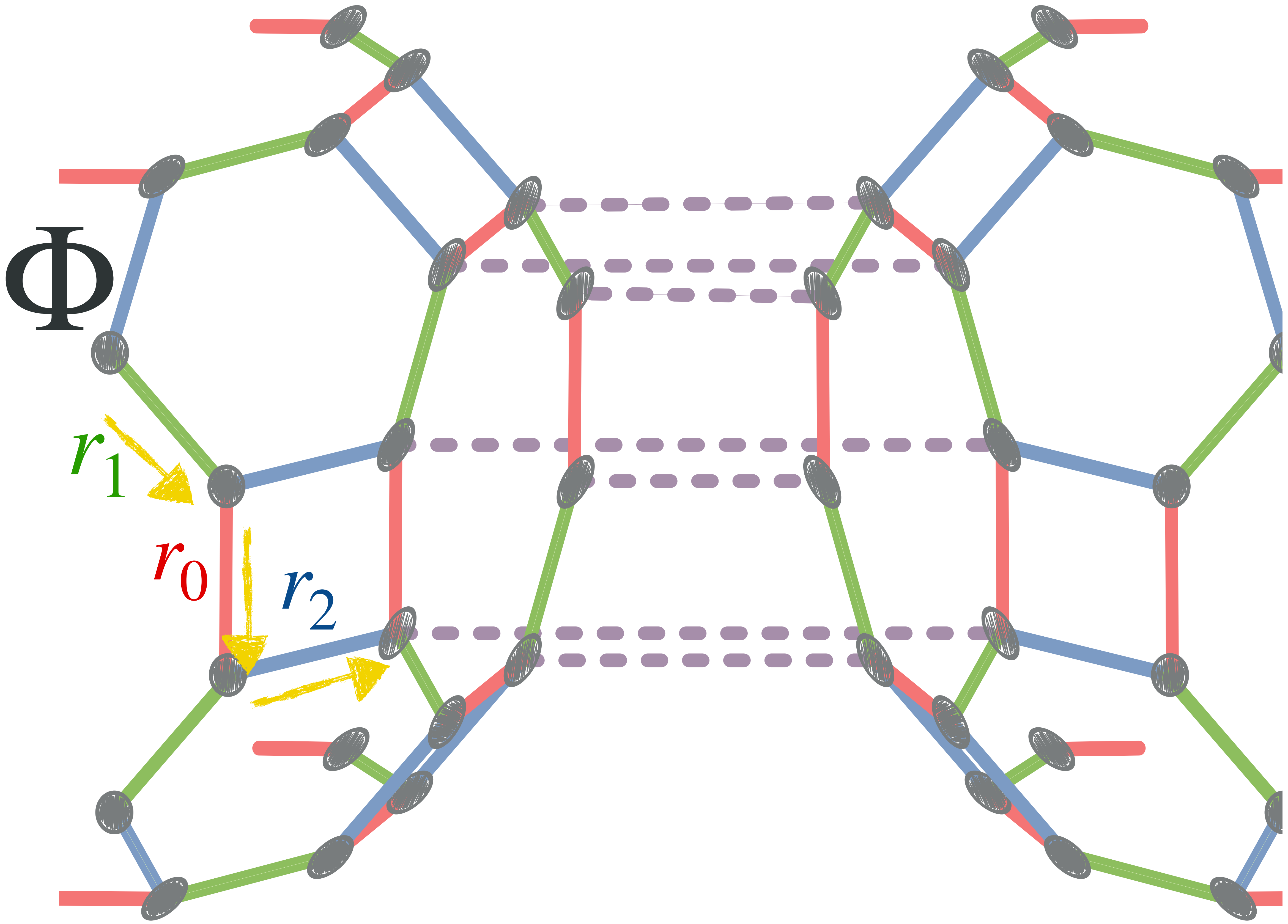

$r_2 \, r_0 \, r_1 \, \Phi$

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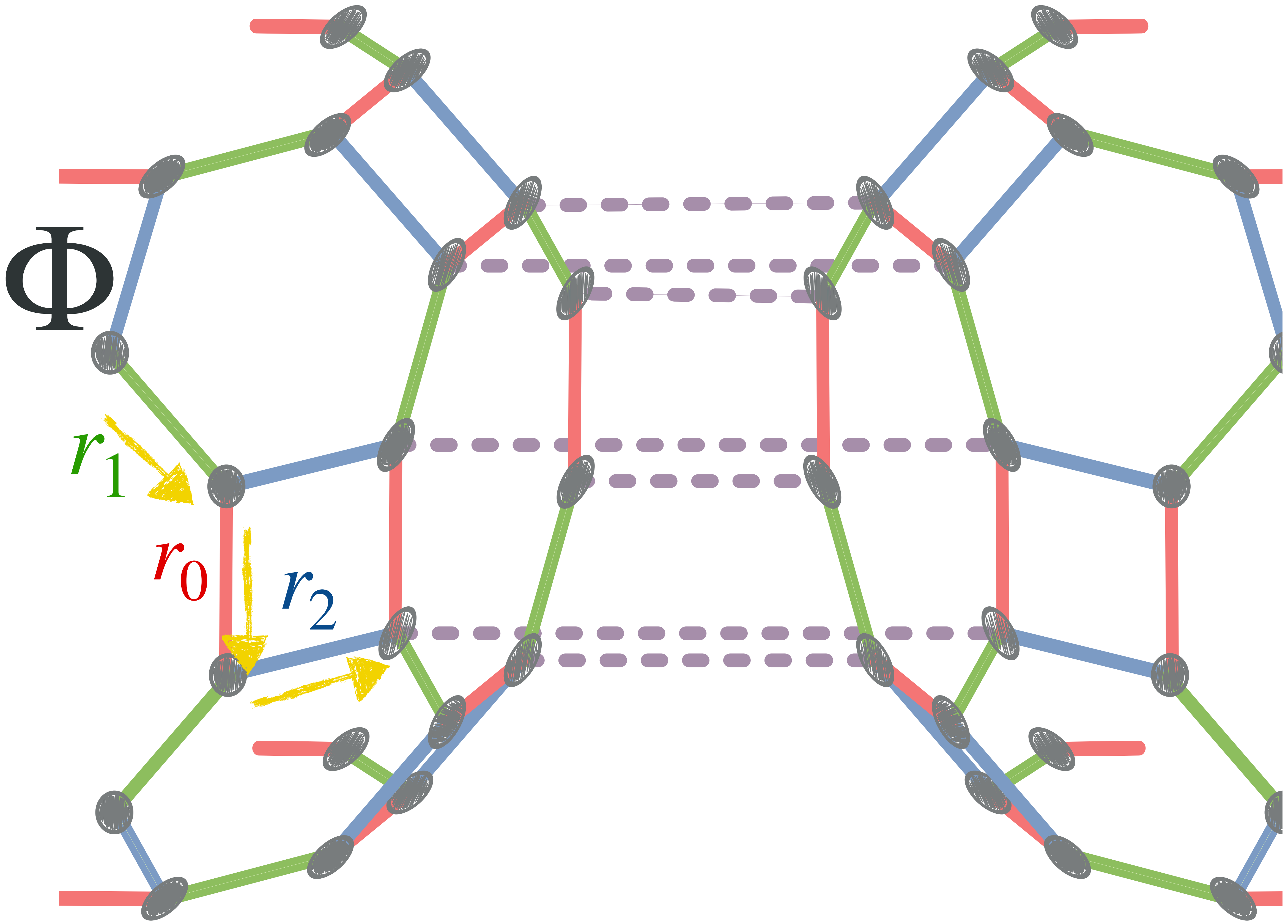

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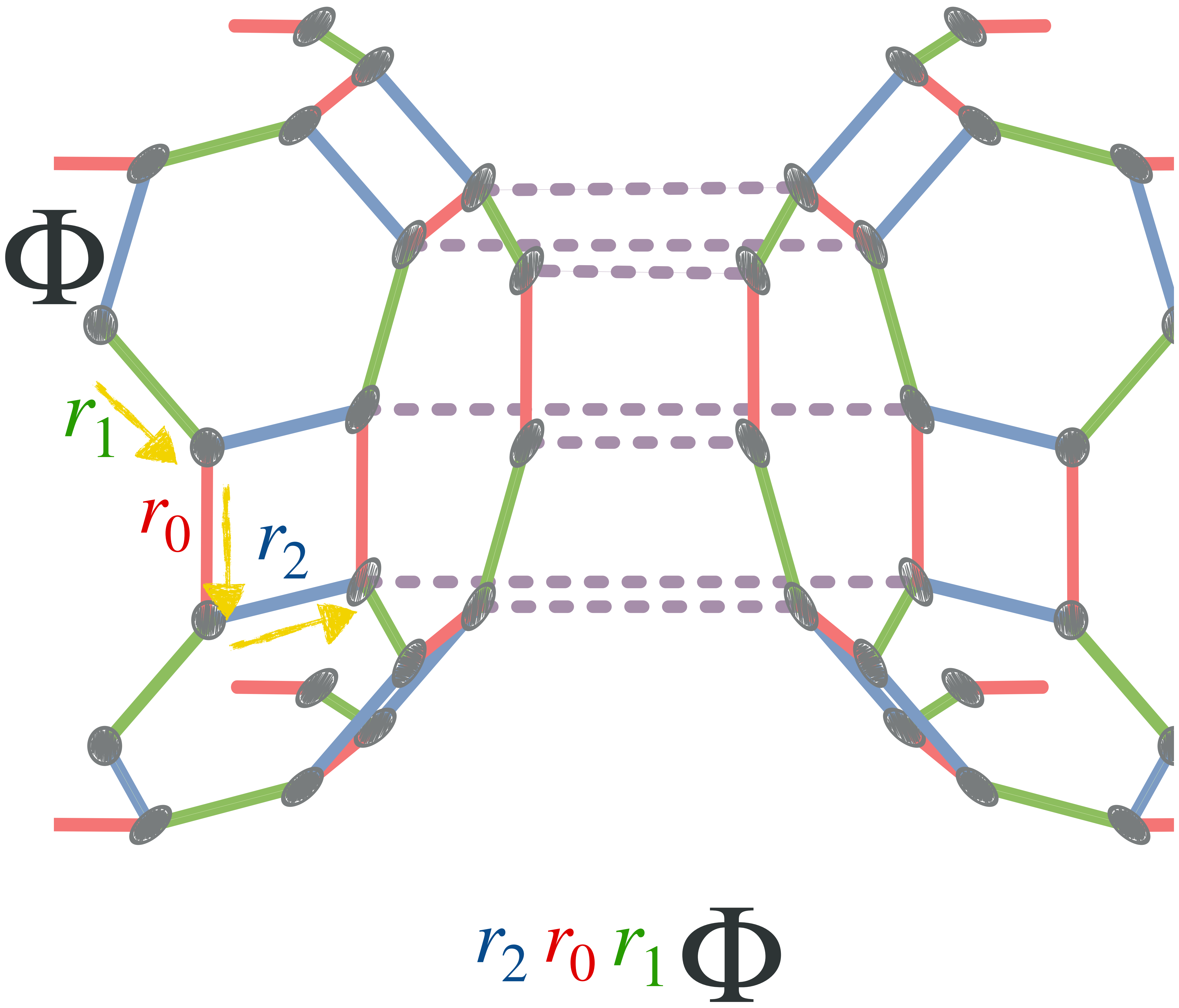
$$\mathrm{Aut}(\mathscr{U}^n) = W_n$$

Politopos y maniplexes



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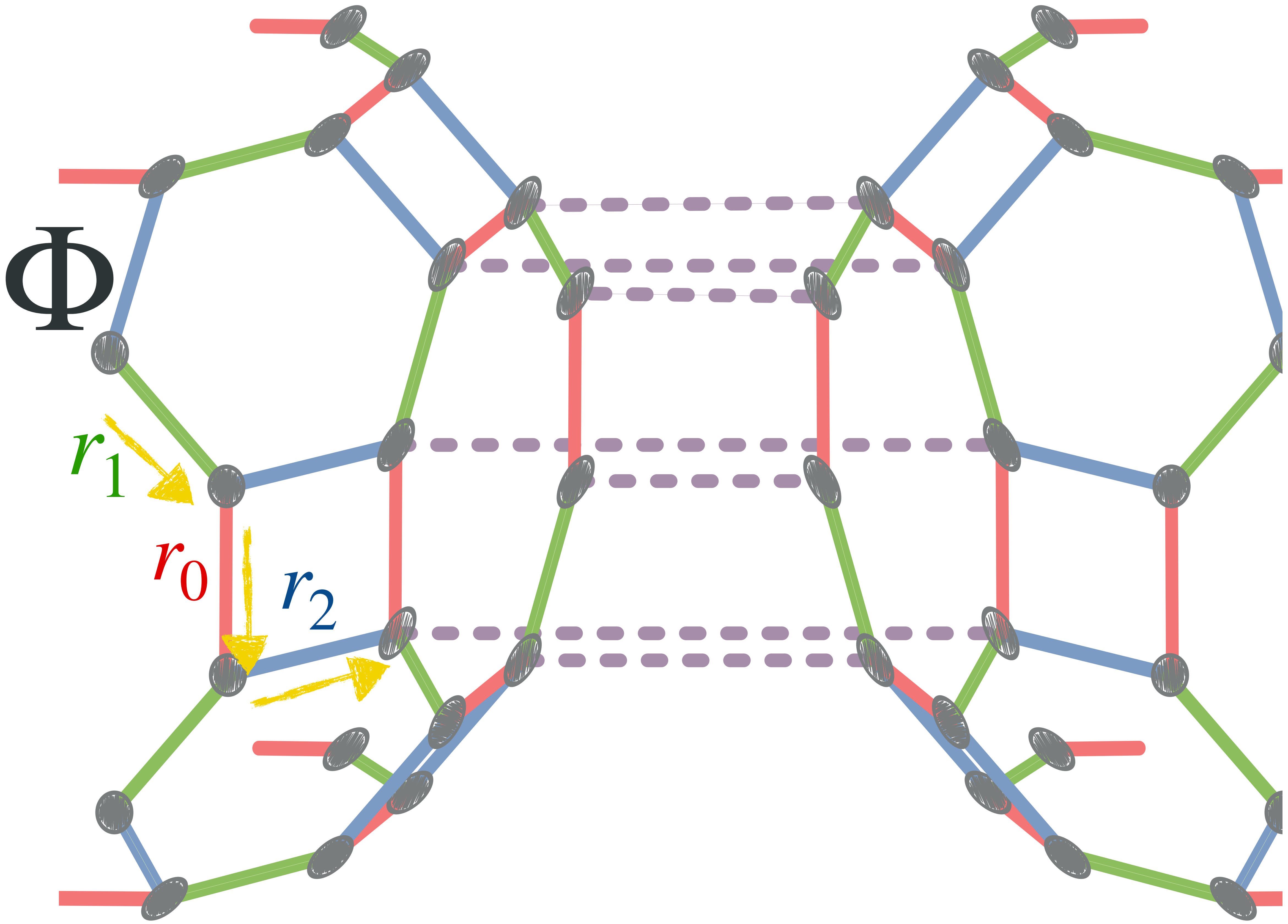

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Hartley,... (1999, ...):
Todo n-maniplex es un cociente de $\mathscr{U}^n$.

$$\mathscr{U}^n/M \searrow \mathscr{U}^n/N \iff M \leq N$$

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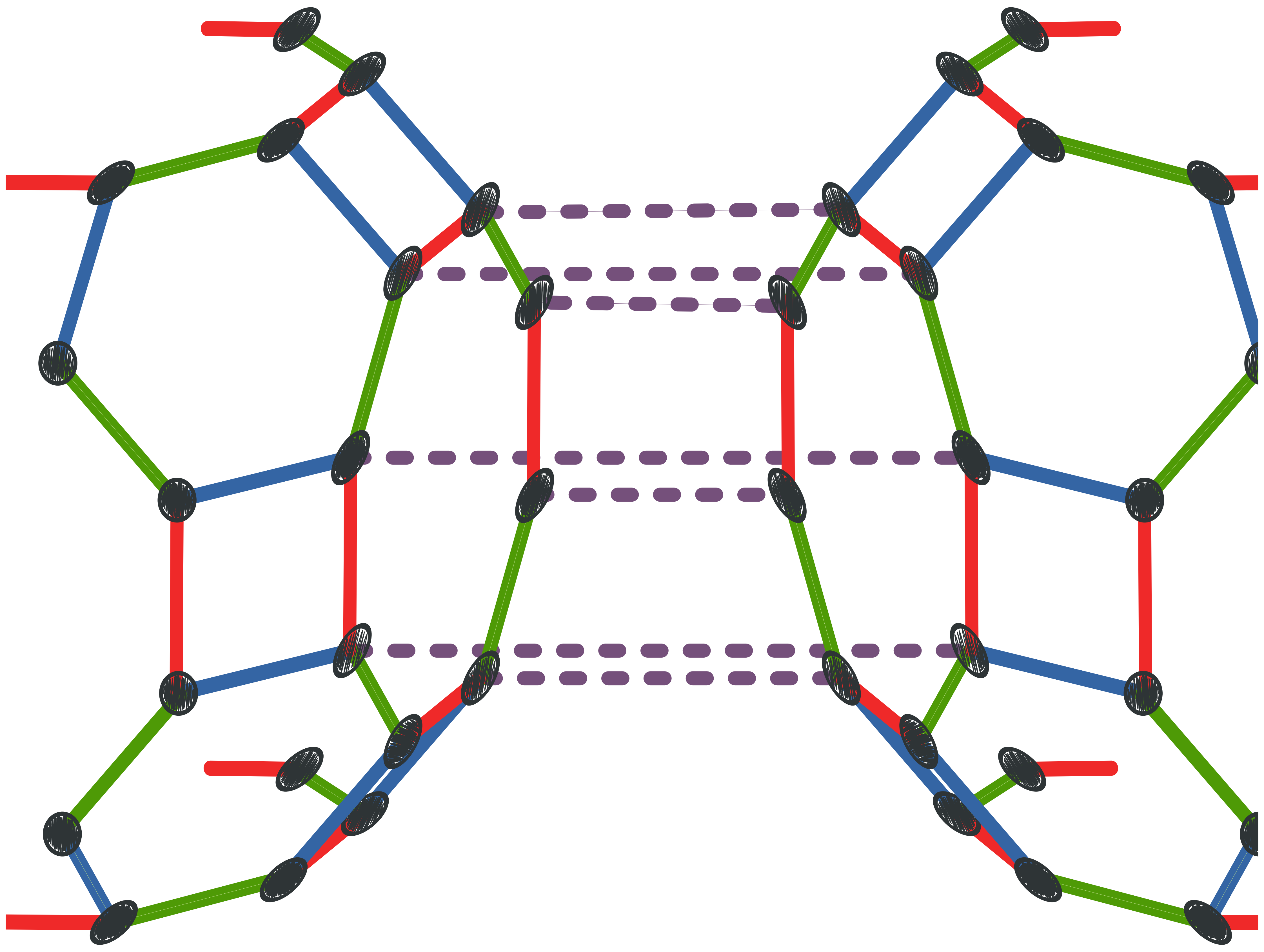

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semi-aristas ✓
aristas paralelas ✓
lazos ✗

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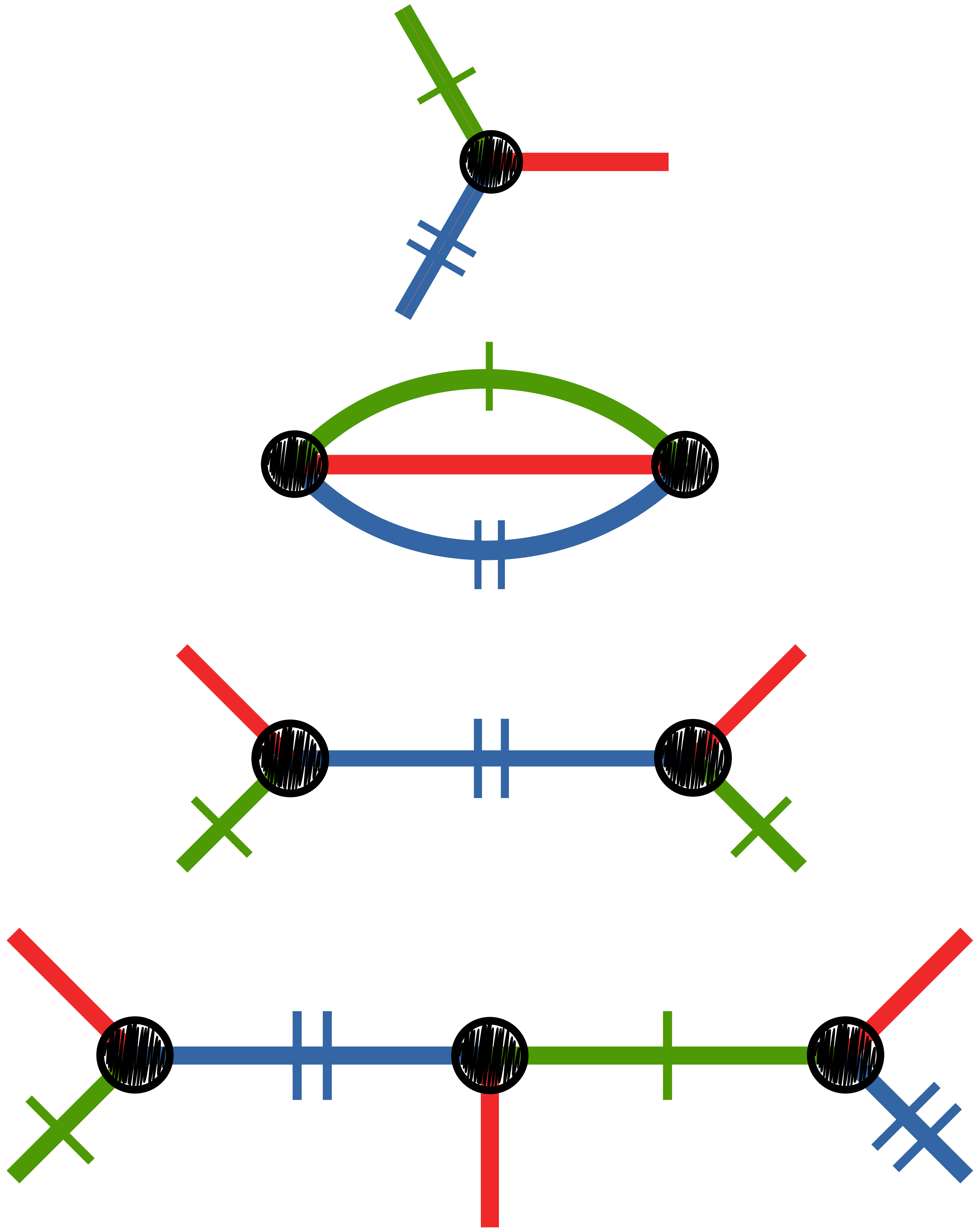

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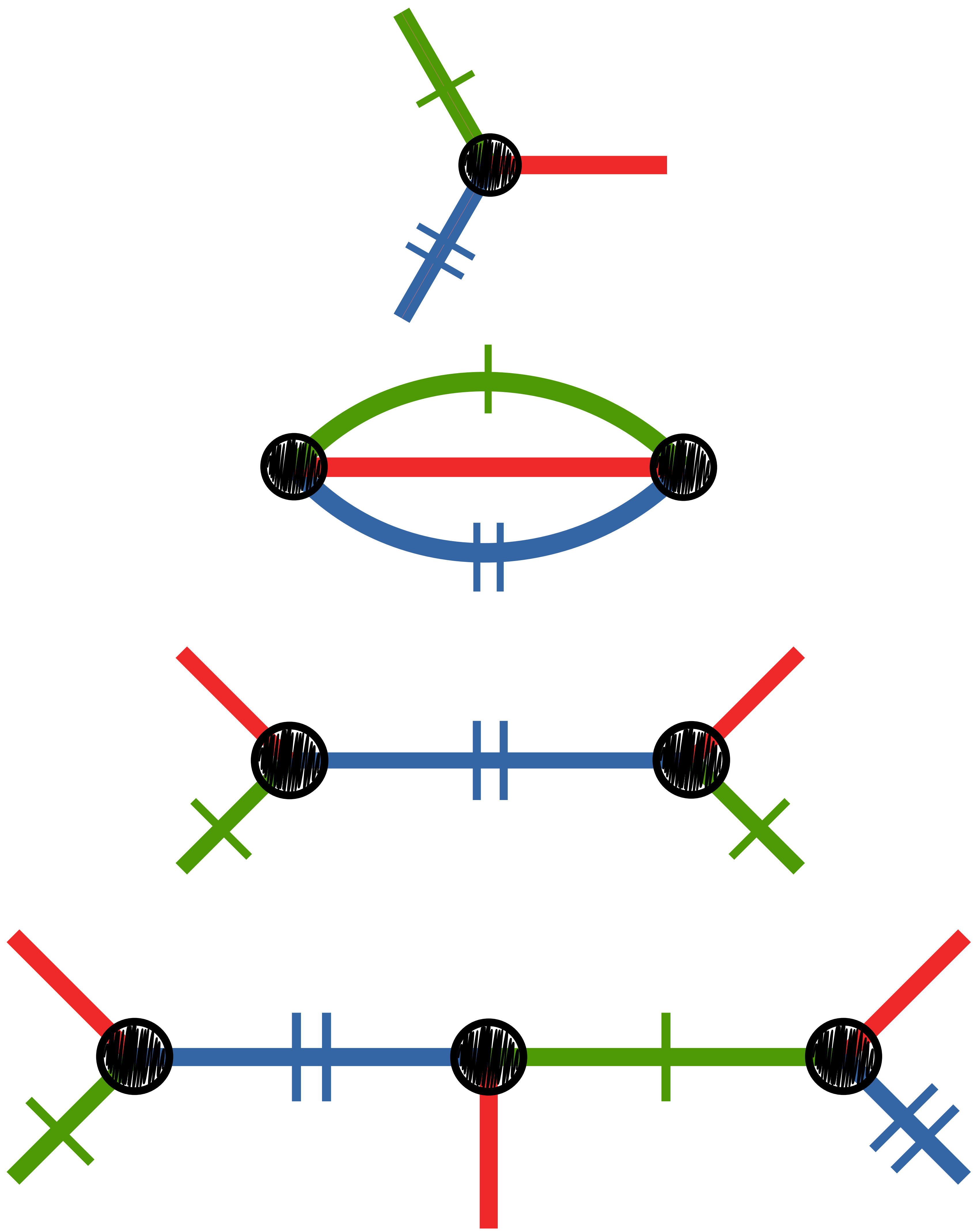
Los conceptos de cubrientes, cocientes y automorfismos se extienden de manera natural de maniplexes a premaniplexes



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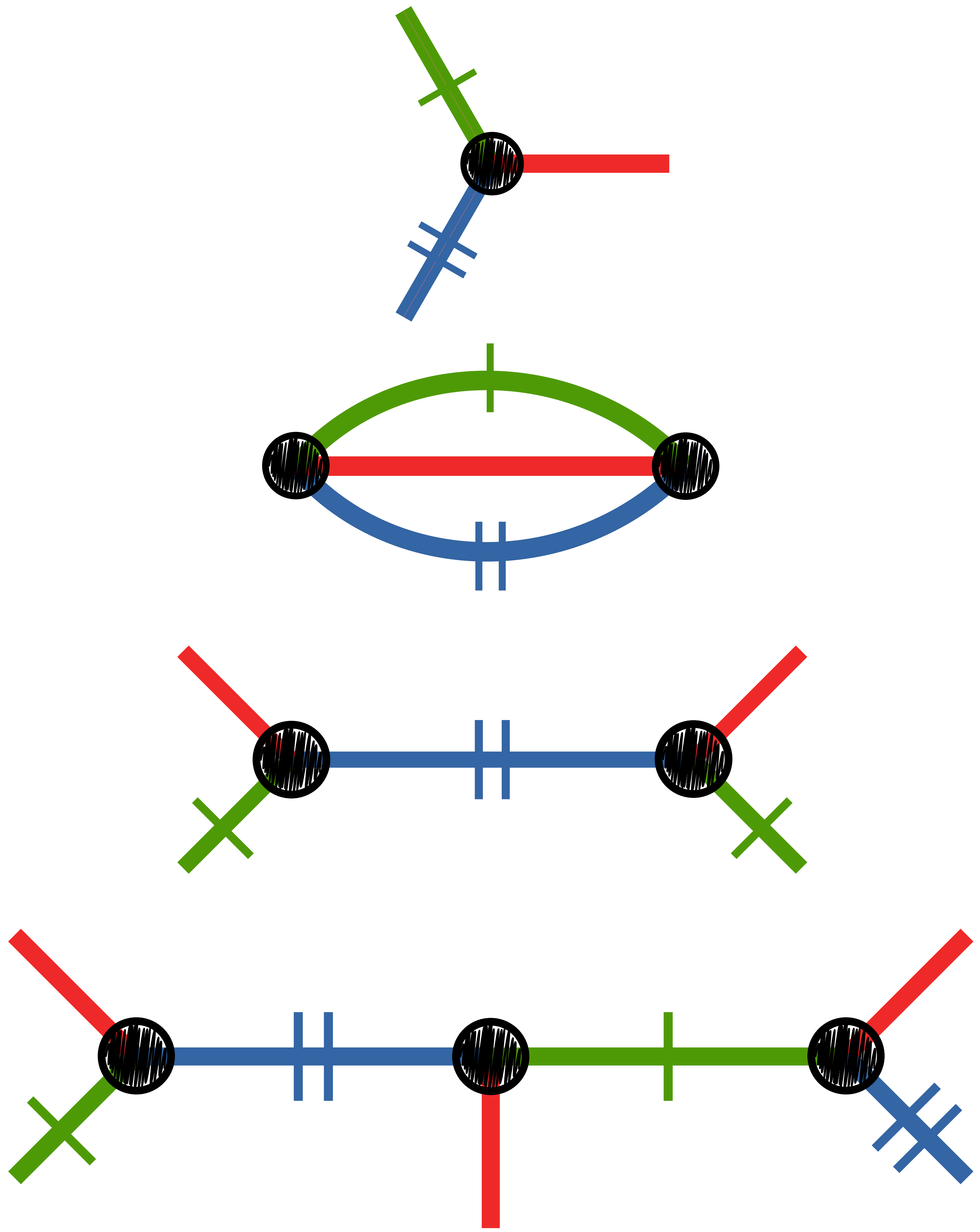

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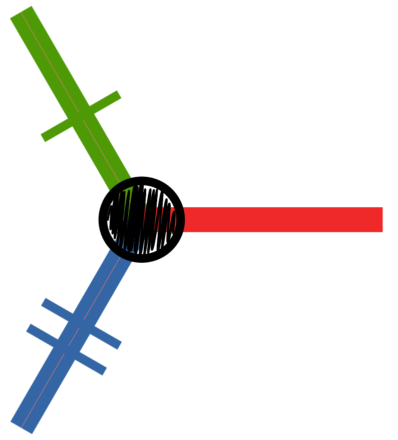
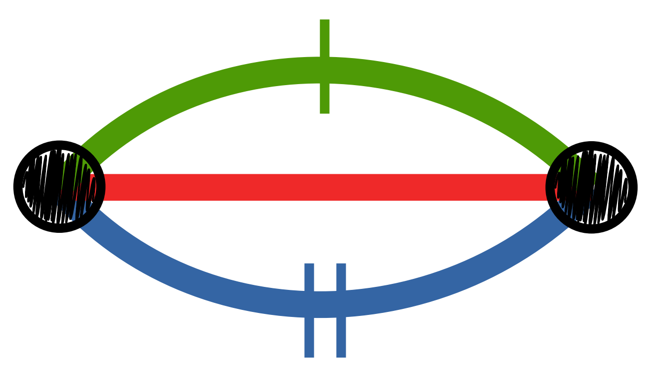
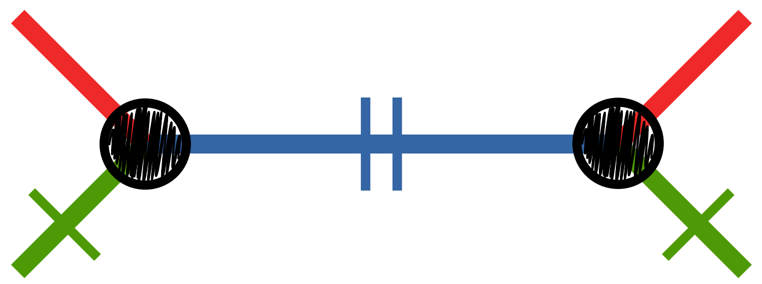
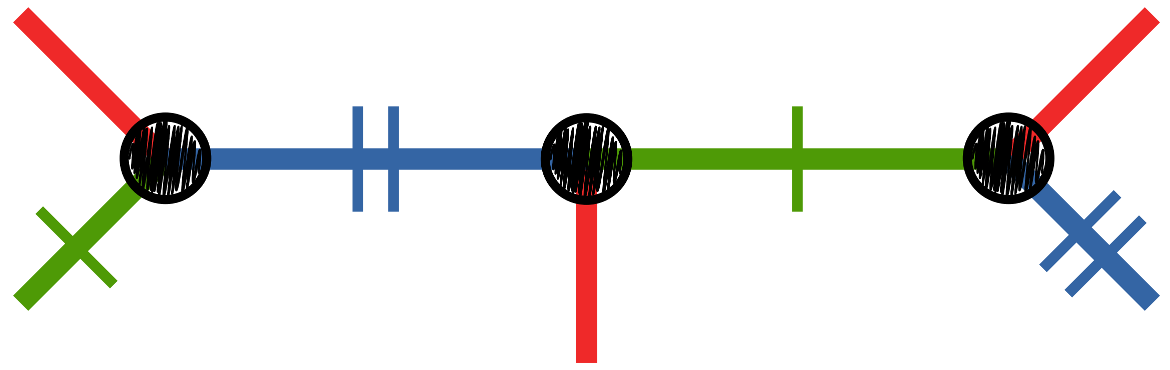

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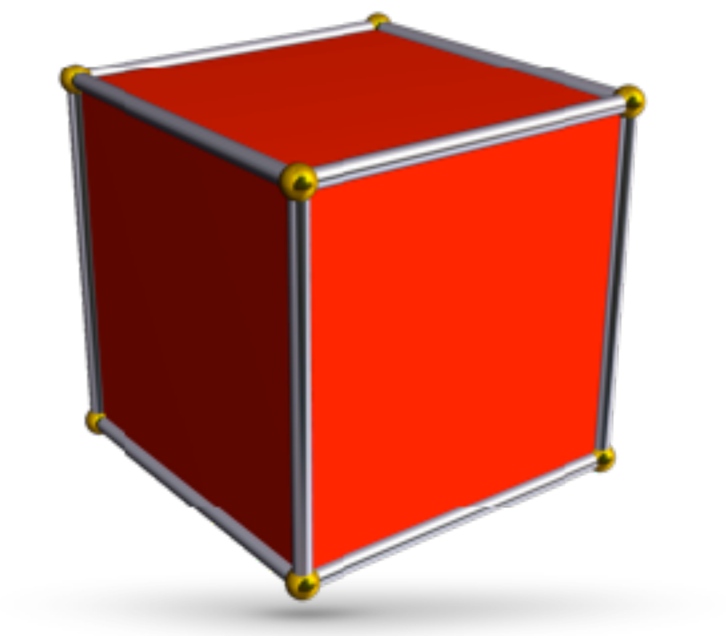

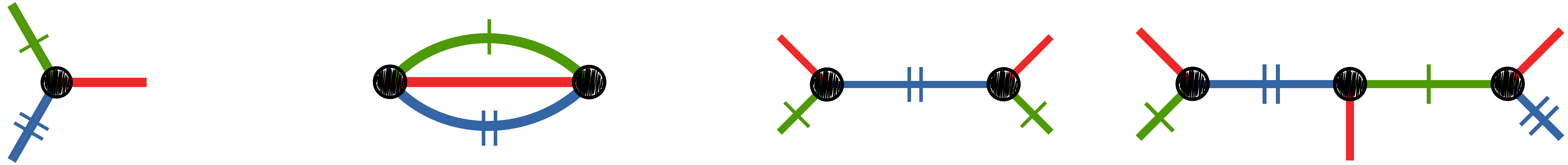

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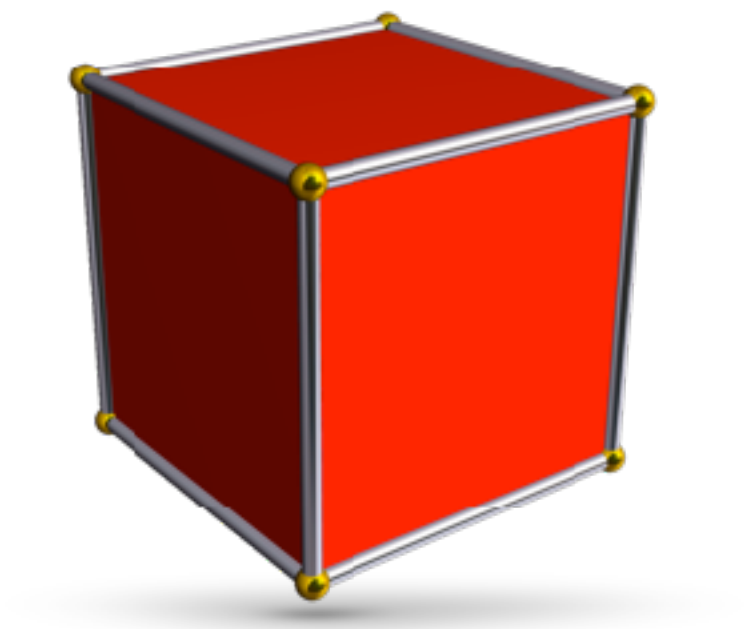

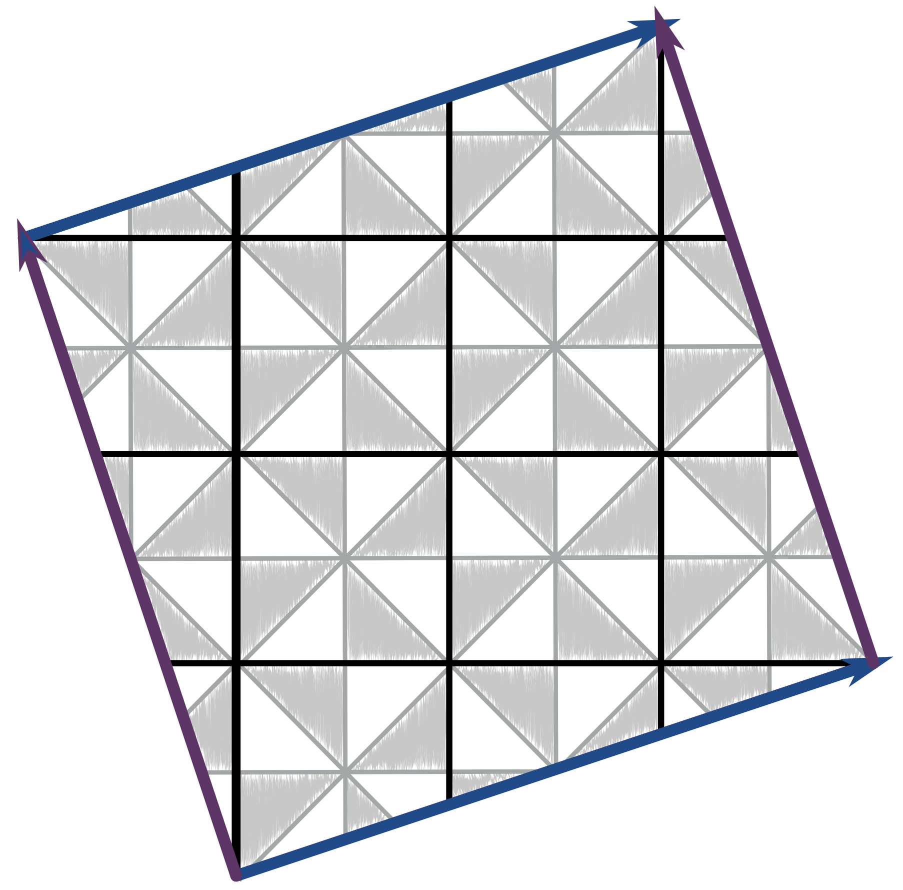

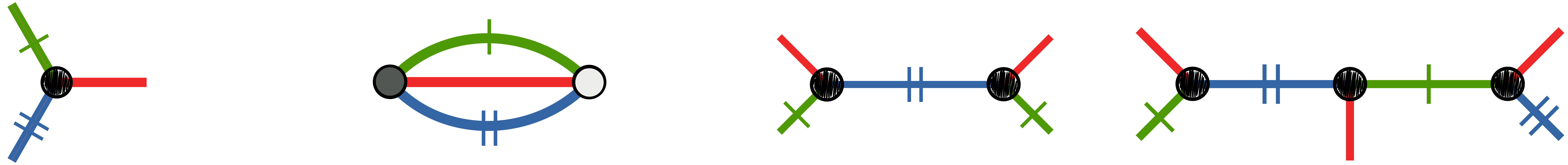

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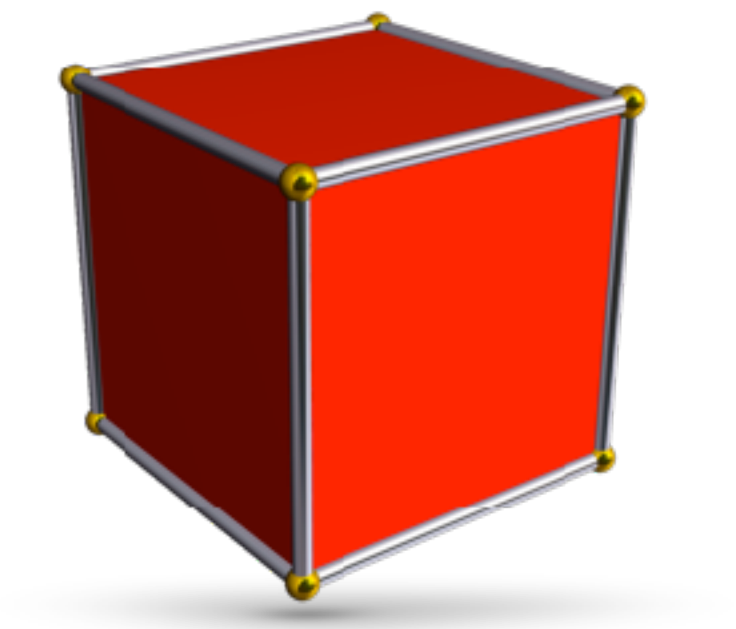
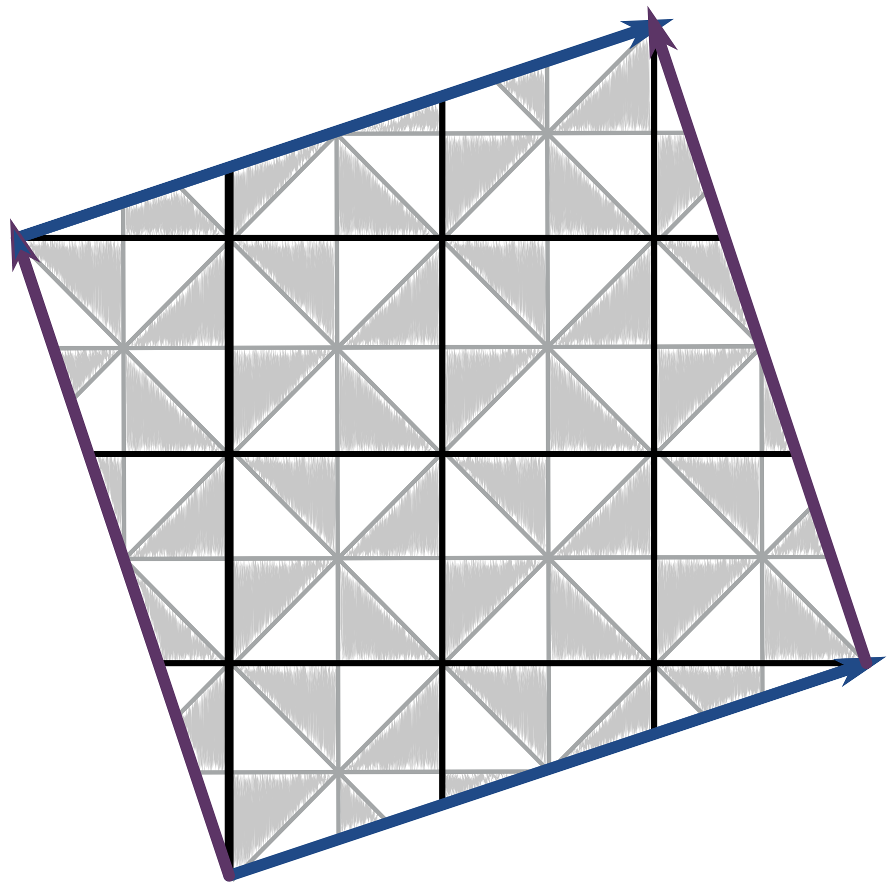
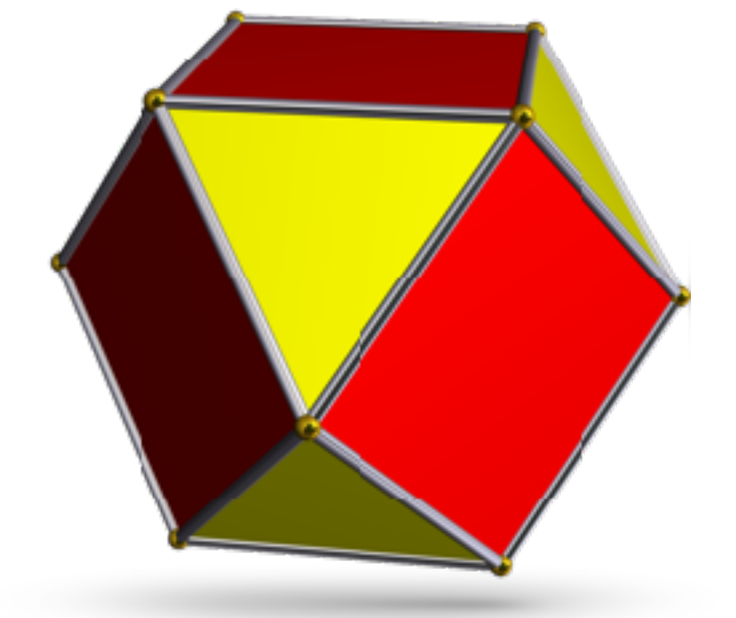
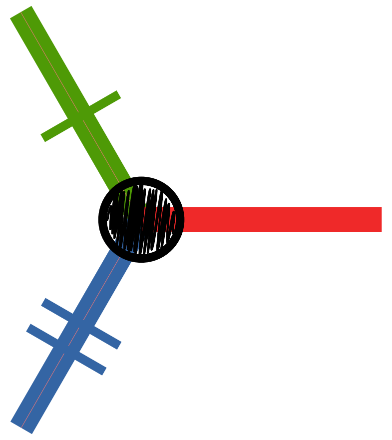
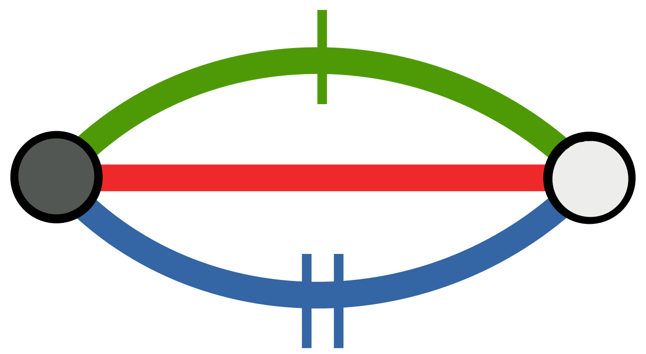
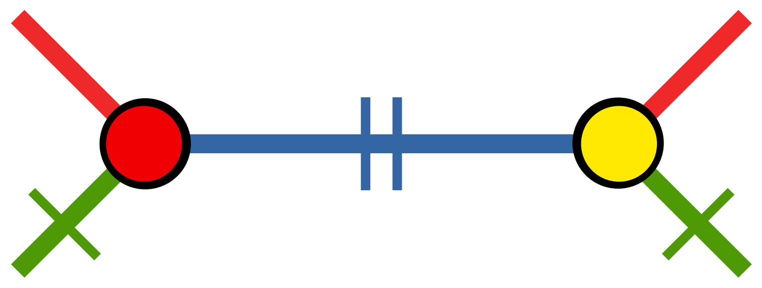
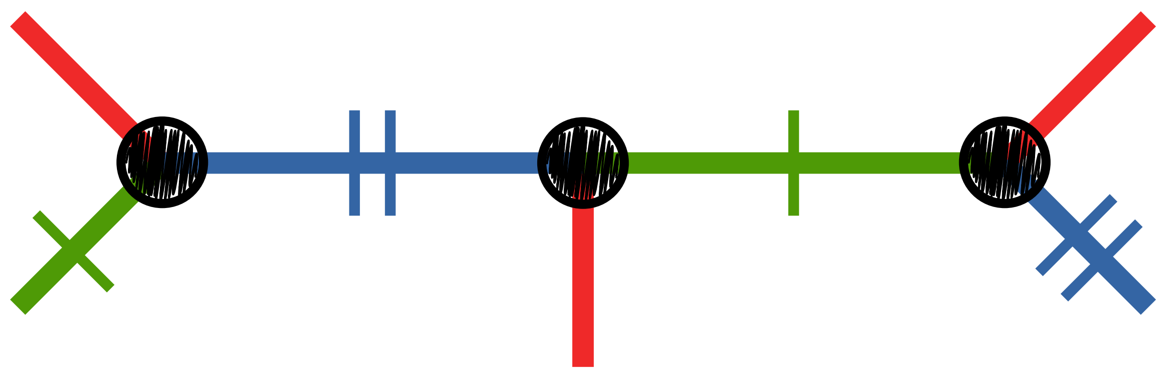

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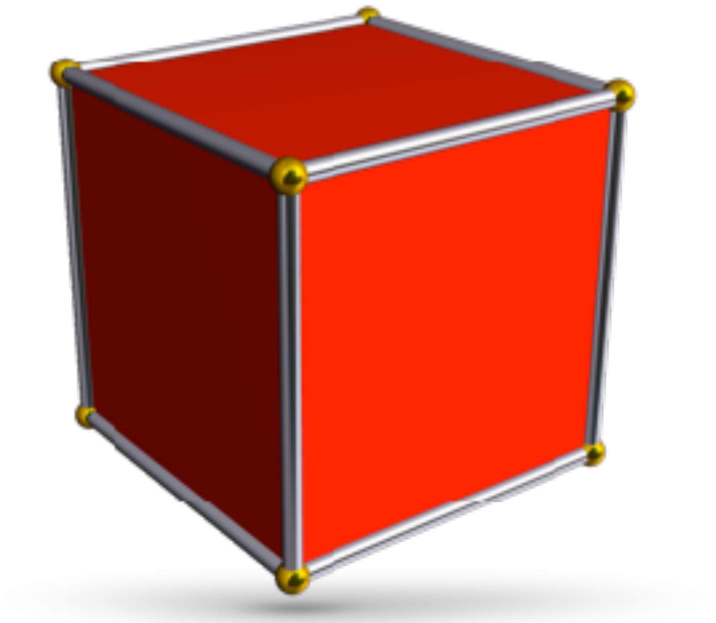
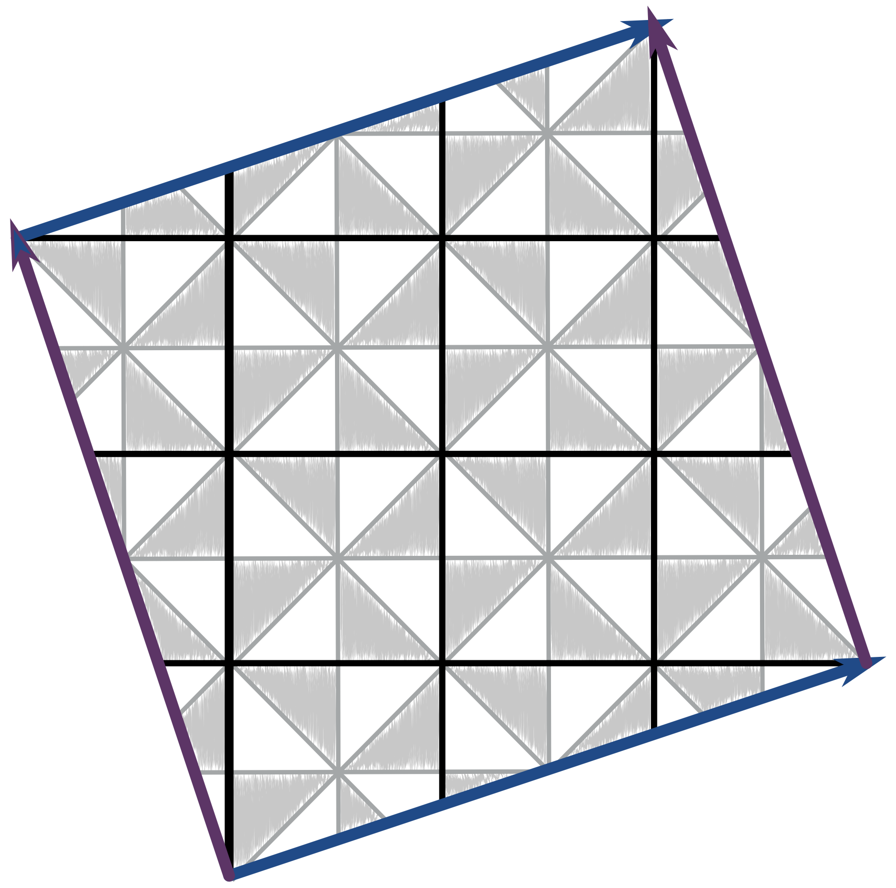
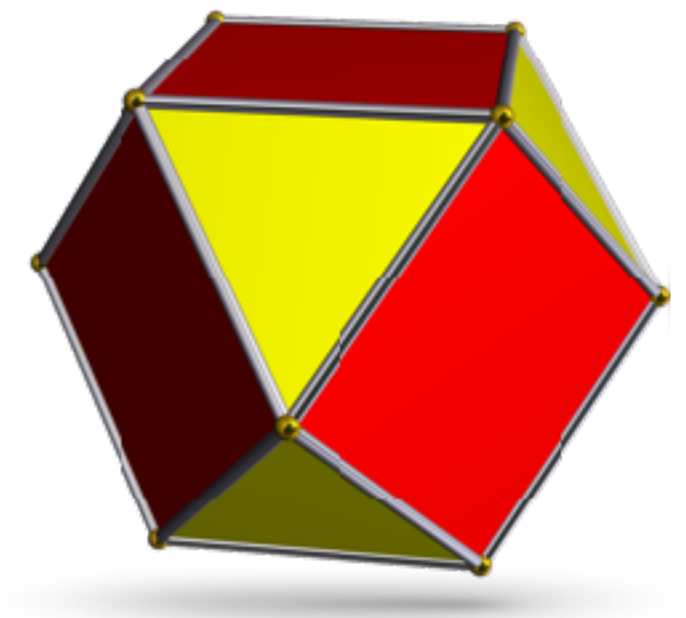
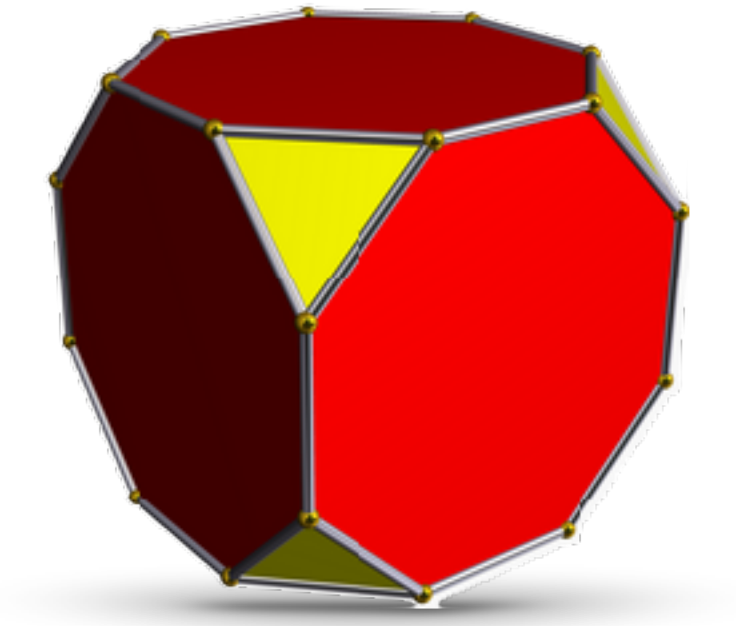
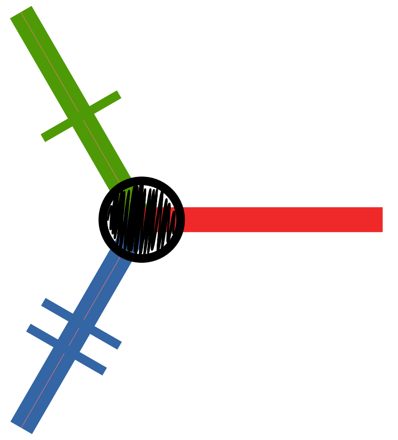
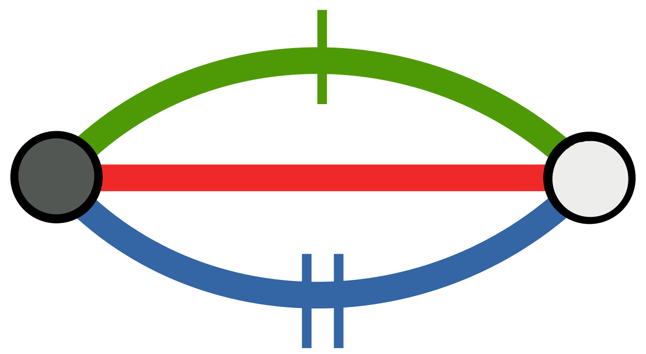
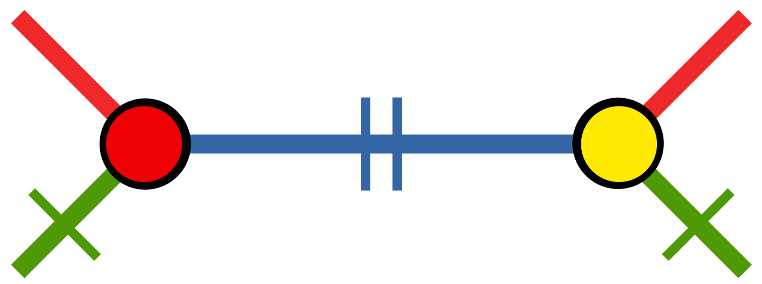
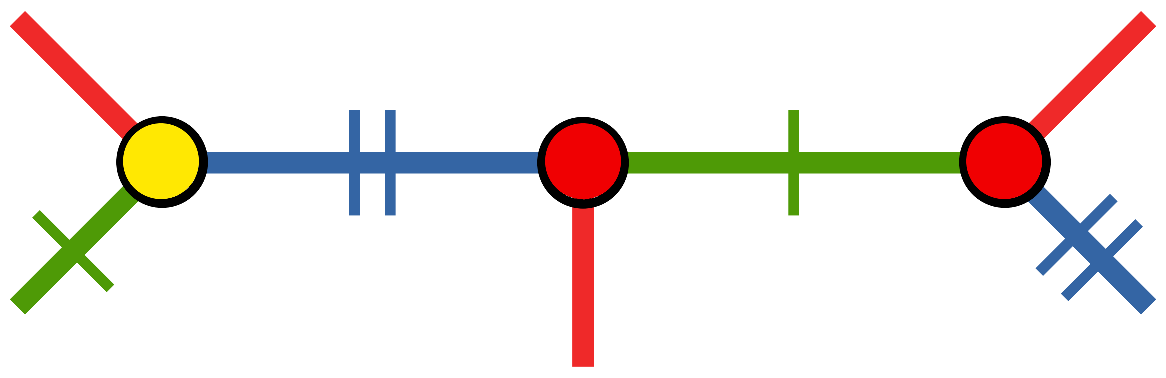





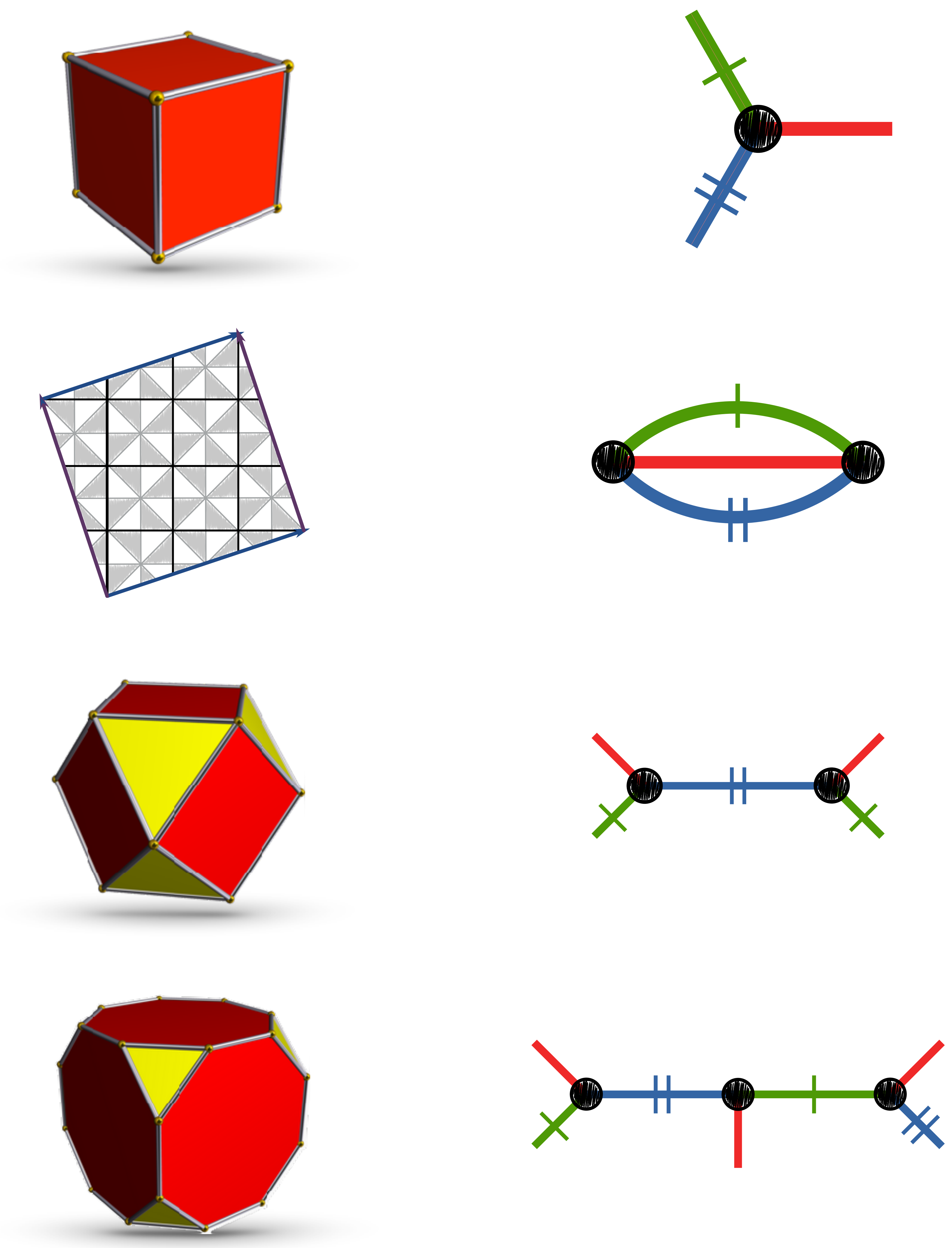





Premaniplexes

Conjetura del tipo de simetría:

Dado un n-premaniplex conexo $\mathcal{T}$, existe un n-maniplex (politopo) $\mathcal{M}$ tal que

$$\mathrm{STG}(\mathcal{M}) = \mathcal{T}$$



Operaciones de voltaje

- Un (m, n)- operador de voltaje es un par $(\mathscr{Y}, \eta)$

Operaciones de voltaje

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$\mathscr{Y}$

n-premaniplex

Operaciones de voltaje

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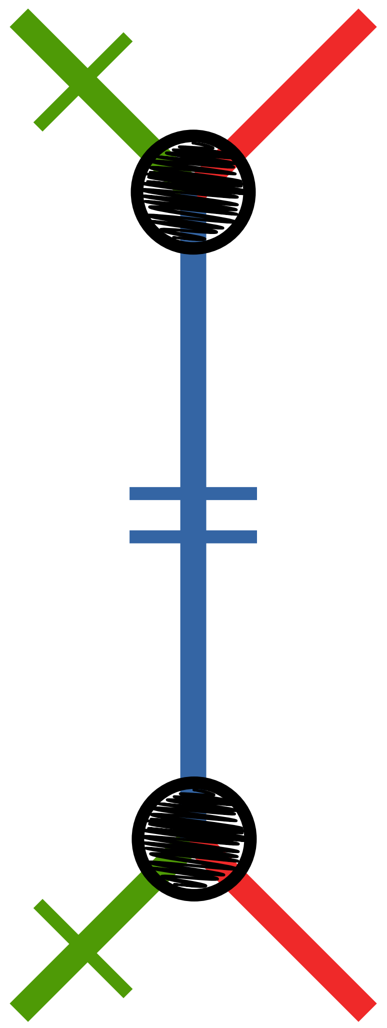
  $(\mathscr{Y}, \eta)$



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n-premaniplex

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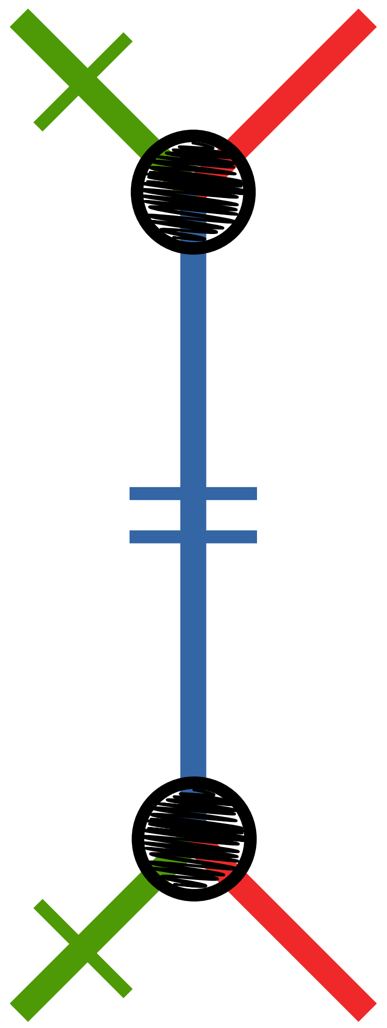


$\mathscr{Y}$

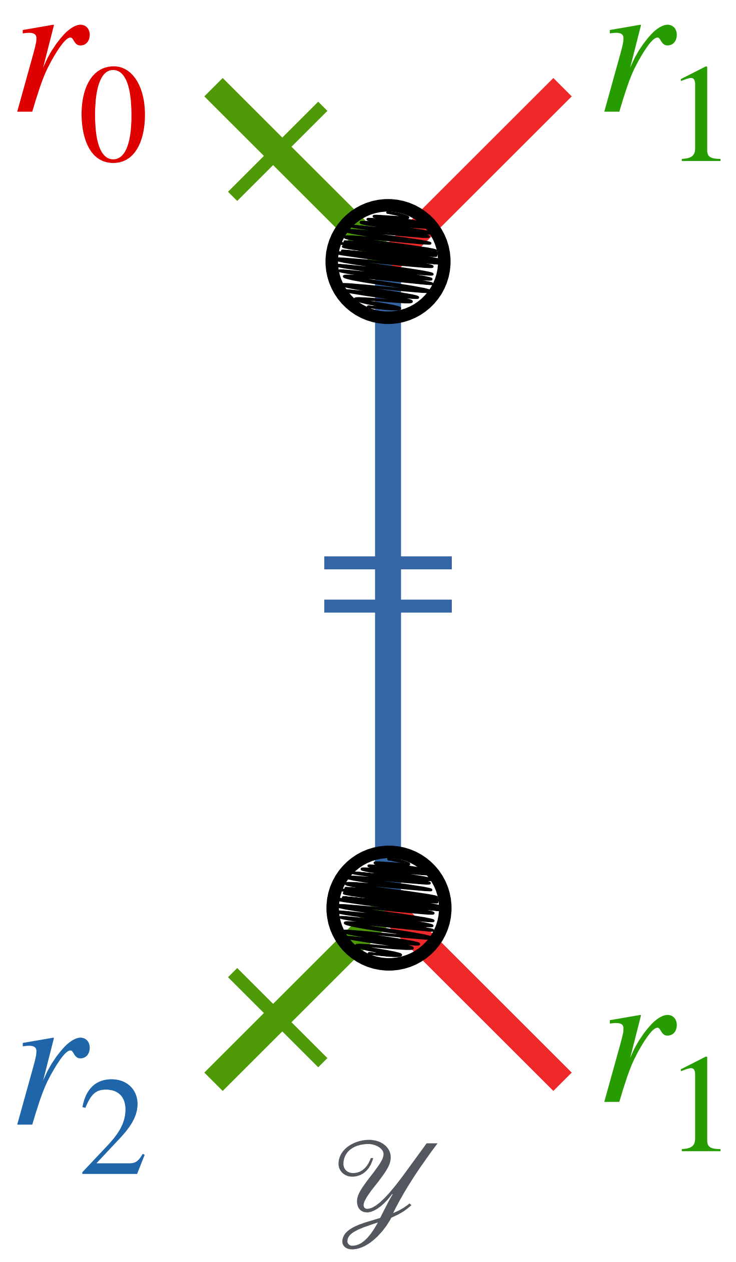
n-premaniplex

$\eta : W_m \to \mathscr{Y}$

Asignación de voltajes

Operaciones de voltaje

- Un (m, n)- operador de voltaje es un par $(\mathcal{Y}, \eta)$



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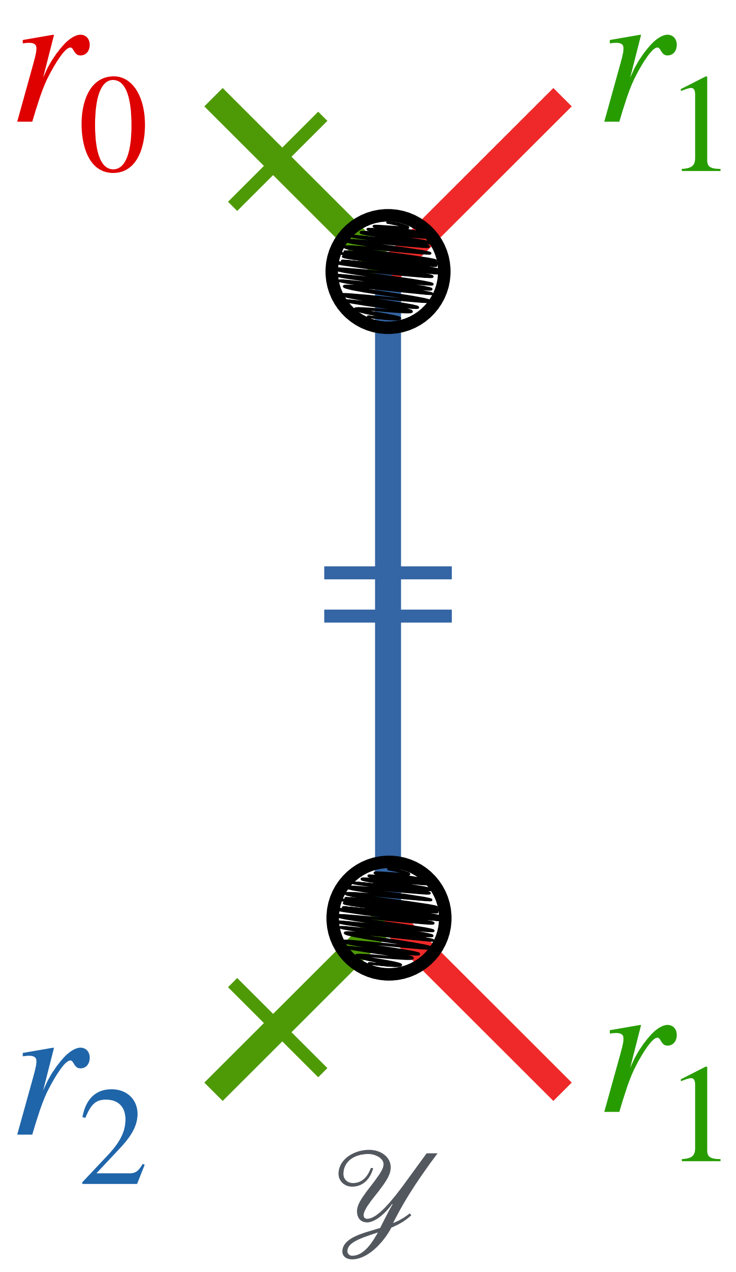

n-premaniplex

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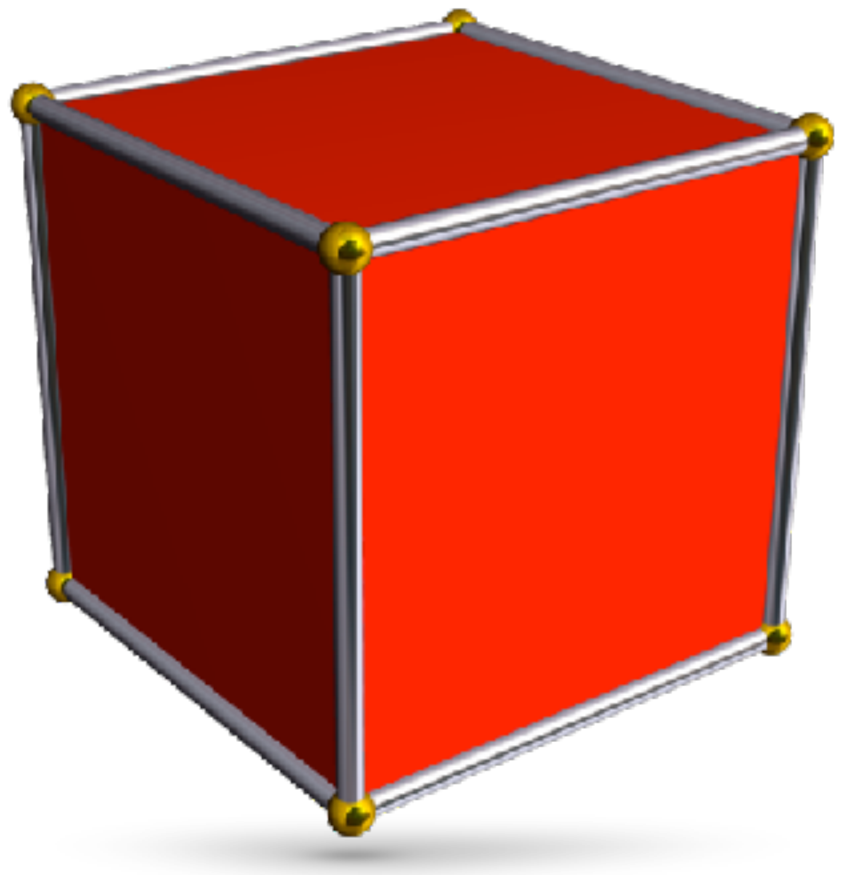
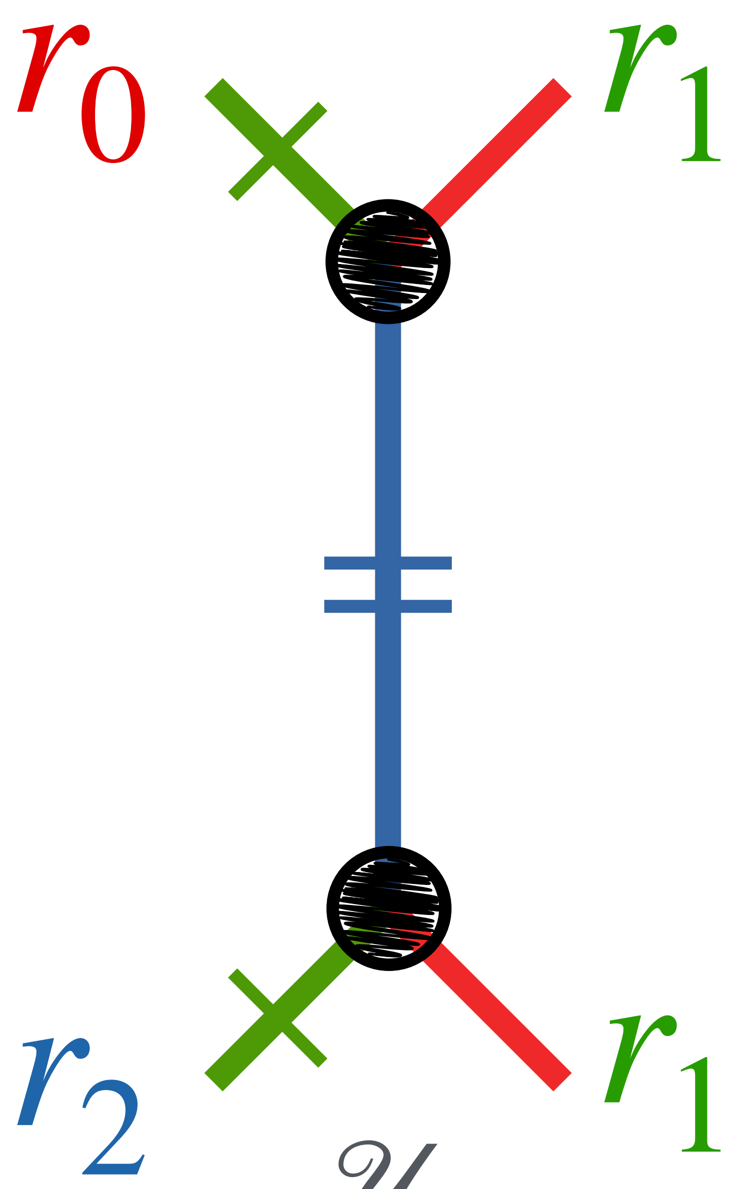


$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

n-premaniplex

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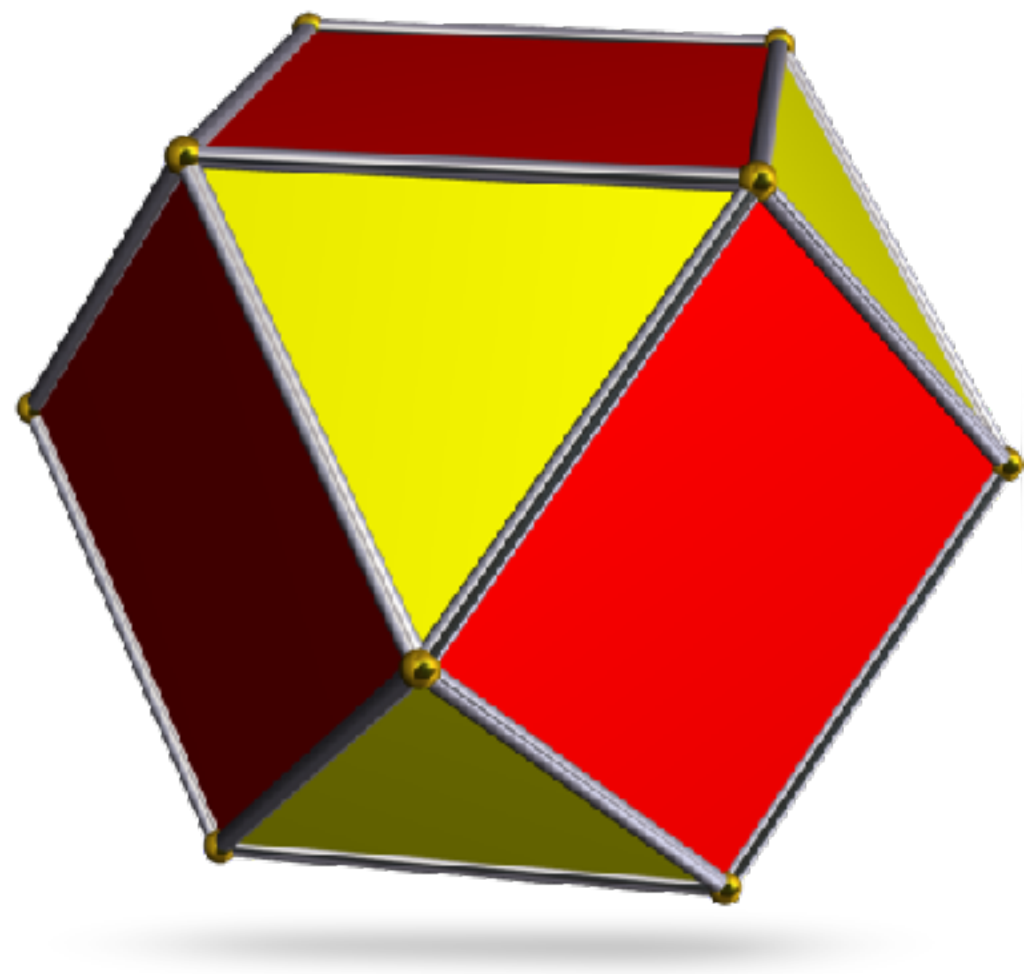
Asignación de voltajes

$=$



$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

Operaciones de voltaje

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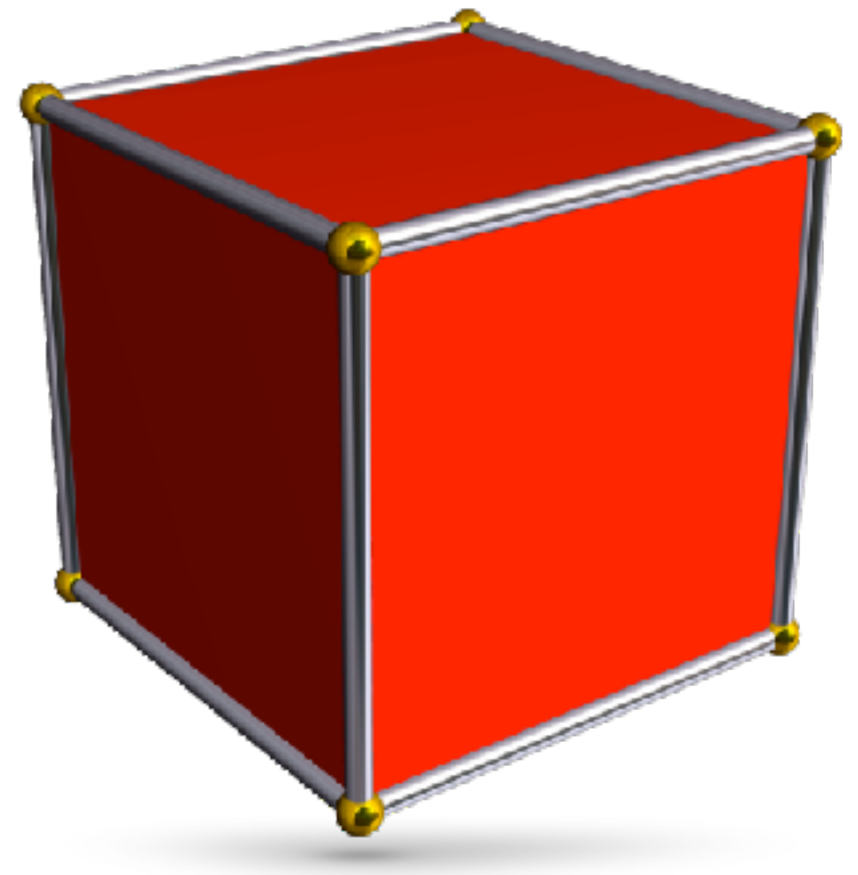

$\mathscr{X}$

m-premaniplex

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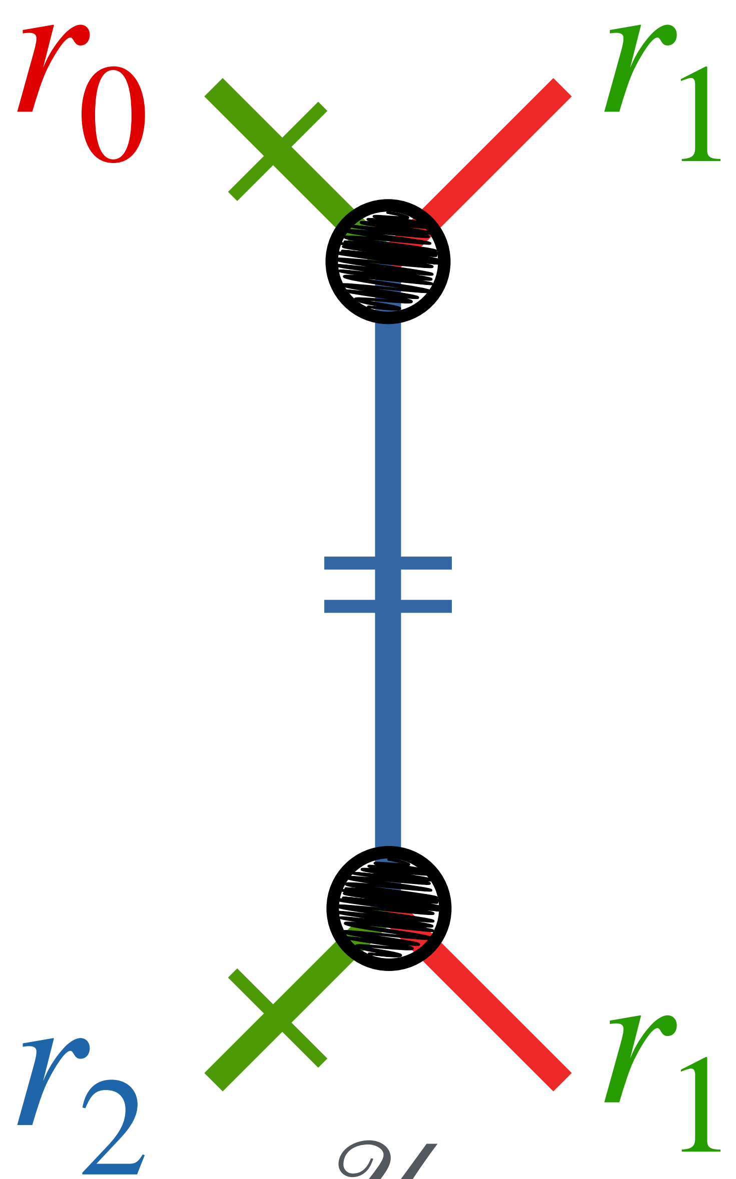

$\mathscr{Y}$

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Asignación de voltajes

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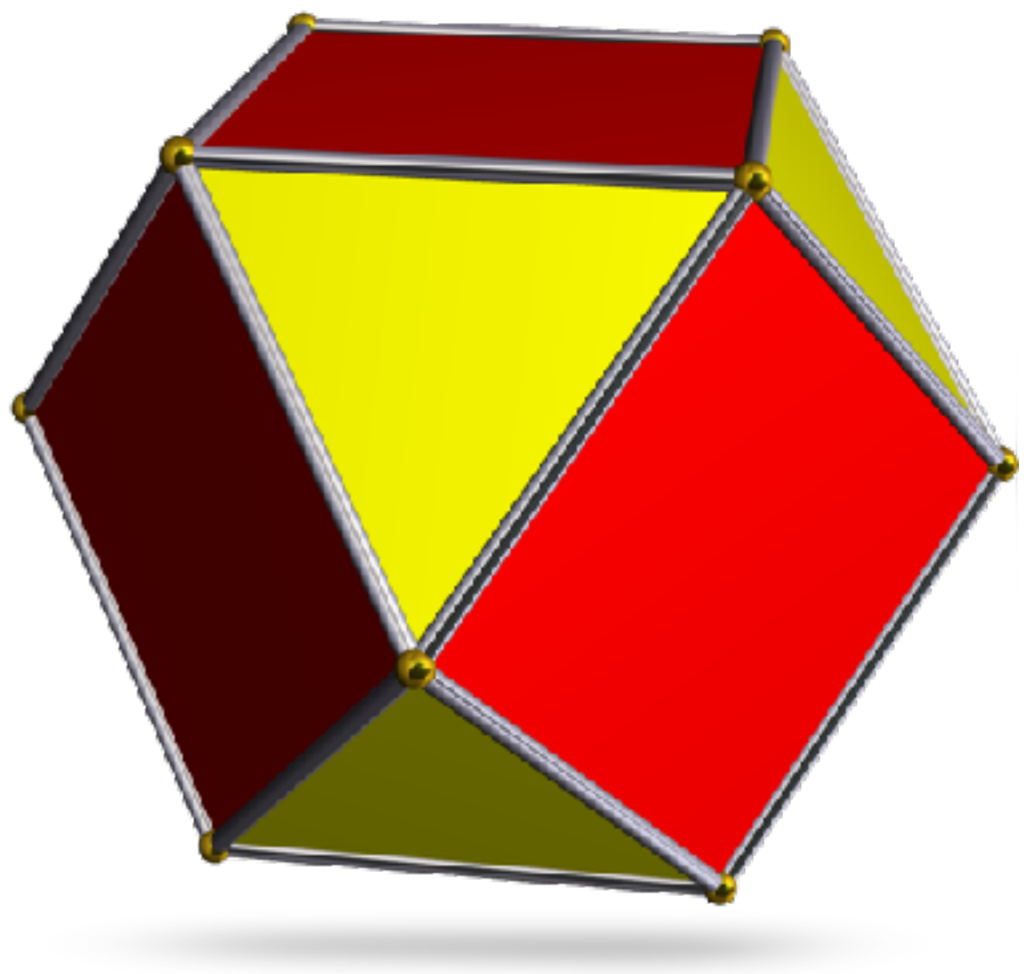

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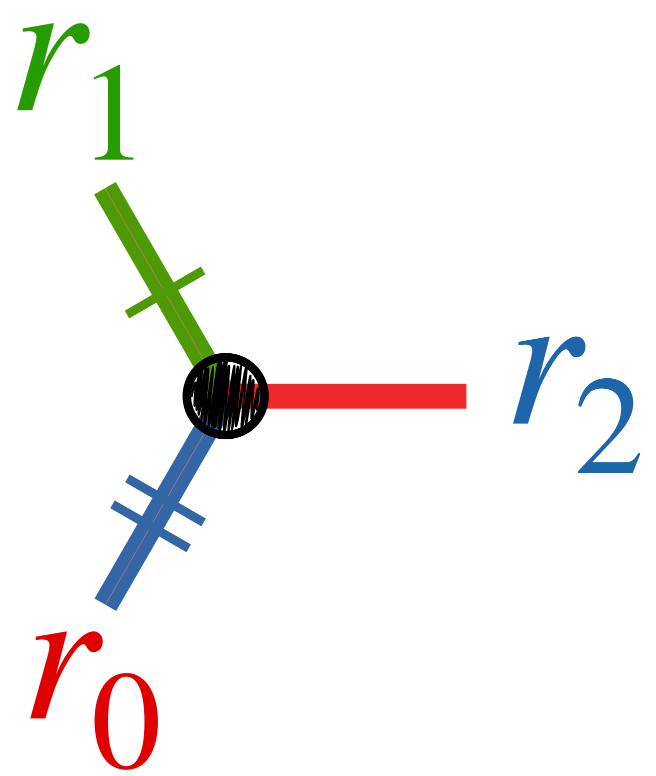

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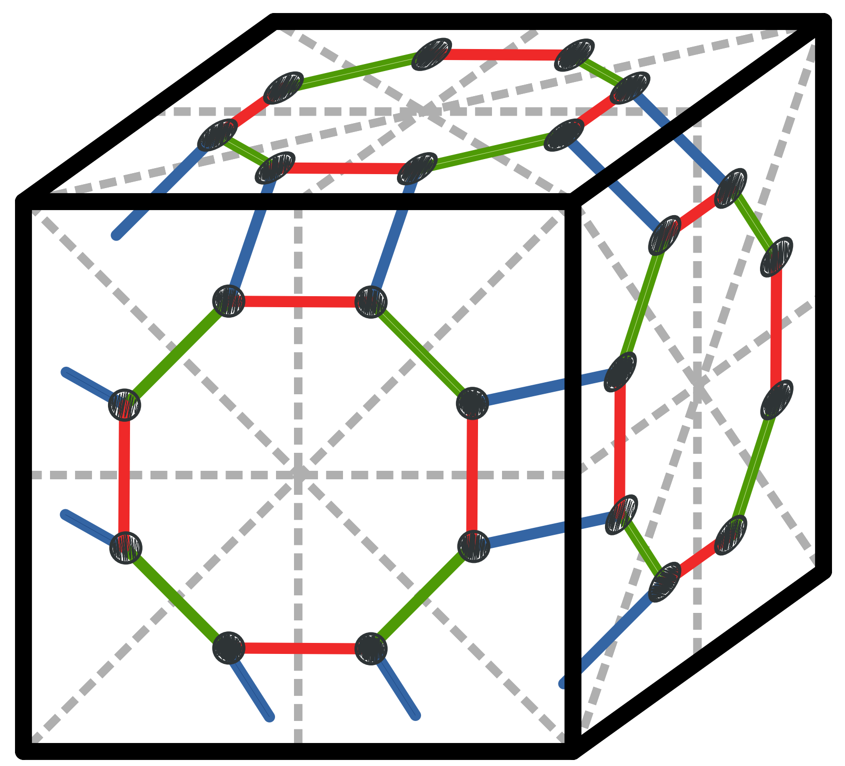

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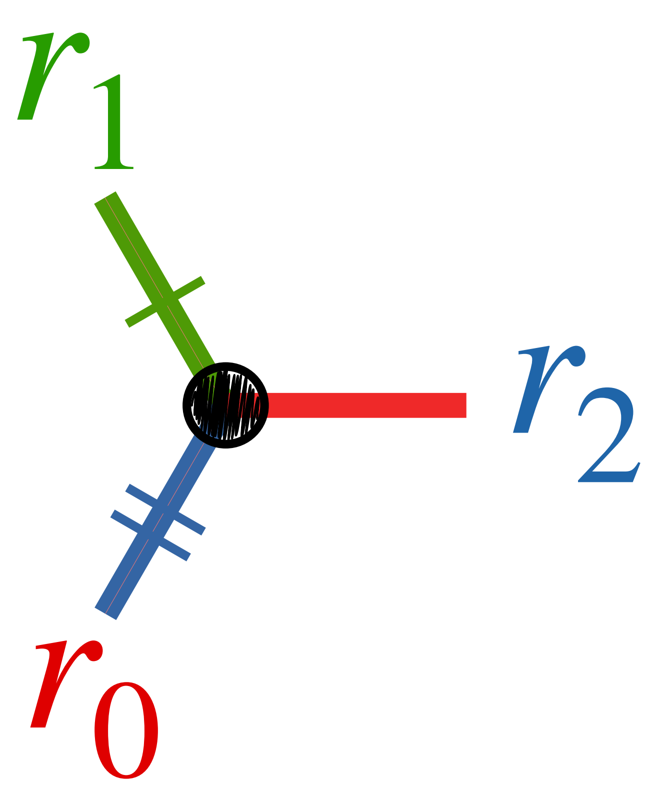

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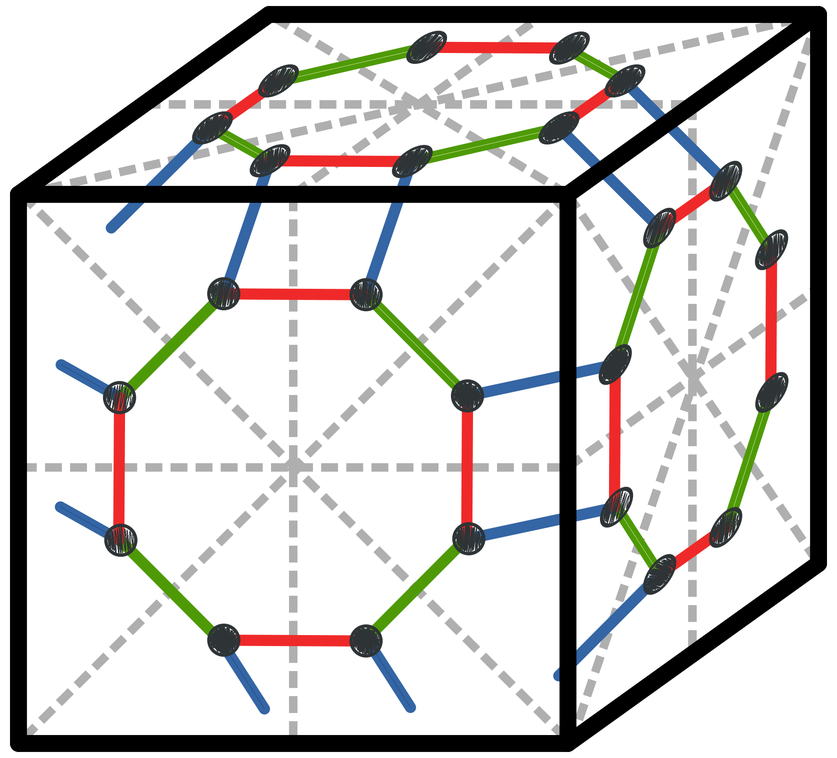

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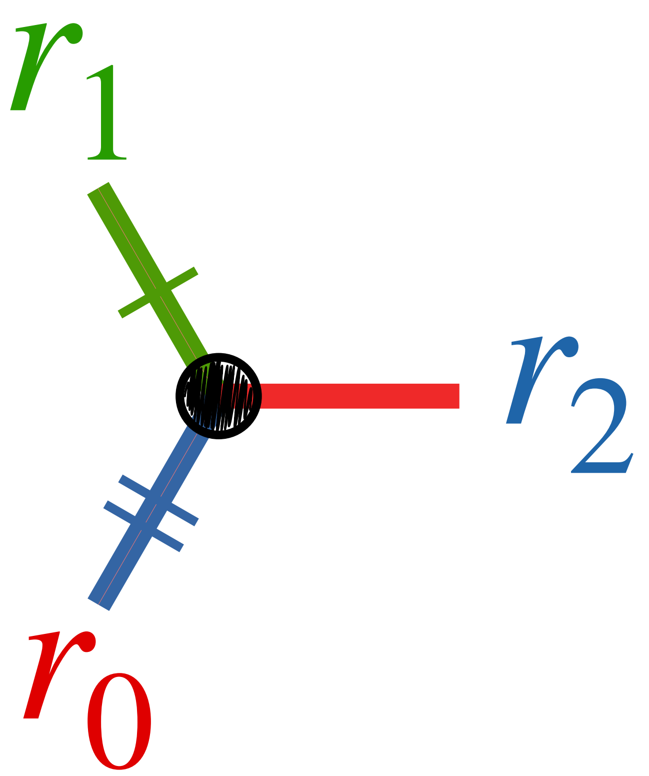

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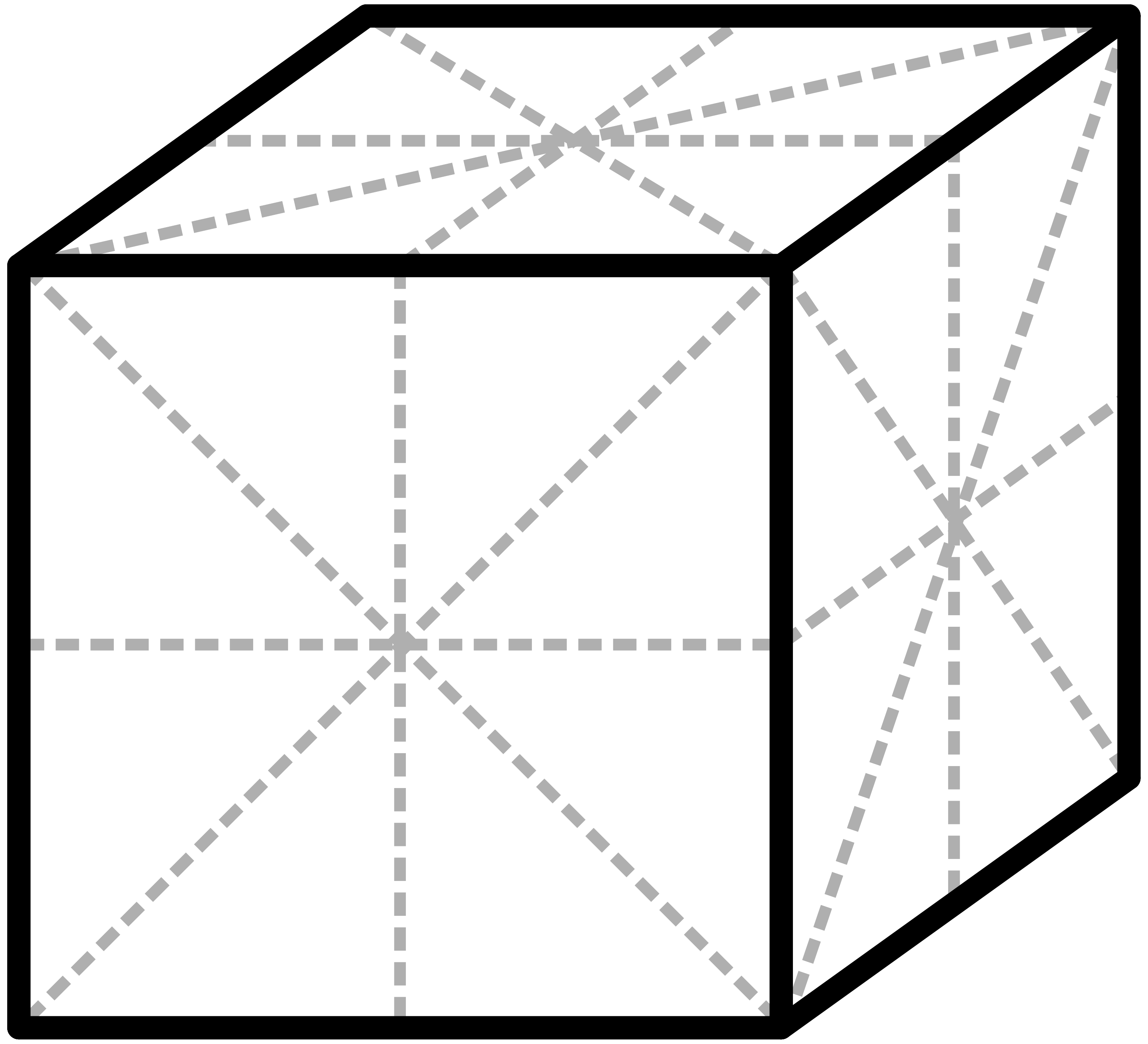

$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \rightarrow \mathscr{Y}$

Asignación de voltajes

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

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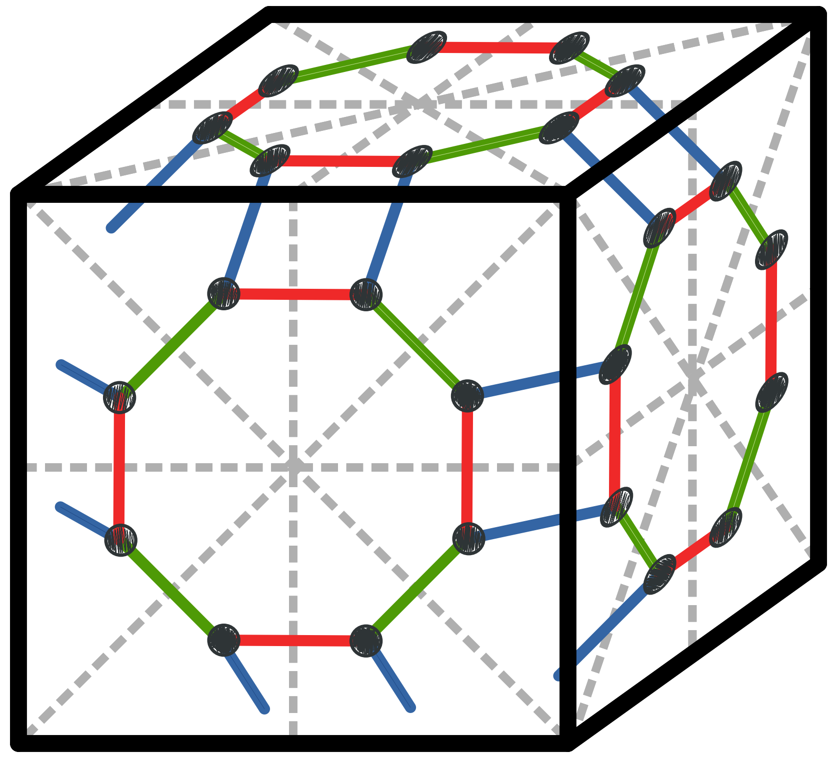
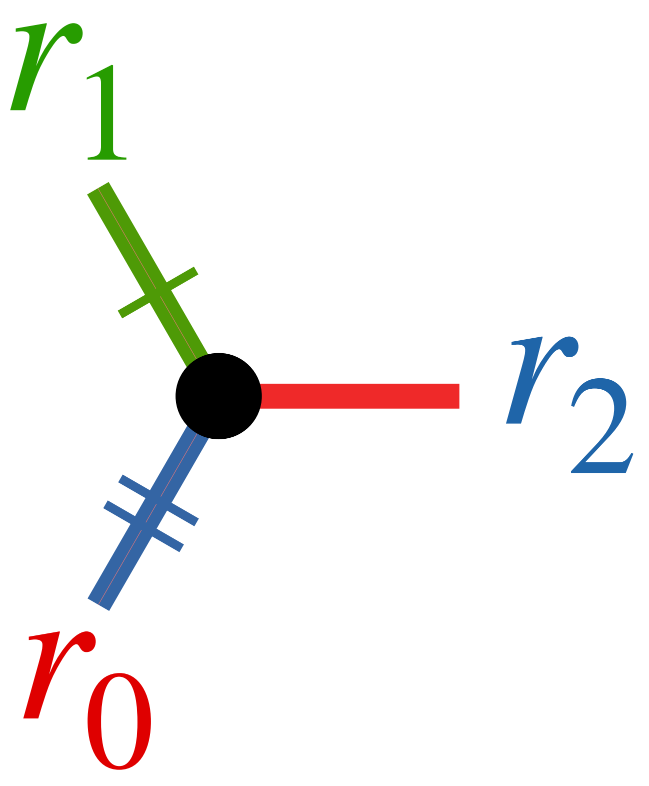


$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

Asignación de voltajes

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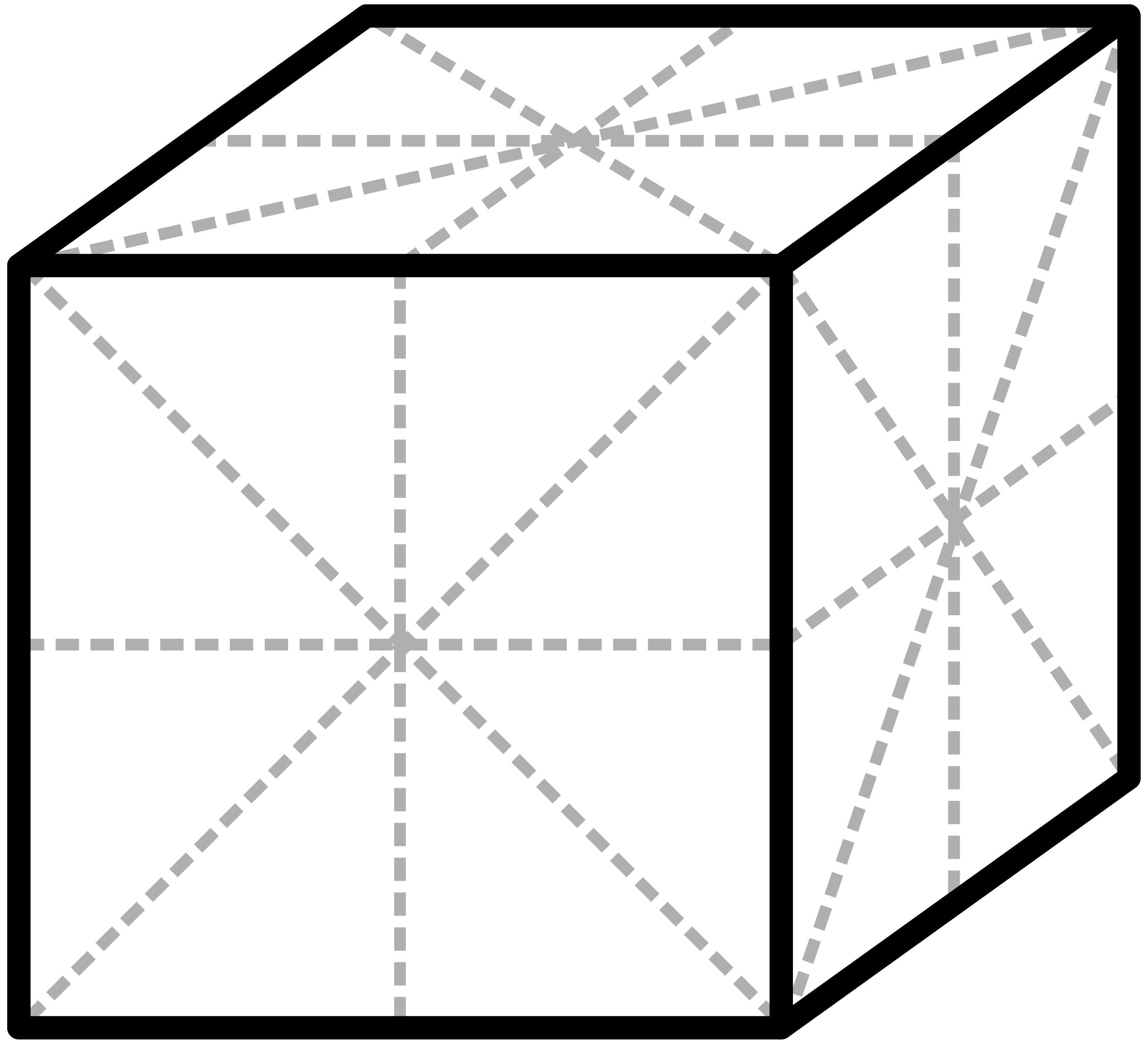

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

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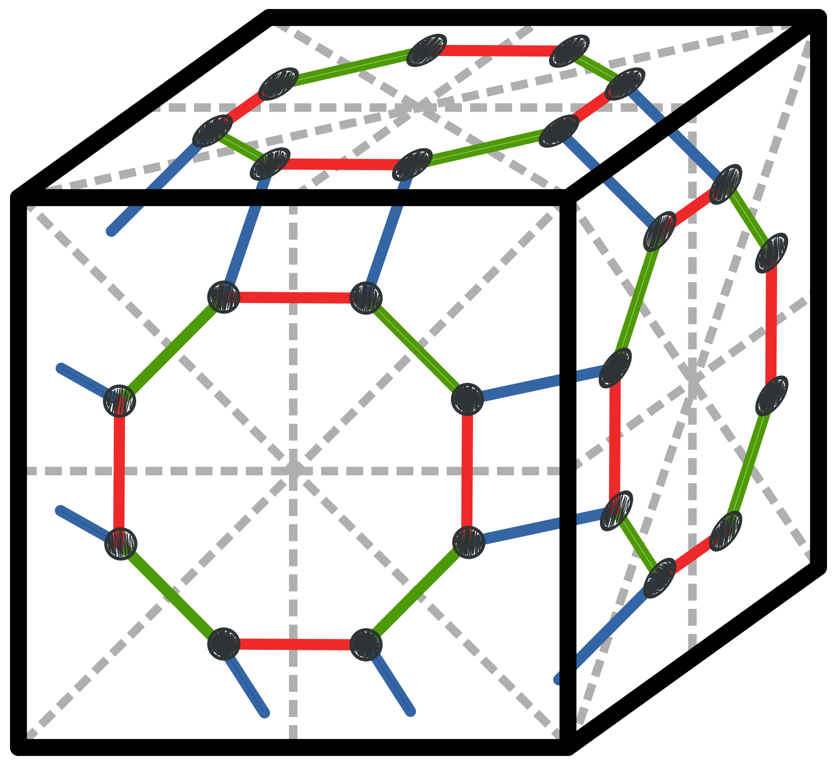

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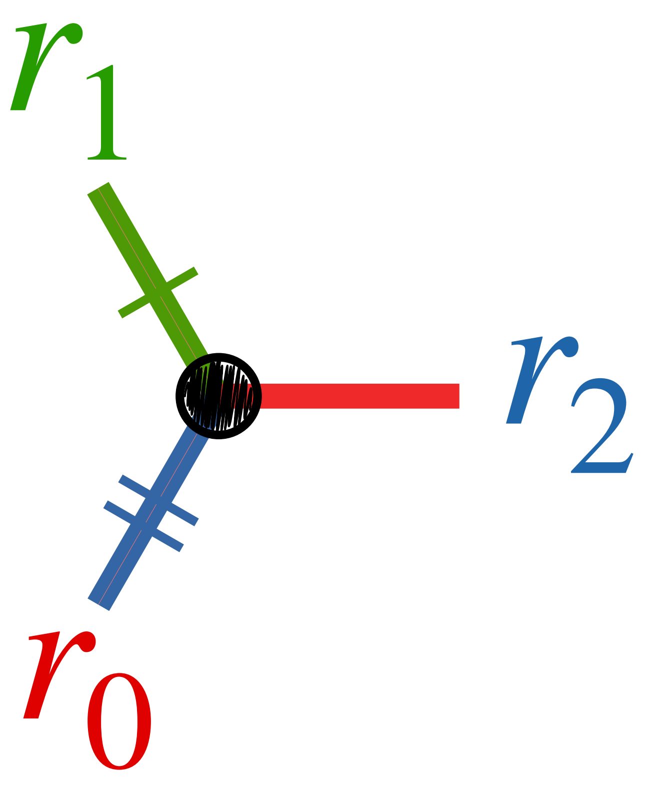

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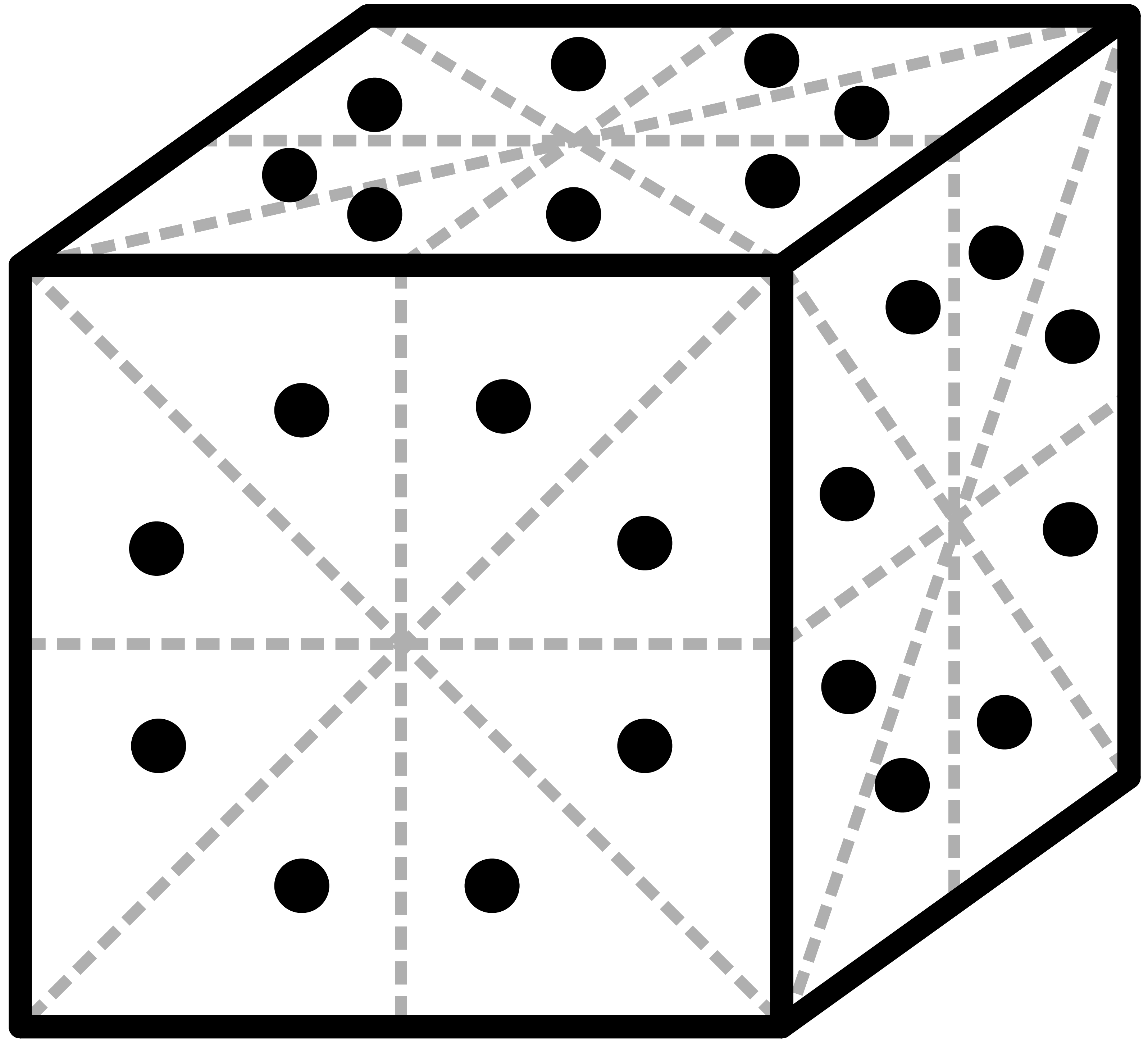

$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

Asignación de voltajes

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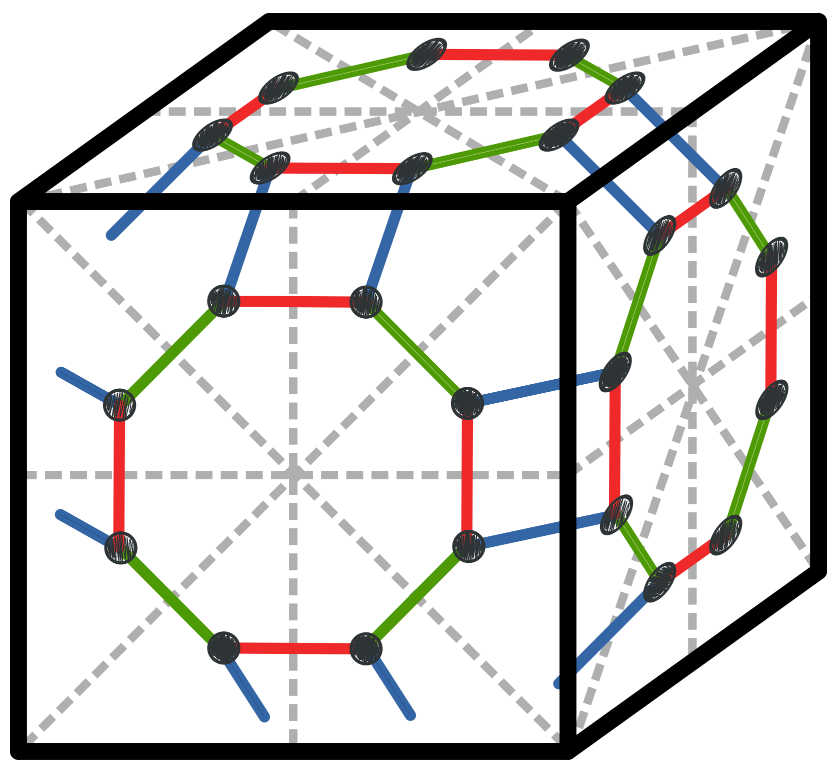
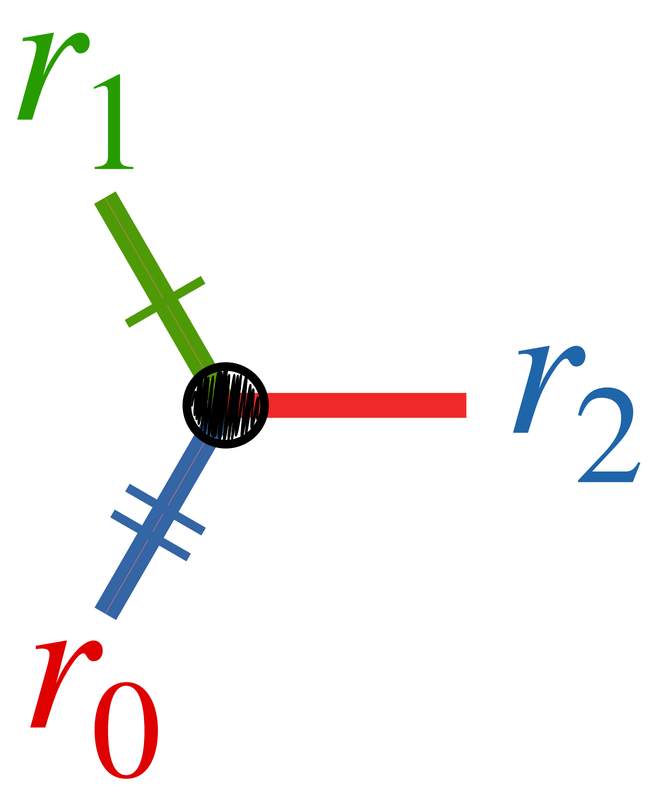


$\mathscr{X}$

m-premaniplex

$\rtimes$



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Asignación de voltajes

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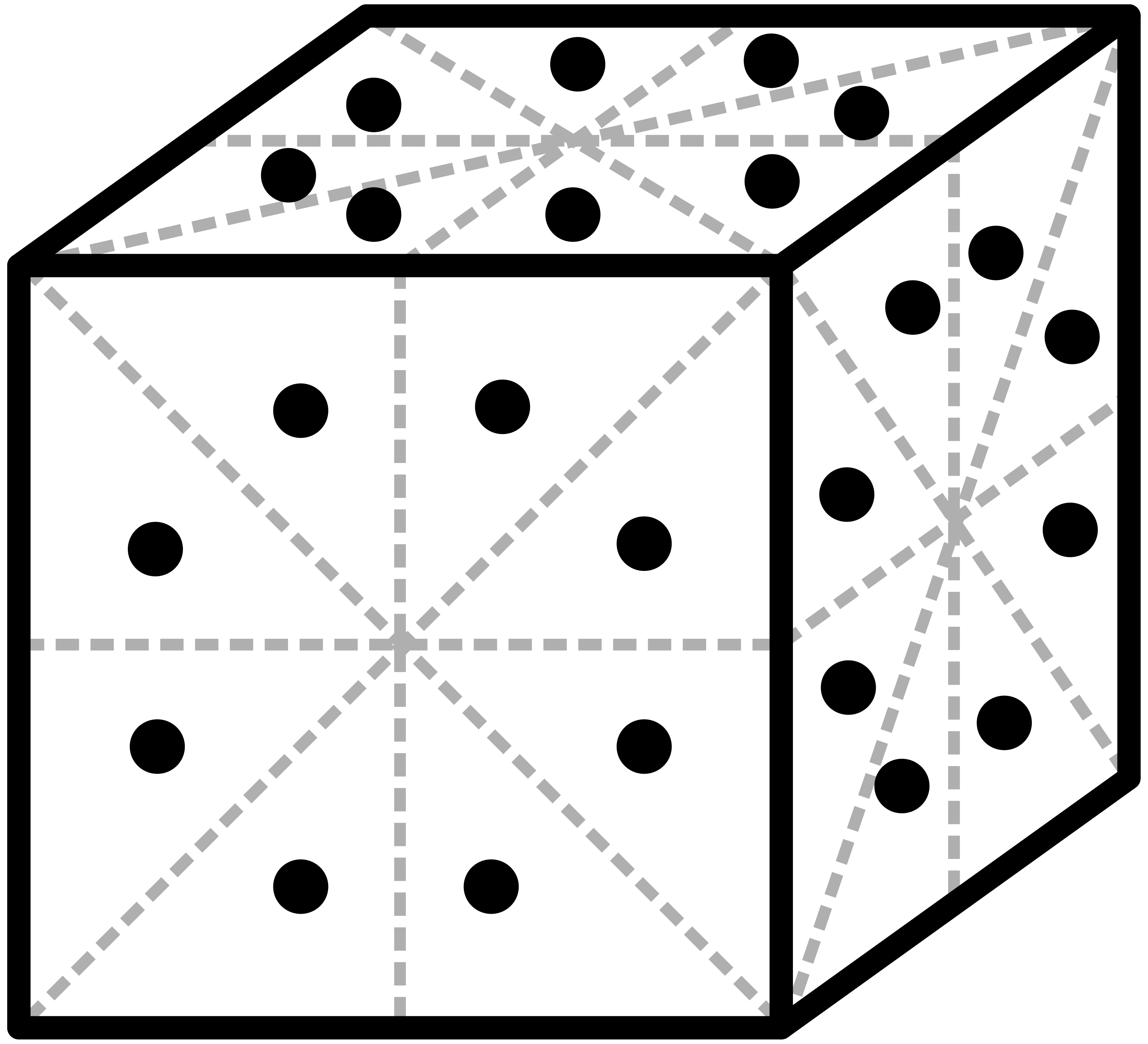

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

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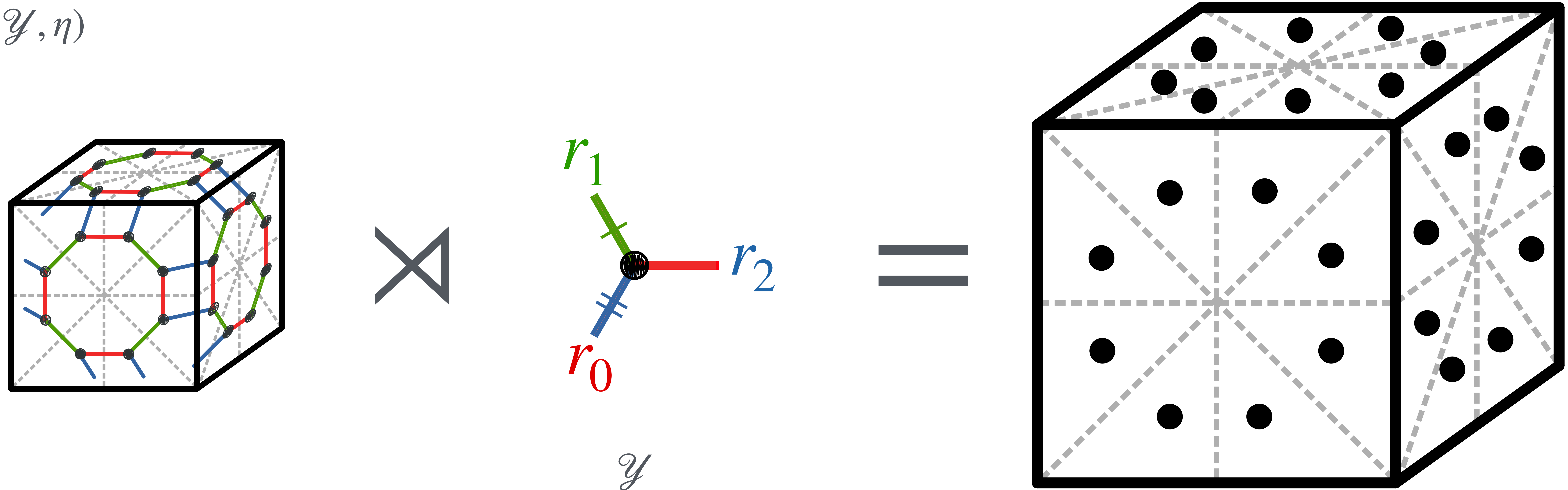

$\mathscr{X}$

m-premaniplex

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Asignación de voltajes

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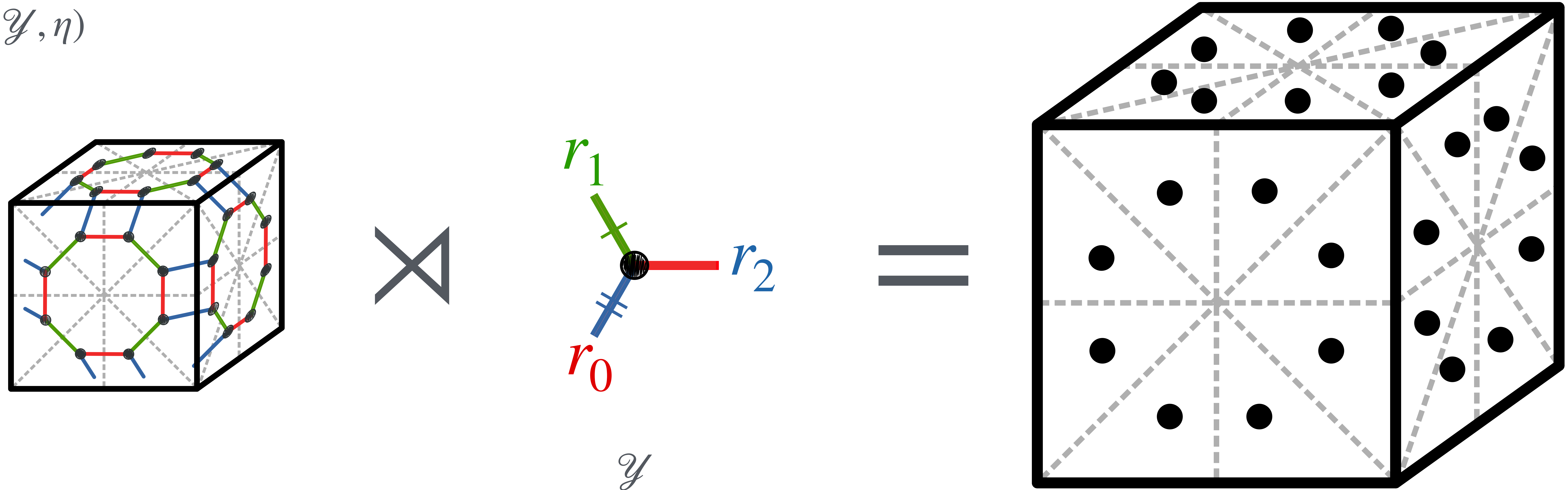

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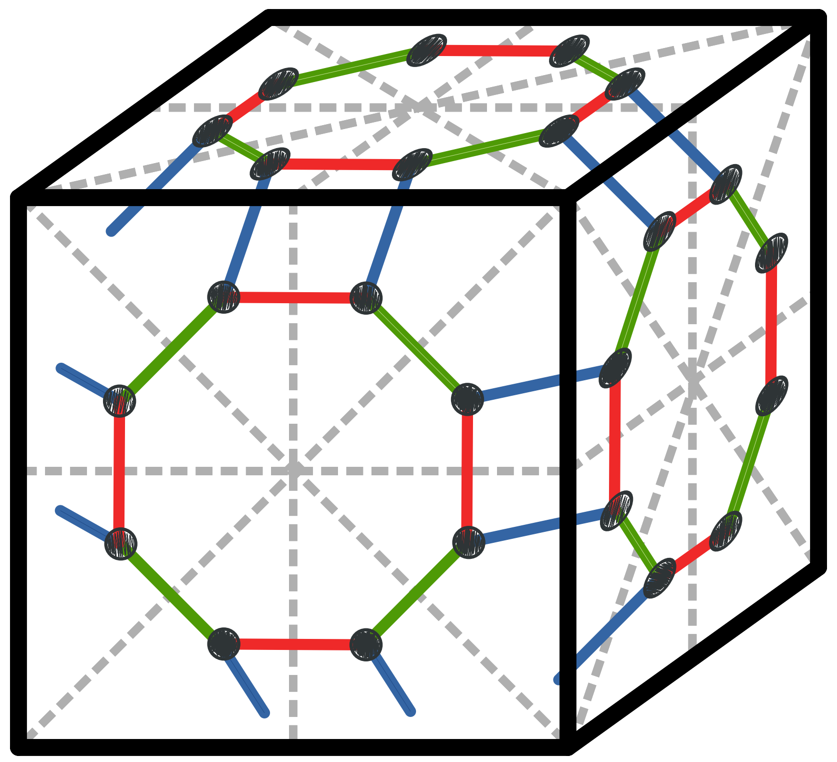

$\mathscr{X}$

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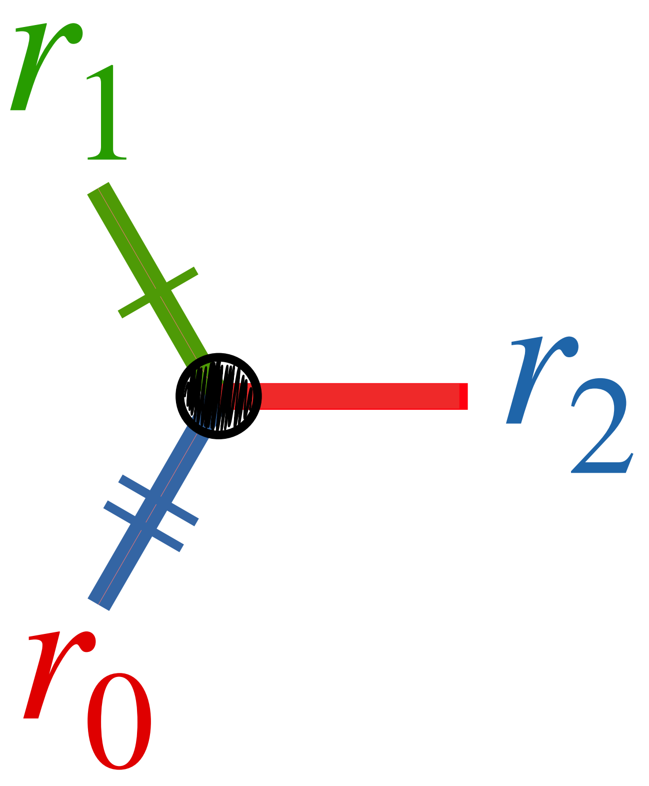

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Asignación de voltajes

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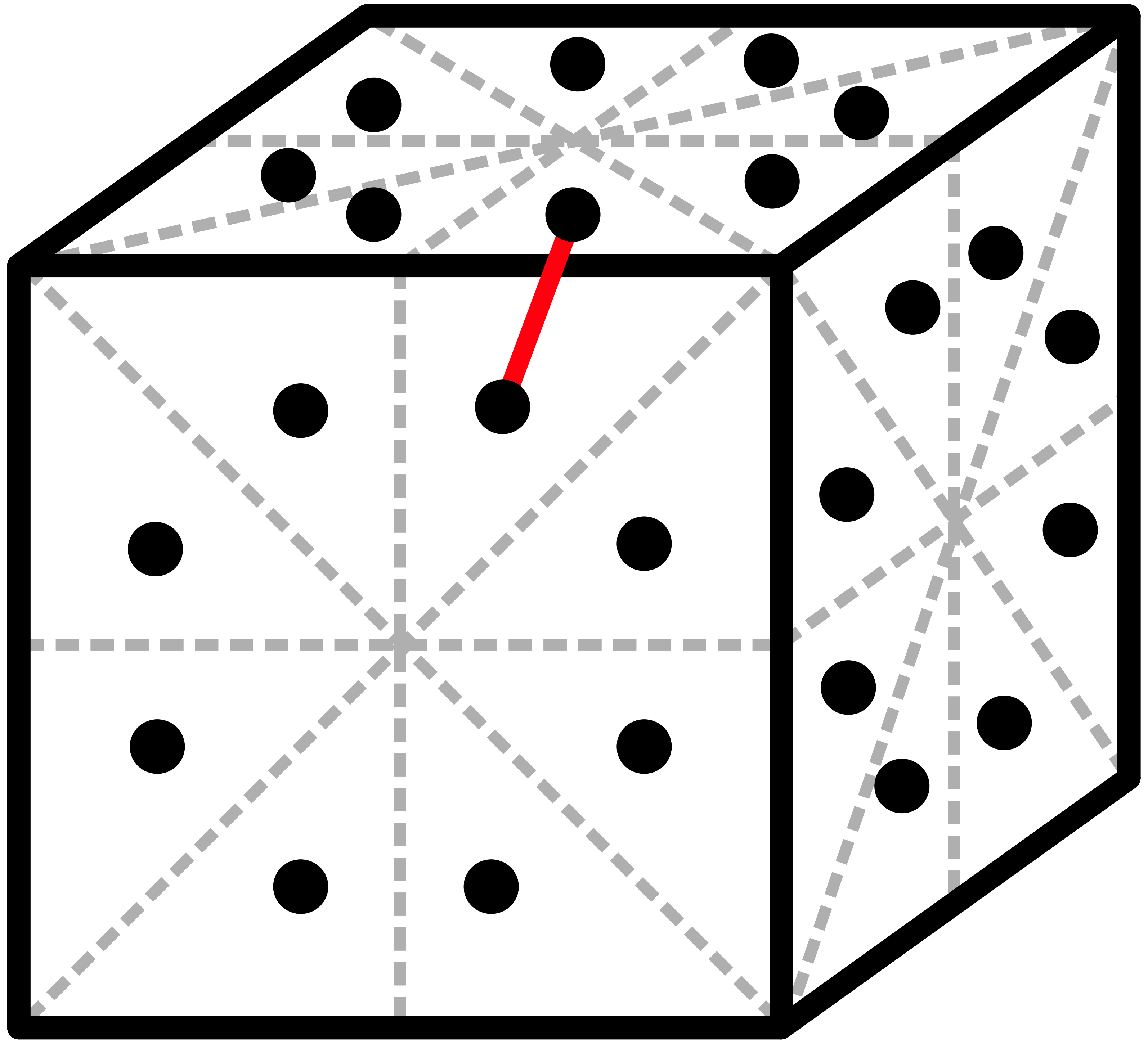

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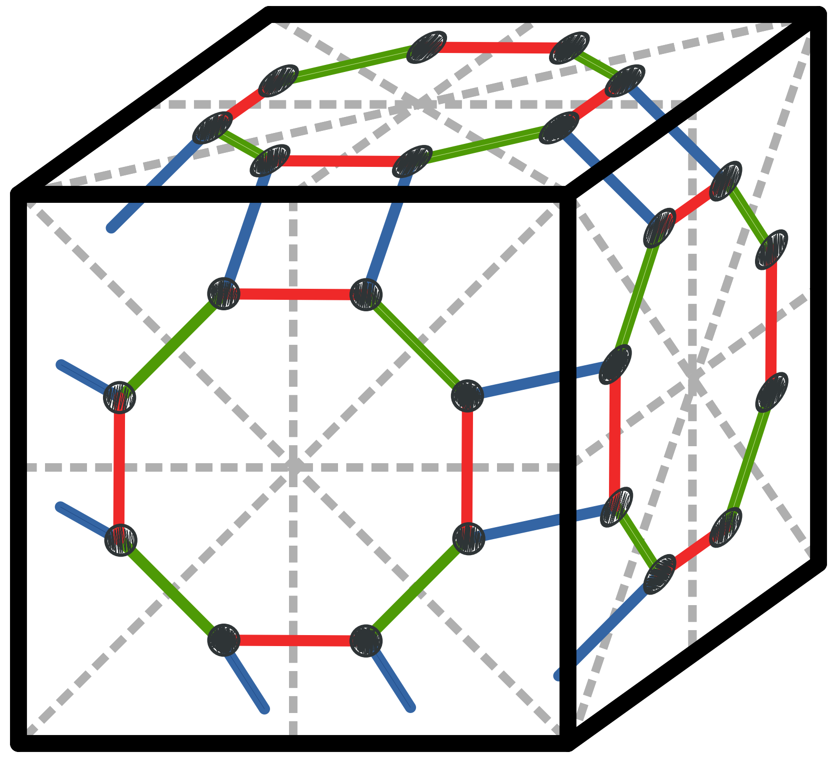

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m-premaniplex

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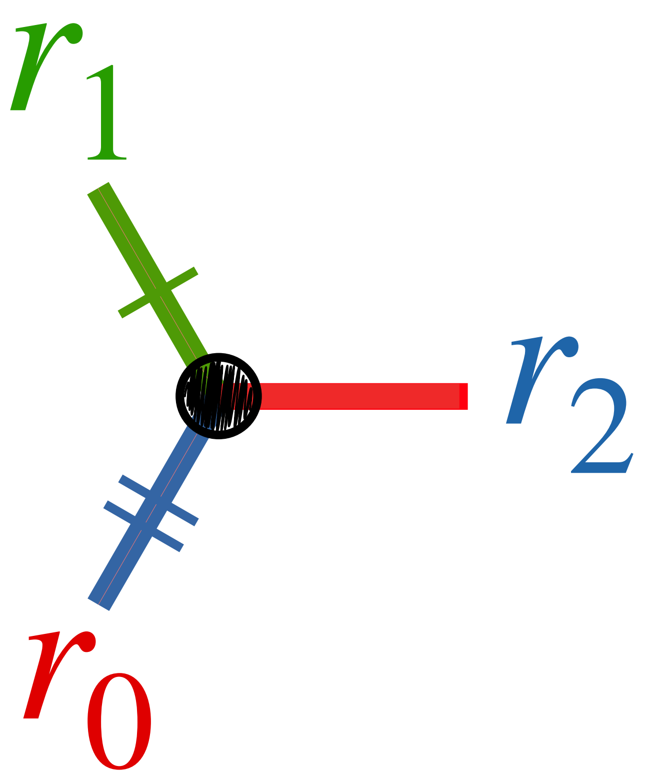

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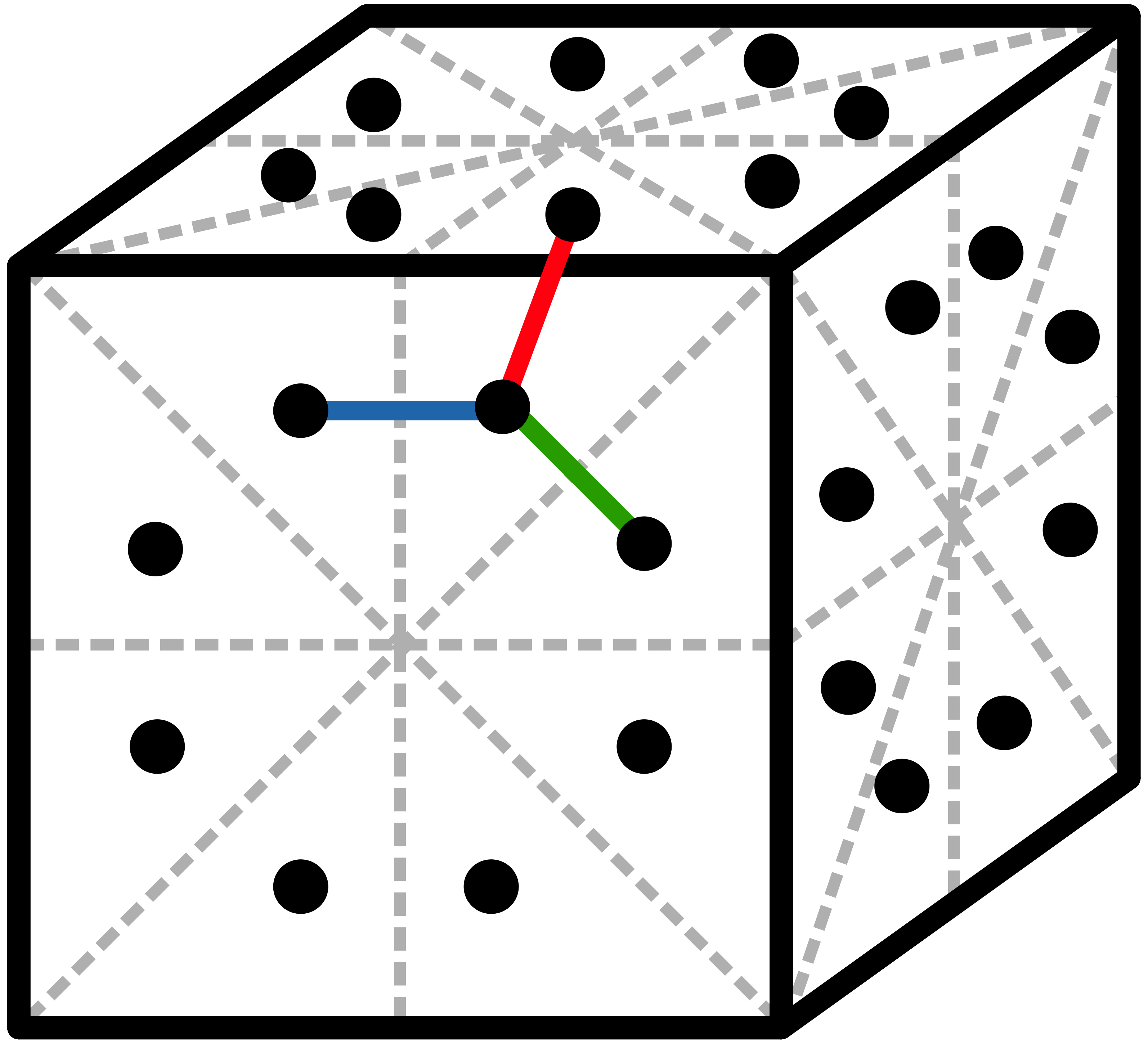

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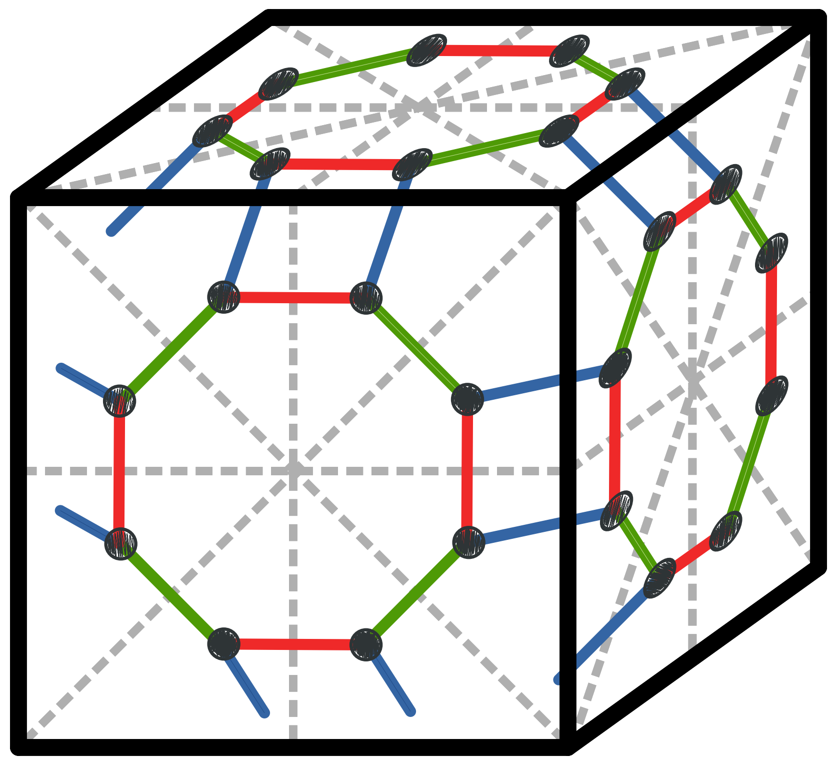

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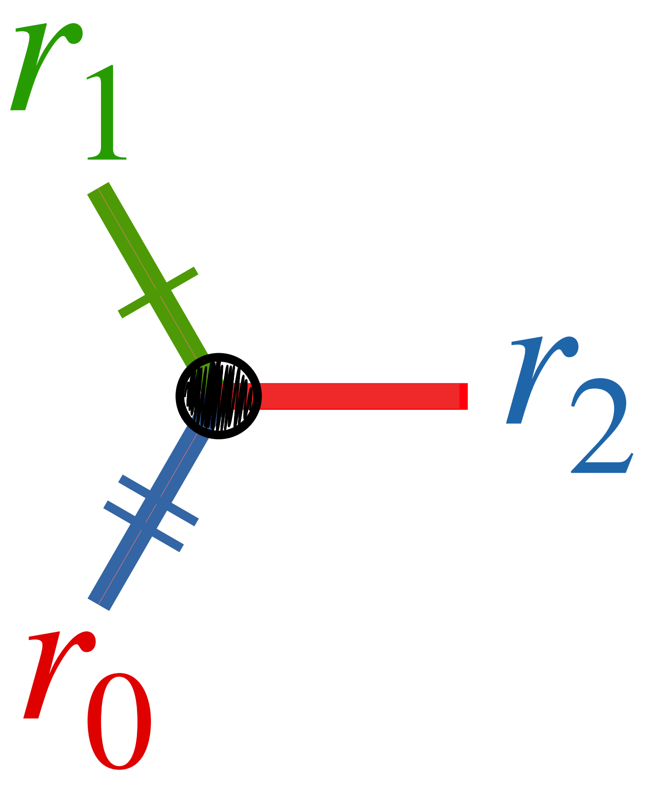

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Asignación de voltajes

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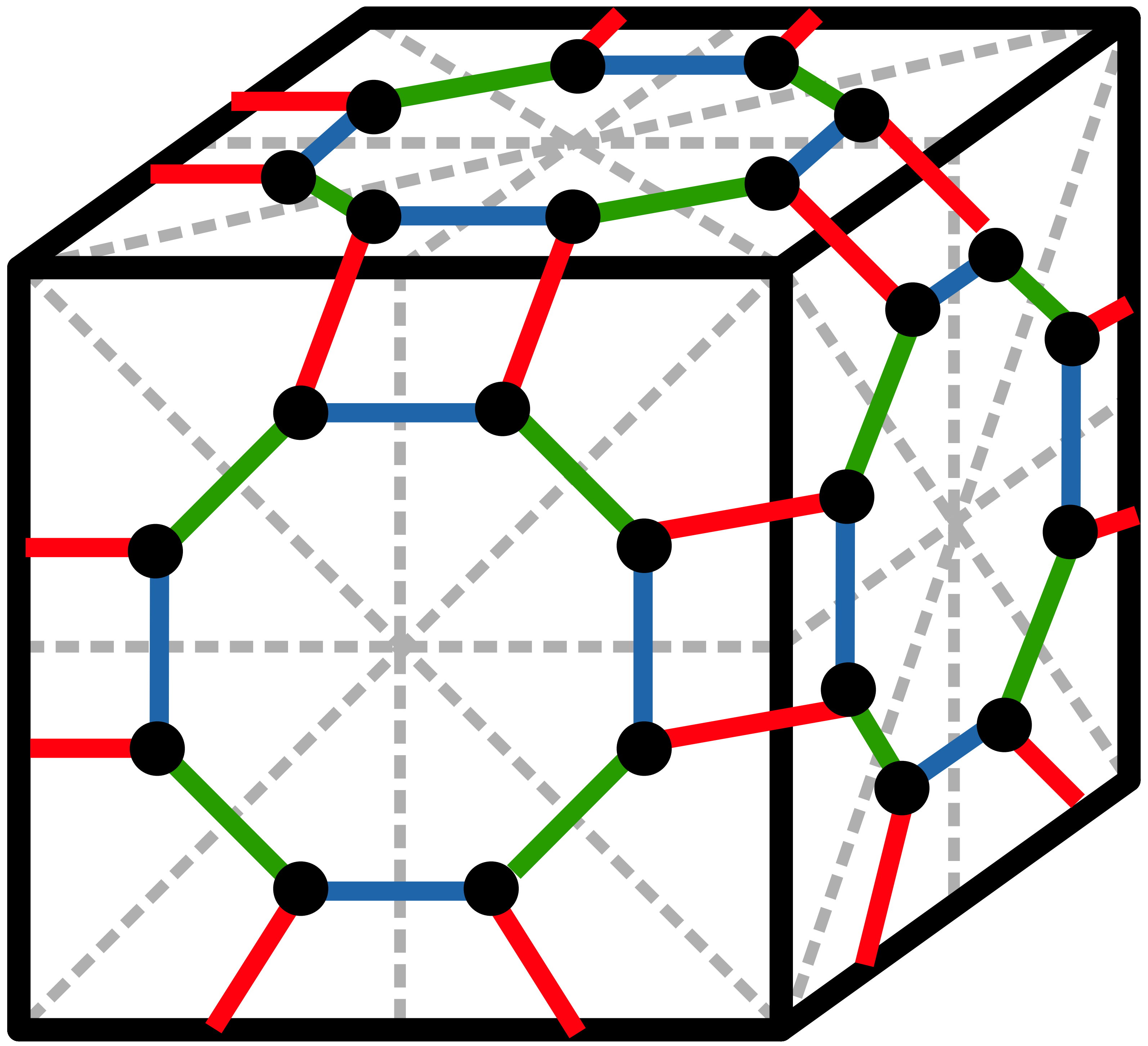

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n-premaniplex

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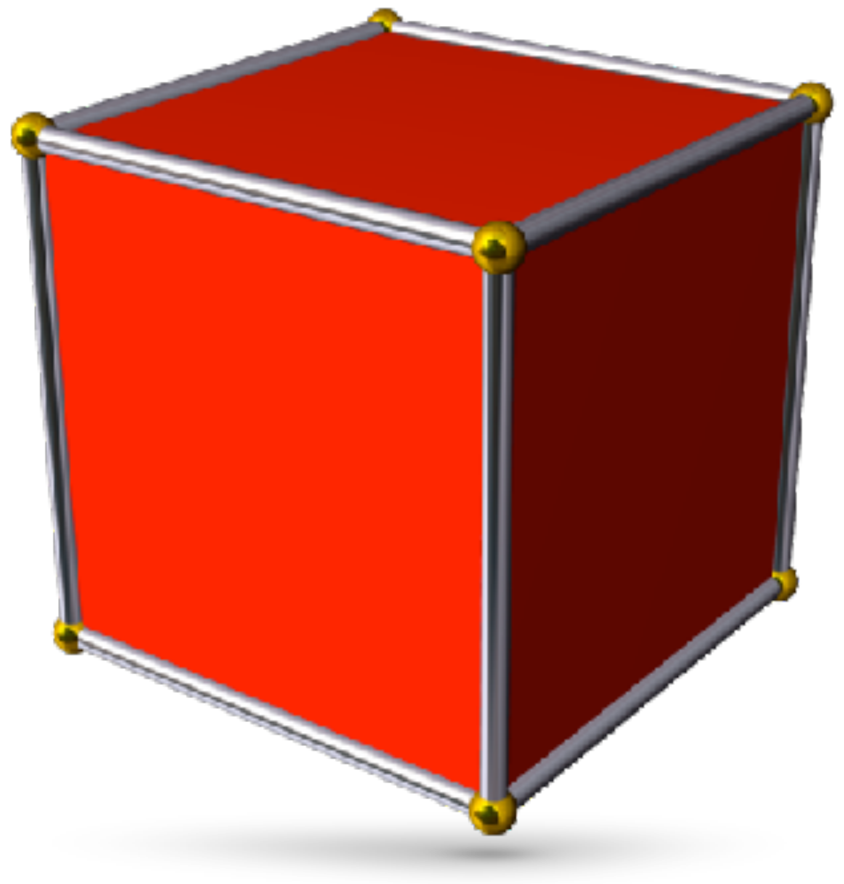

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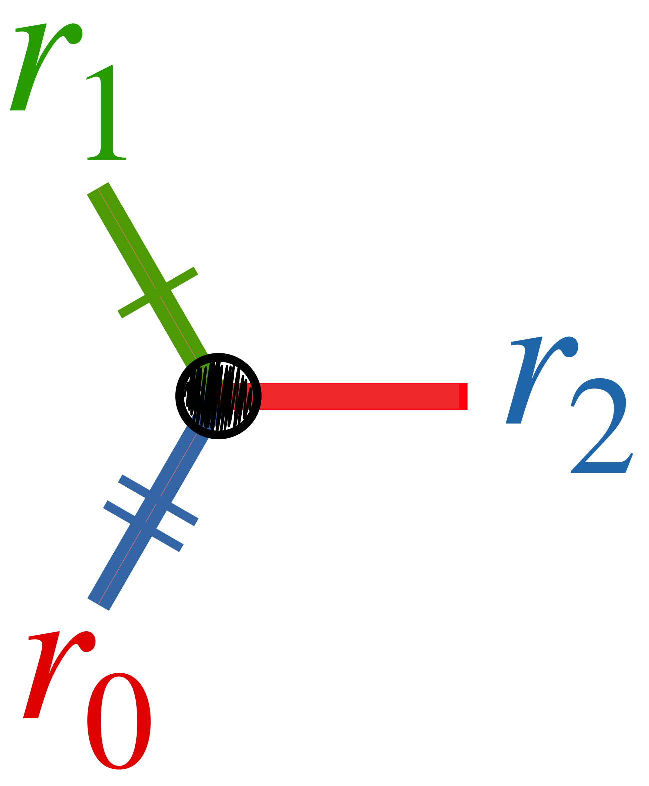

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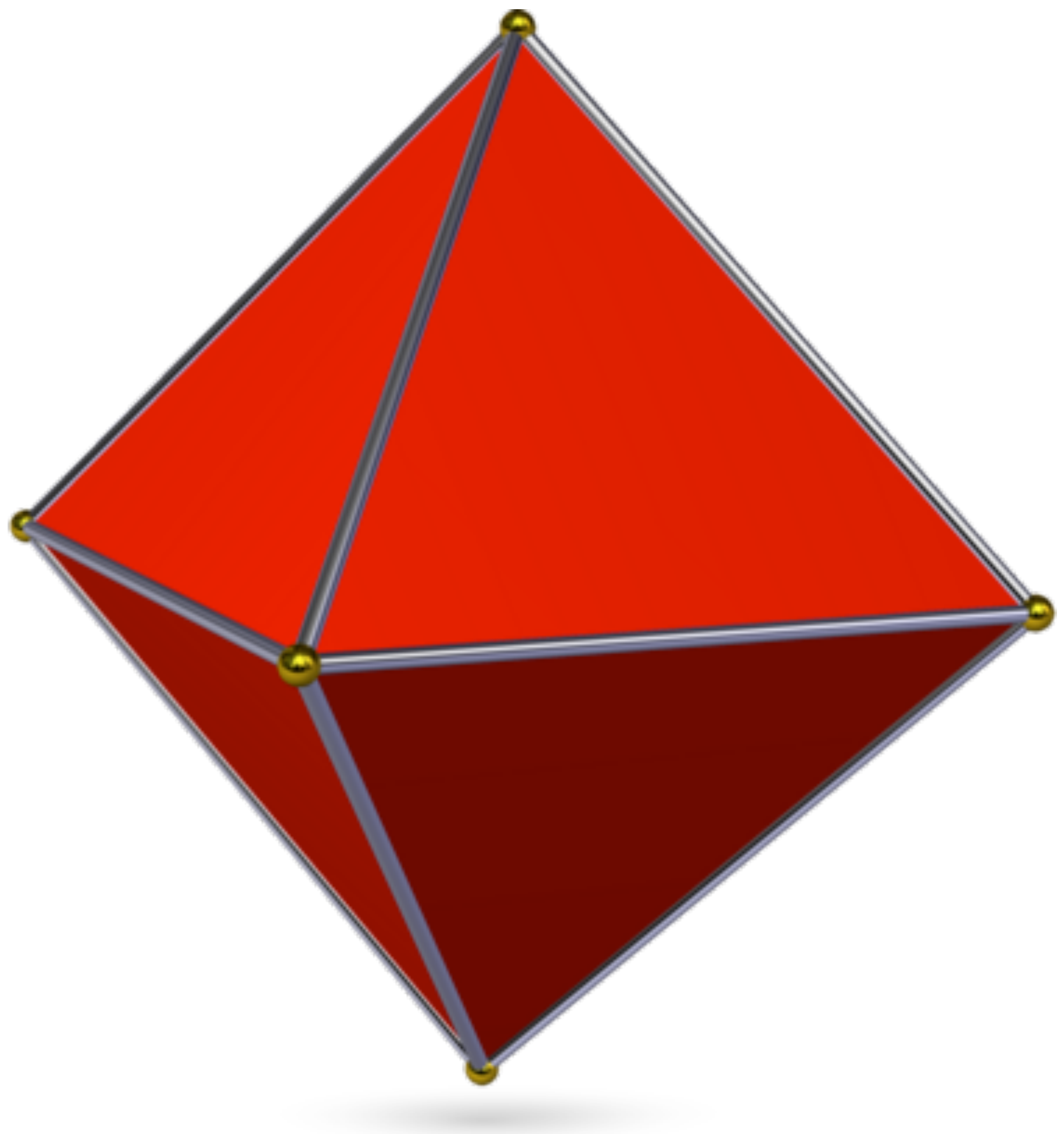

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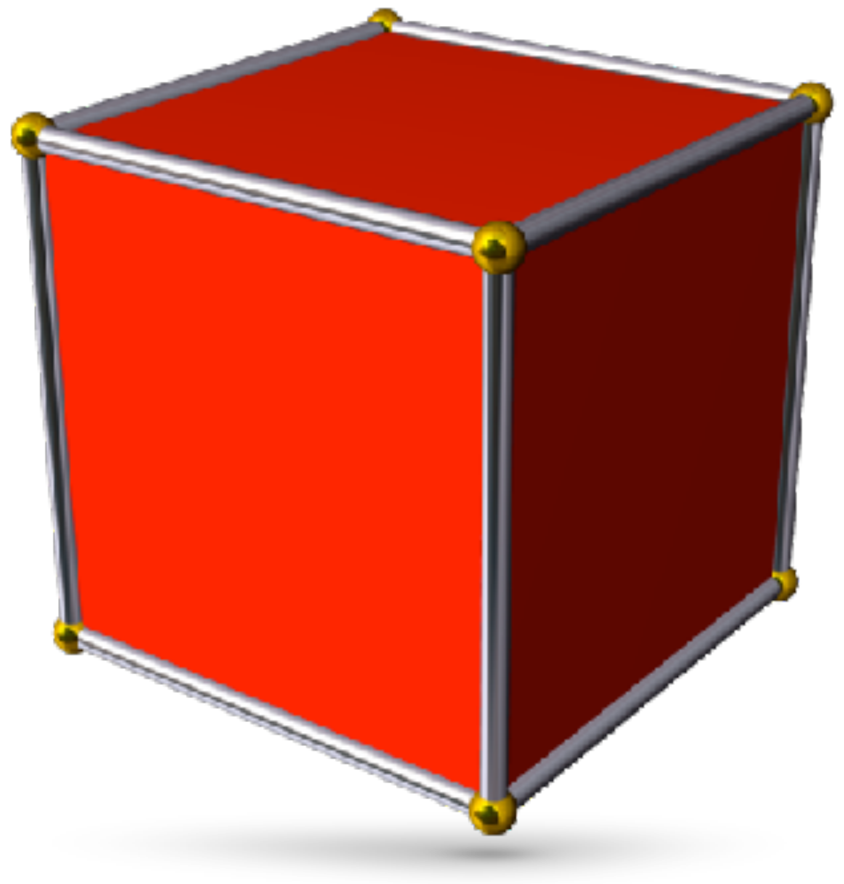

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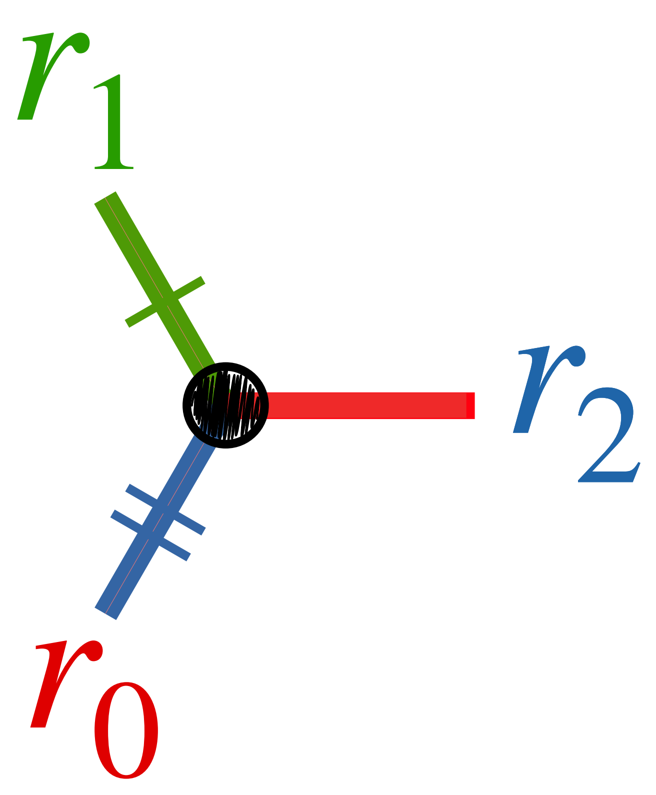

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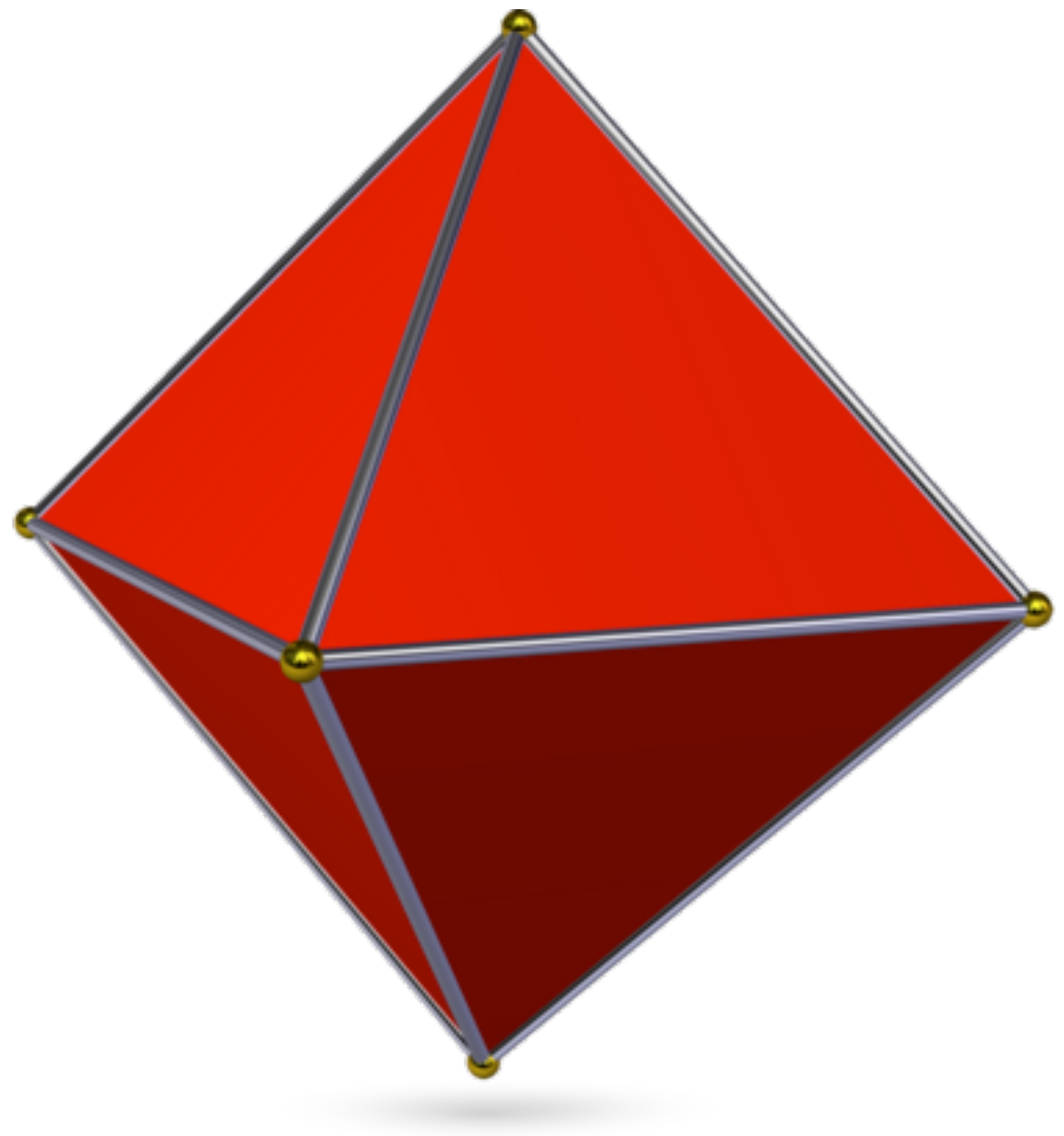

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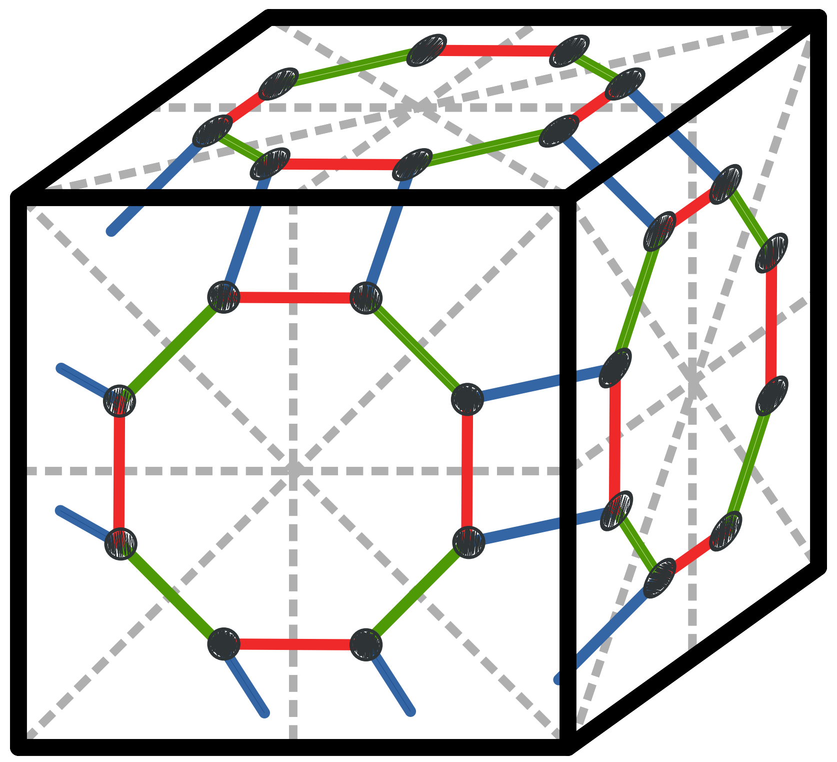

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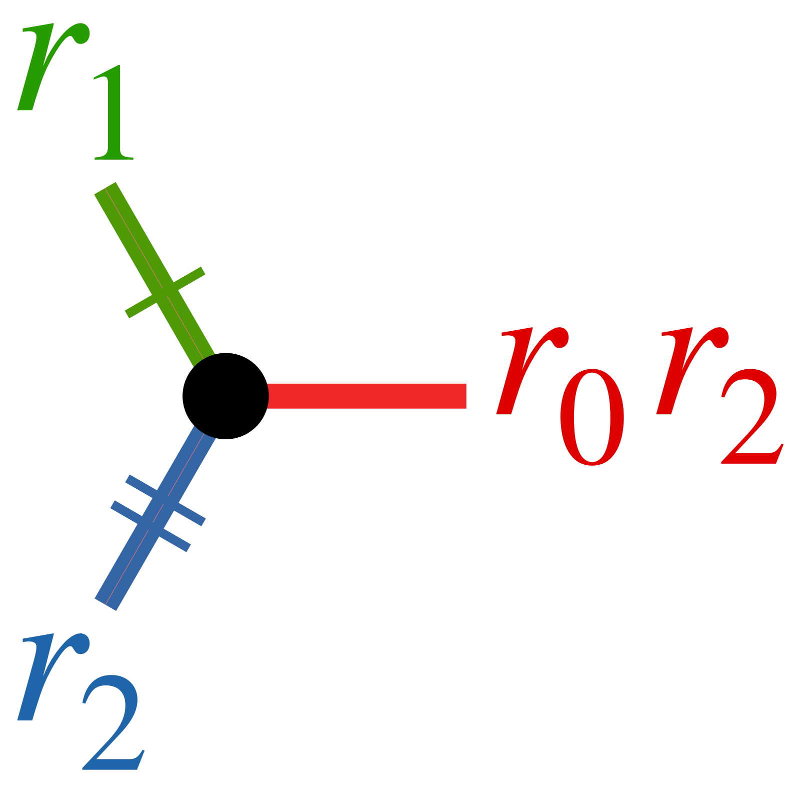

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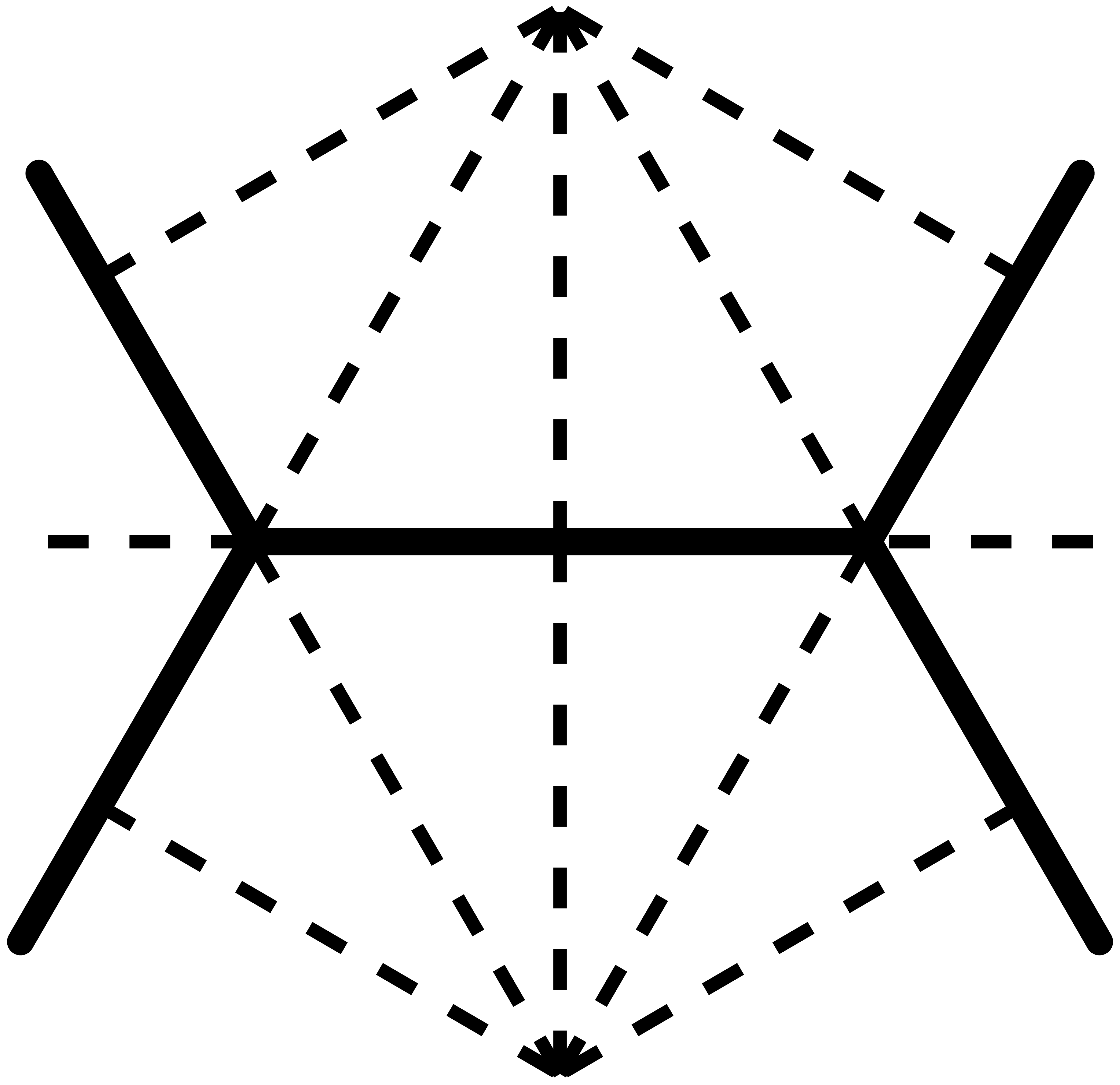

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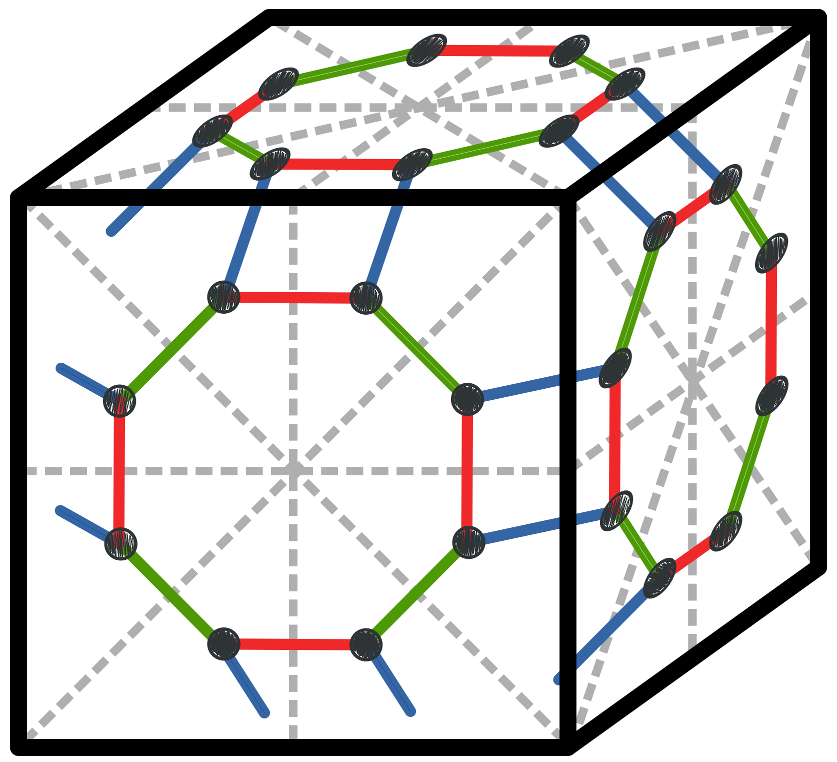
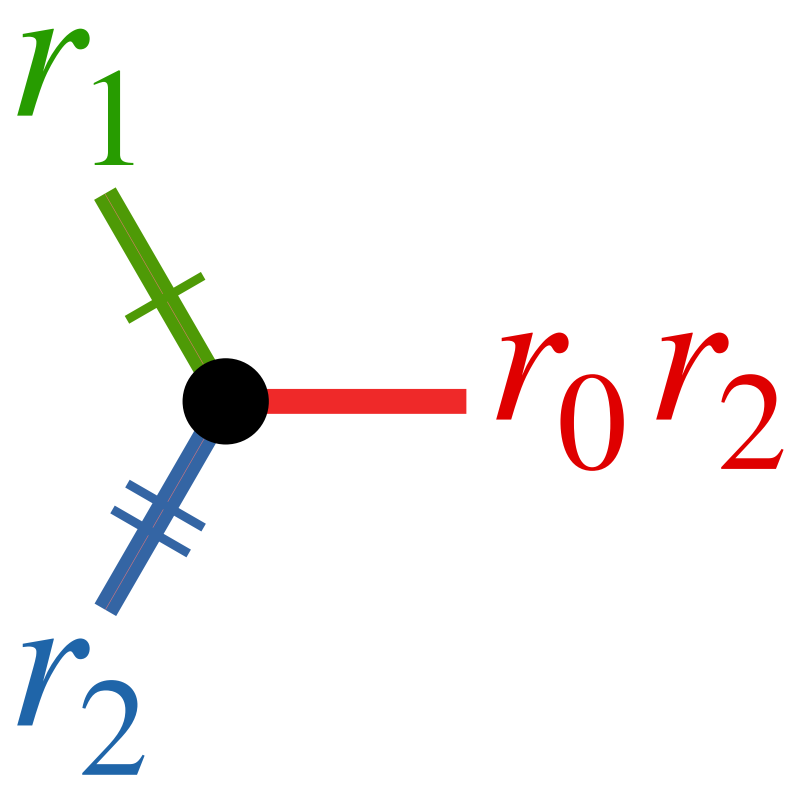


$\rtimes$



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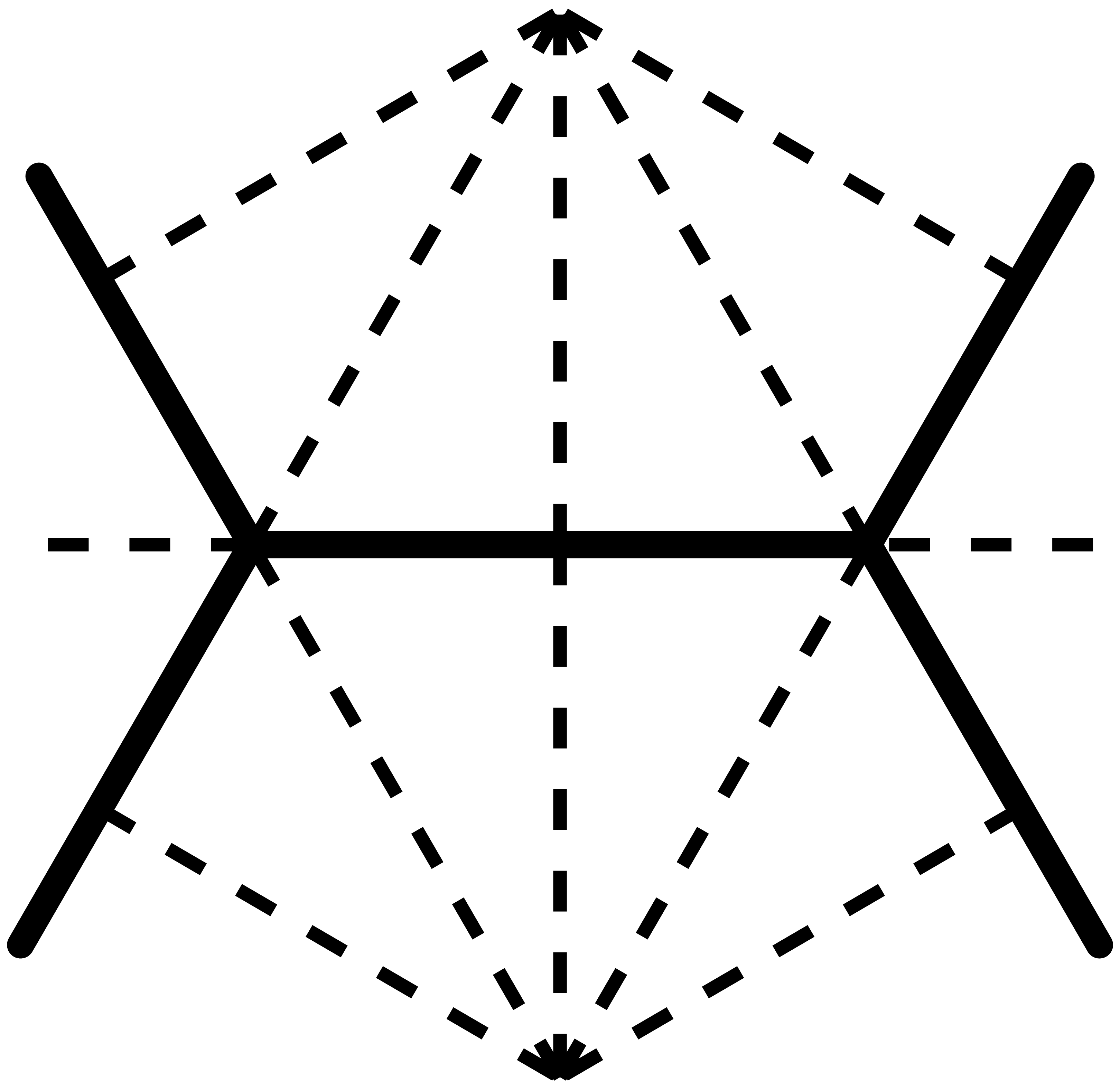

$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

Asignación de voltajes

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

Operaciones de voltaje

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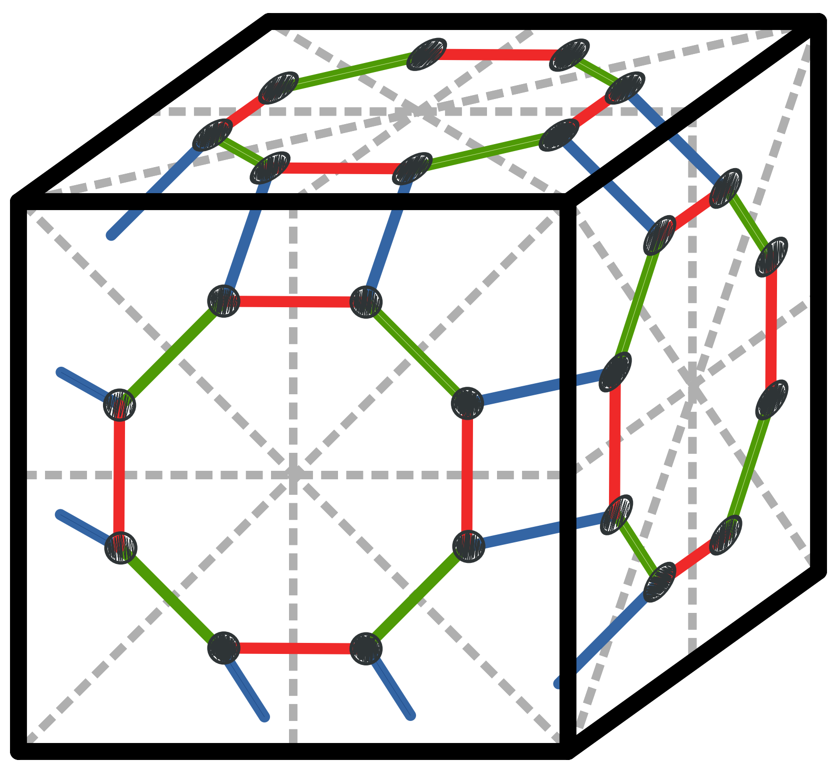

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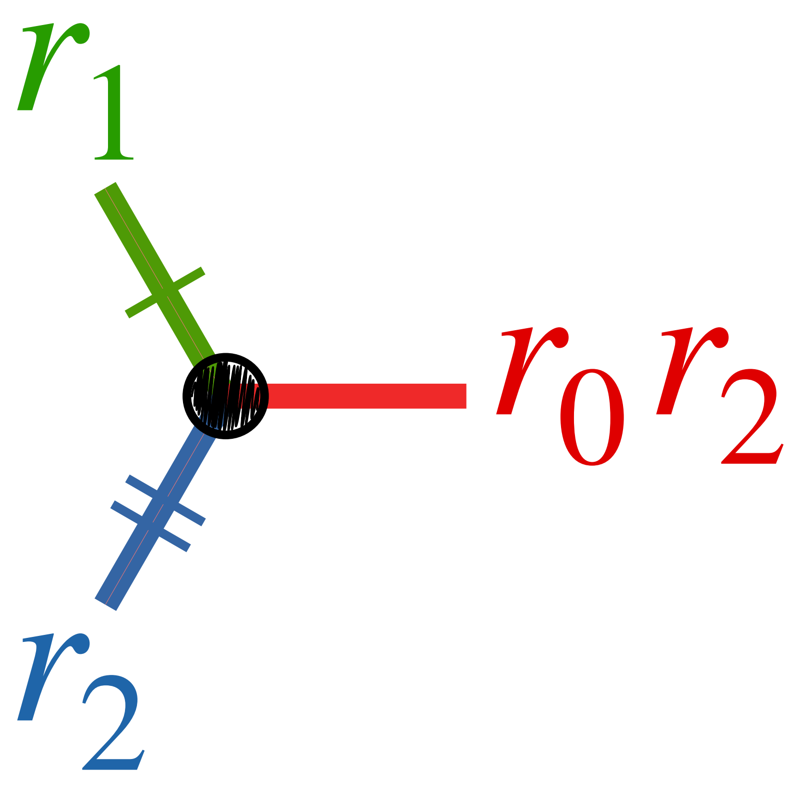

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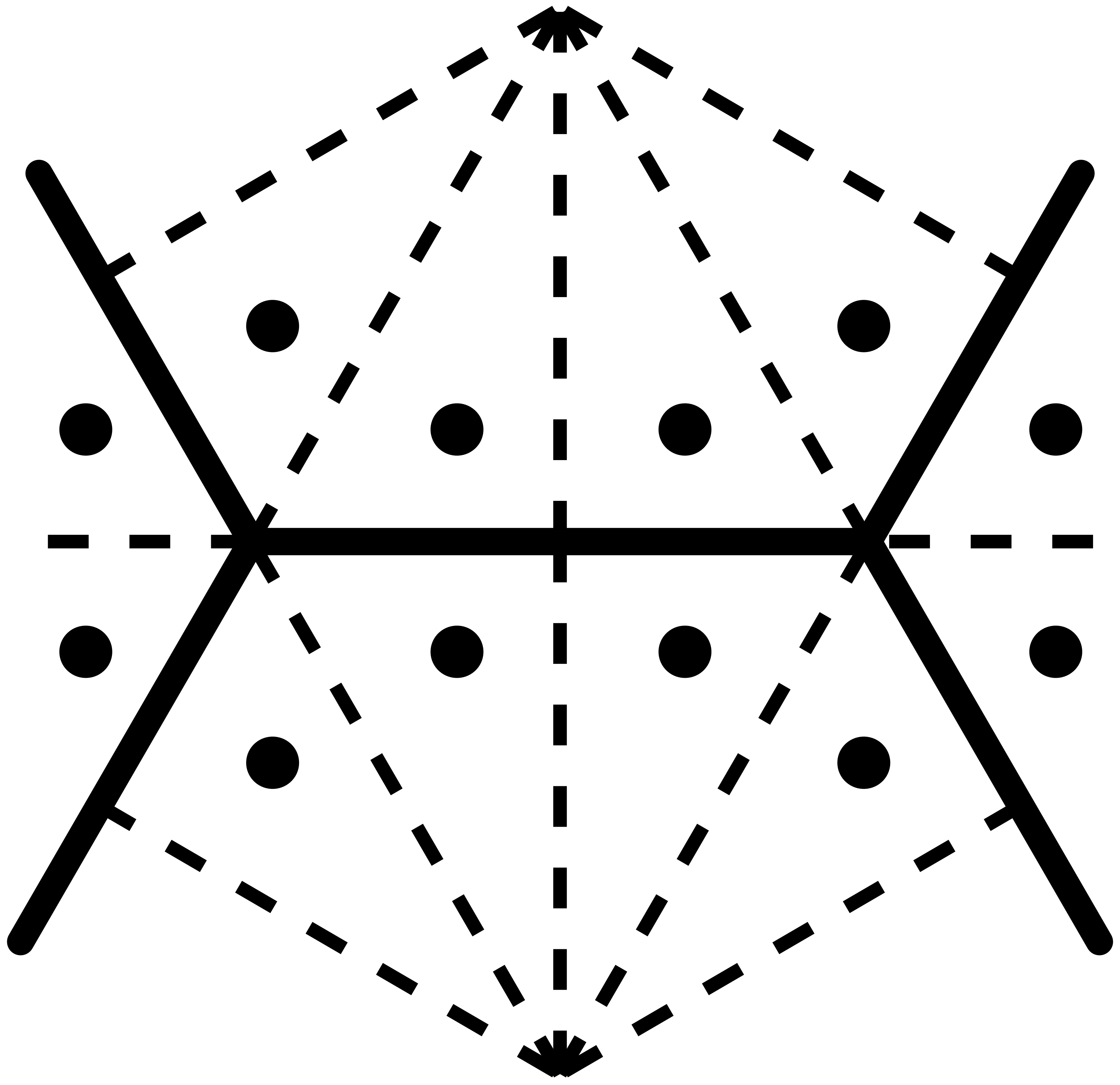

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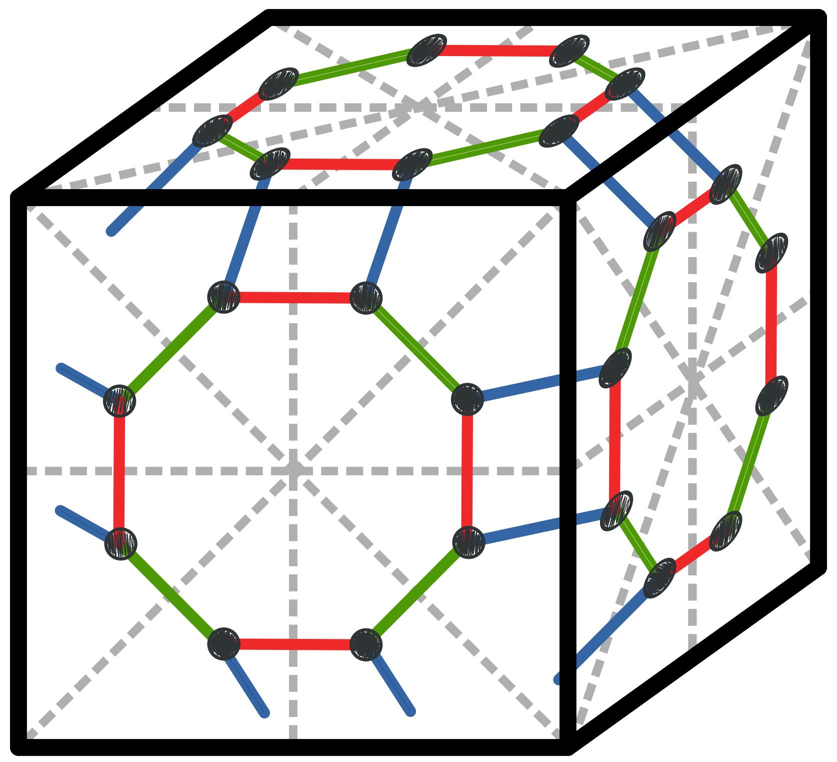

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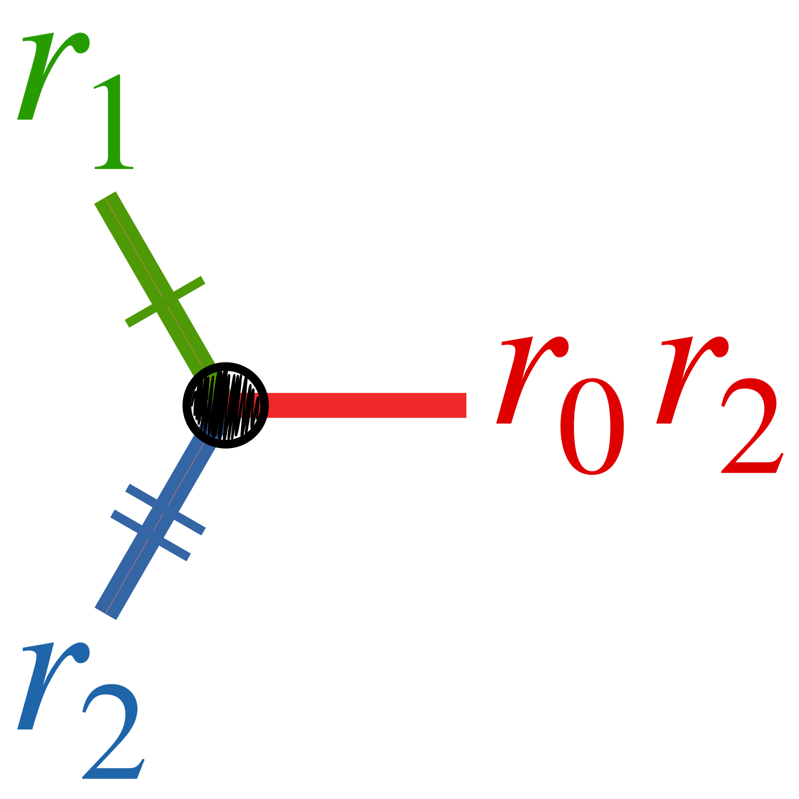

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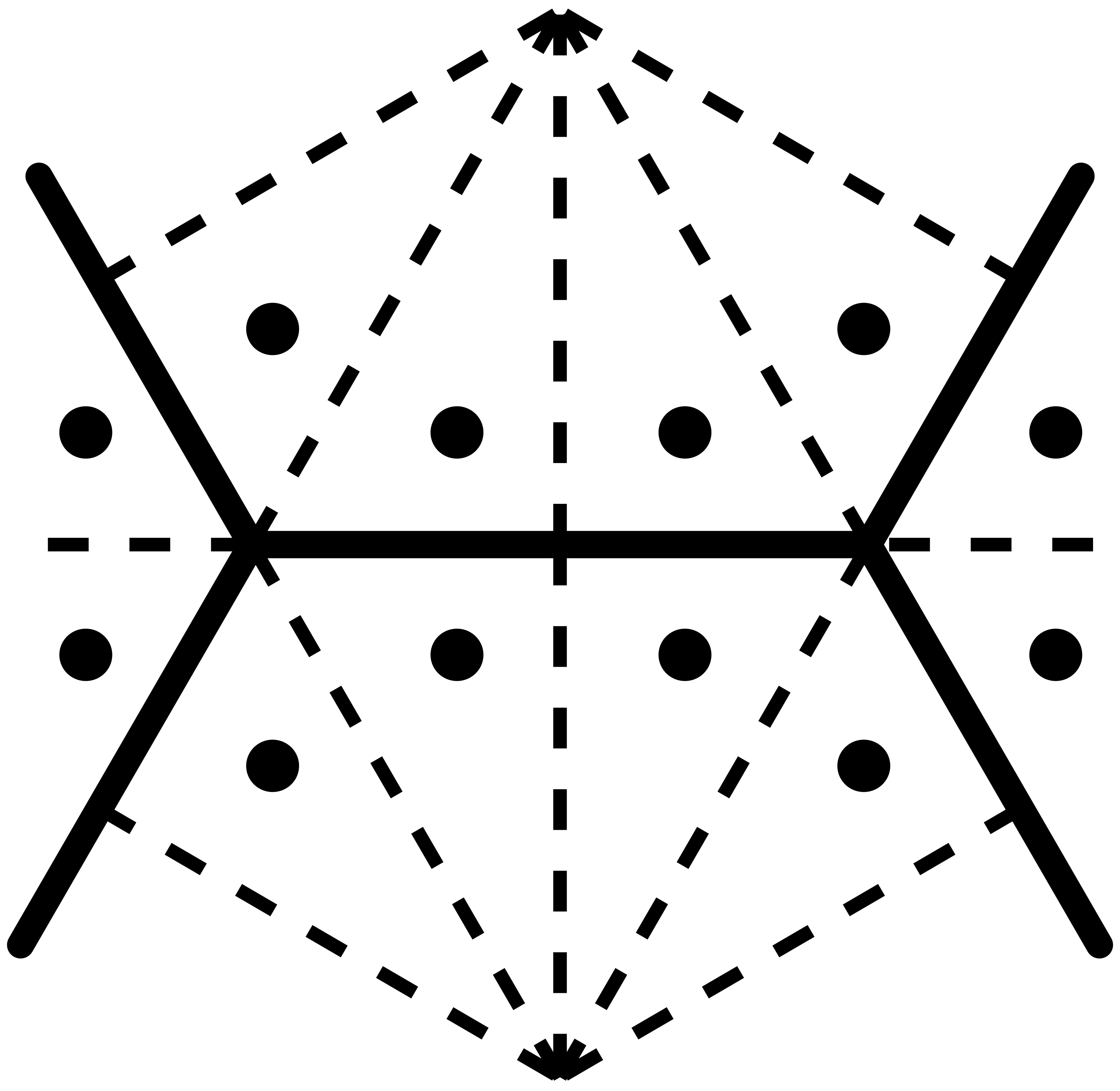

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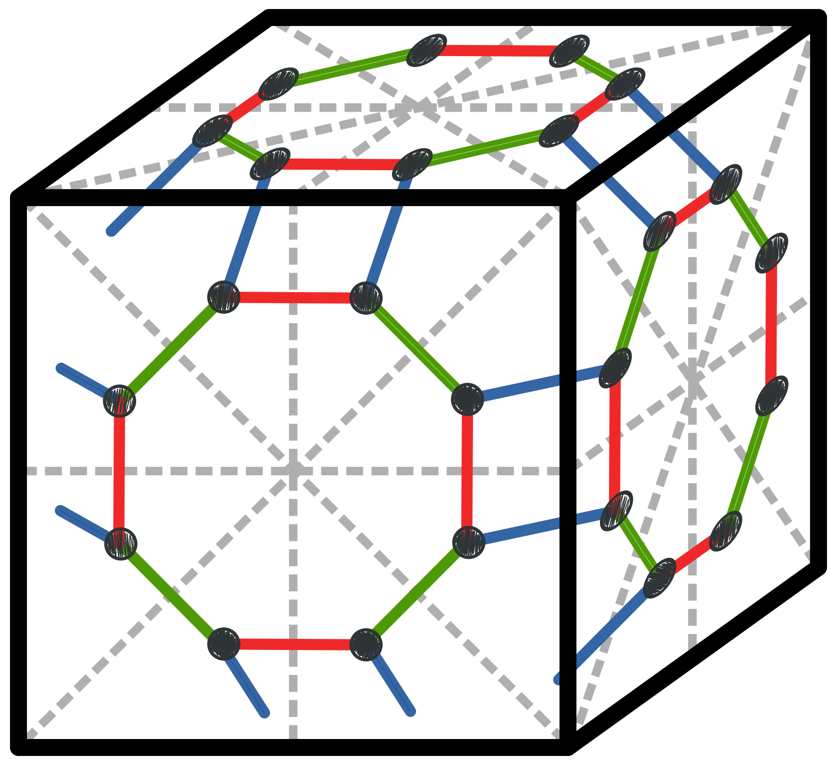

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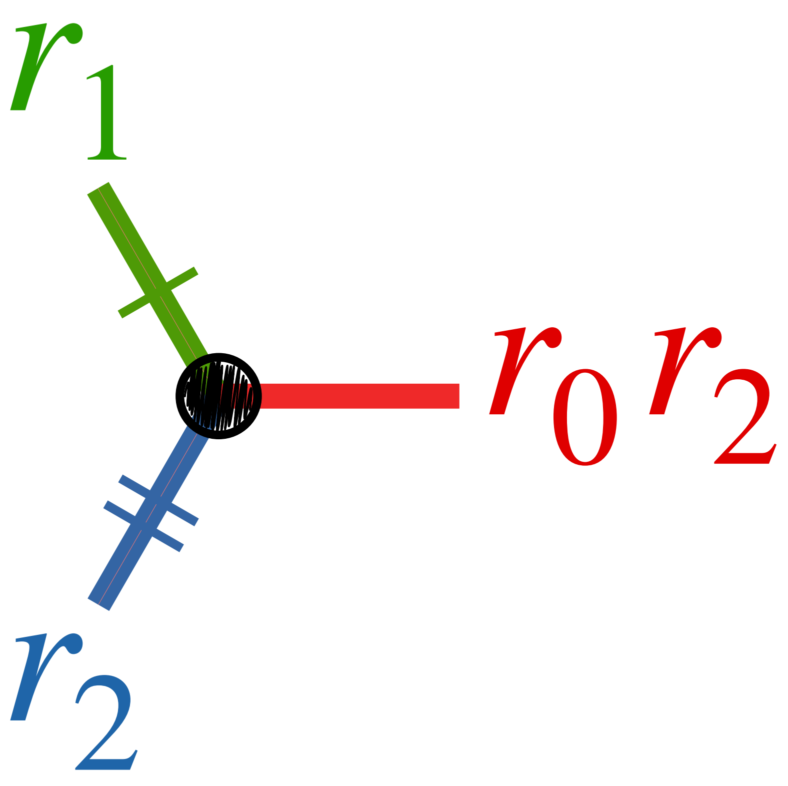

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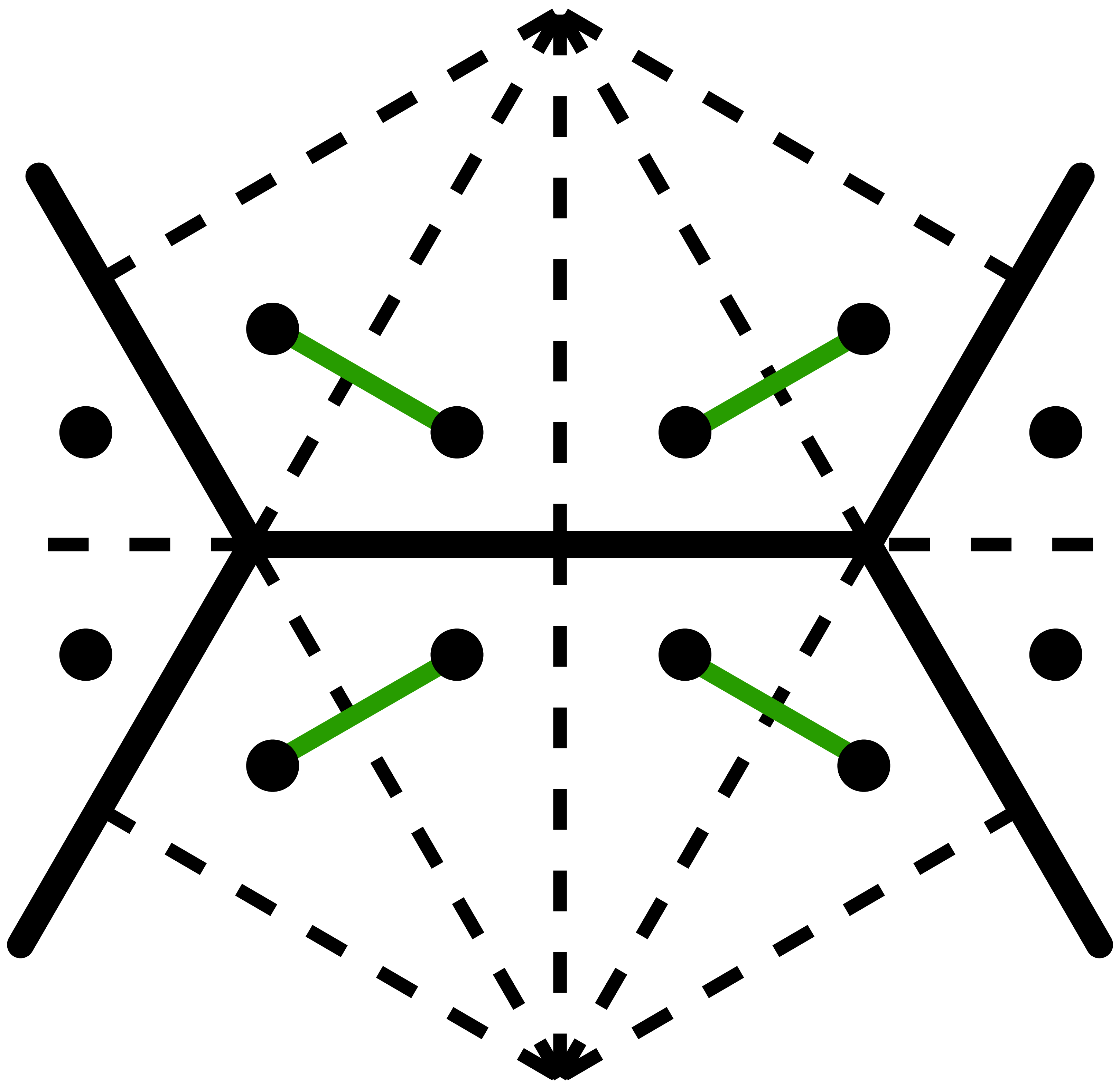

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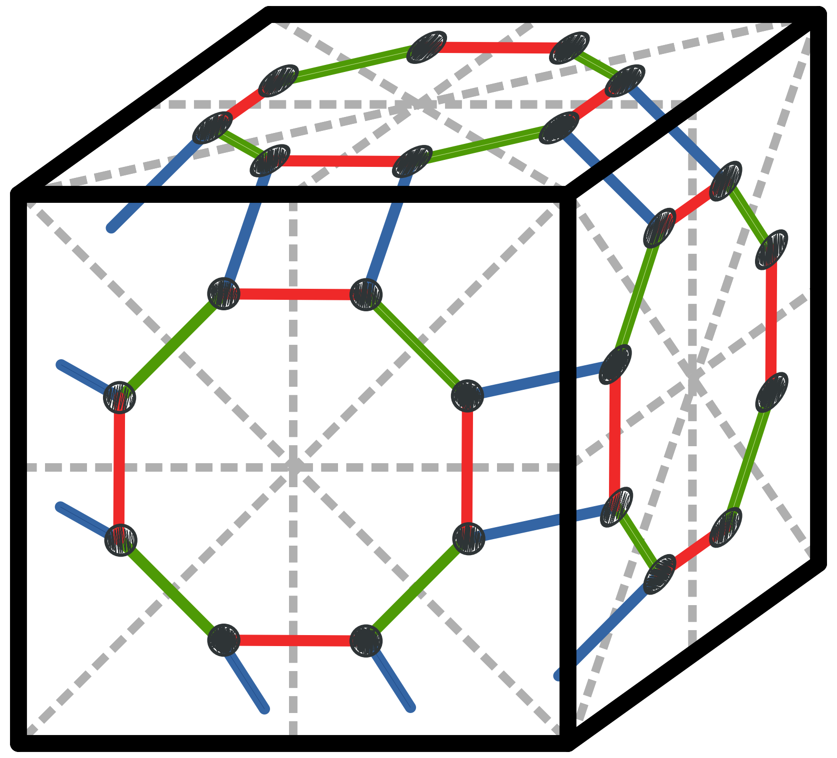

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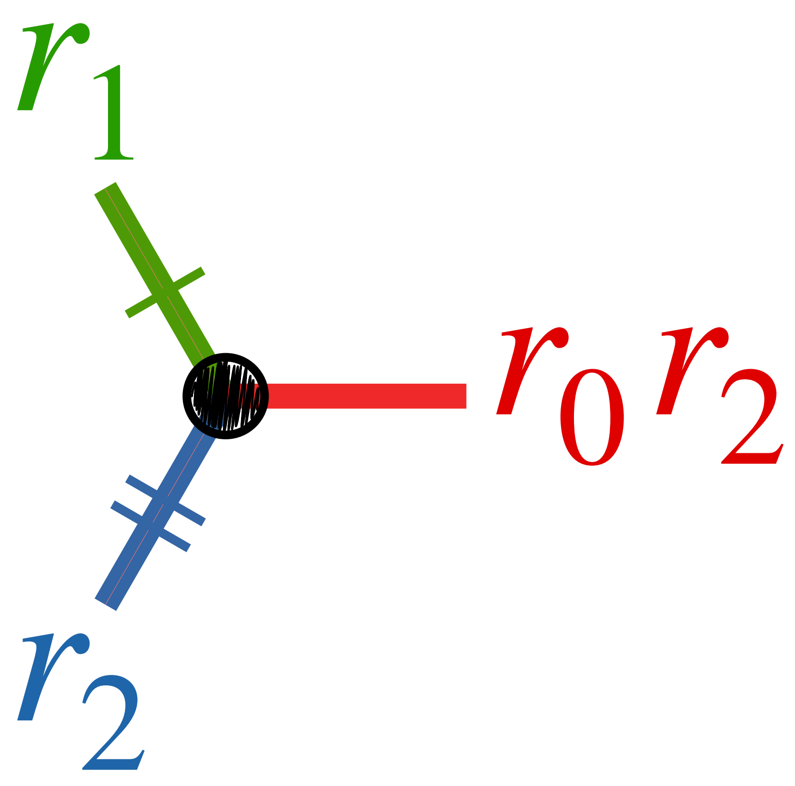

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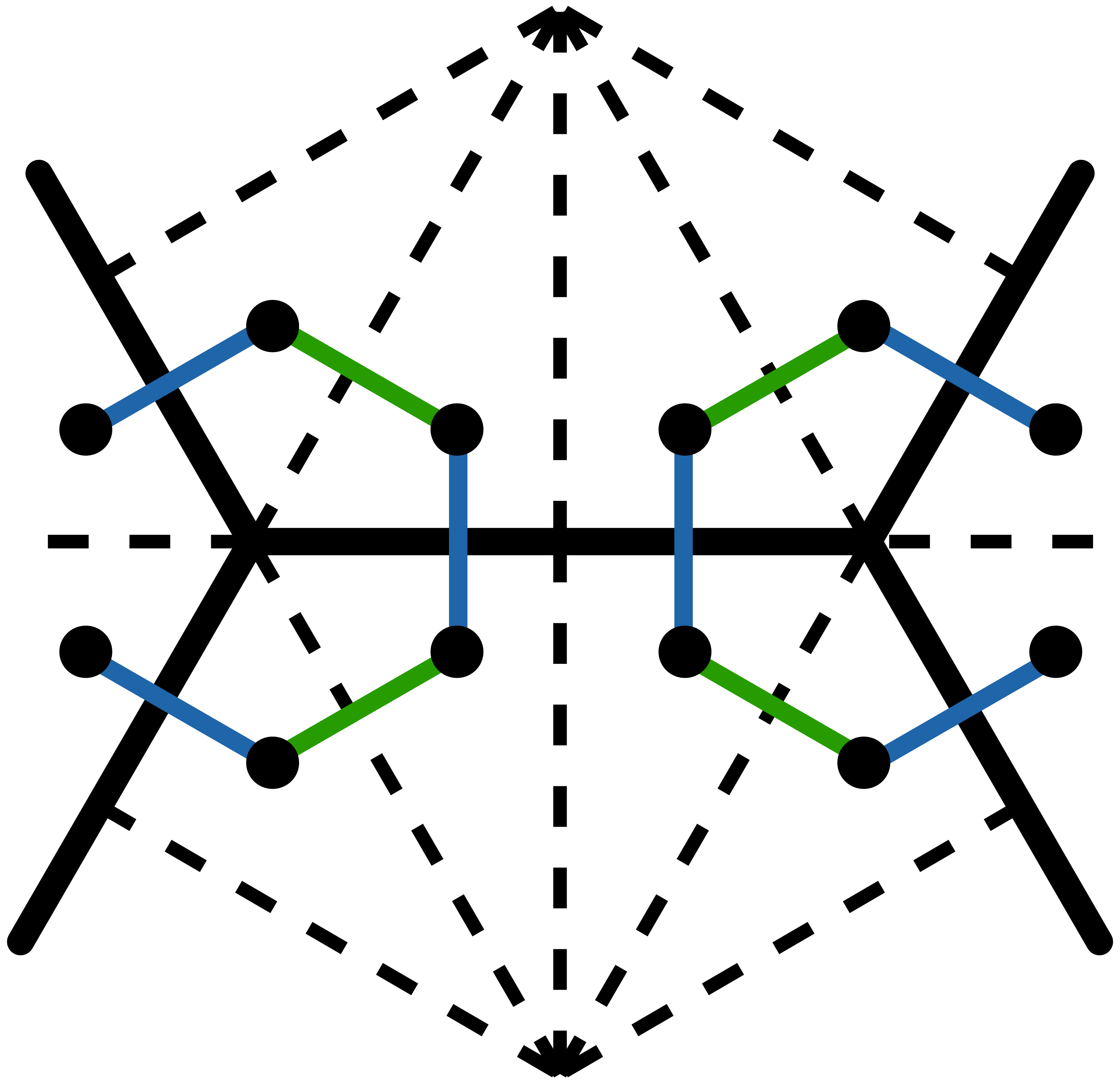

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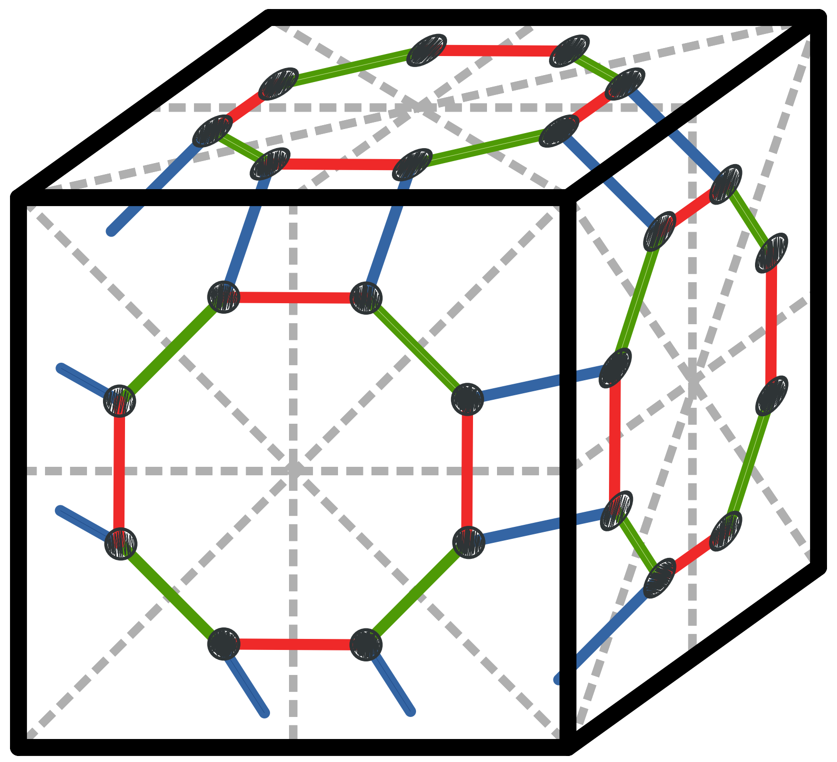

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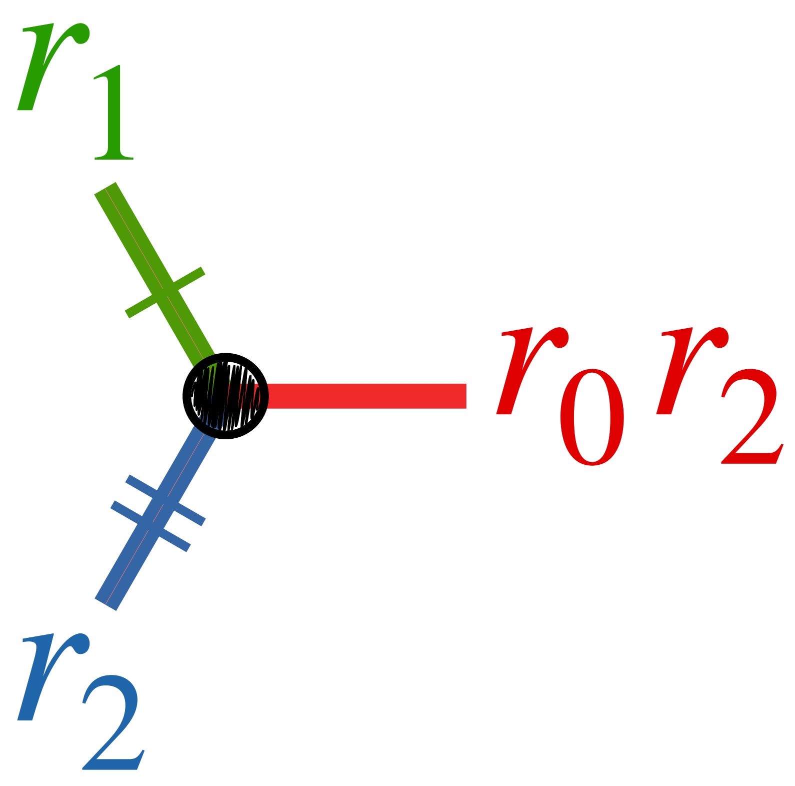

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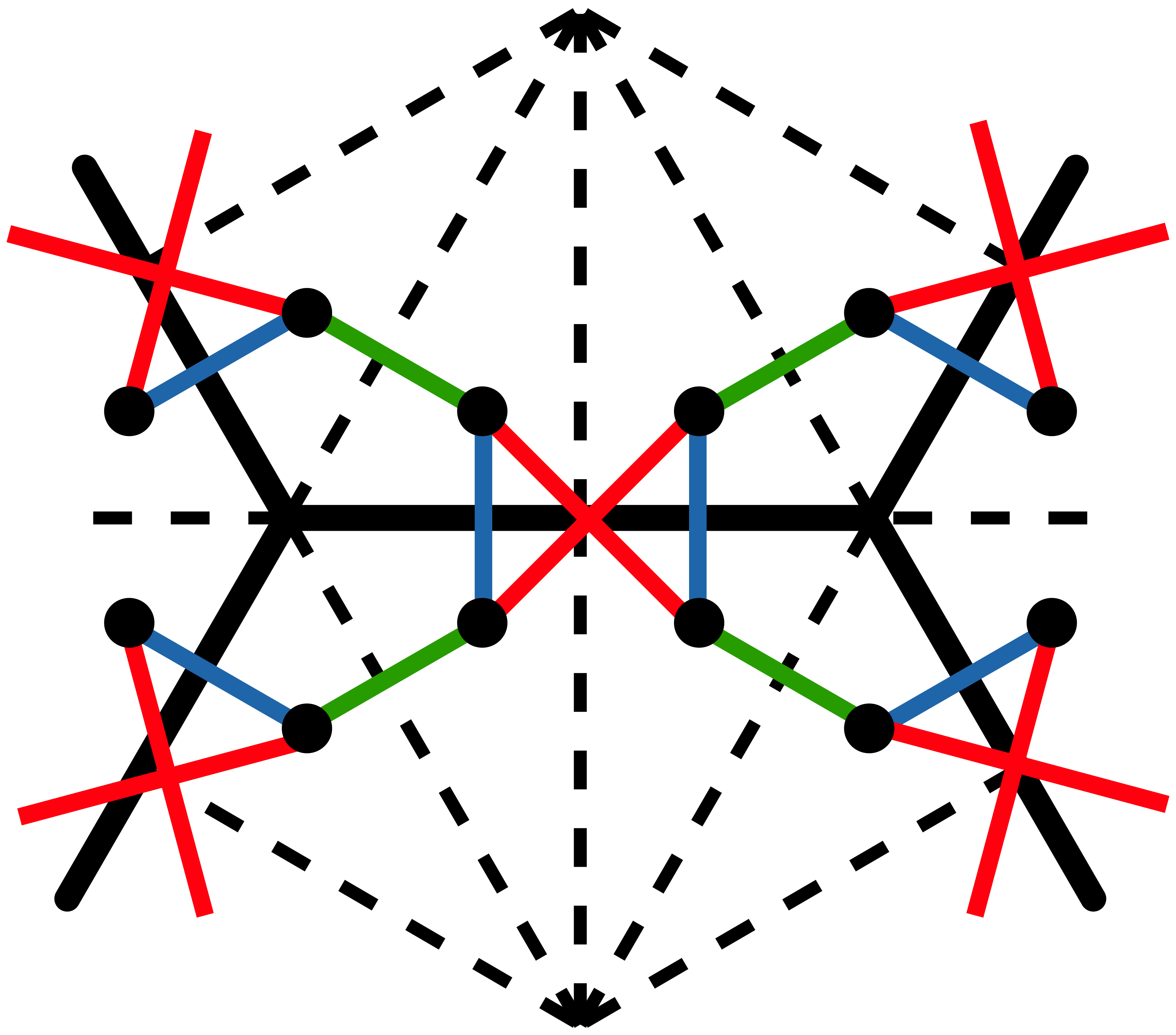

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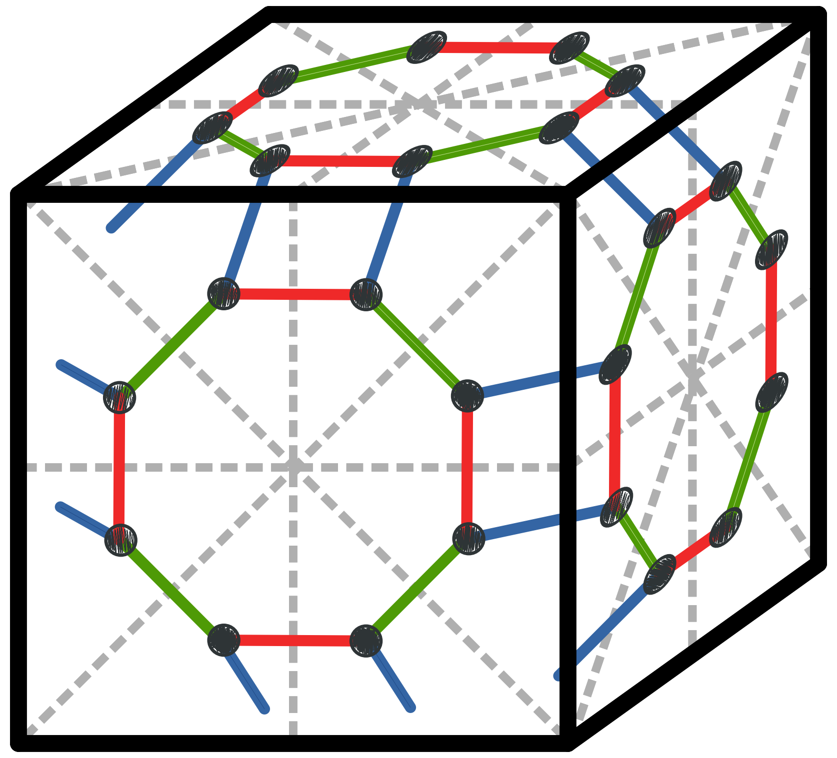

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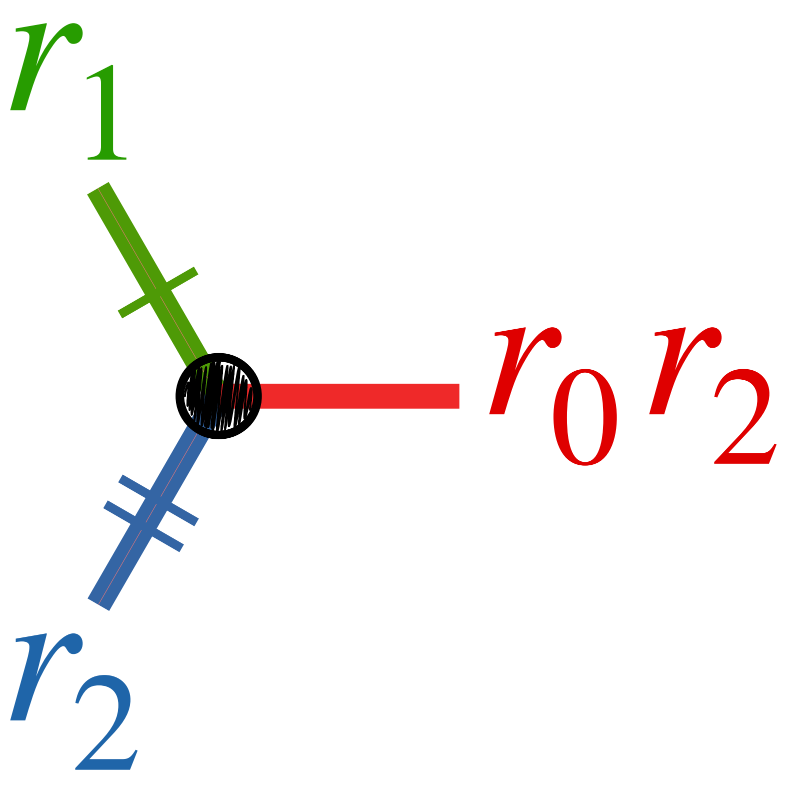

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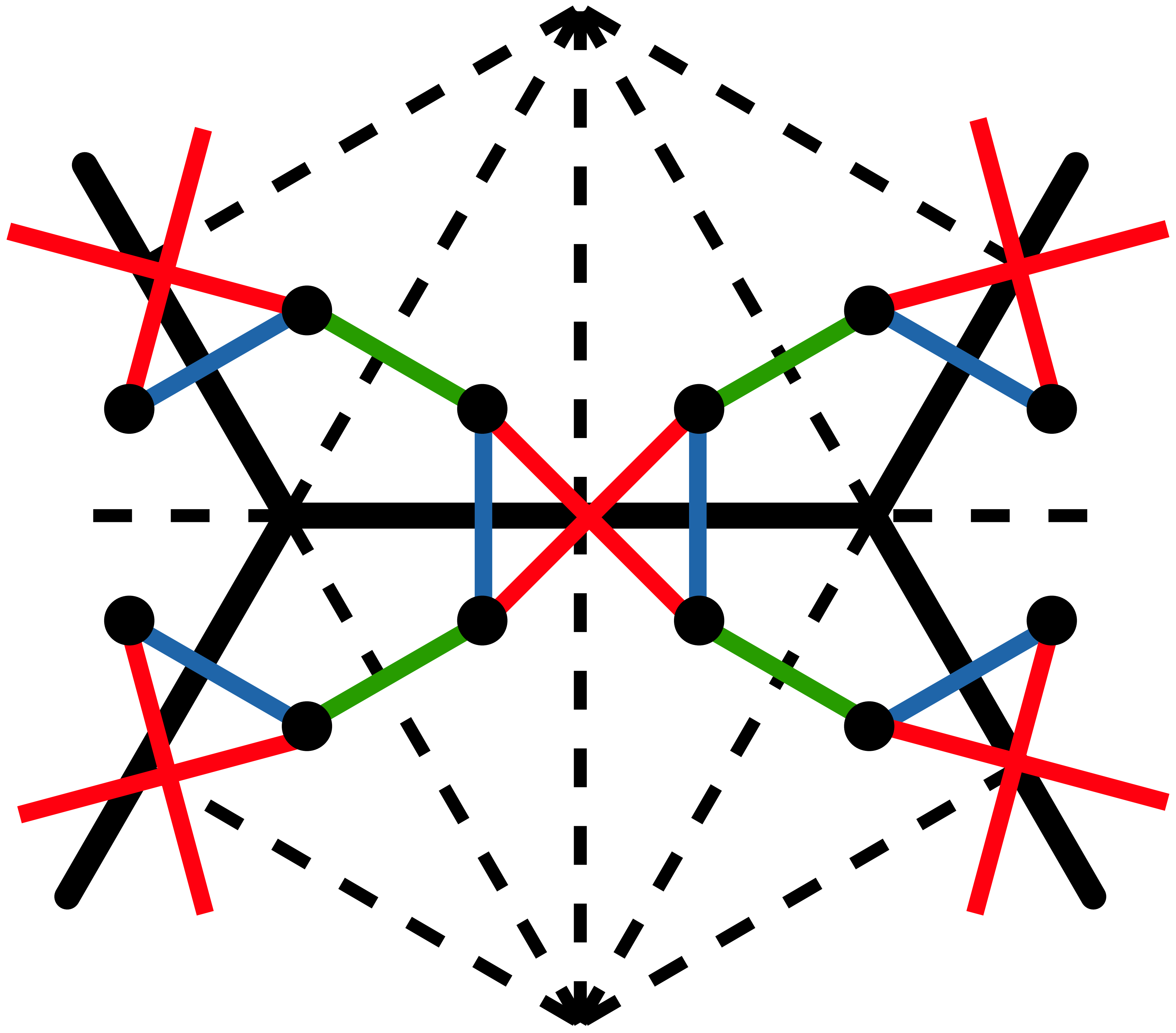

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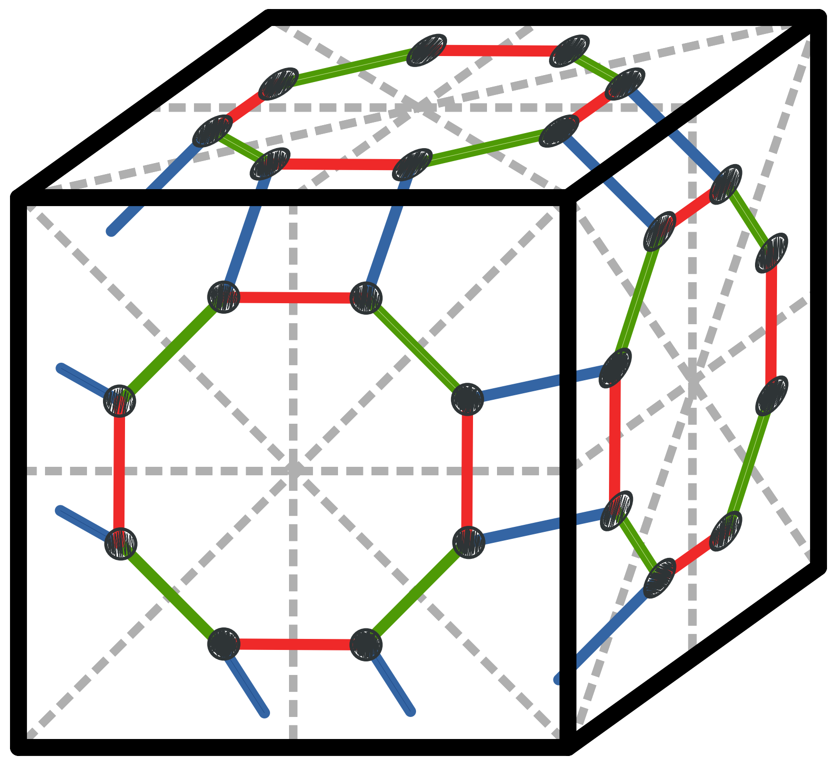

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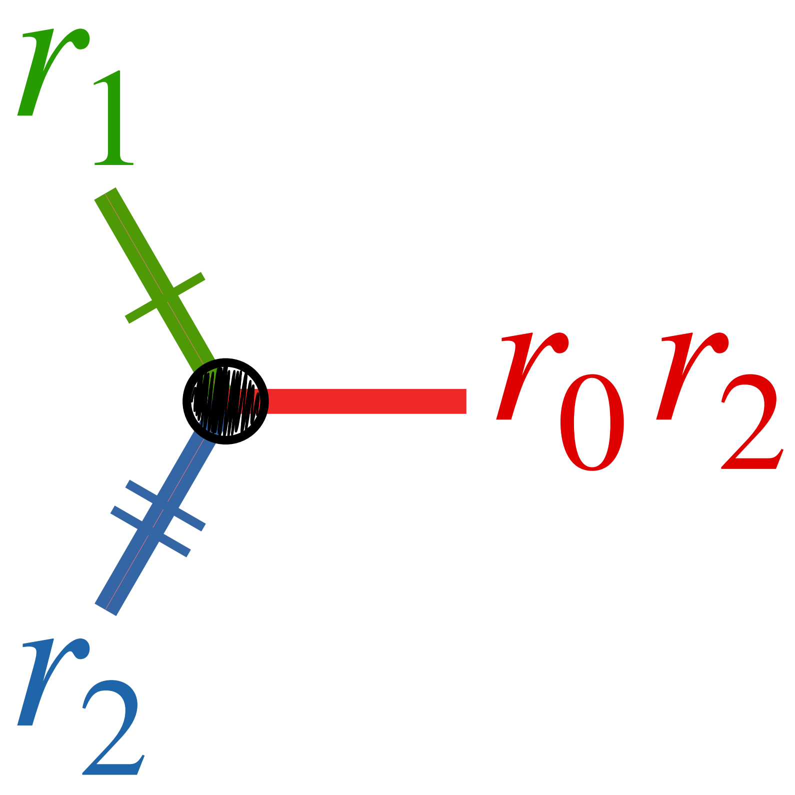

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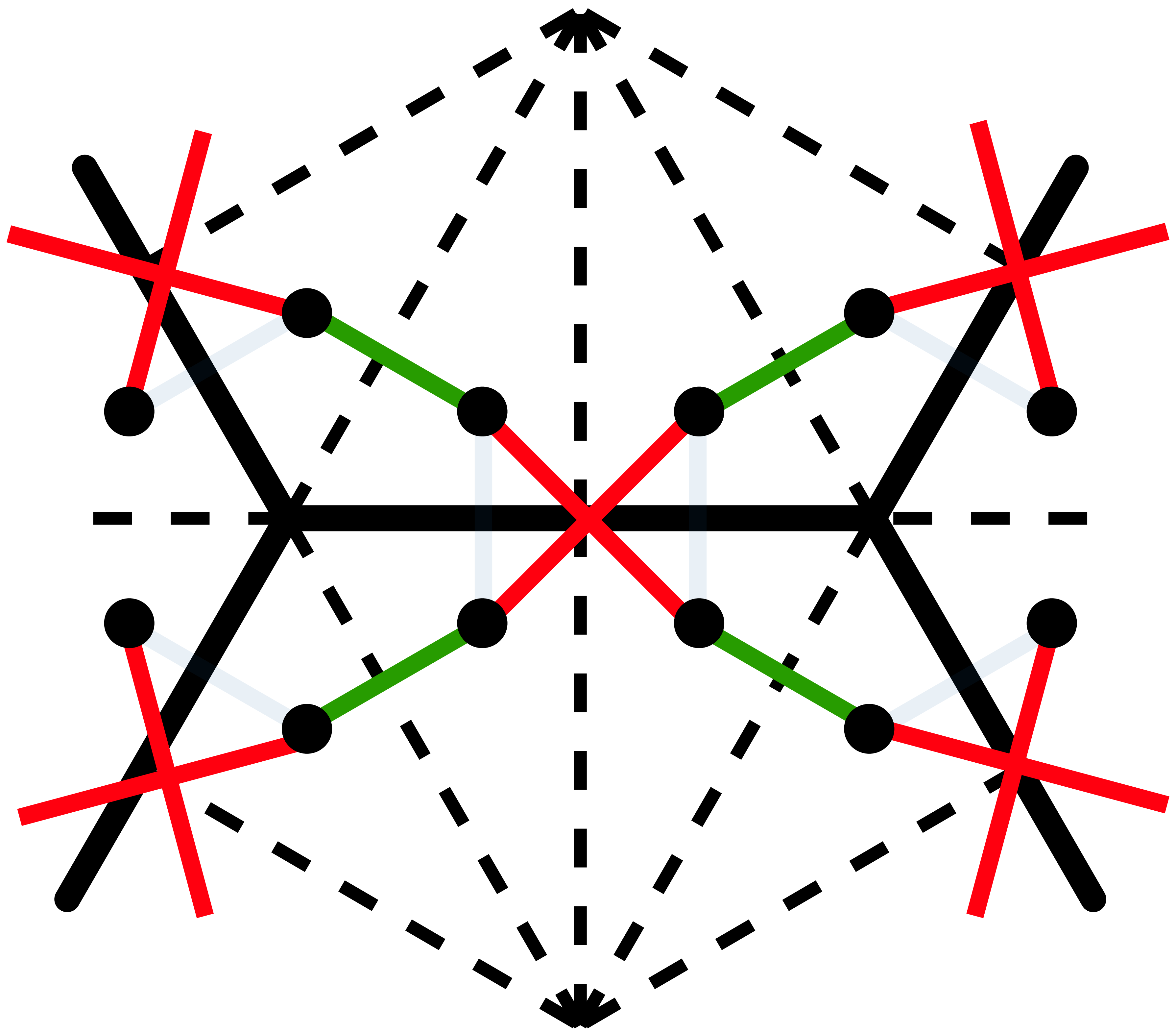

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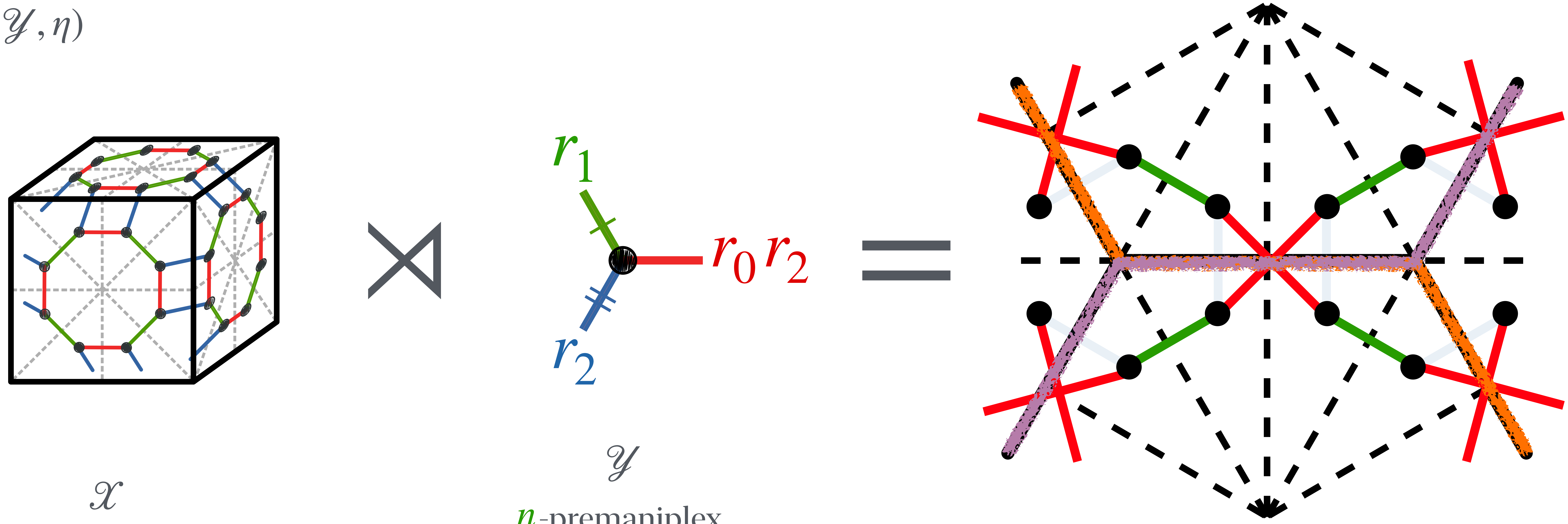

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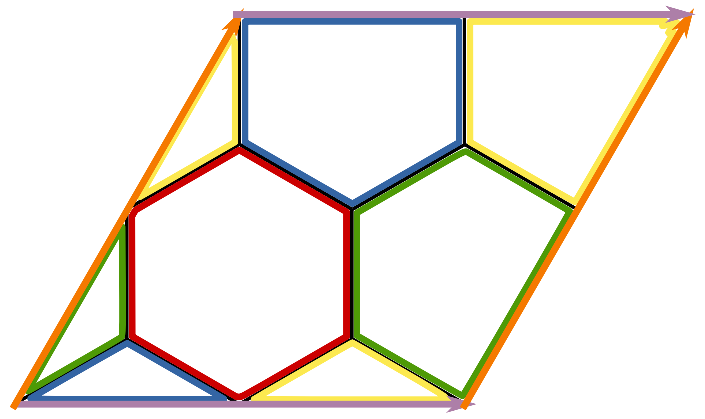

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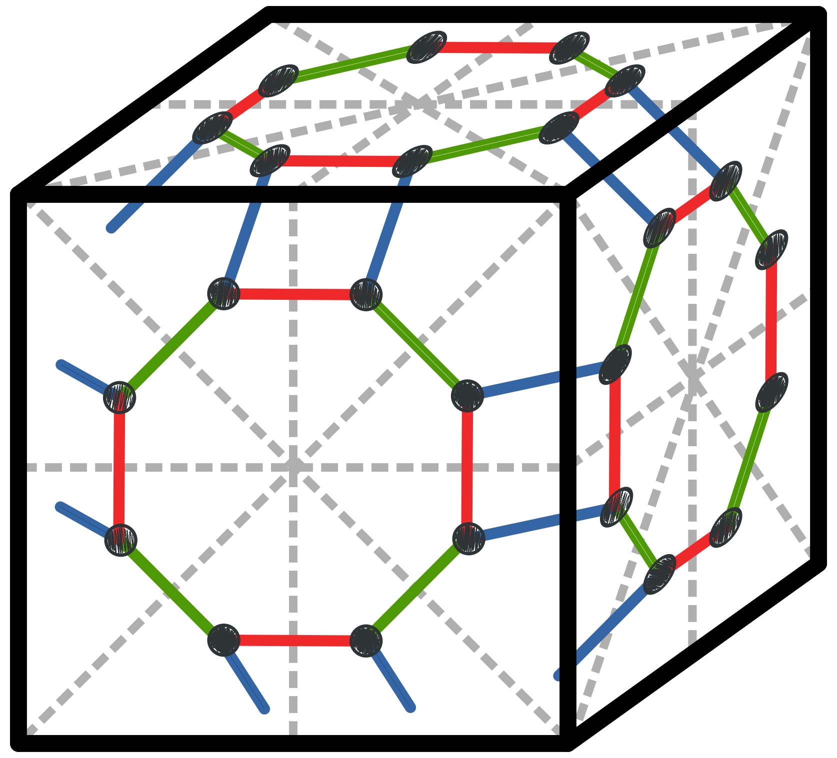

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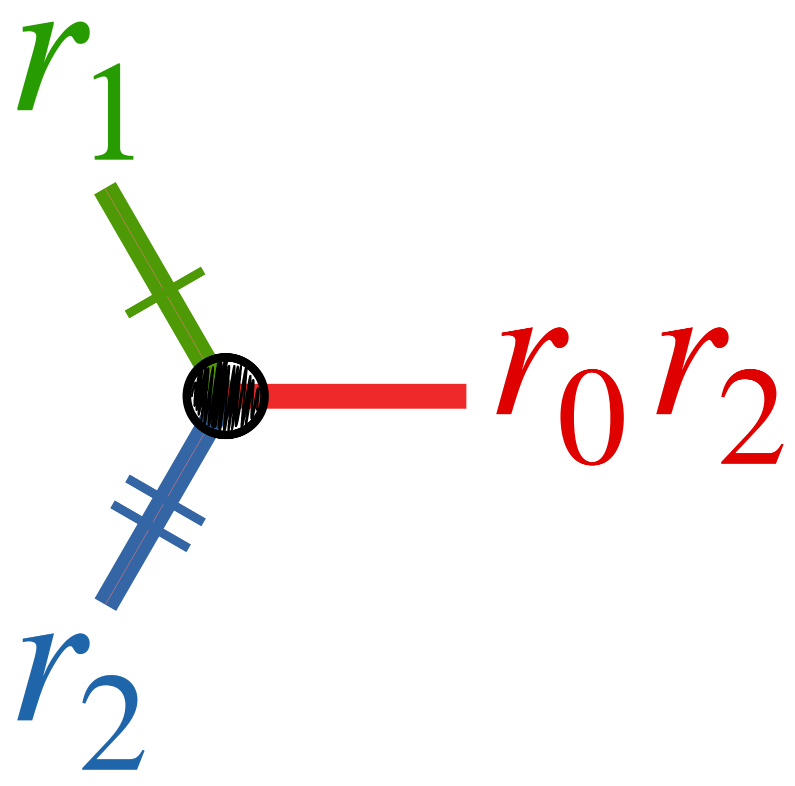

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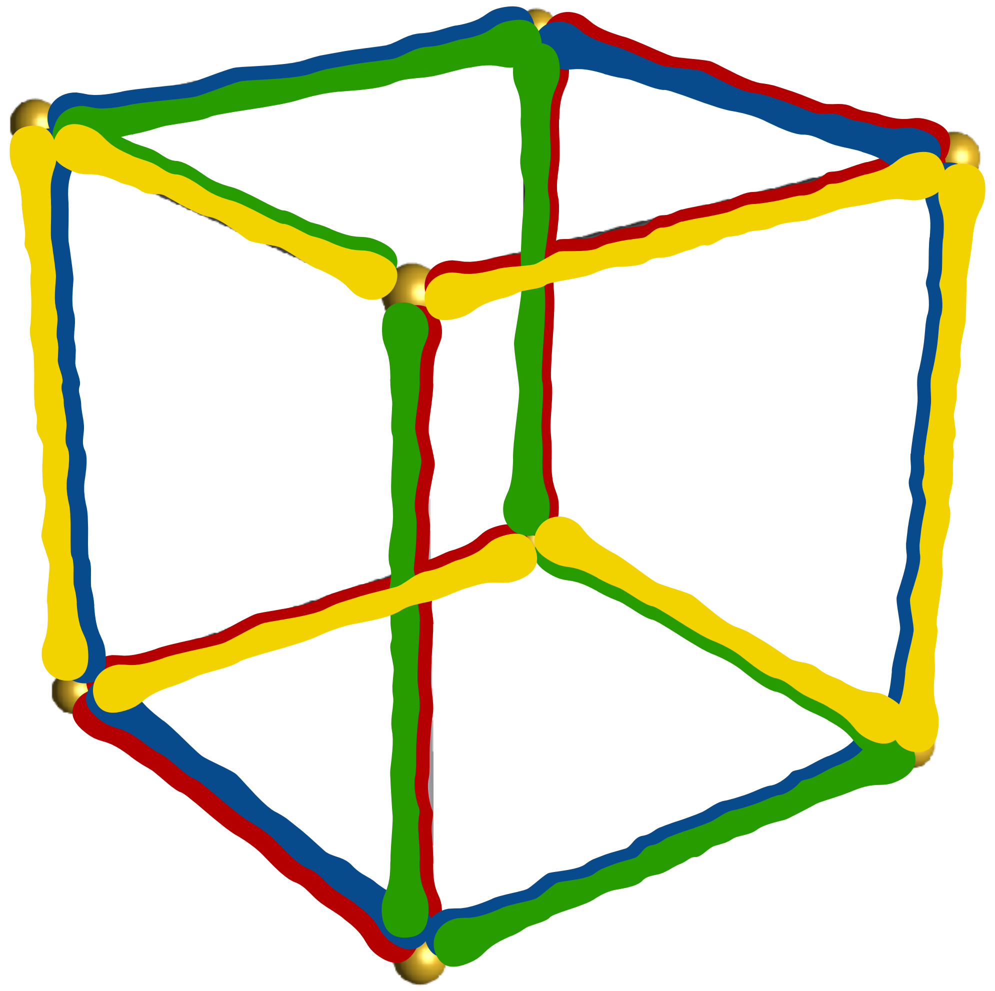

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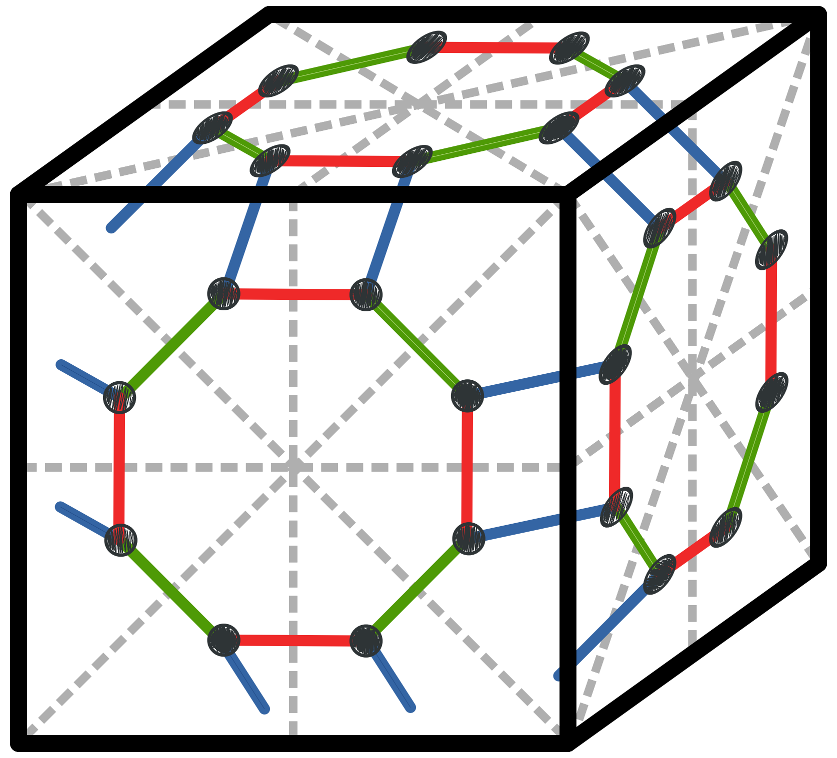

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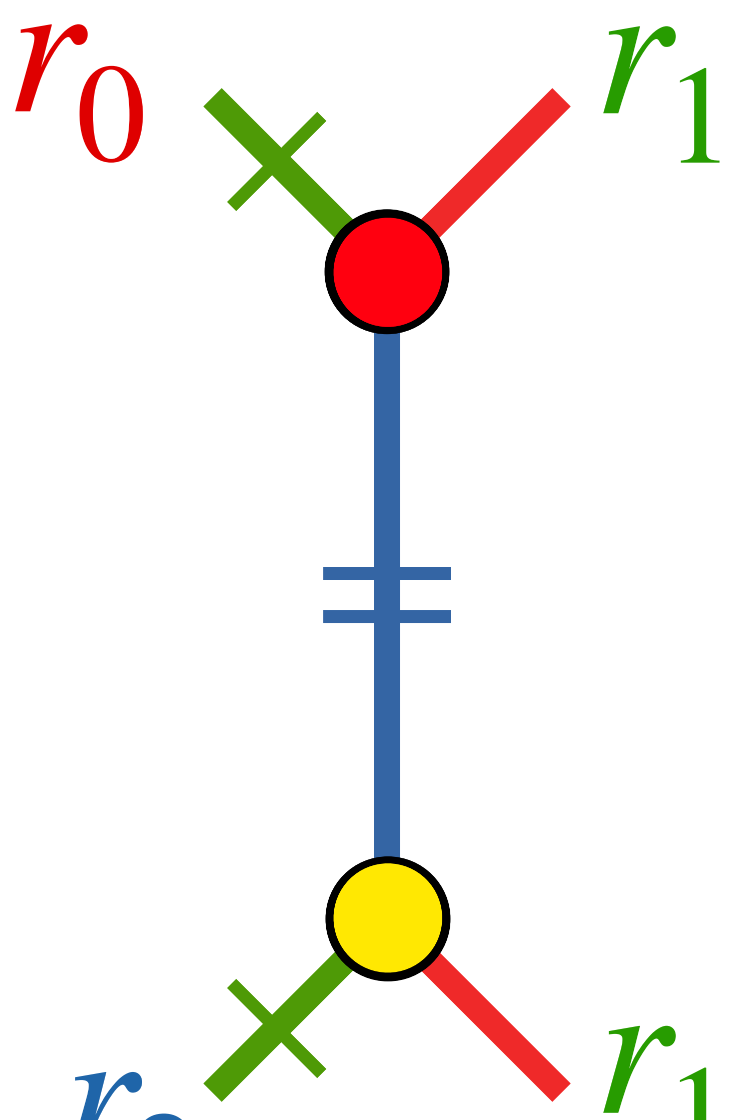
$\rtimes$



$\mathscr{Y}$

n-premaniplex

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Asignación de voltajes

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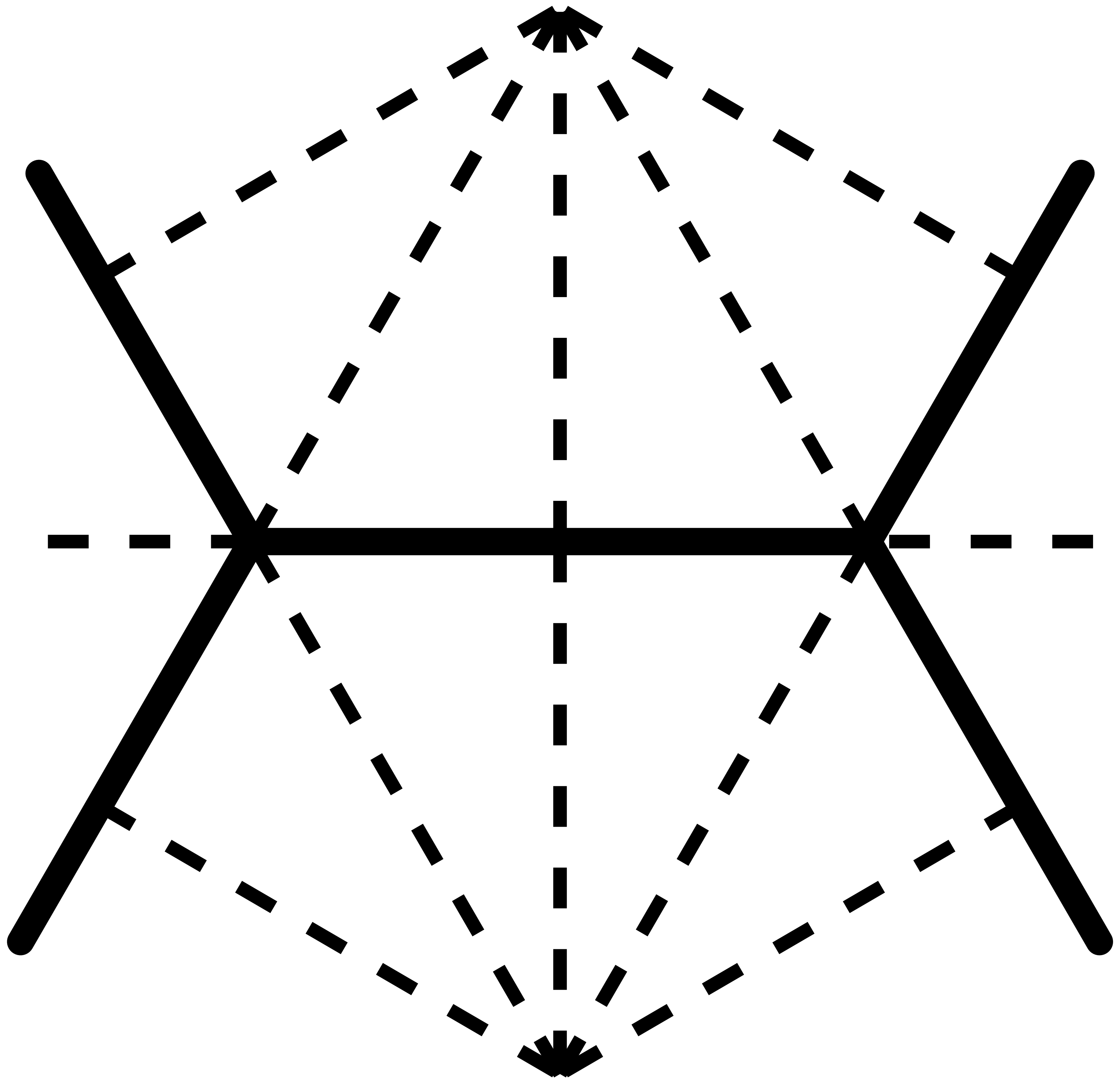

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n-premaniplex

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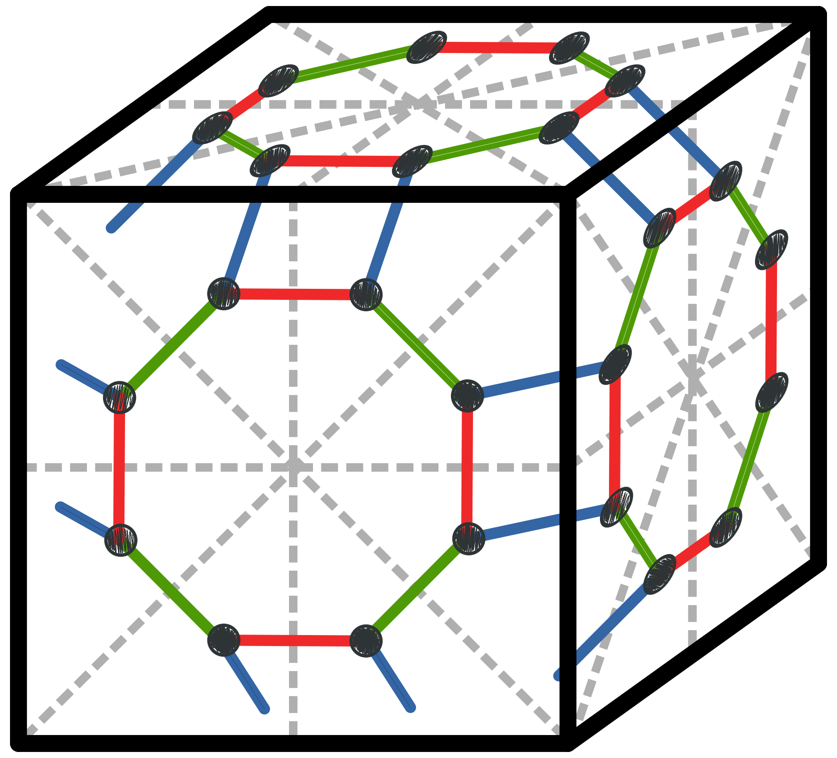

$\mathscr{X}$

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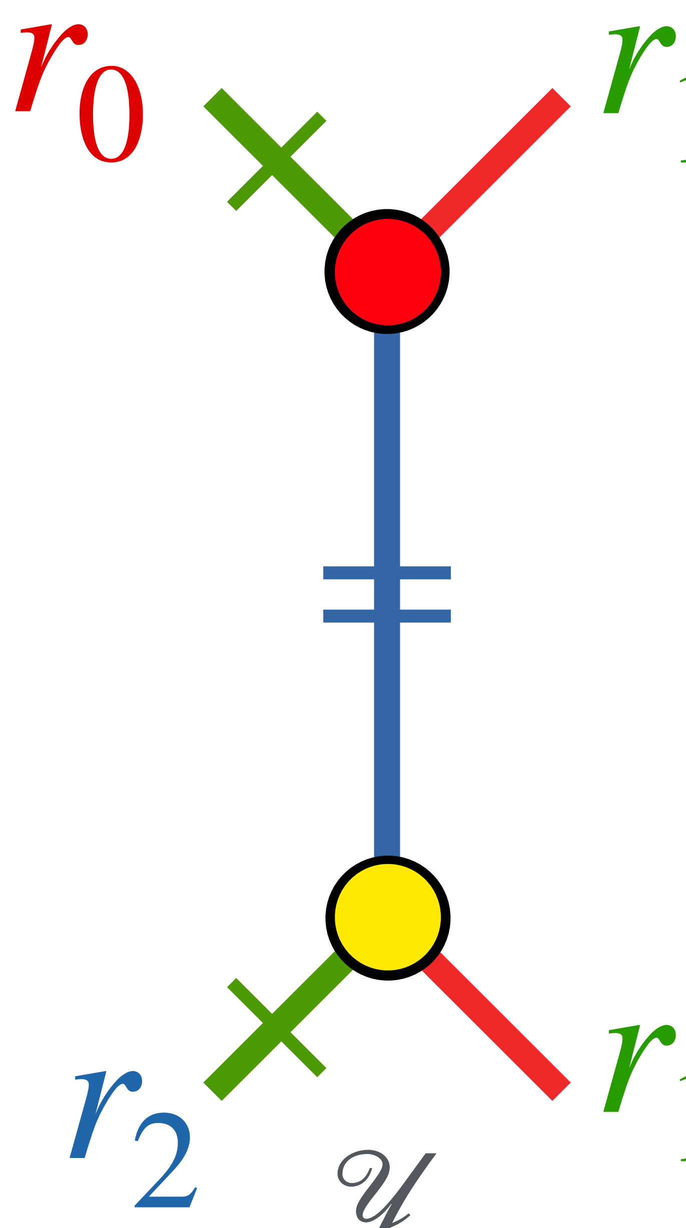

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Asignación de voltajes

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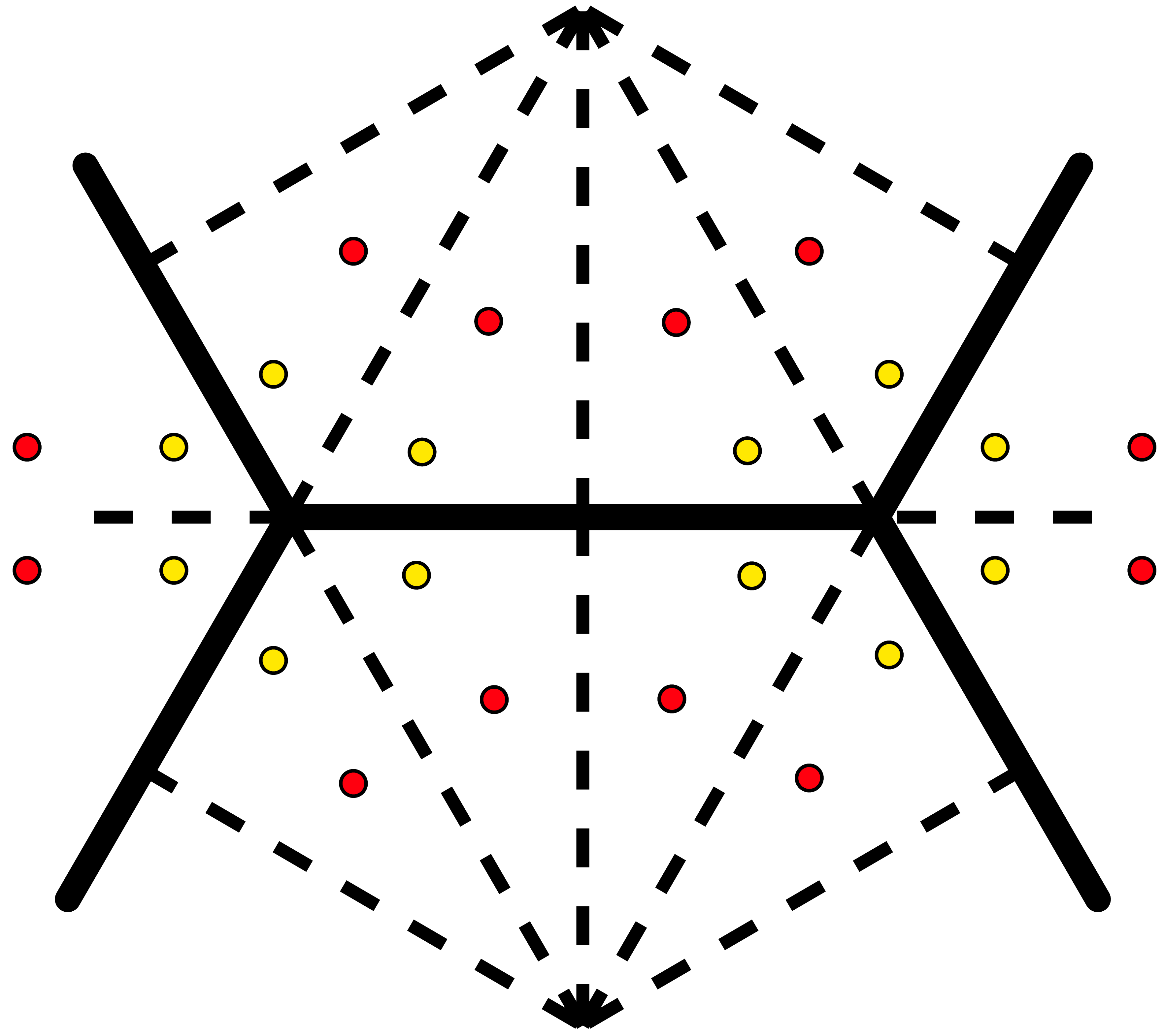

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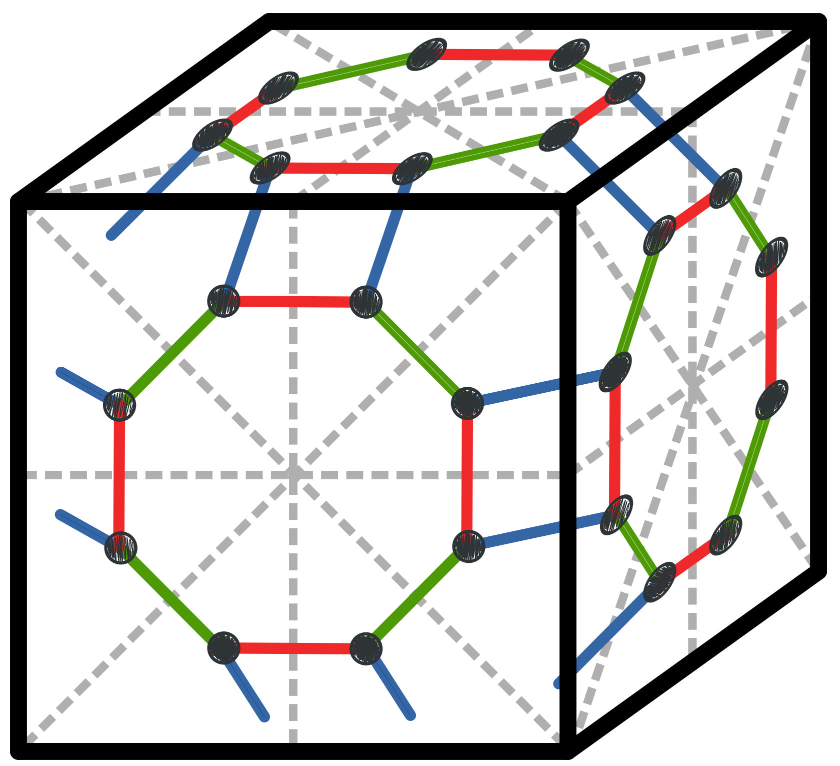

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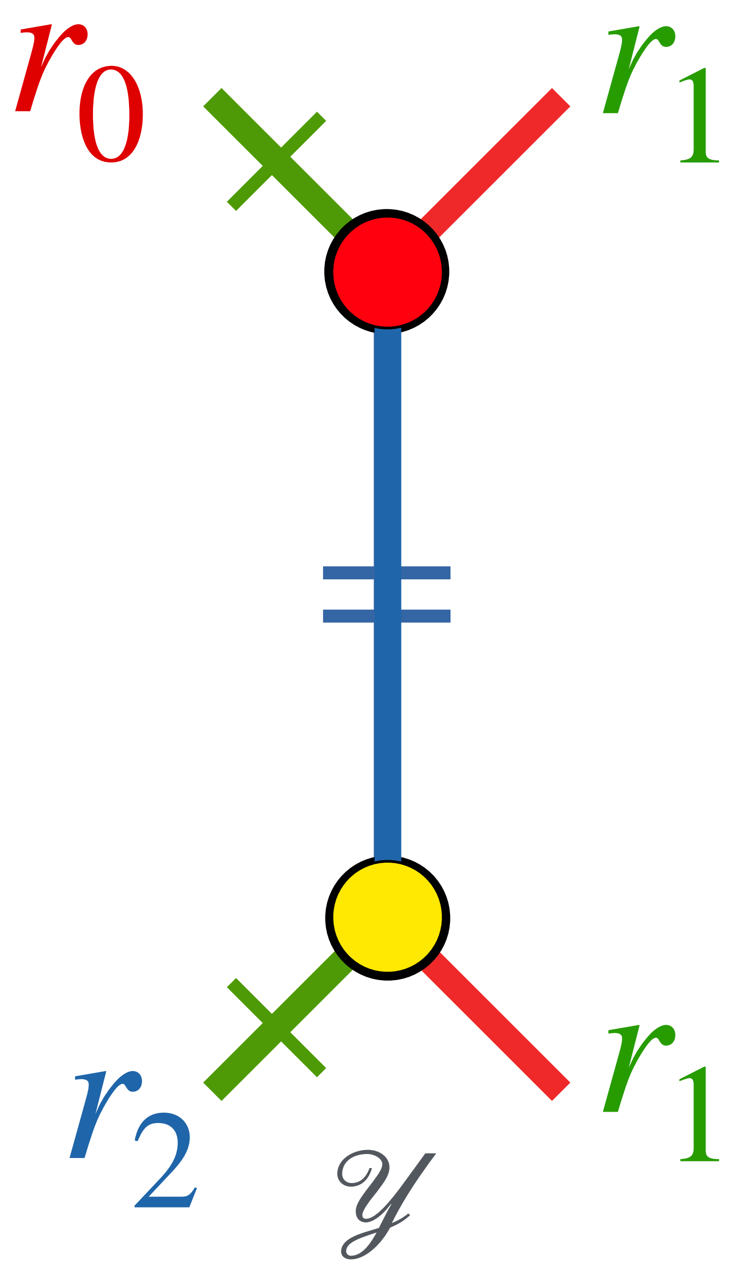

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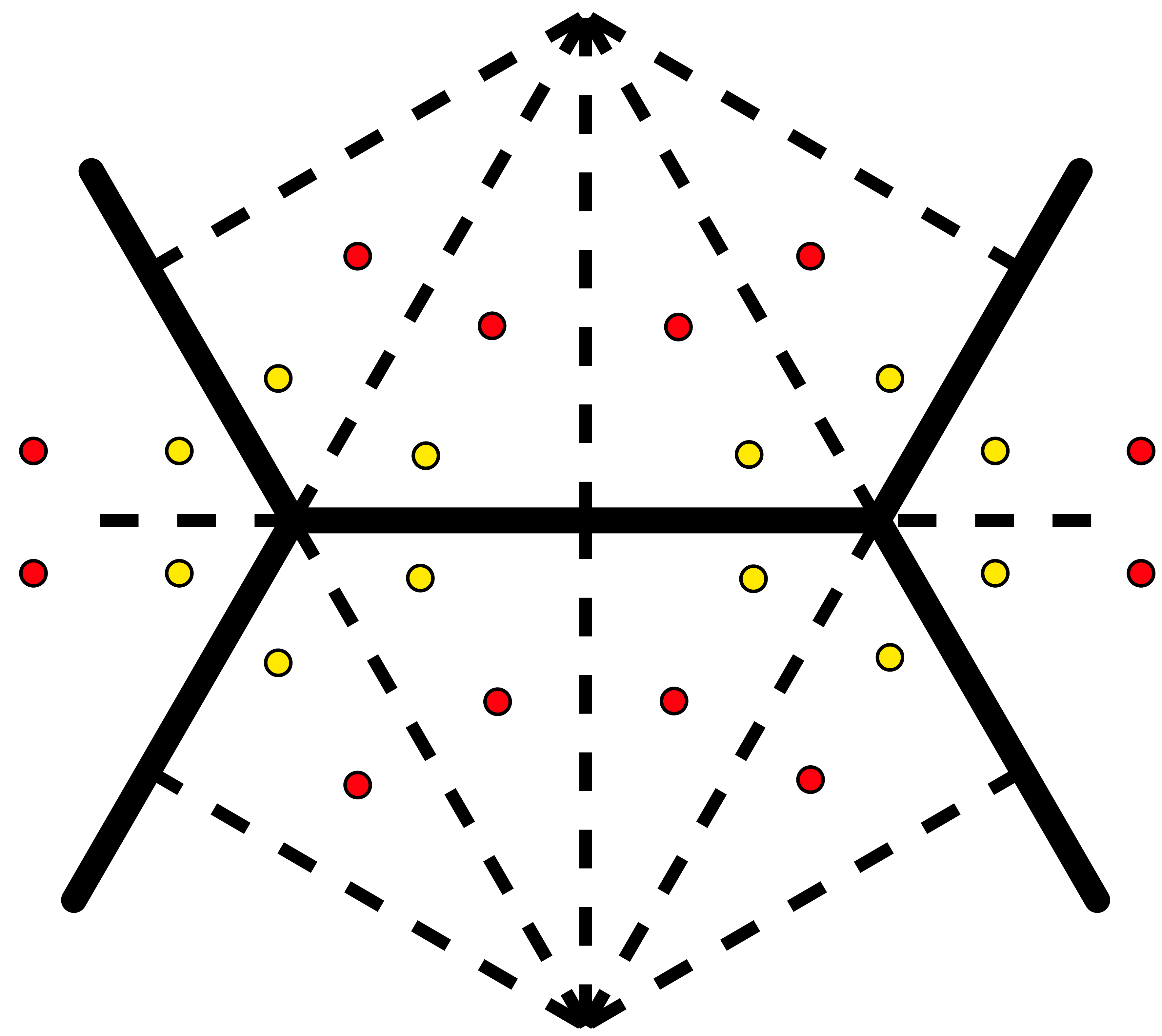

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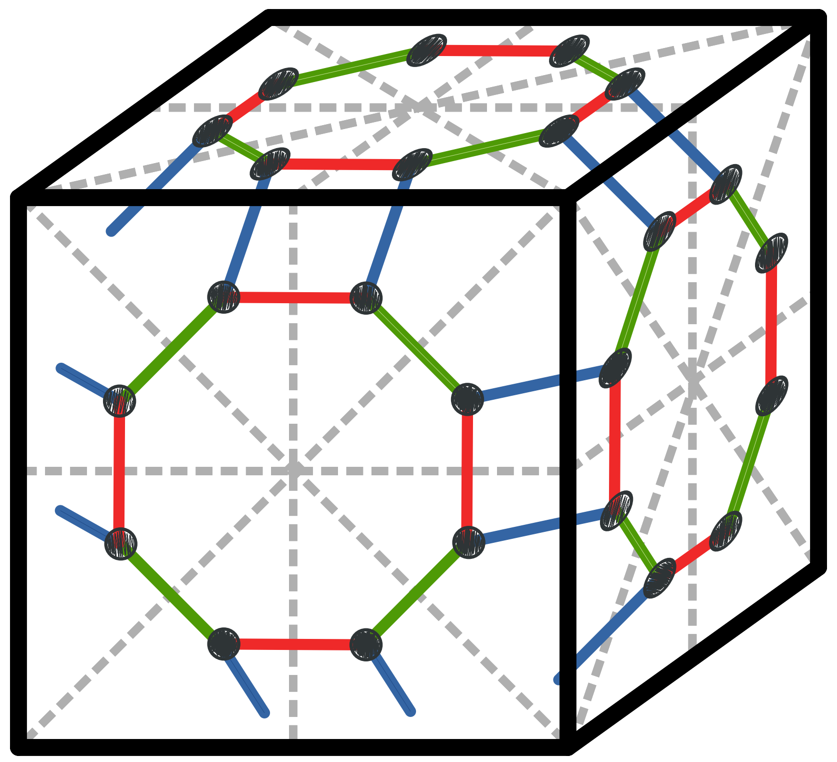

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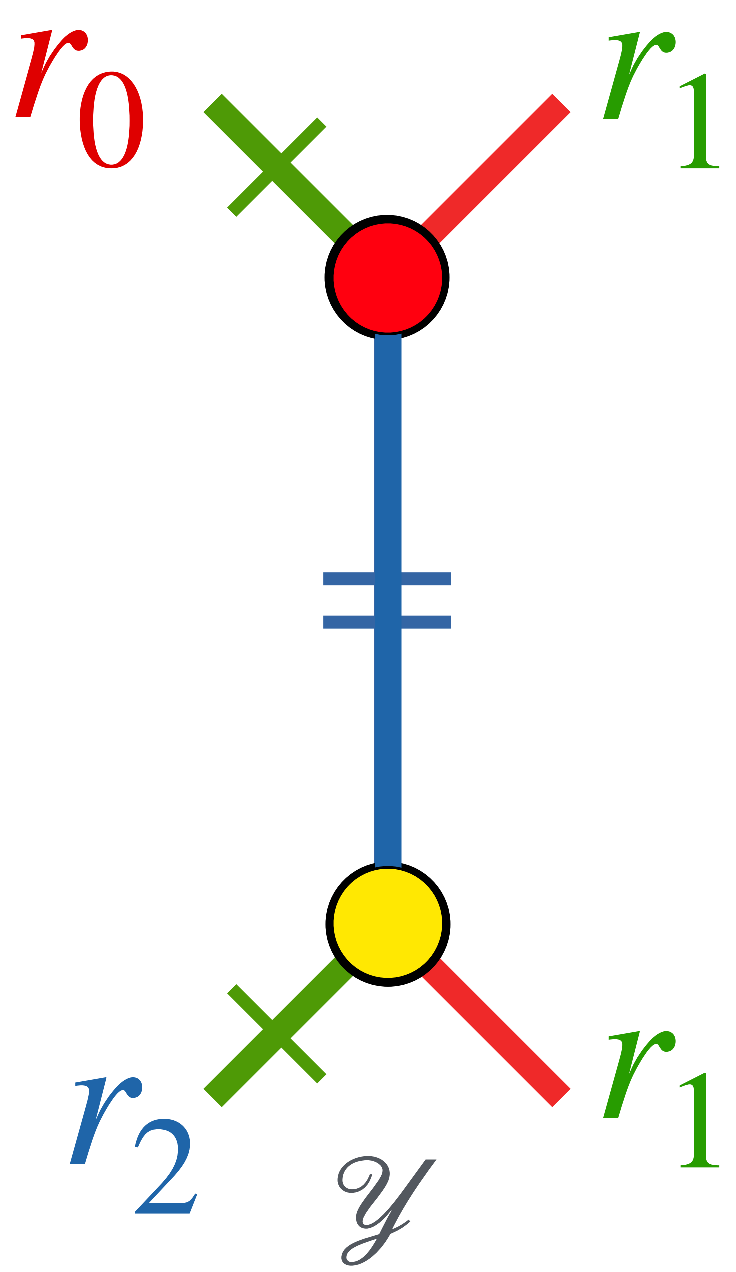

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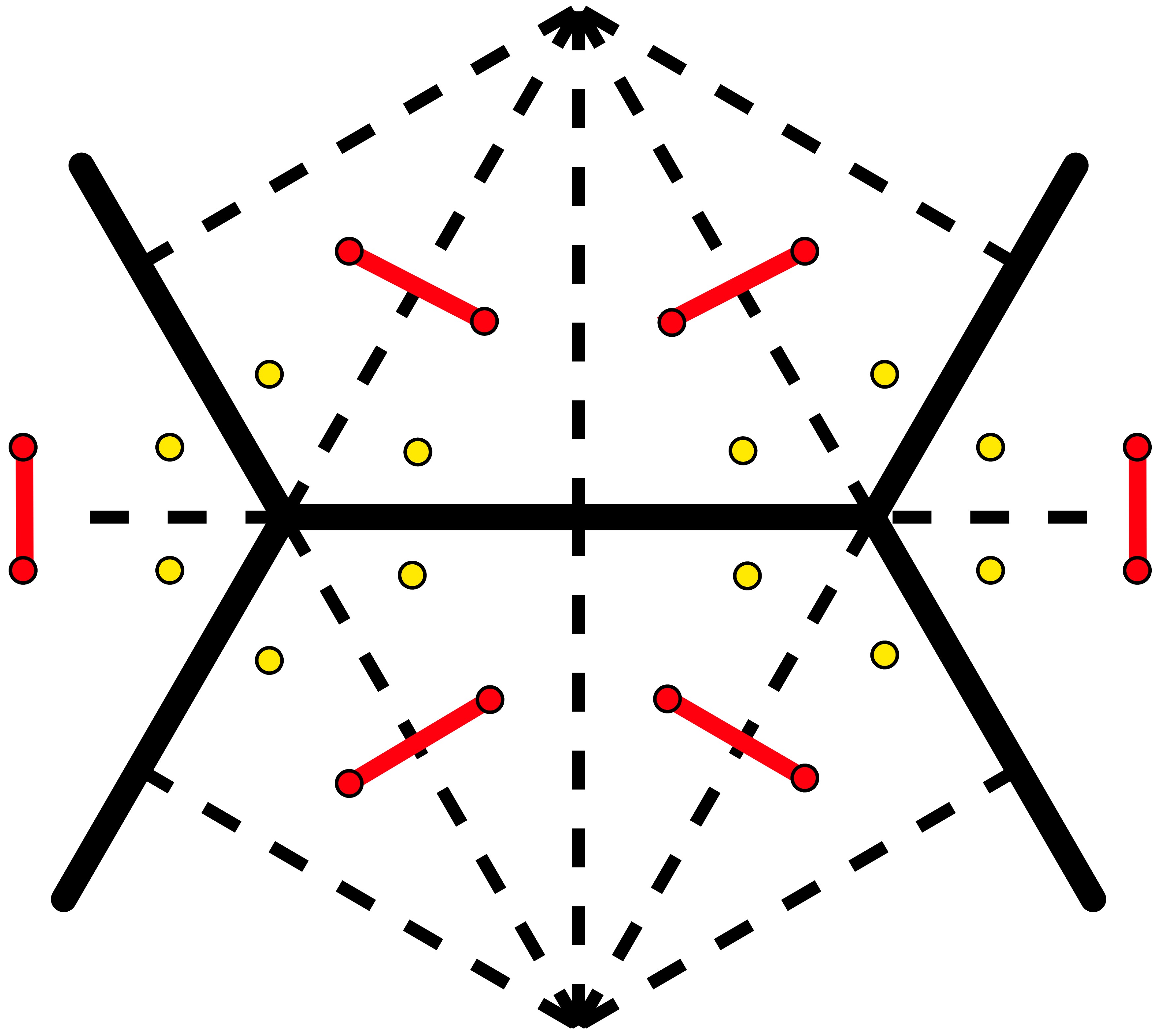

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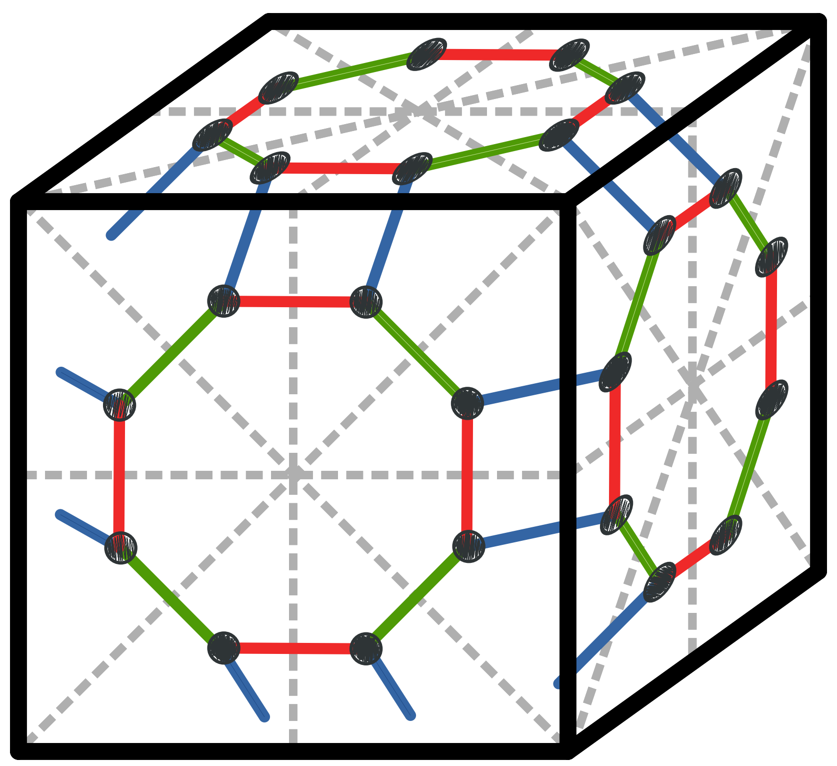

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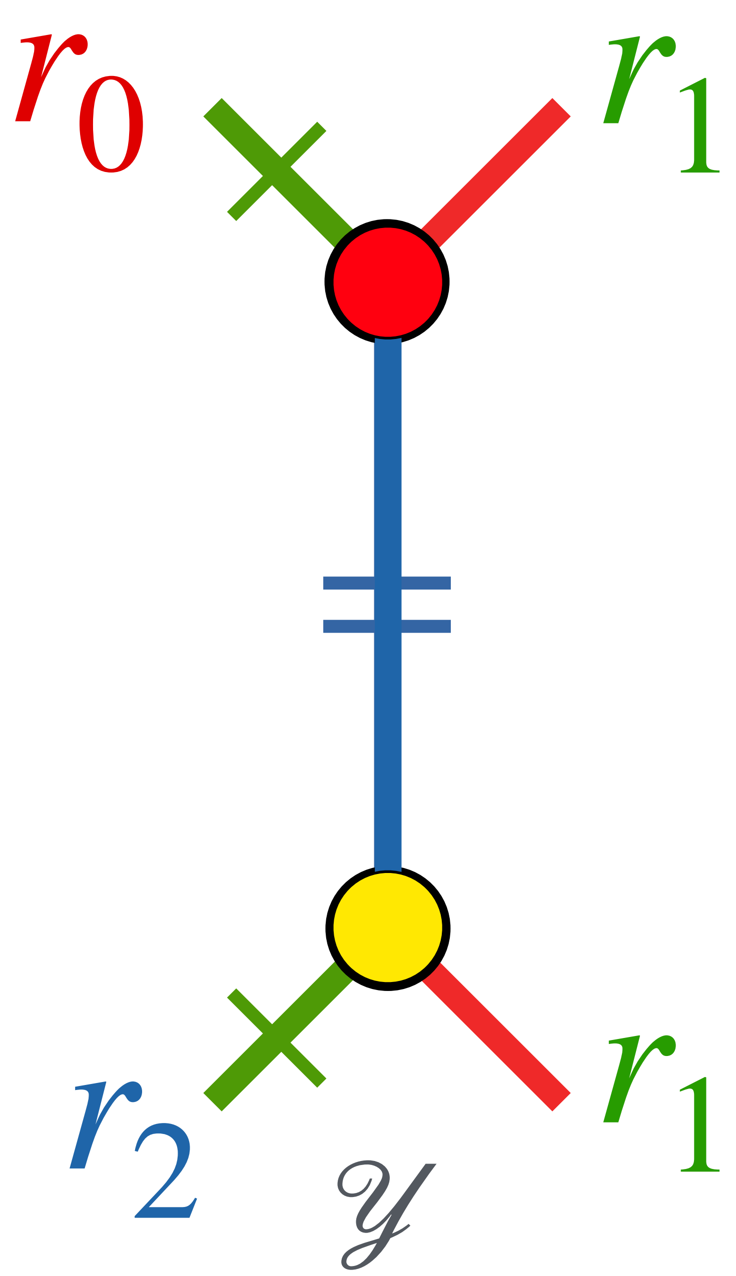

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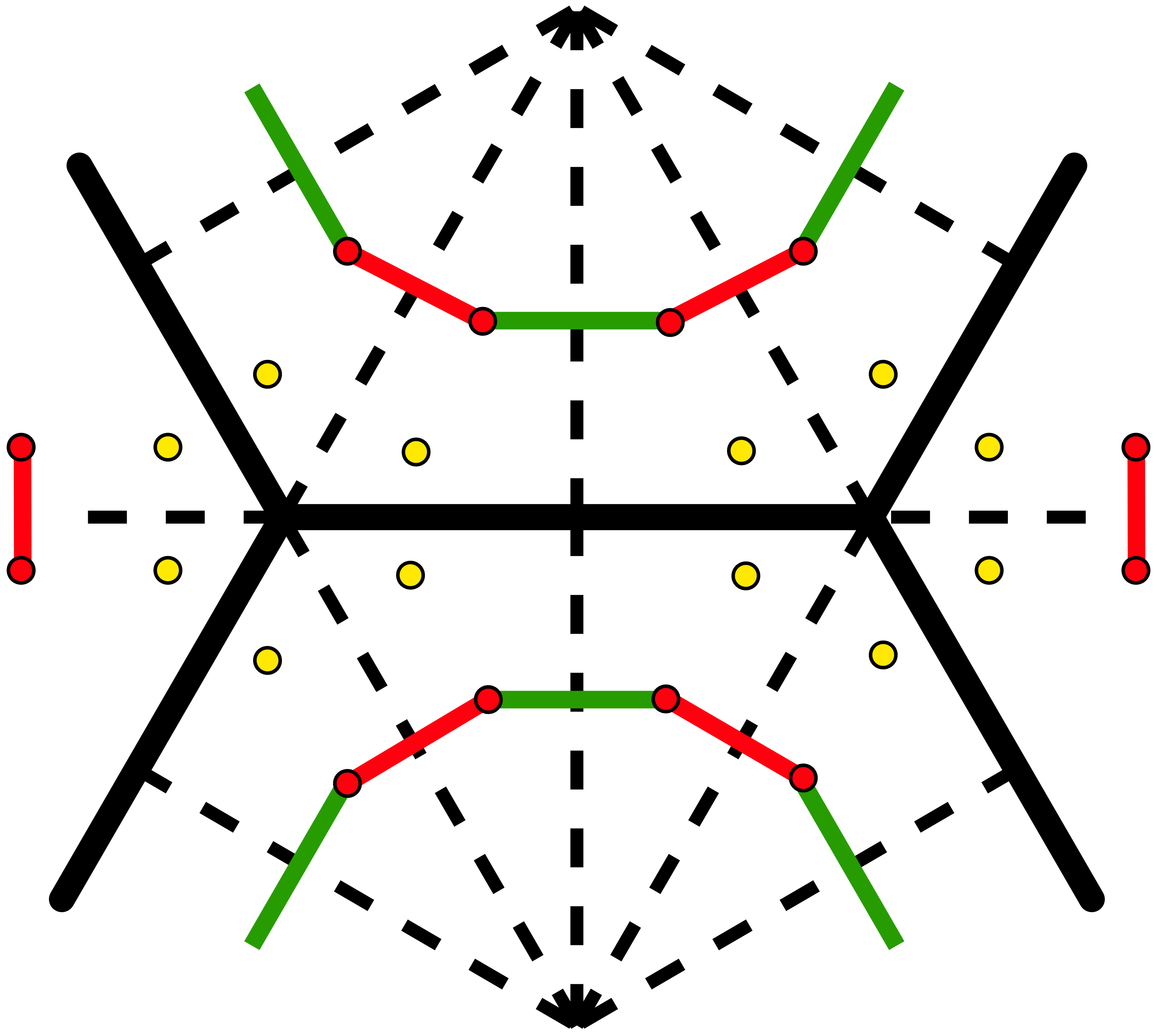

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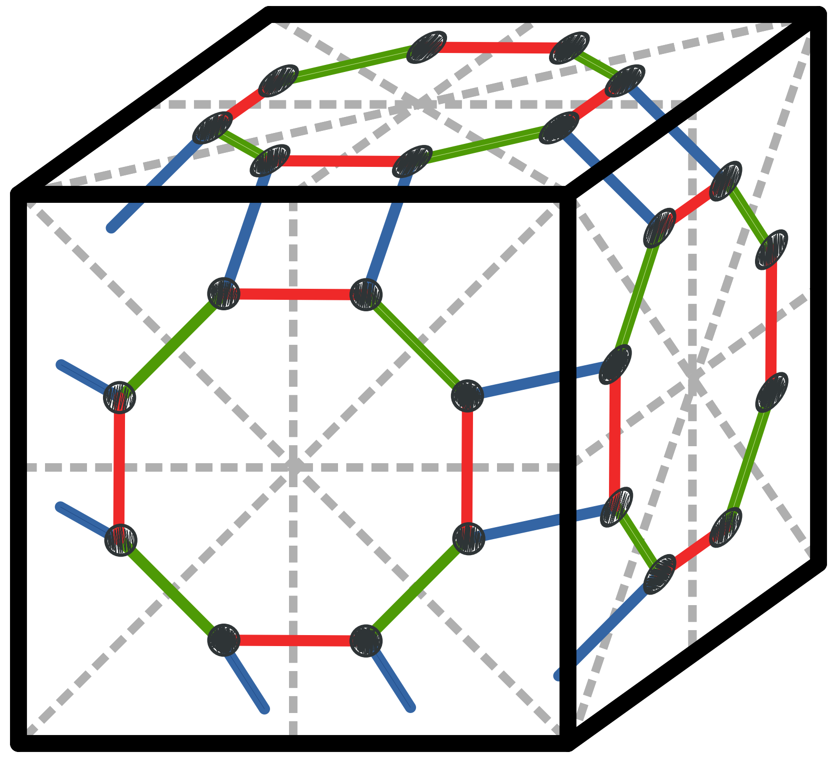

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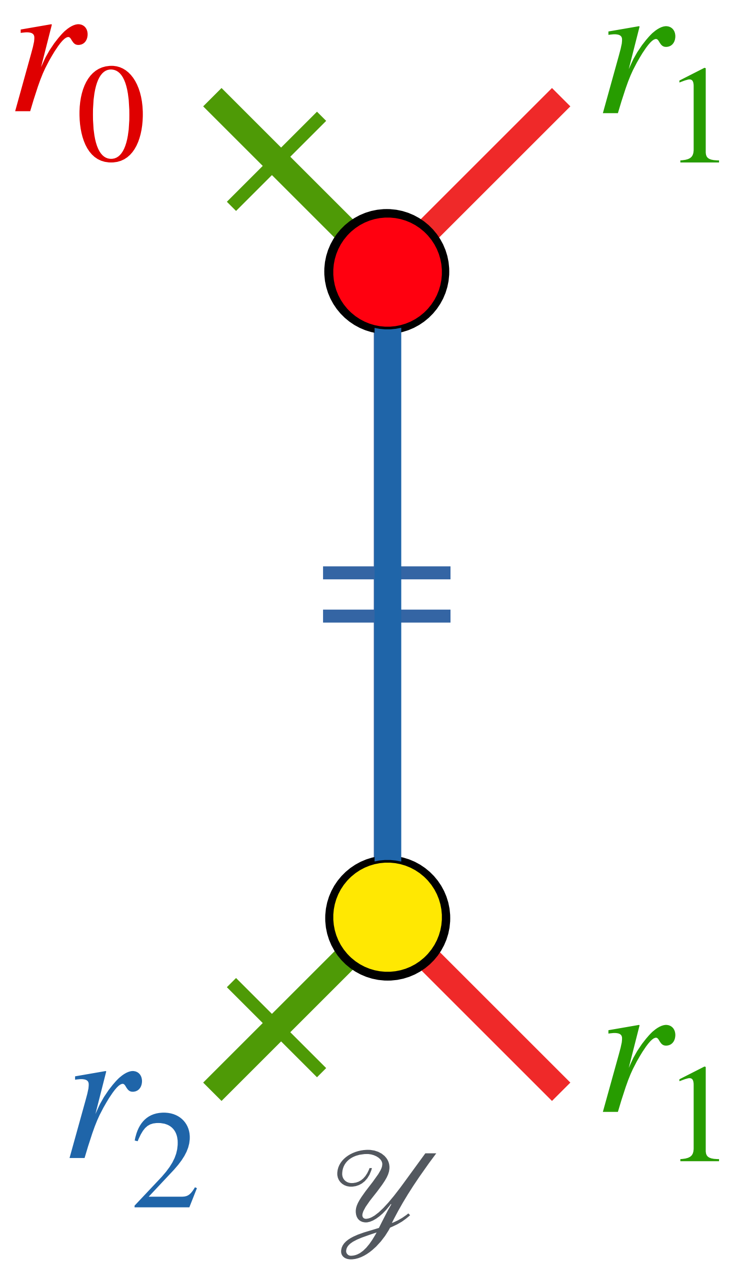

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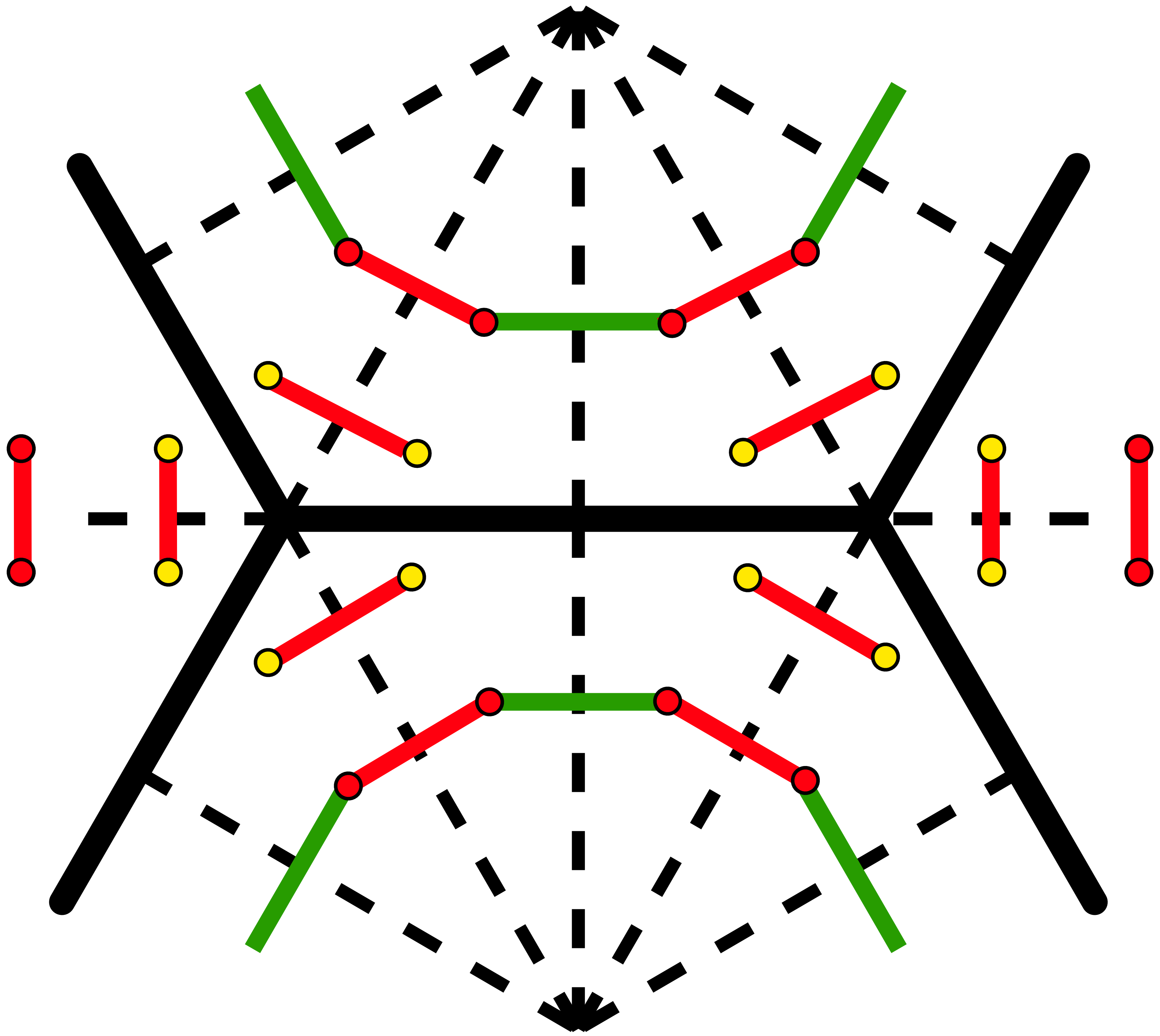

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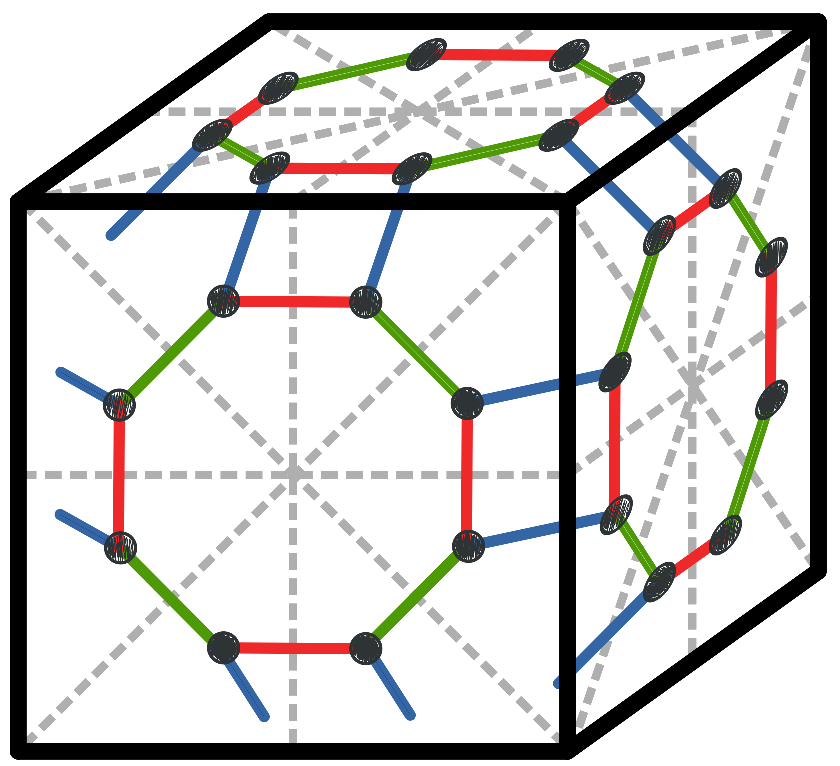

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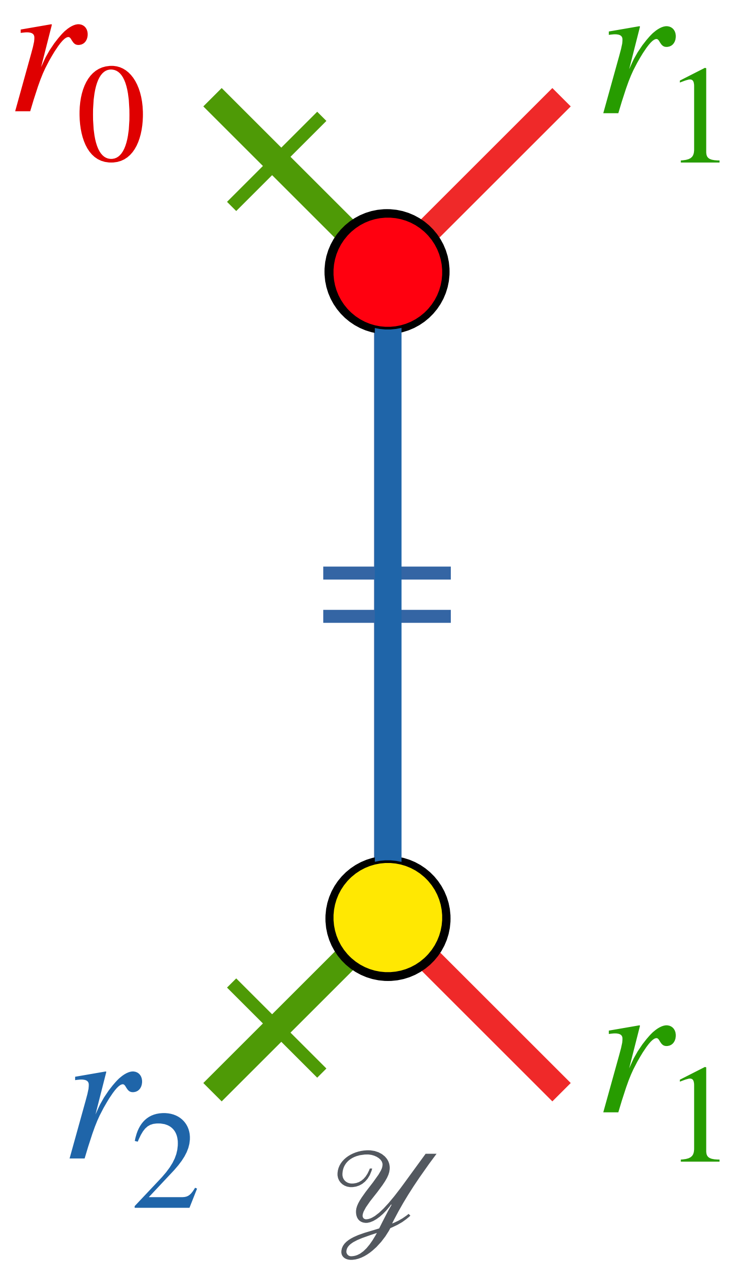

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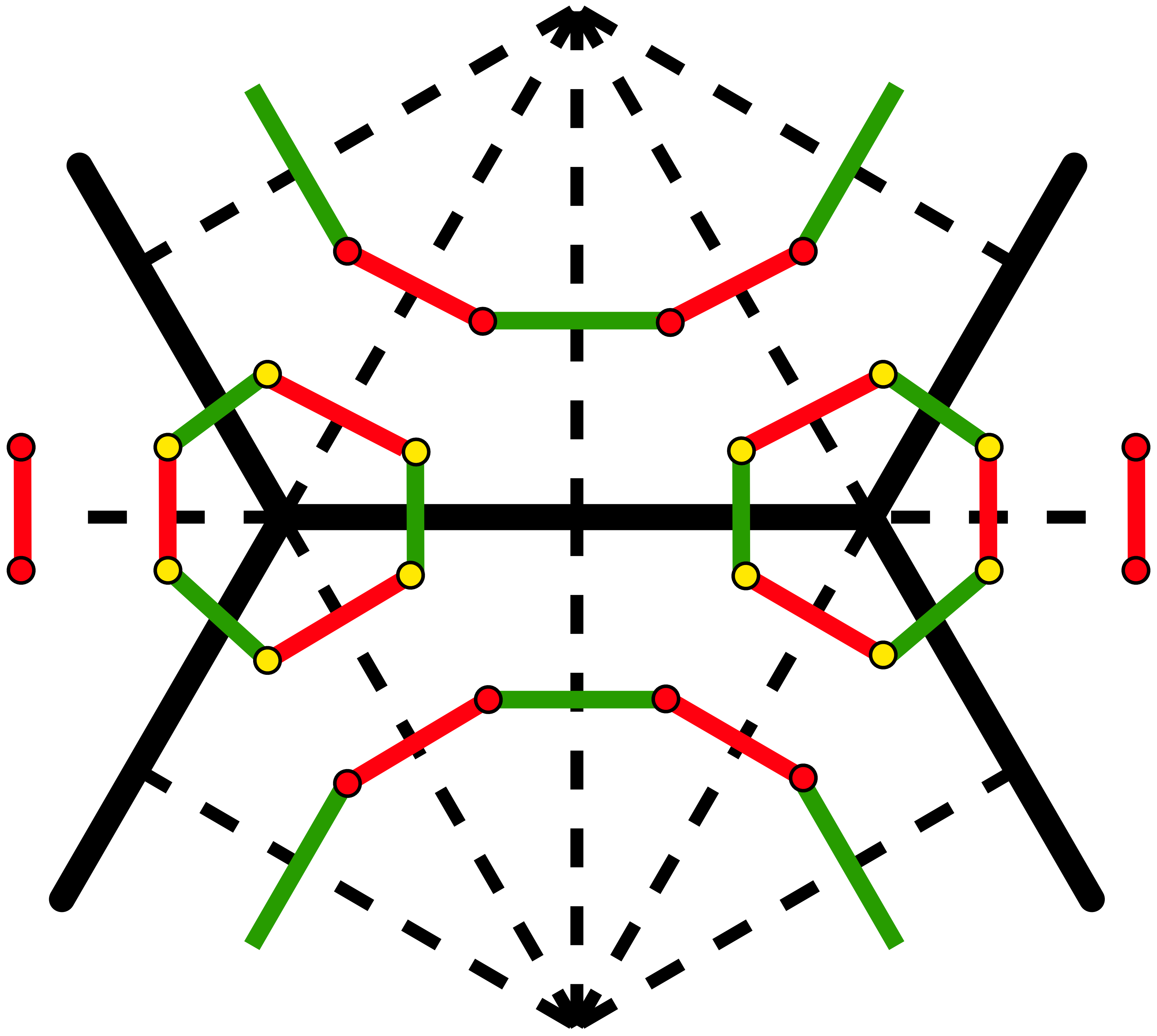

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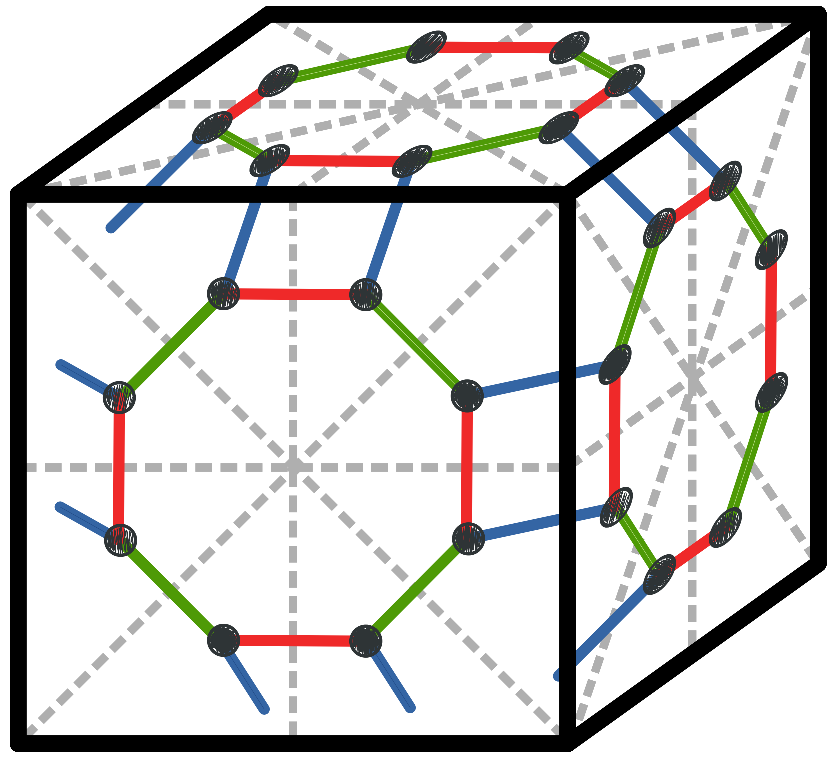

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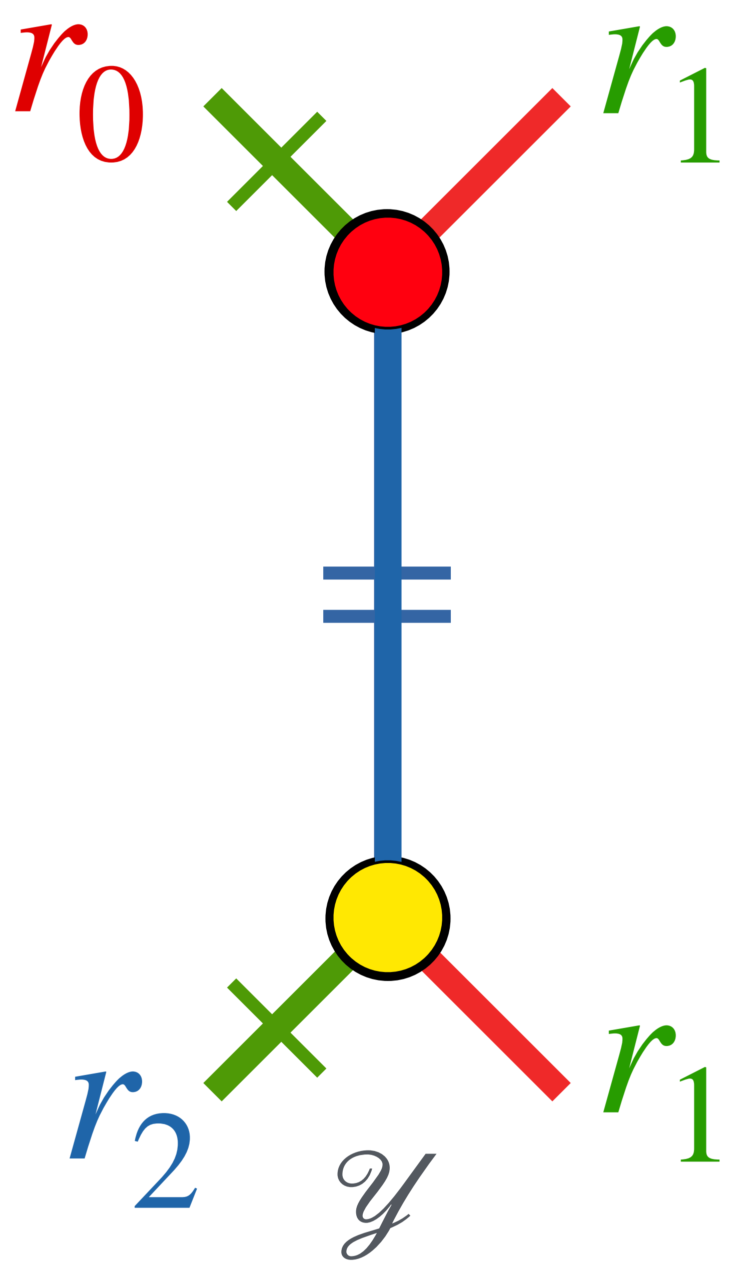

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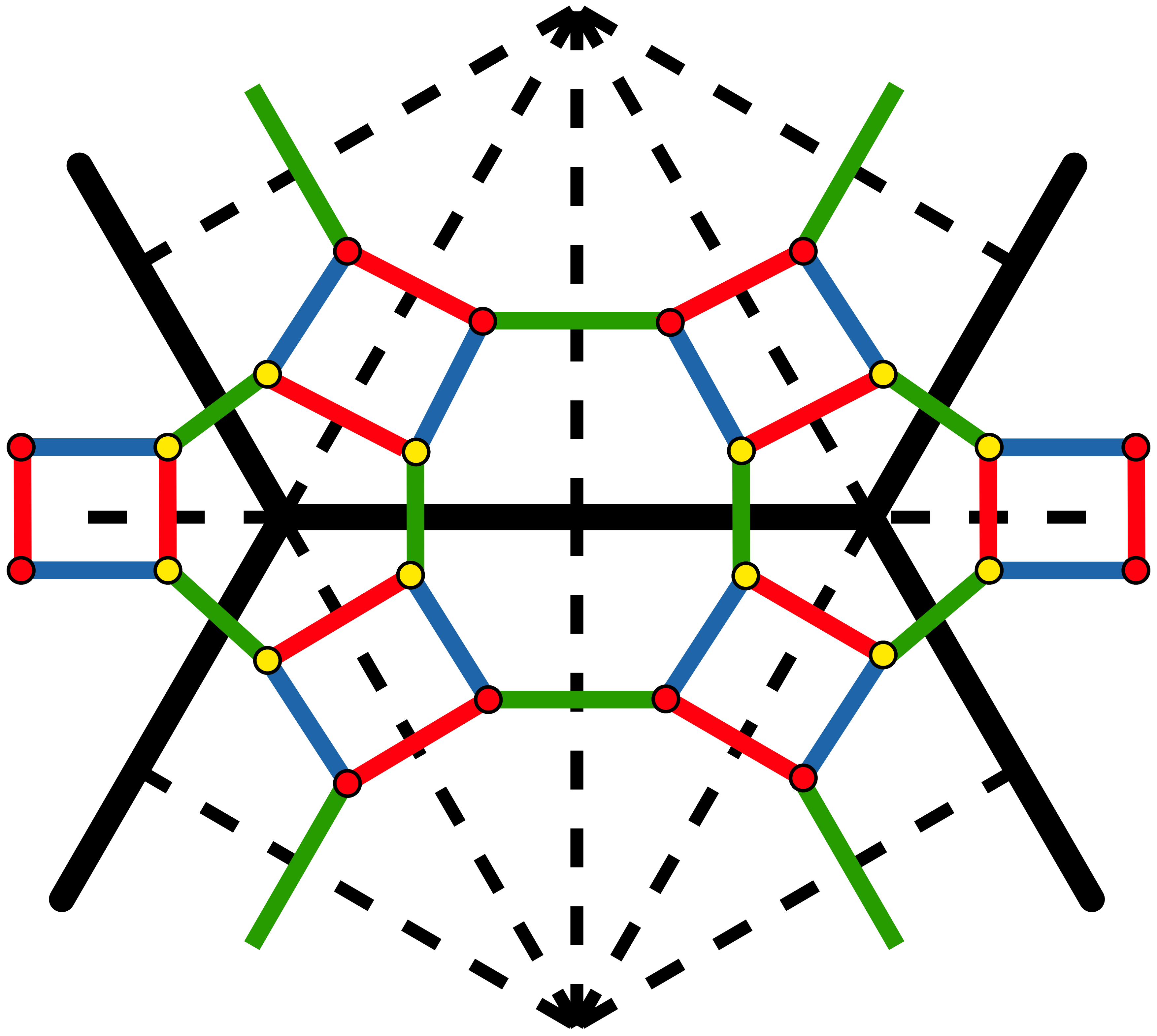

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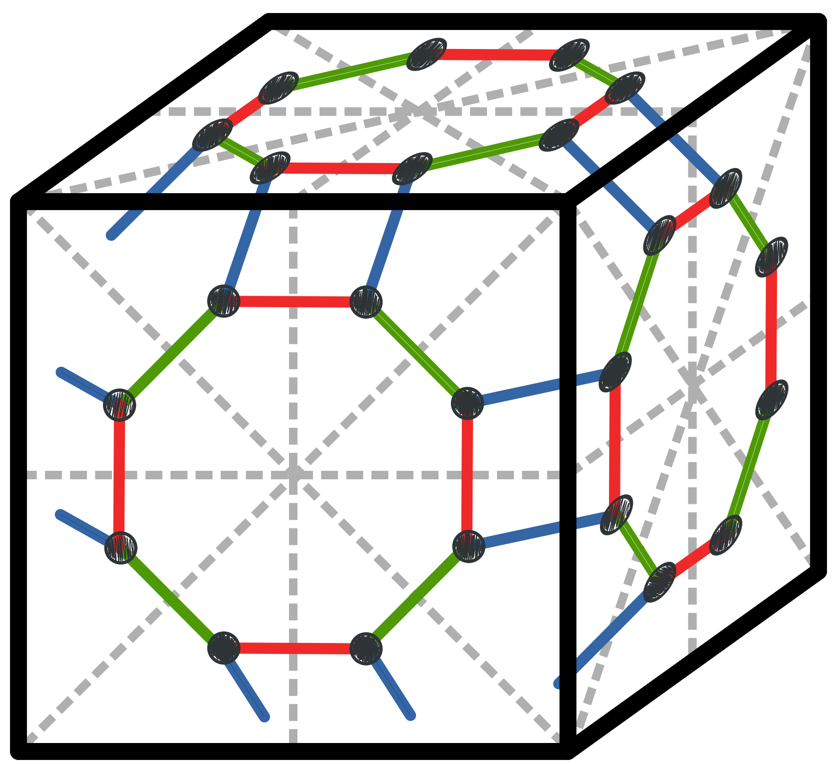

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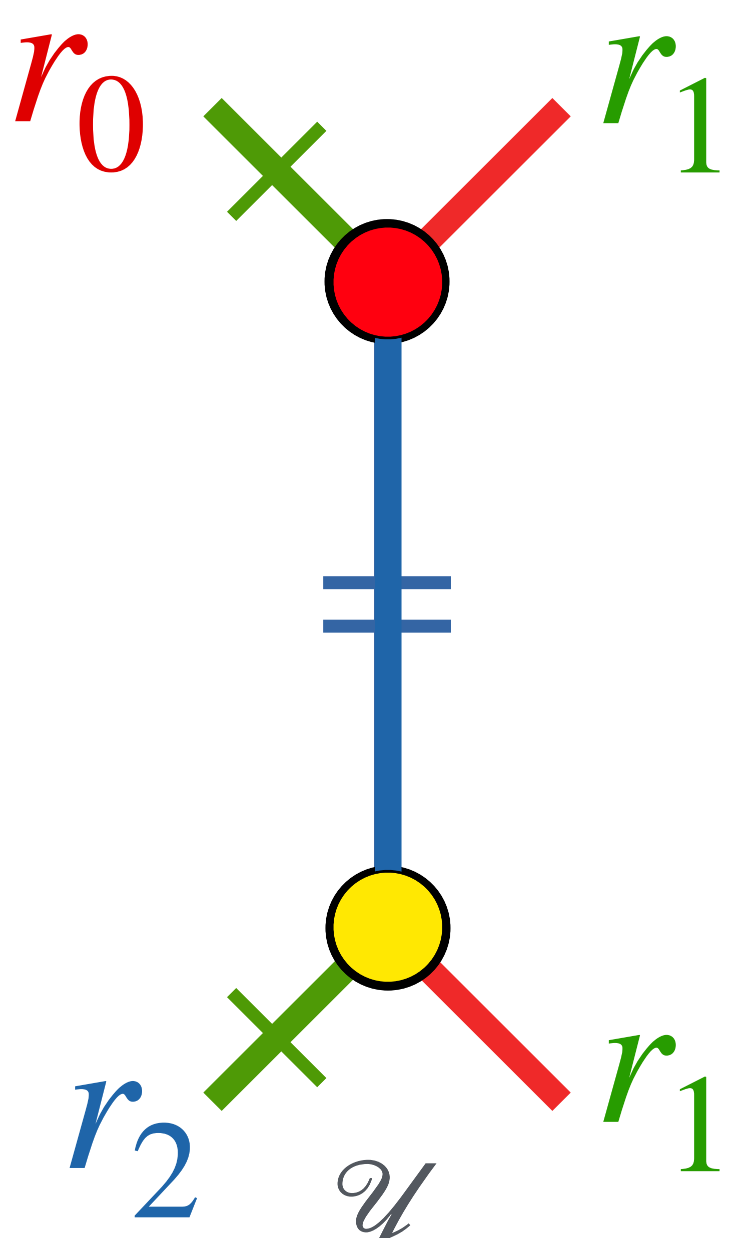

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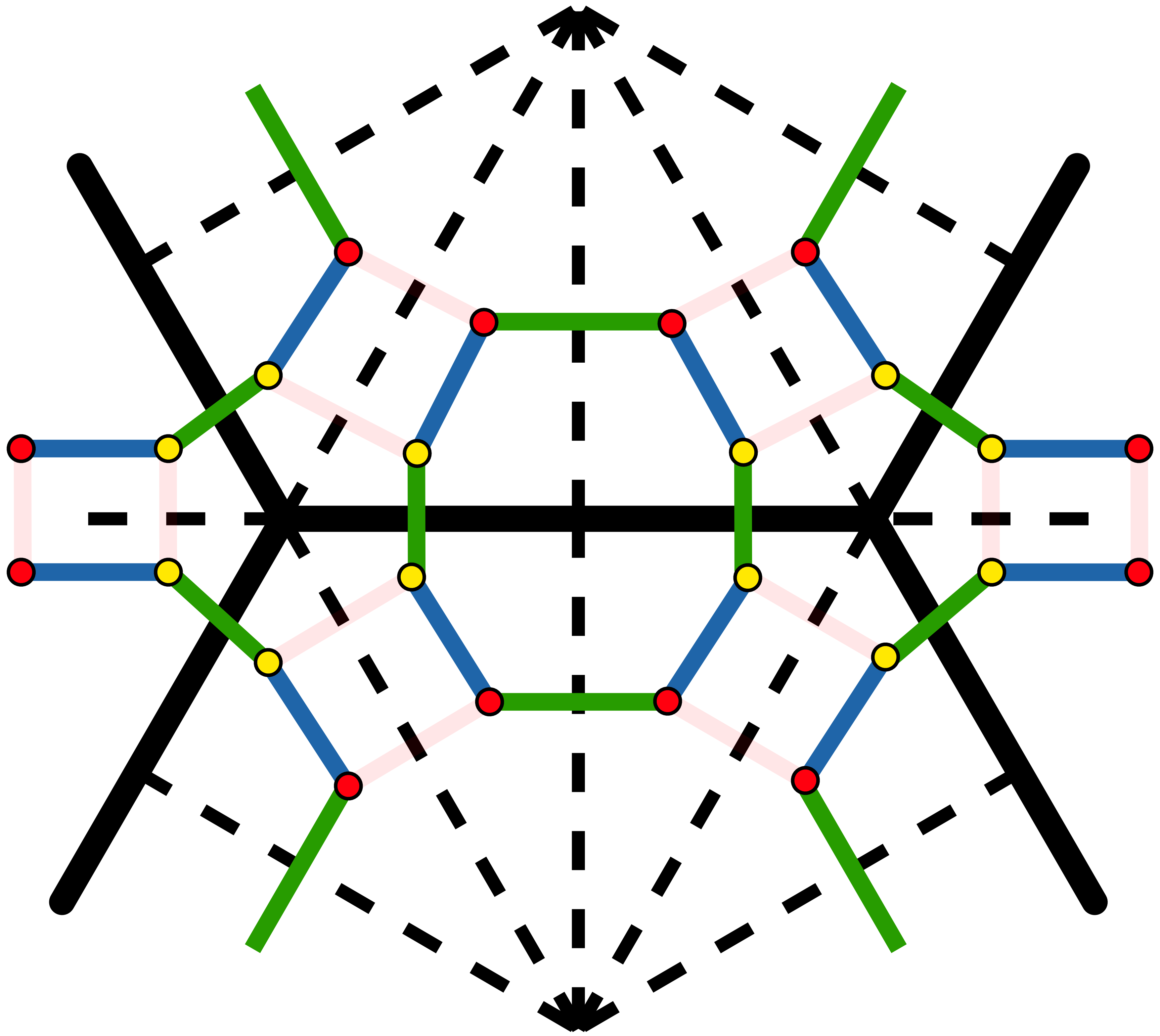

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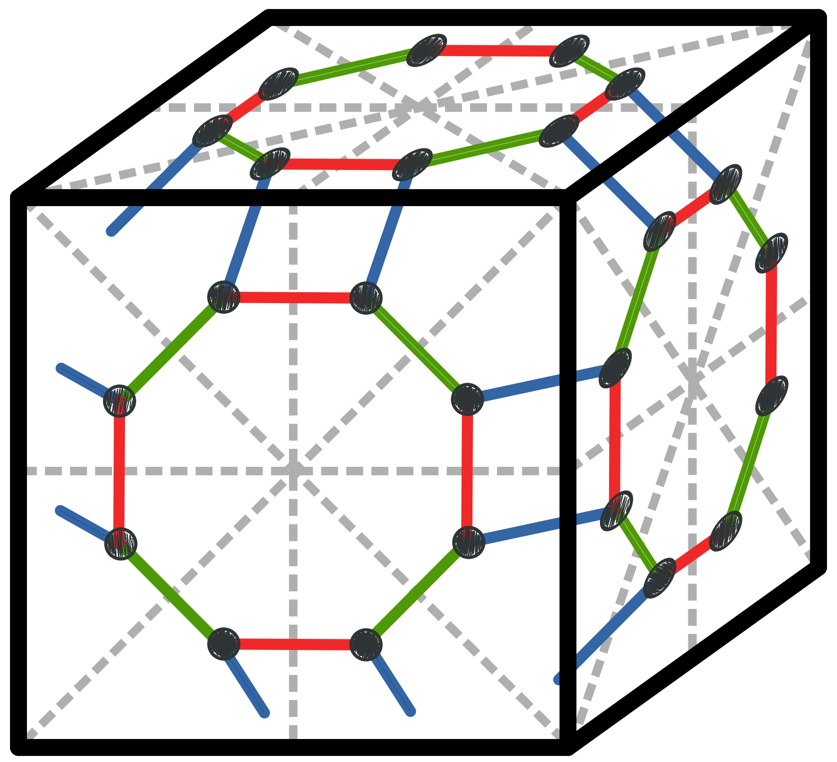

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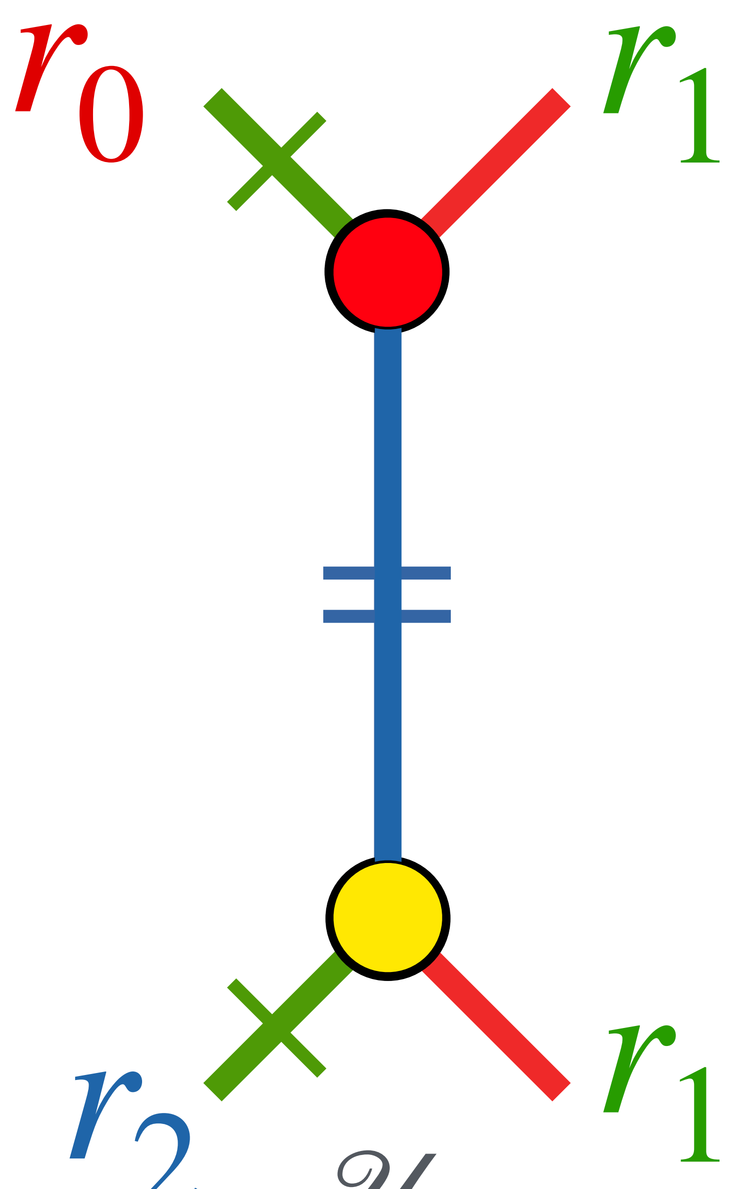


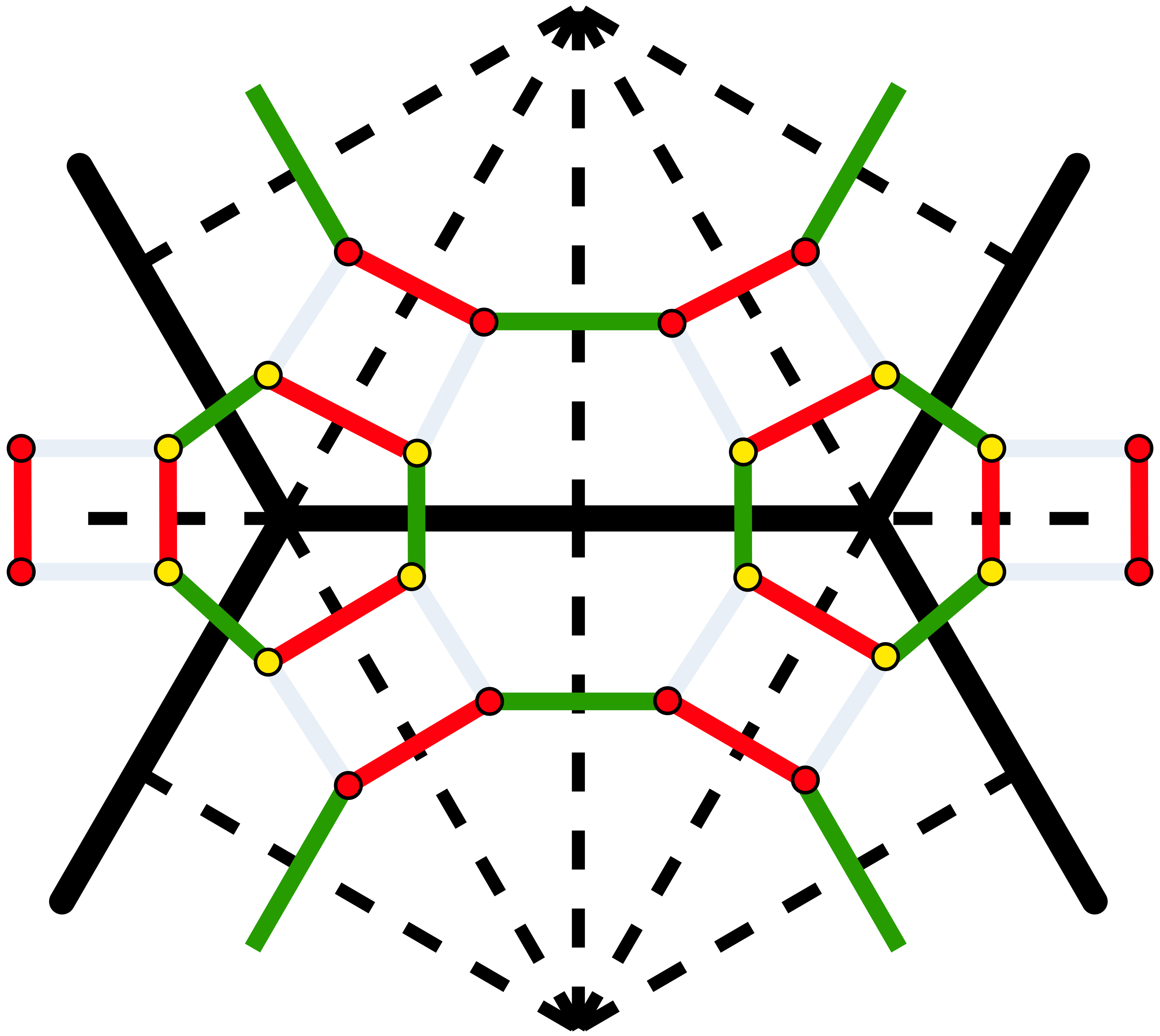
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

Asignación de voltajes

$=$

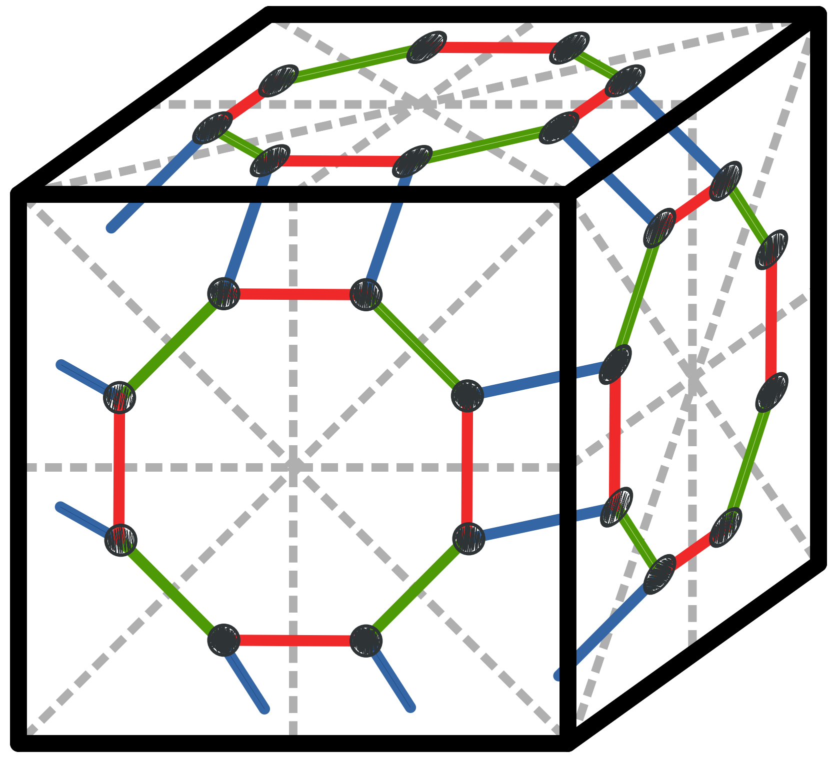


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

Operaciones de voltaje

- Un (m, n)- operador de voltaje es un par $(\mathscr{Y}, \eta)$



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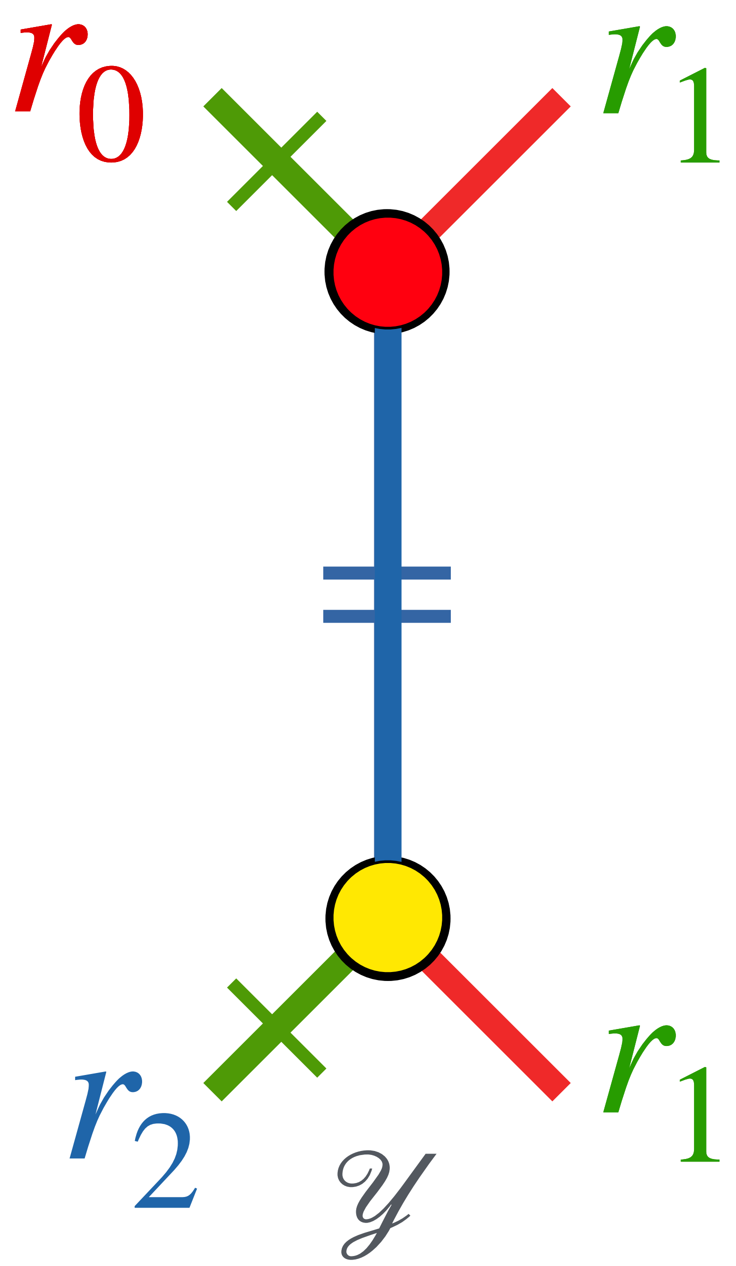

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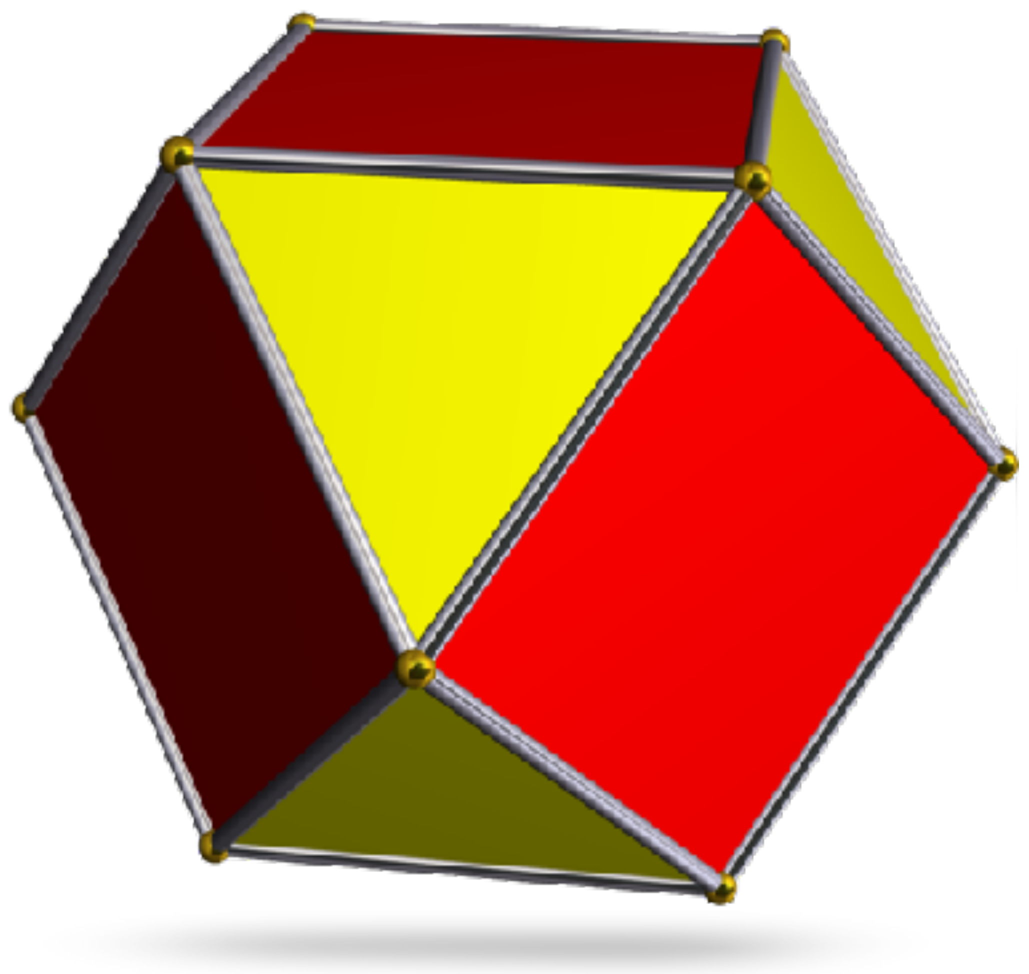

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m-premaniplex

n-premaniplex

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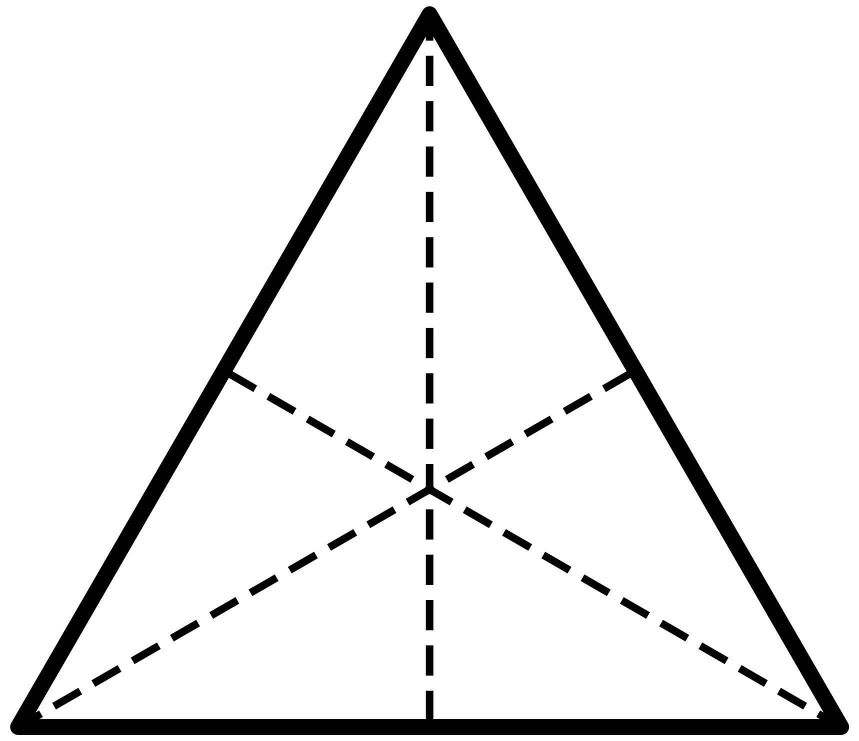
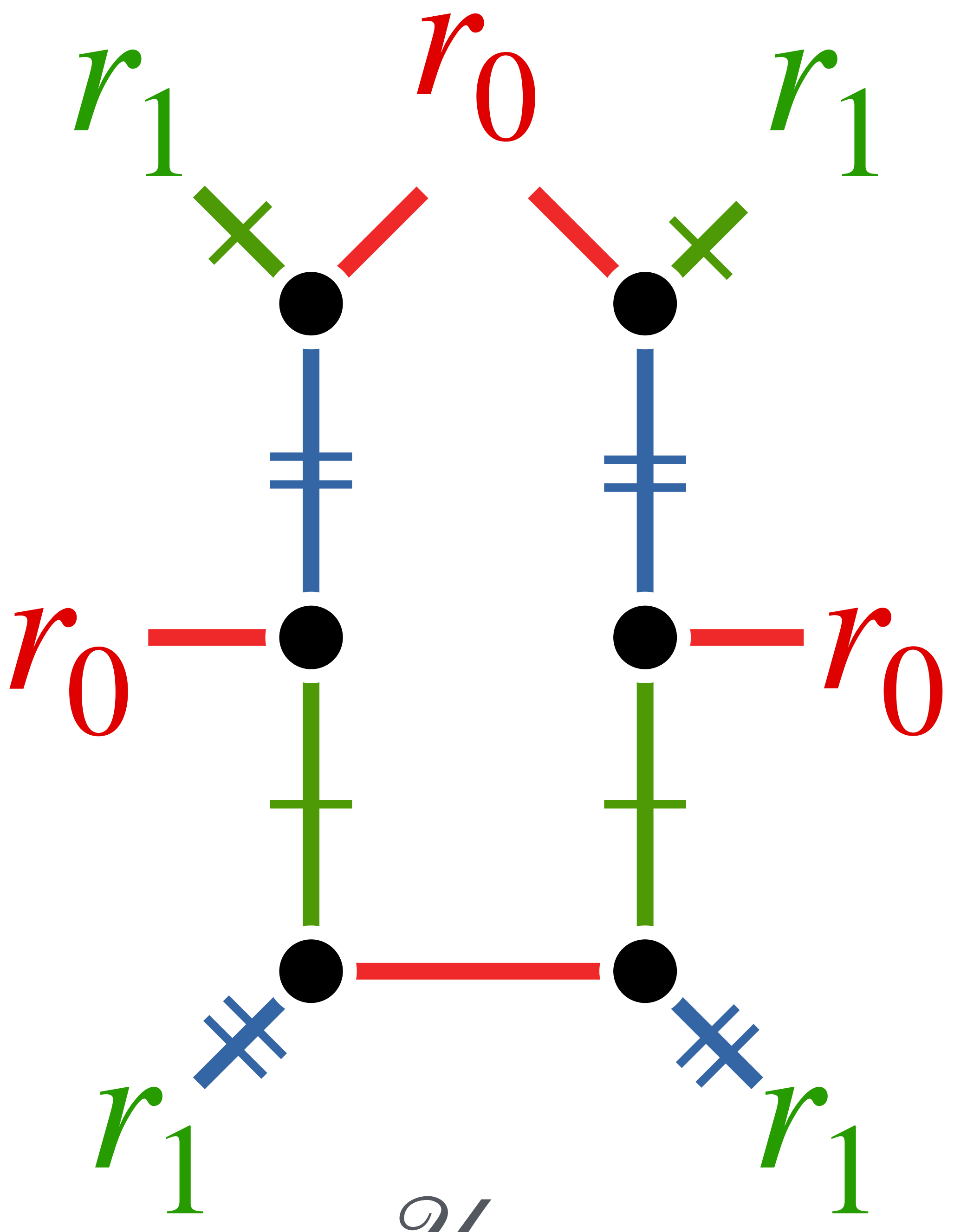


$\mathscr{X}$

m-premaniplex

$\rtimes$



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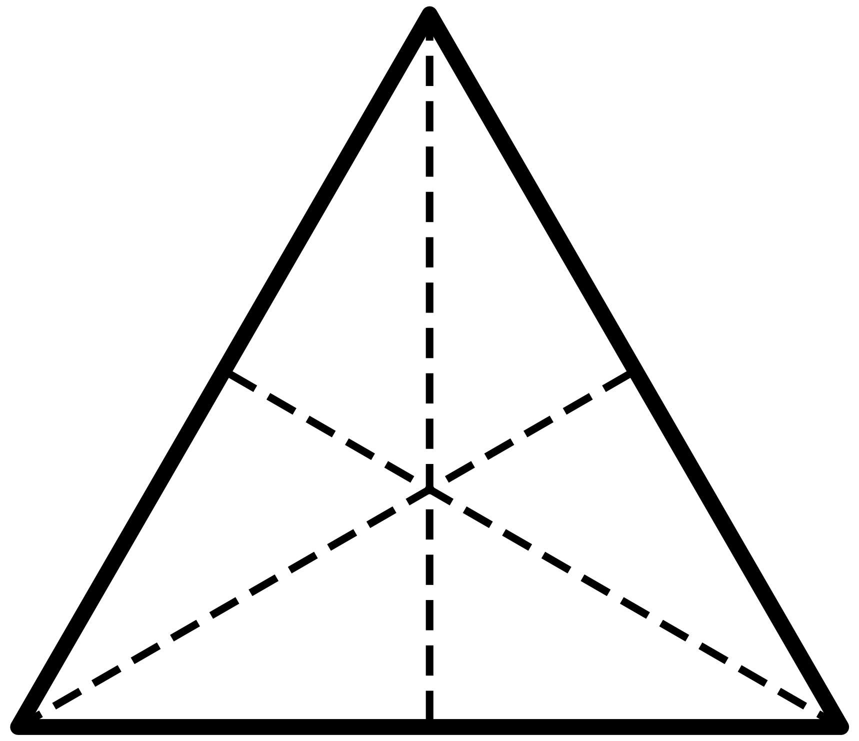

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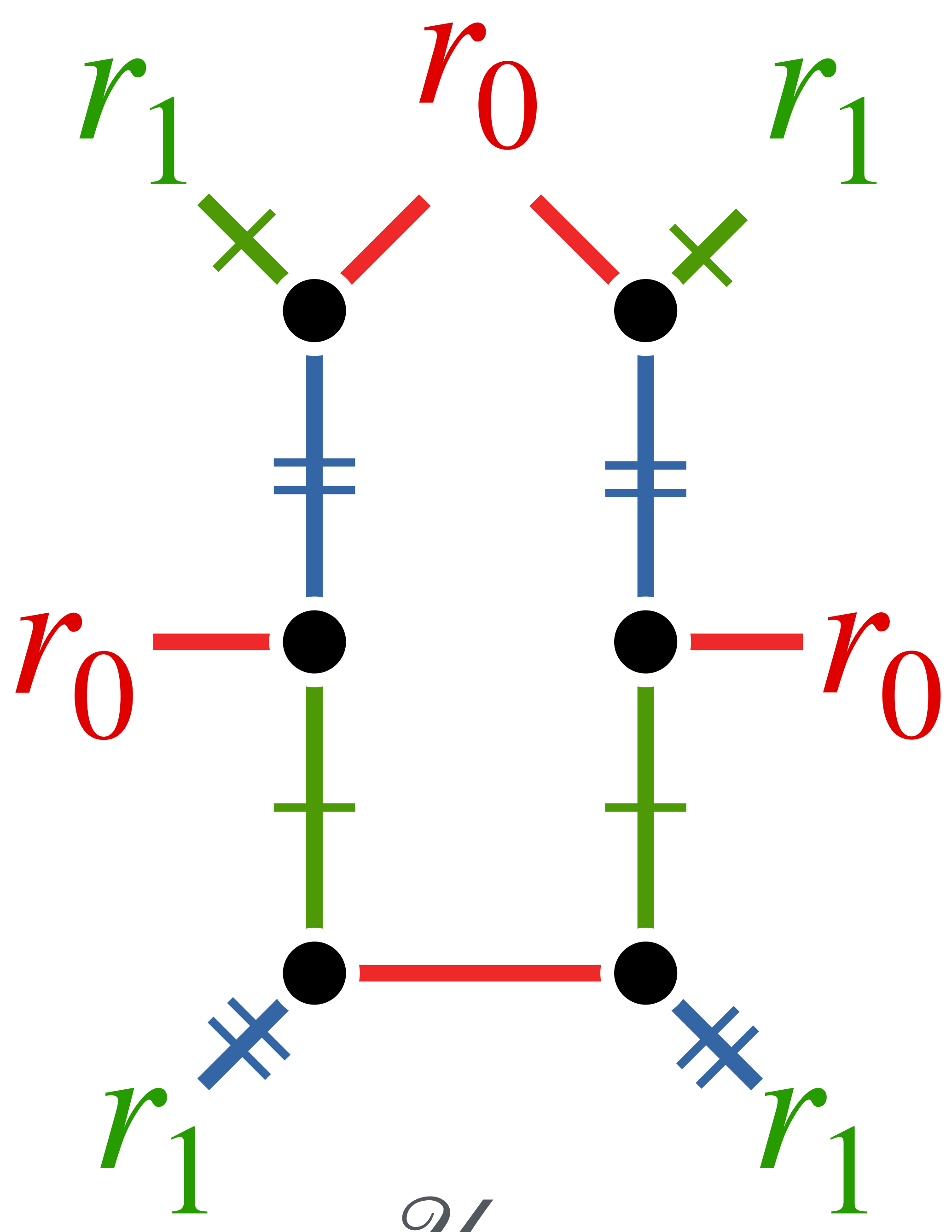

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Asignación de voltajes

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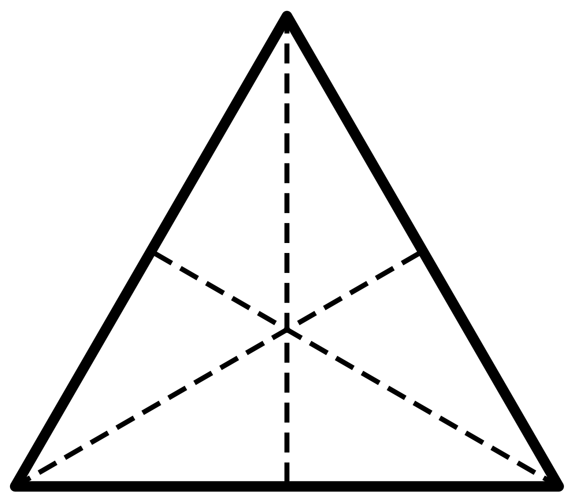

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n-premaniplex

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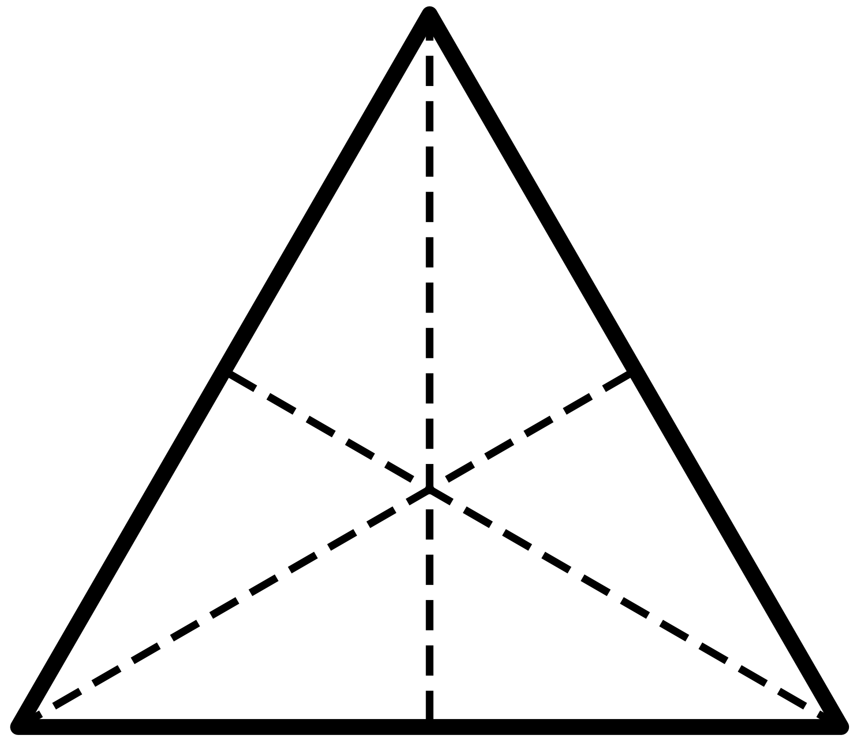
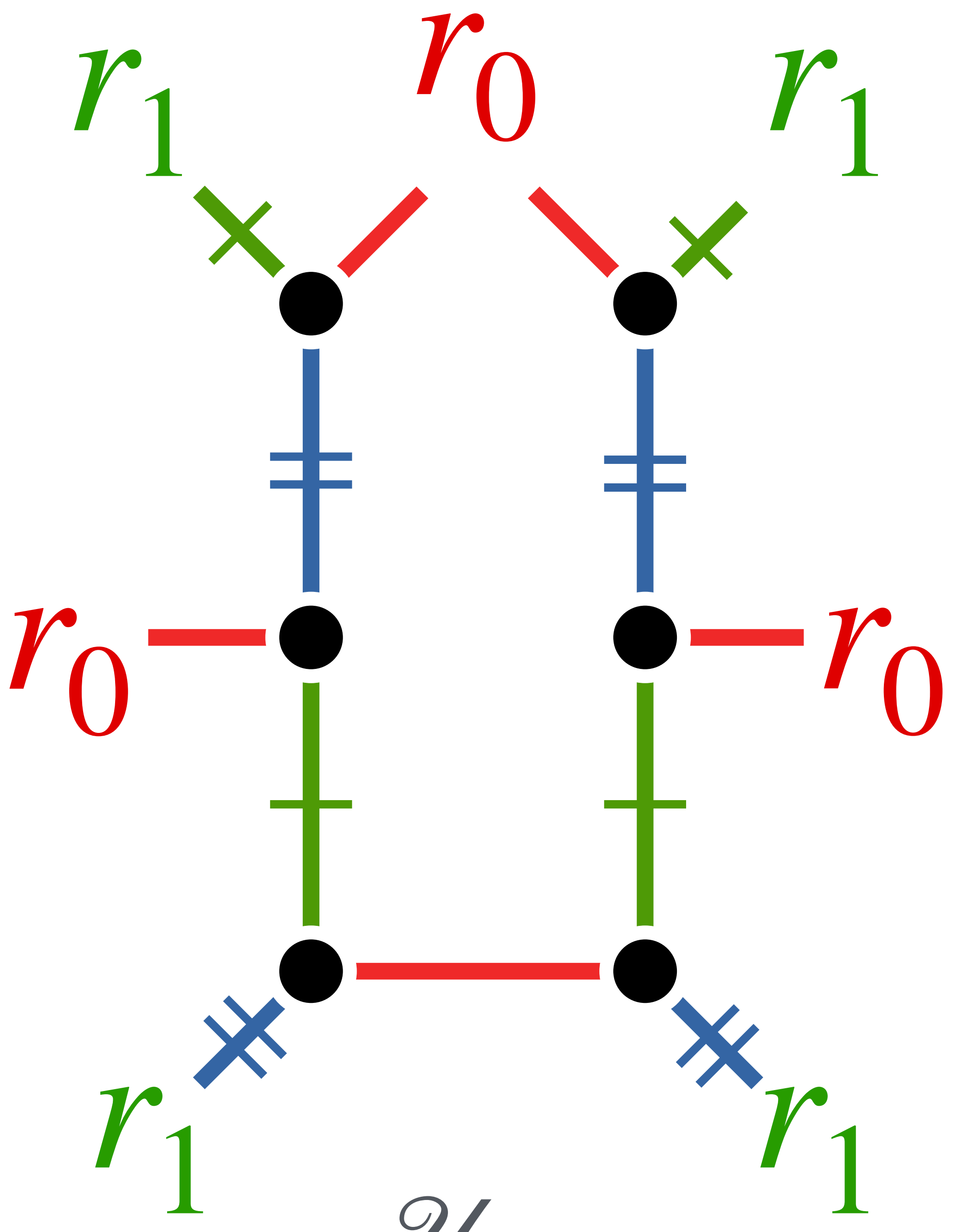


$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

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Asignación de voltajes

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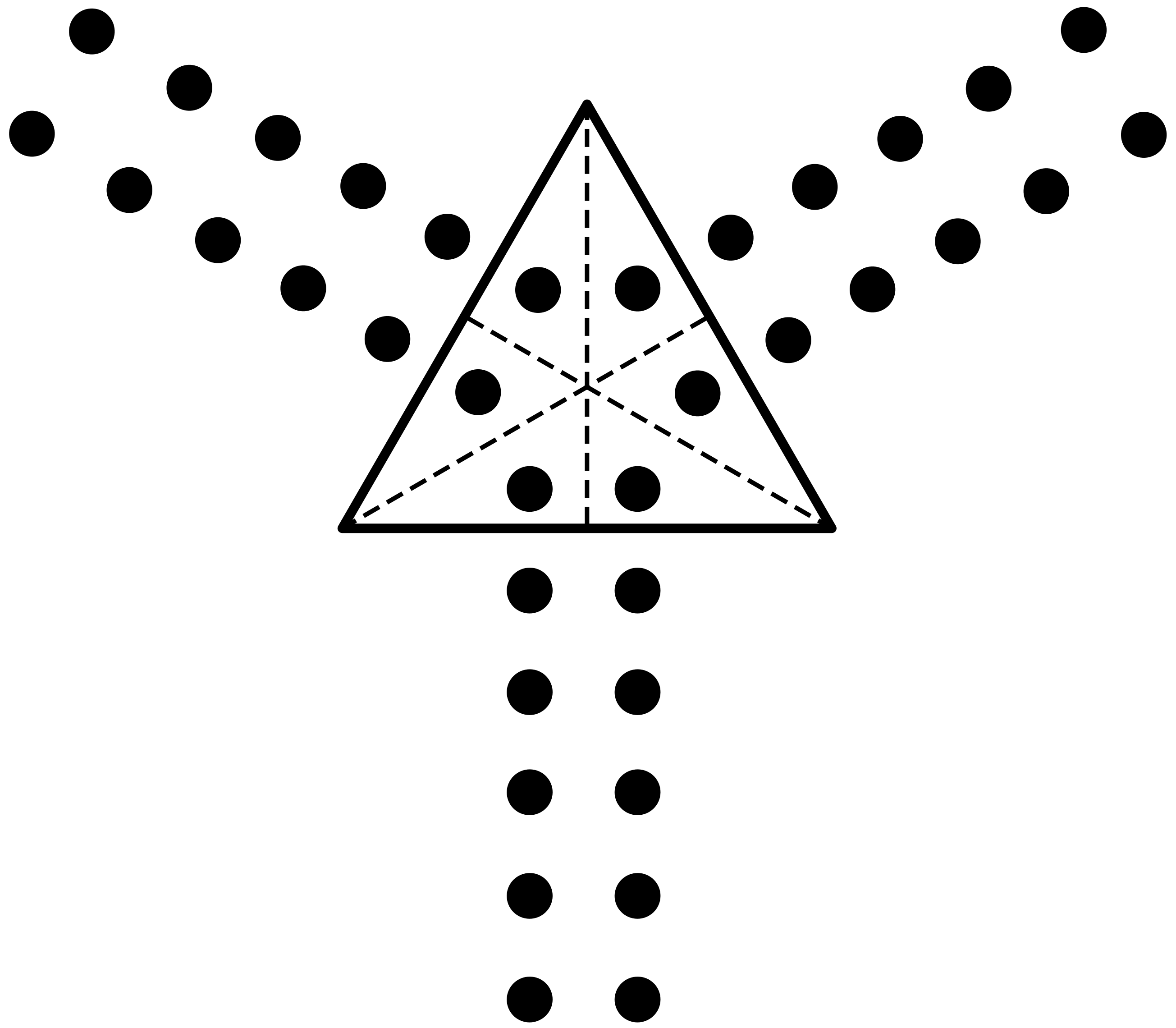

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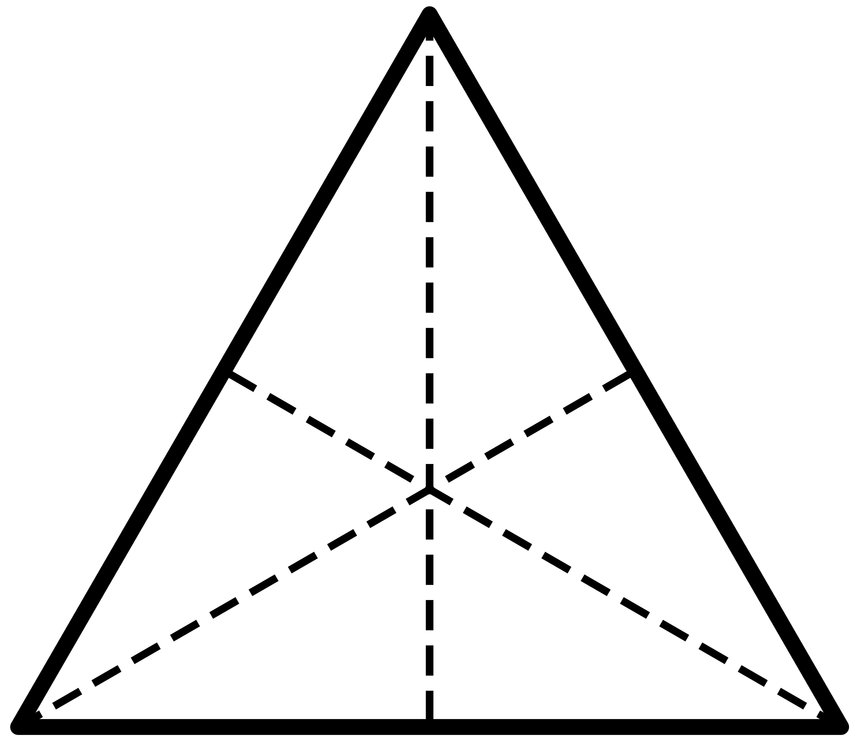
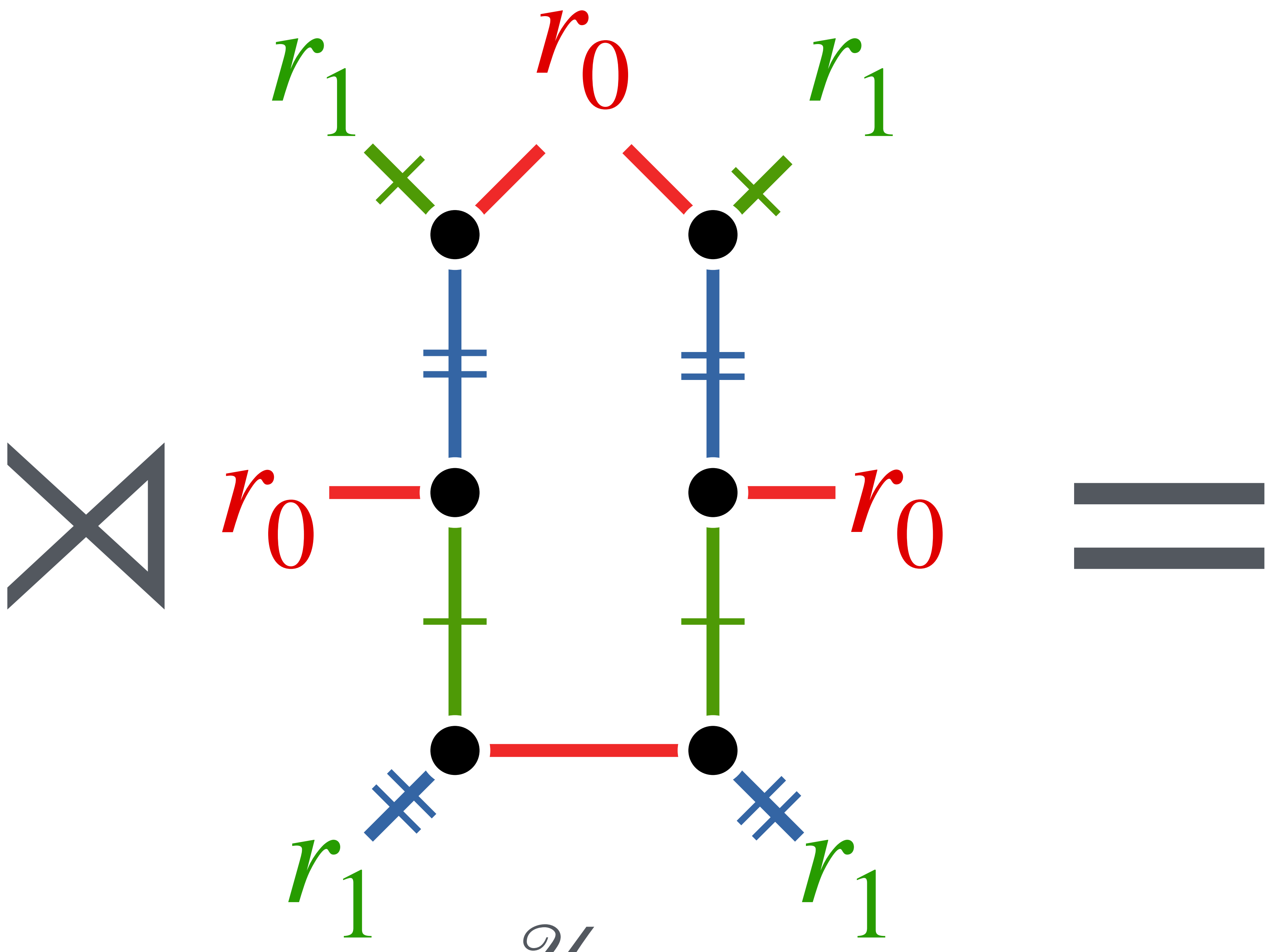


$\mathscr{X}$

m-premaniplex



$\mathscr{Y}$

n-premaniplex

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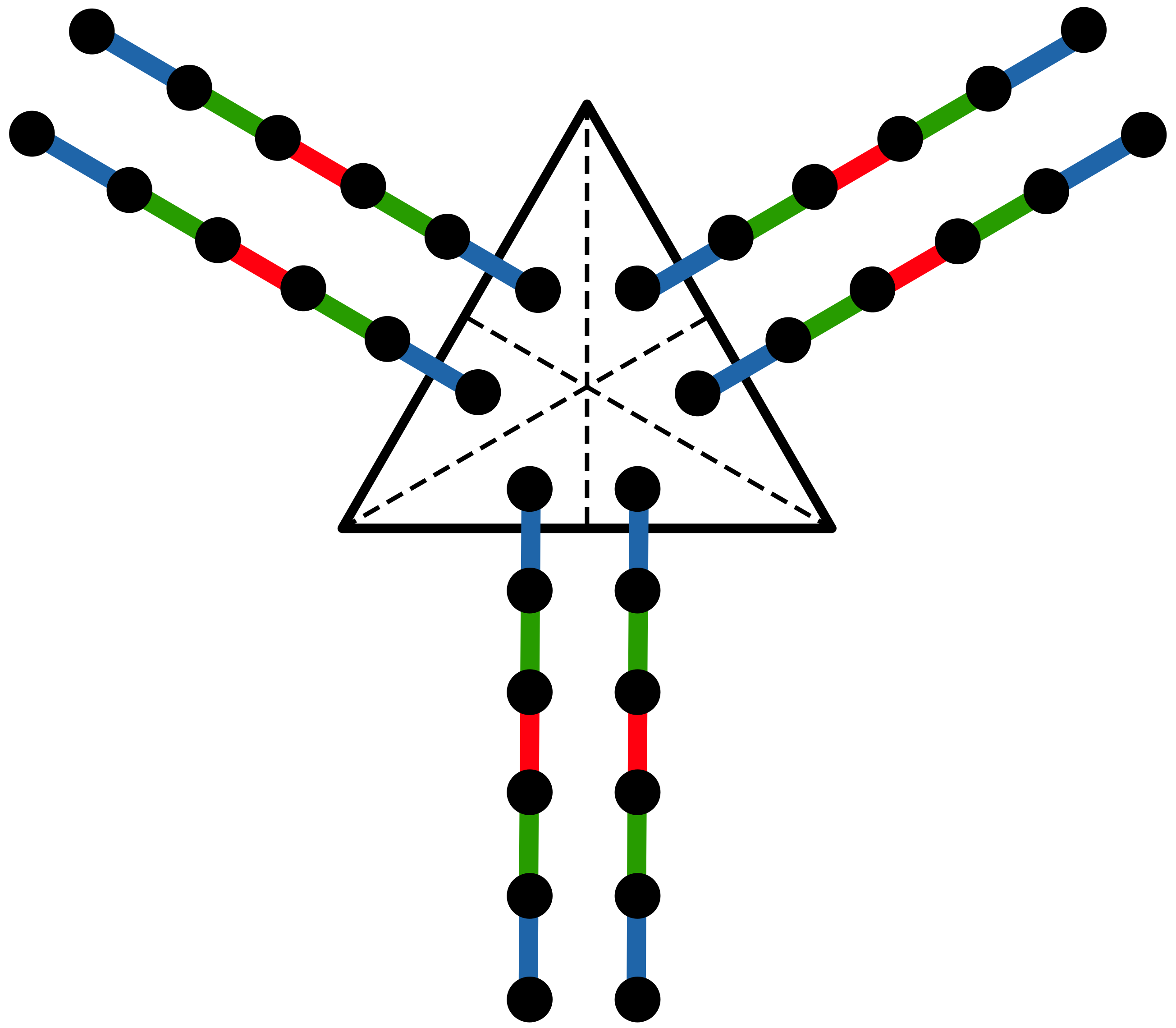
Asignación de voltajes



$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

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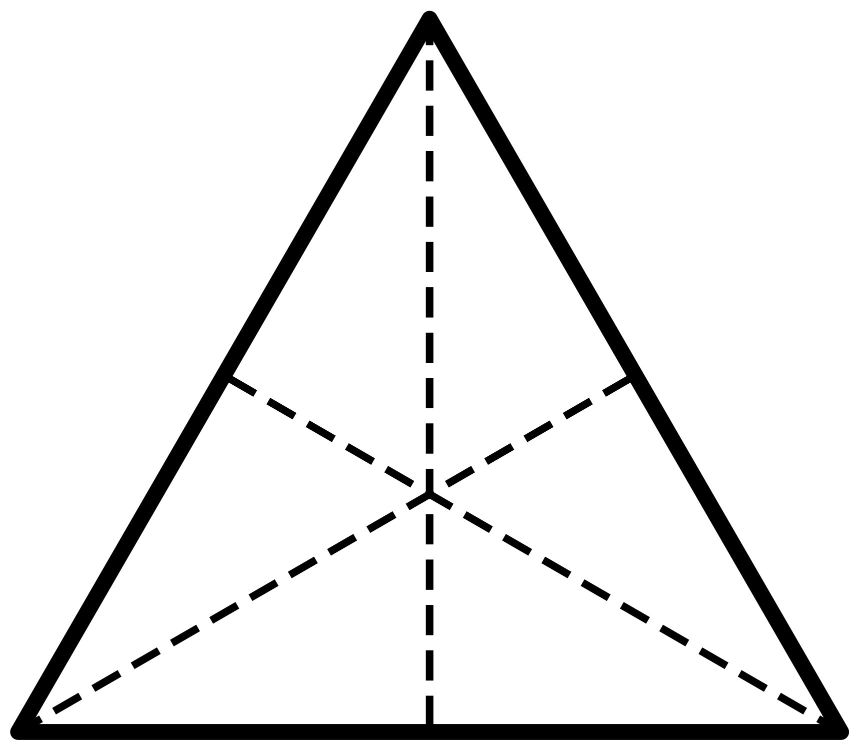
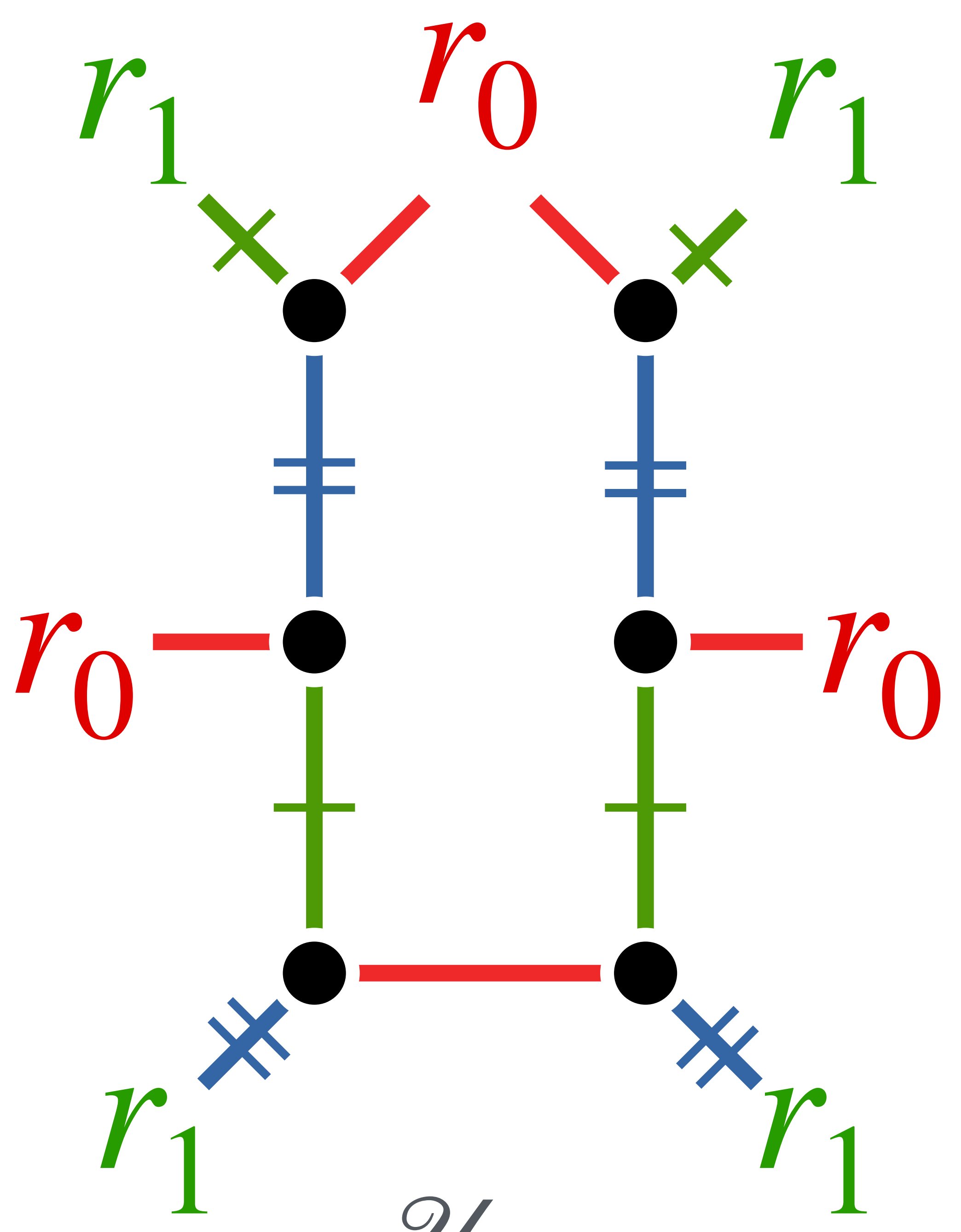


$\mathcal{X}$

m-premaniplex

$\rtimes$



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Asignación de voltajes

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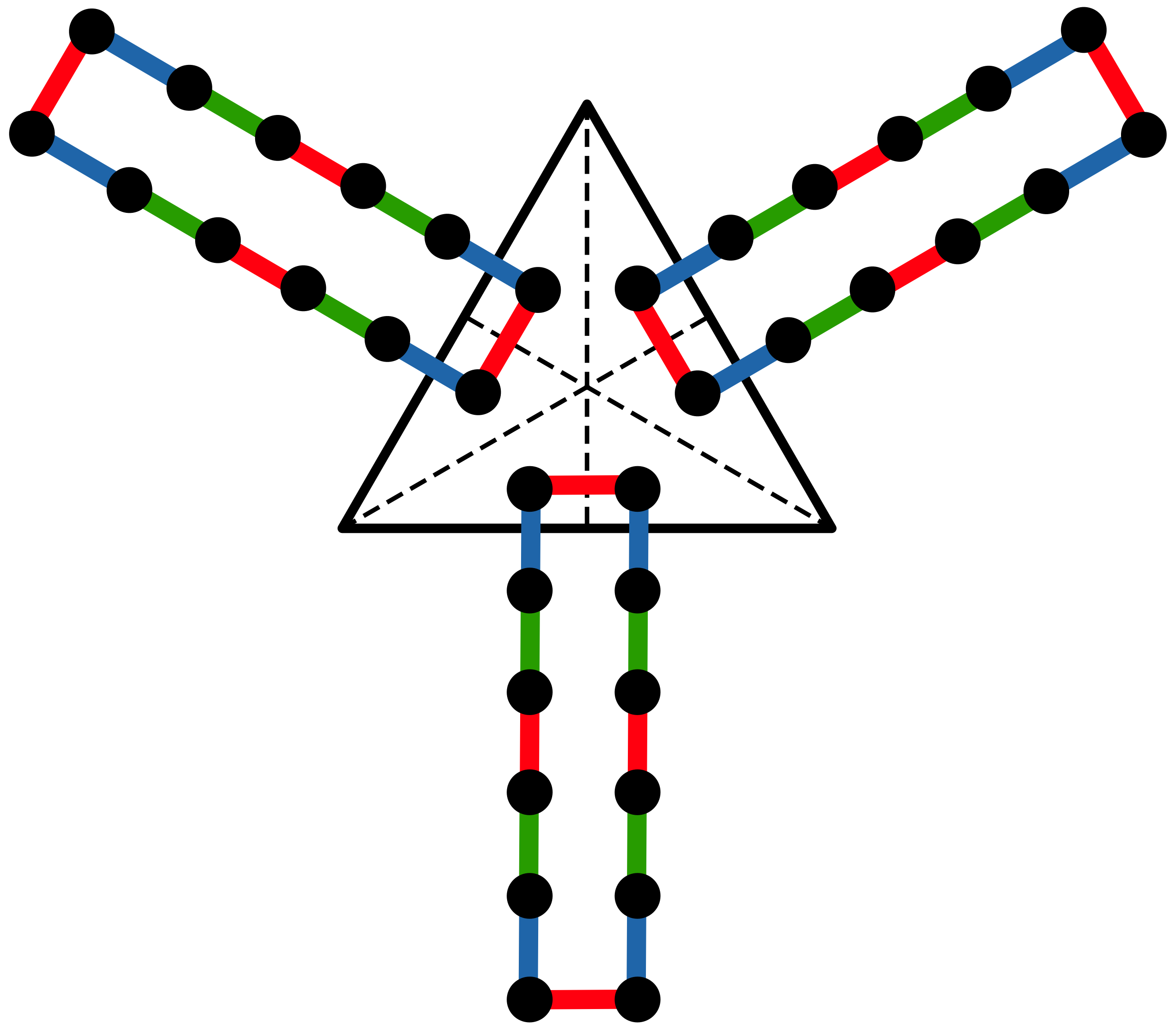

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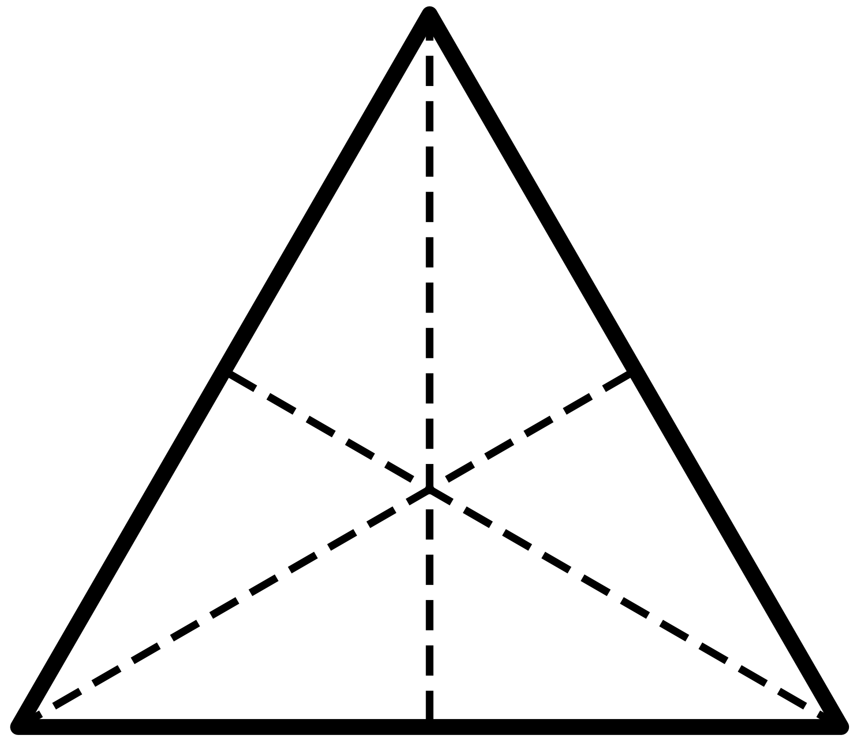

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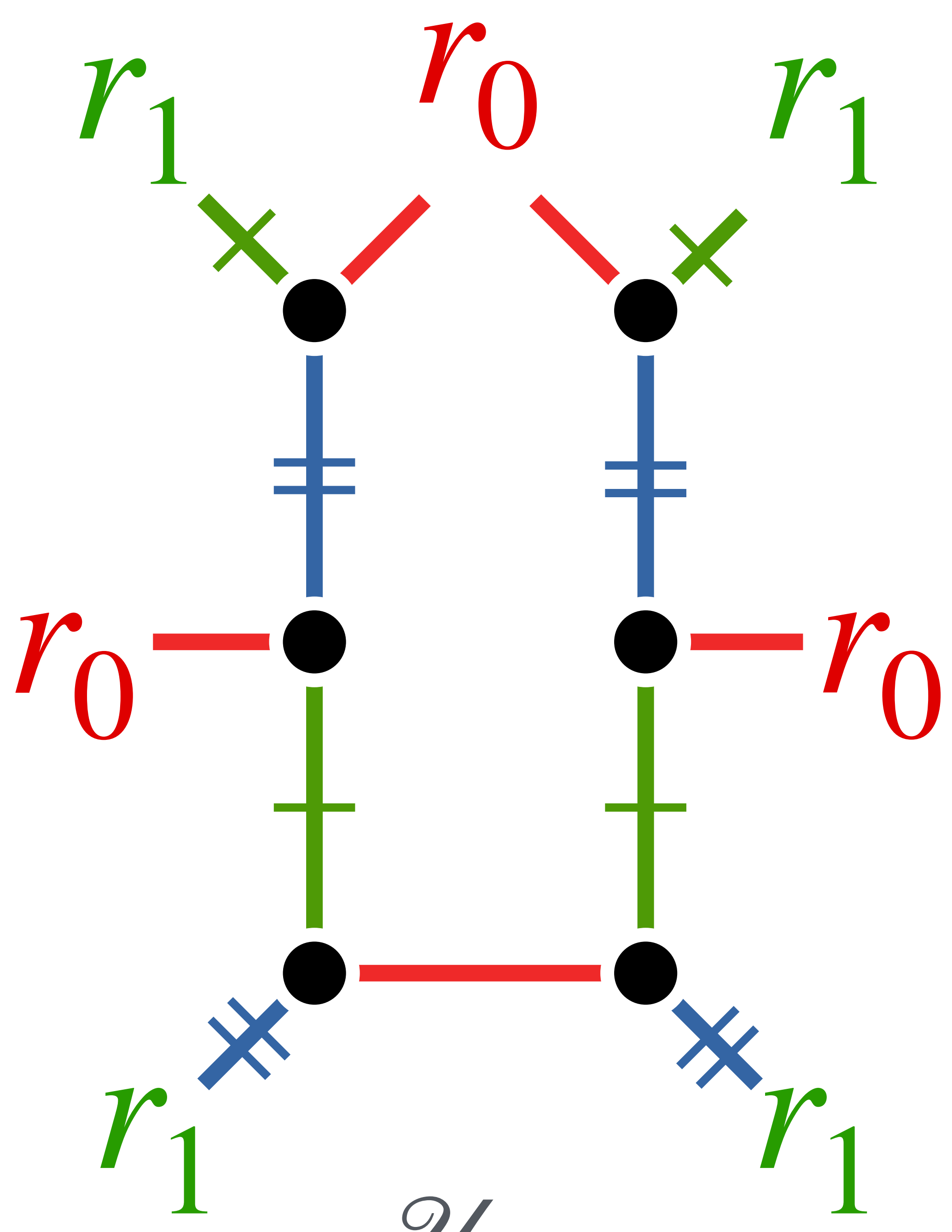

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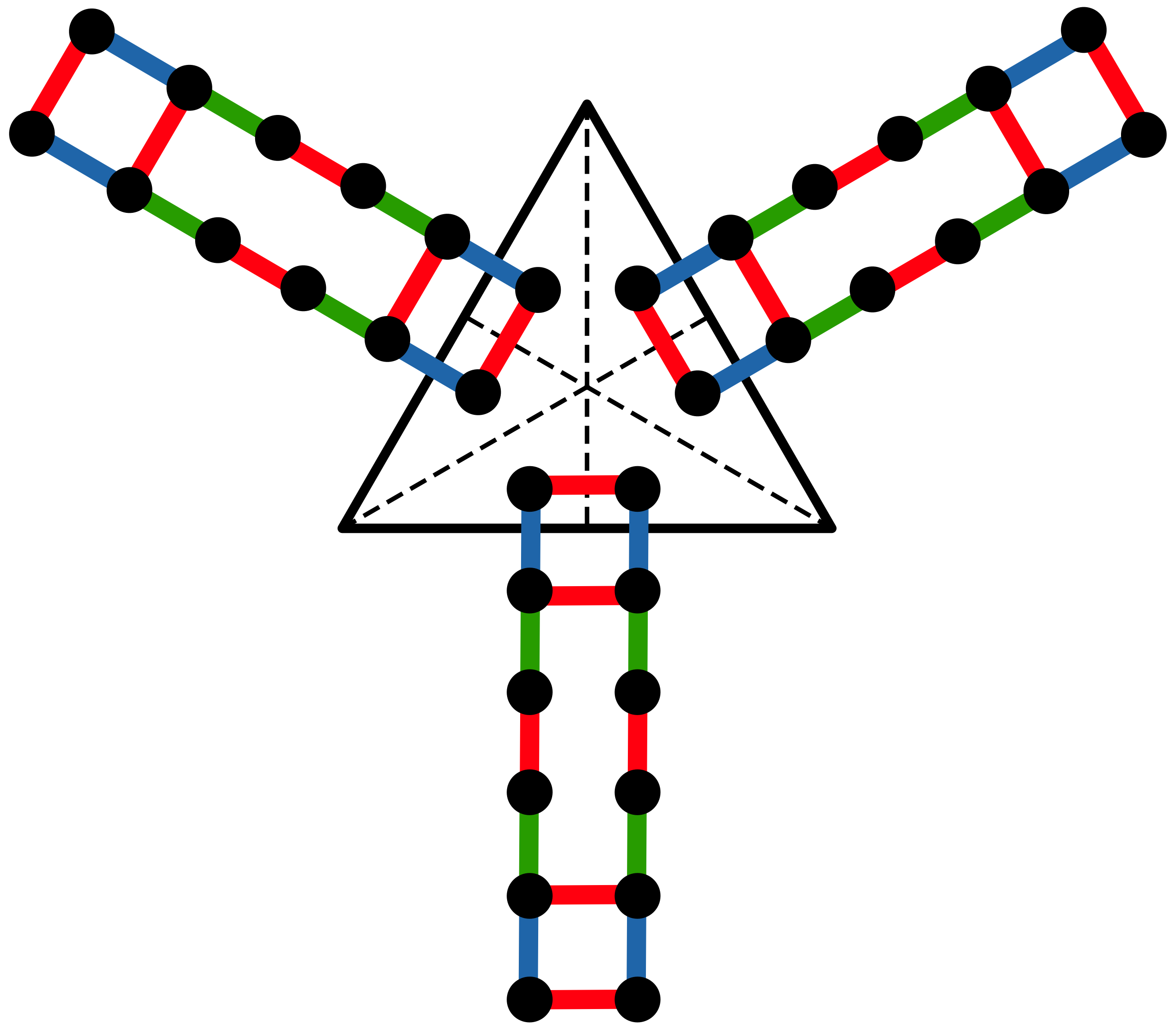

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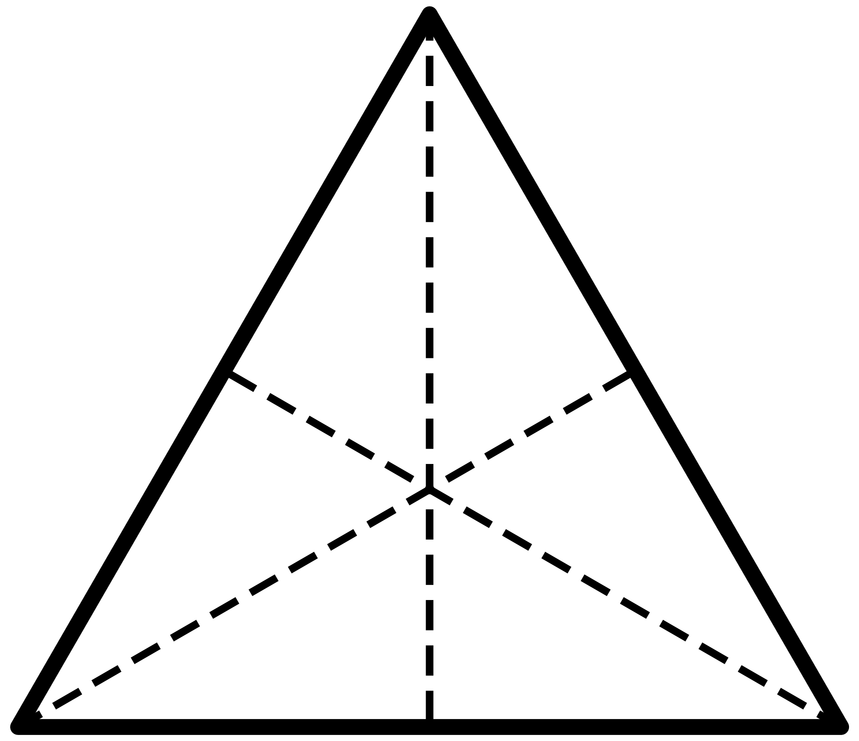

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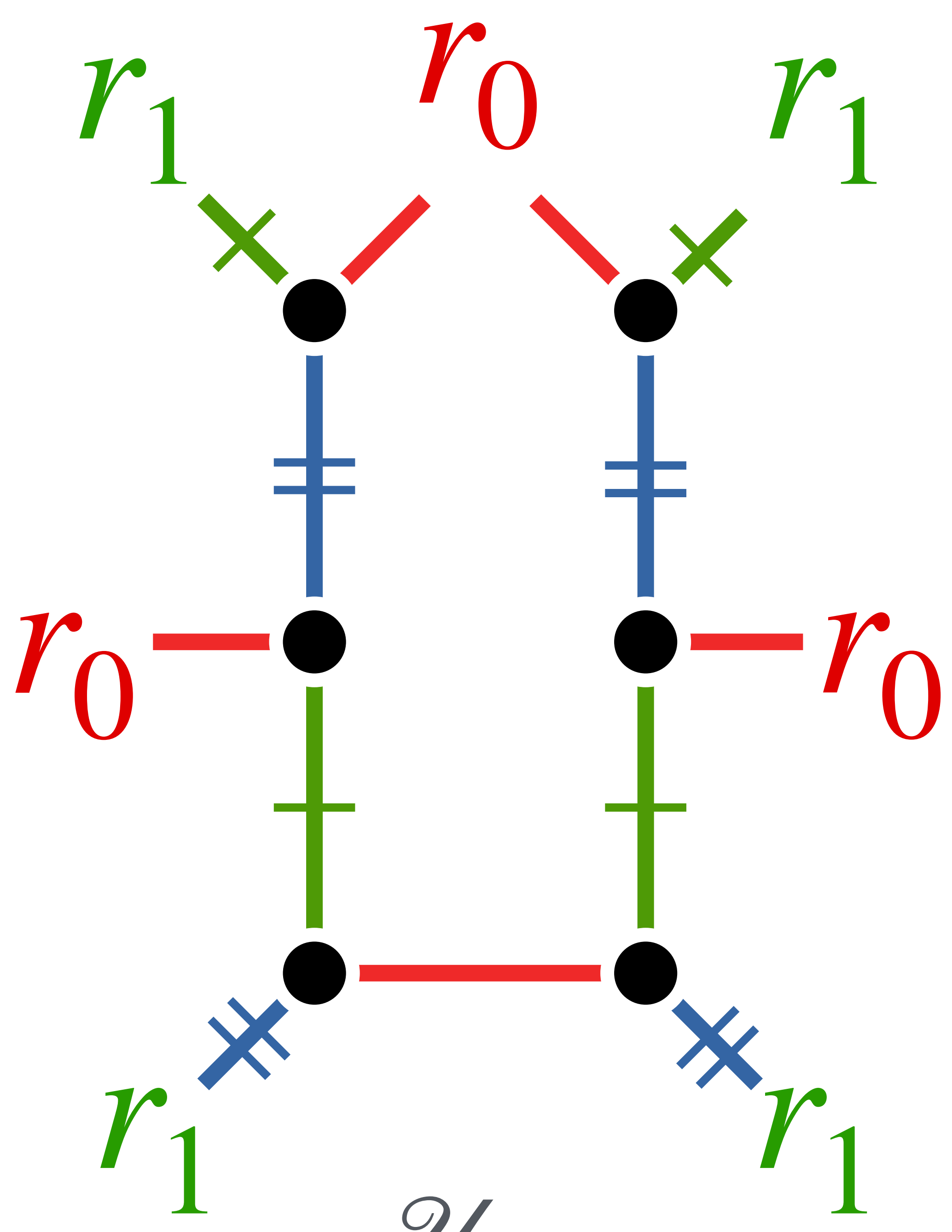

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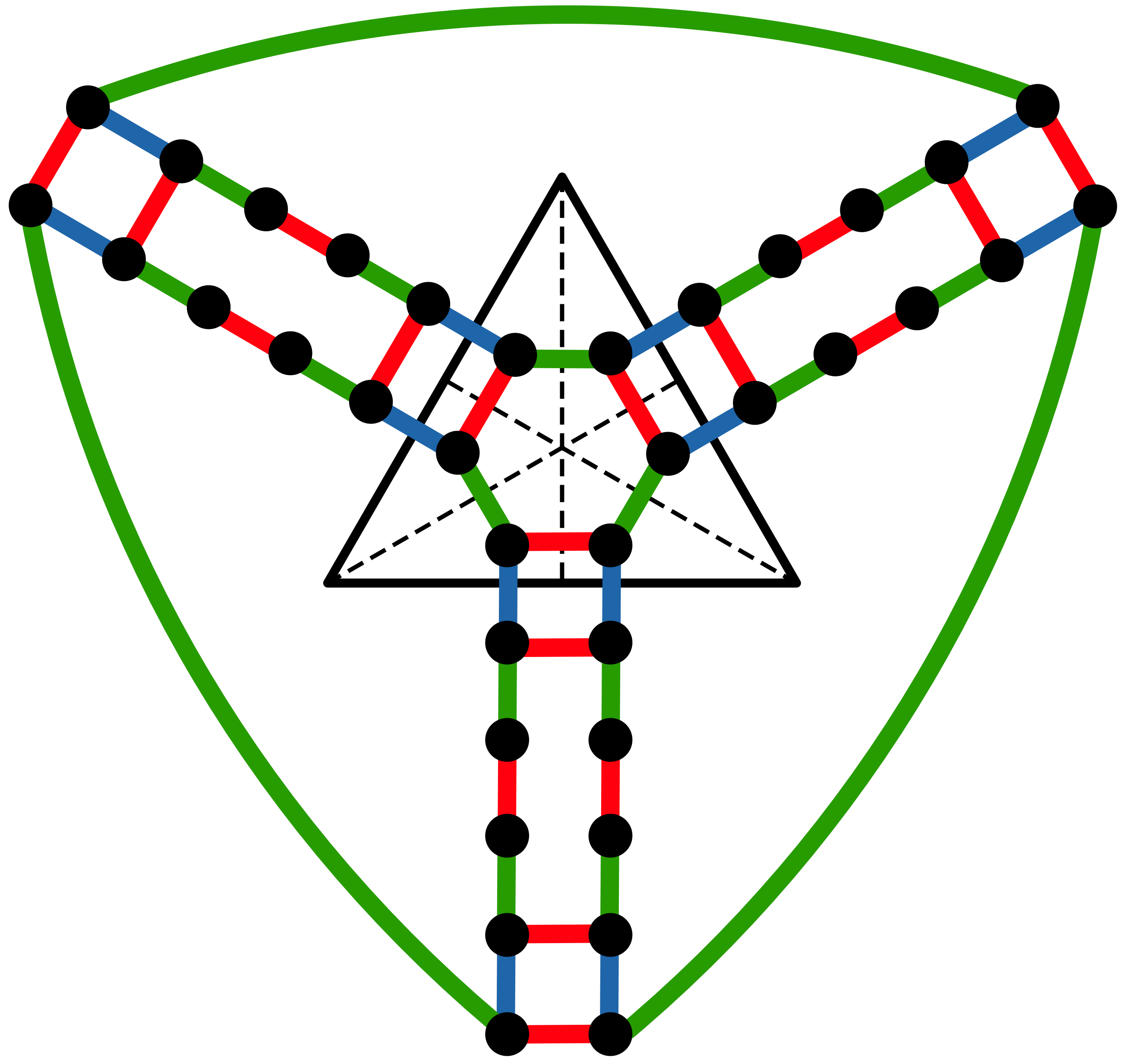

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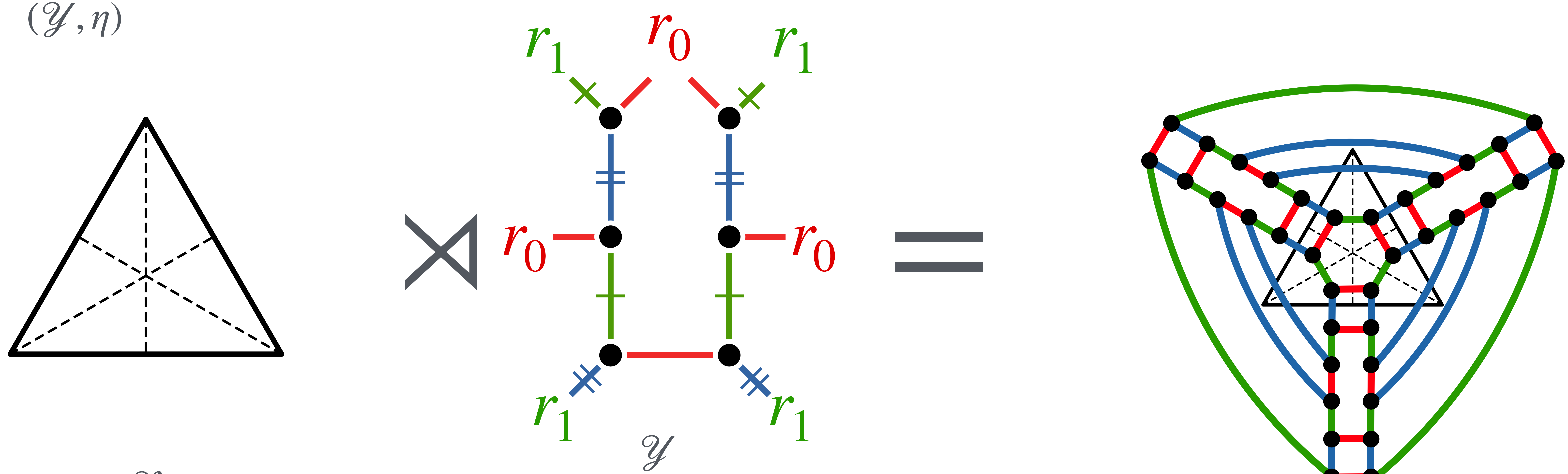

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m-premaniplex

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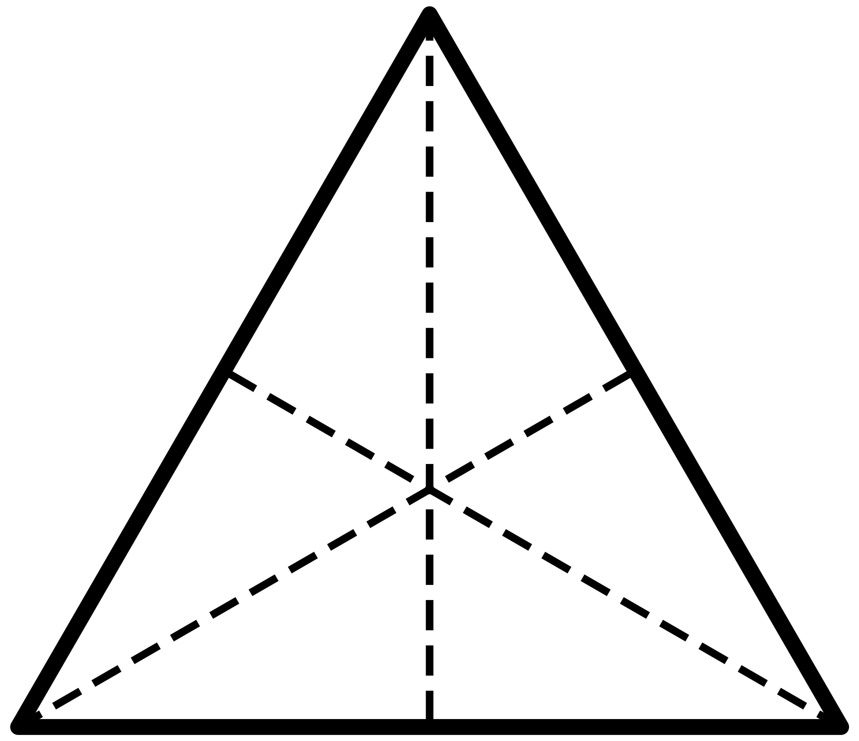

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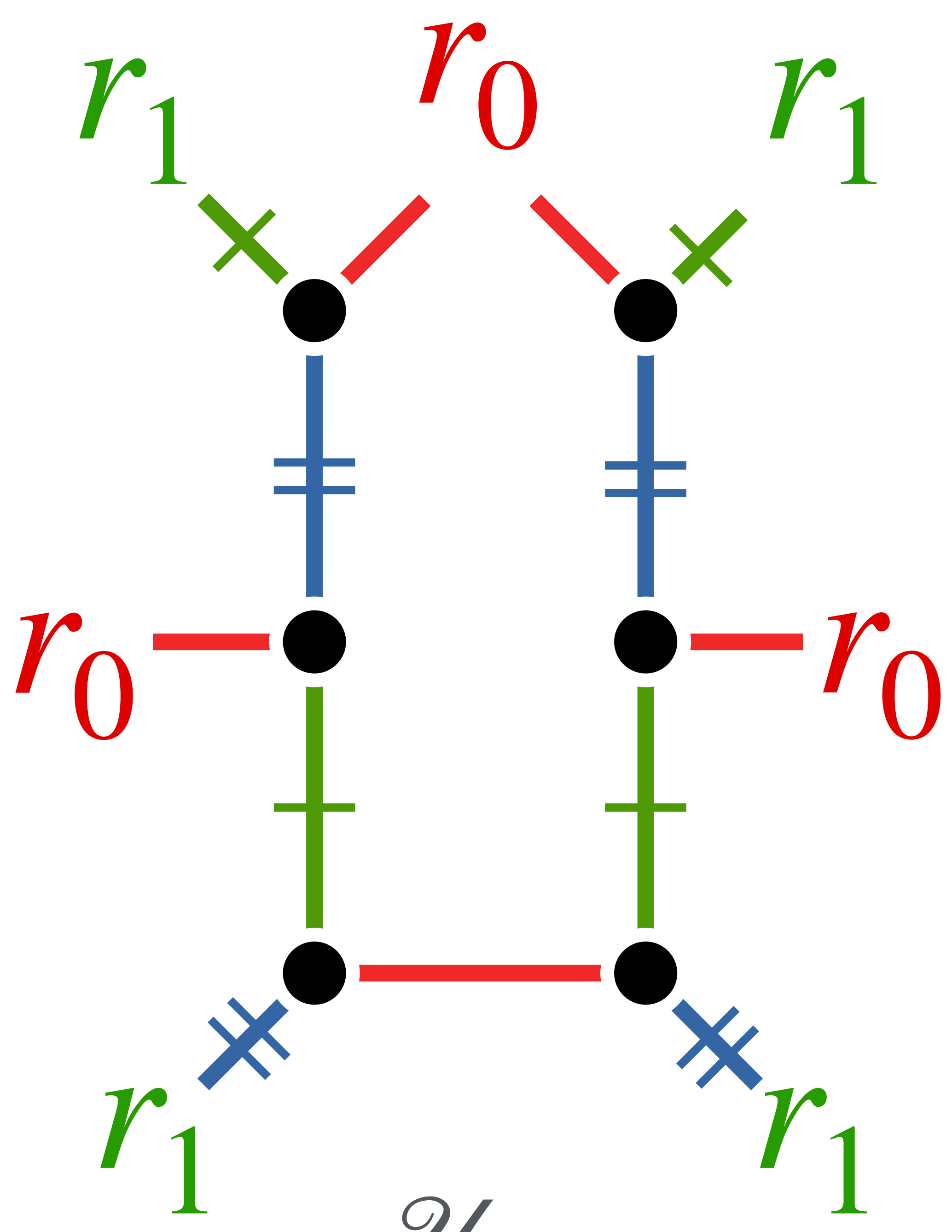

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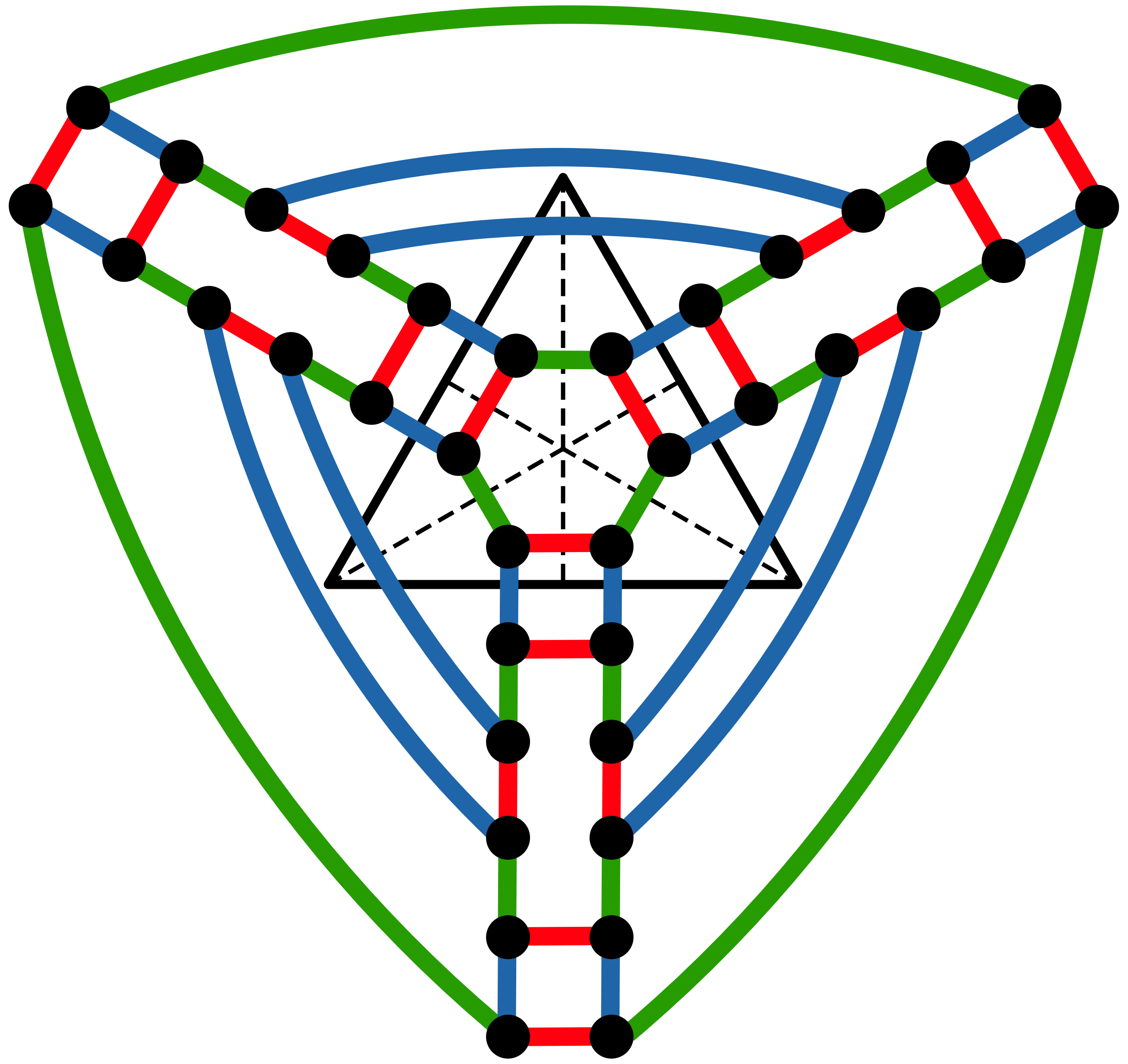

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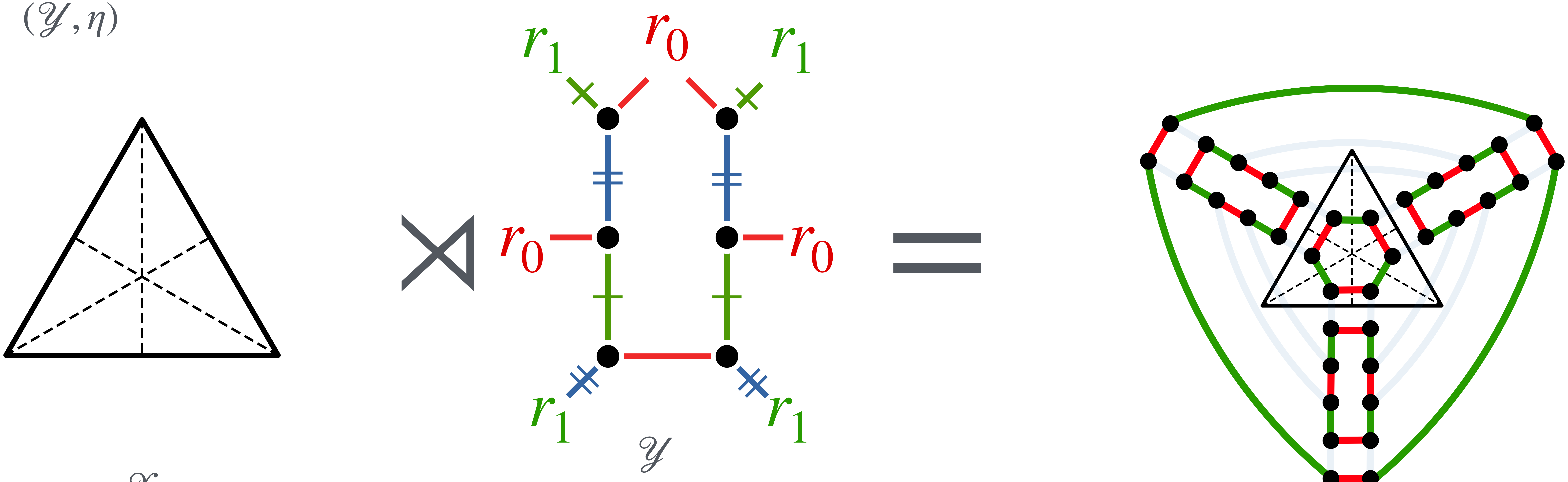

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m-premaniplex

$\mathscr{Y}$

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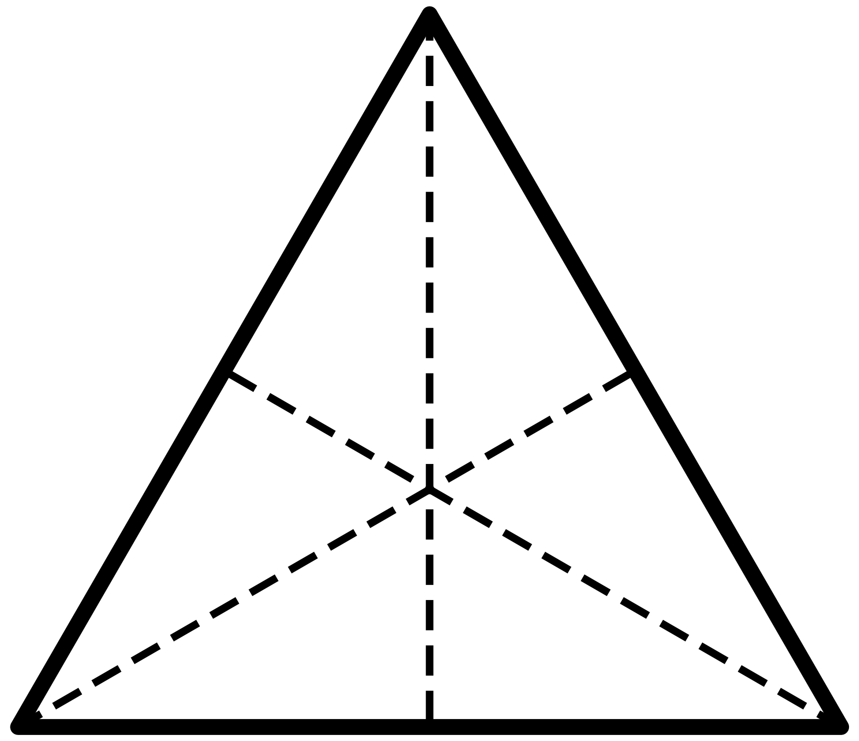

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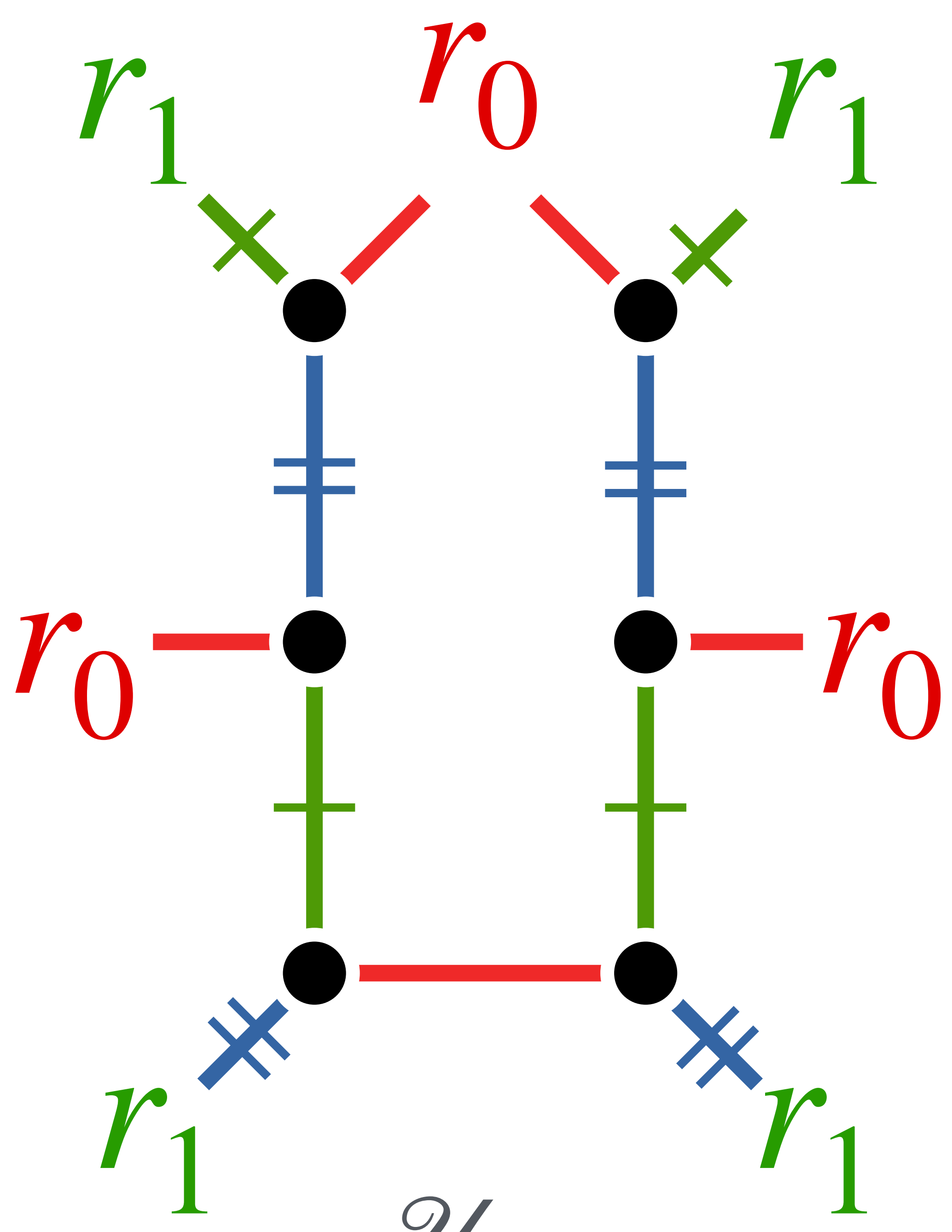

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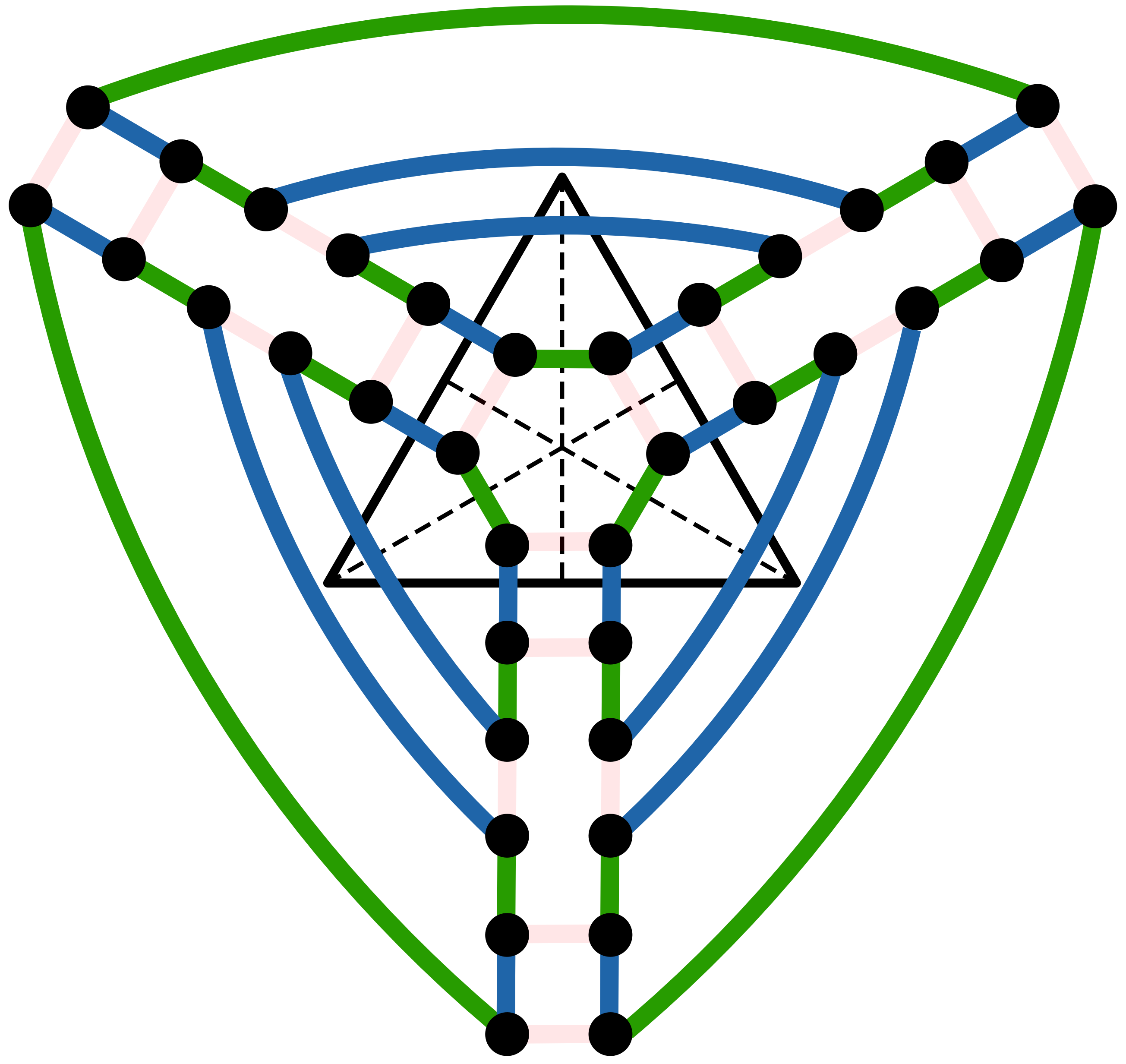

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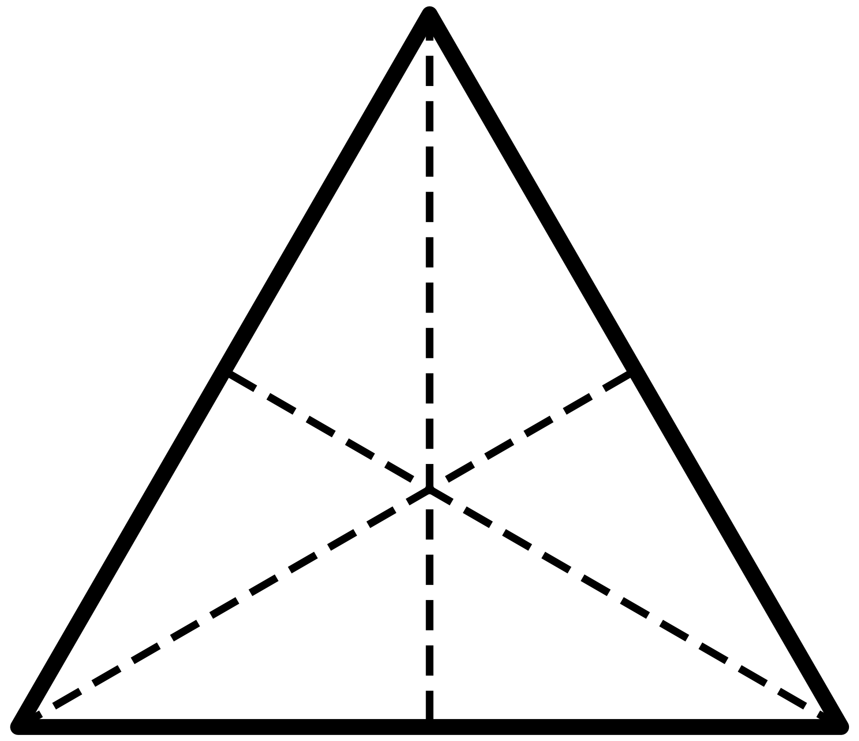

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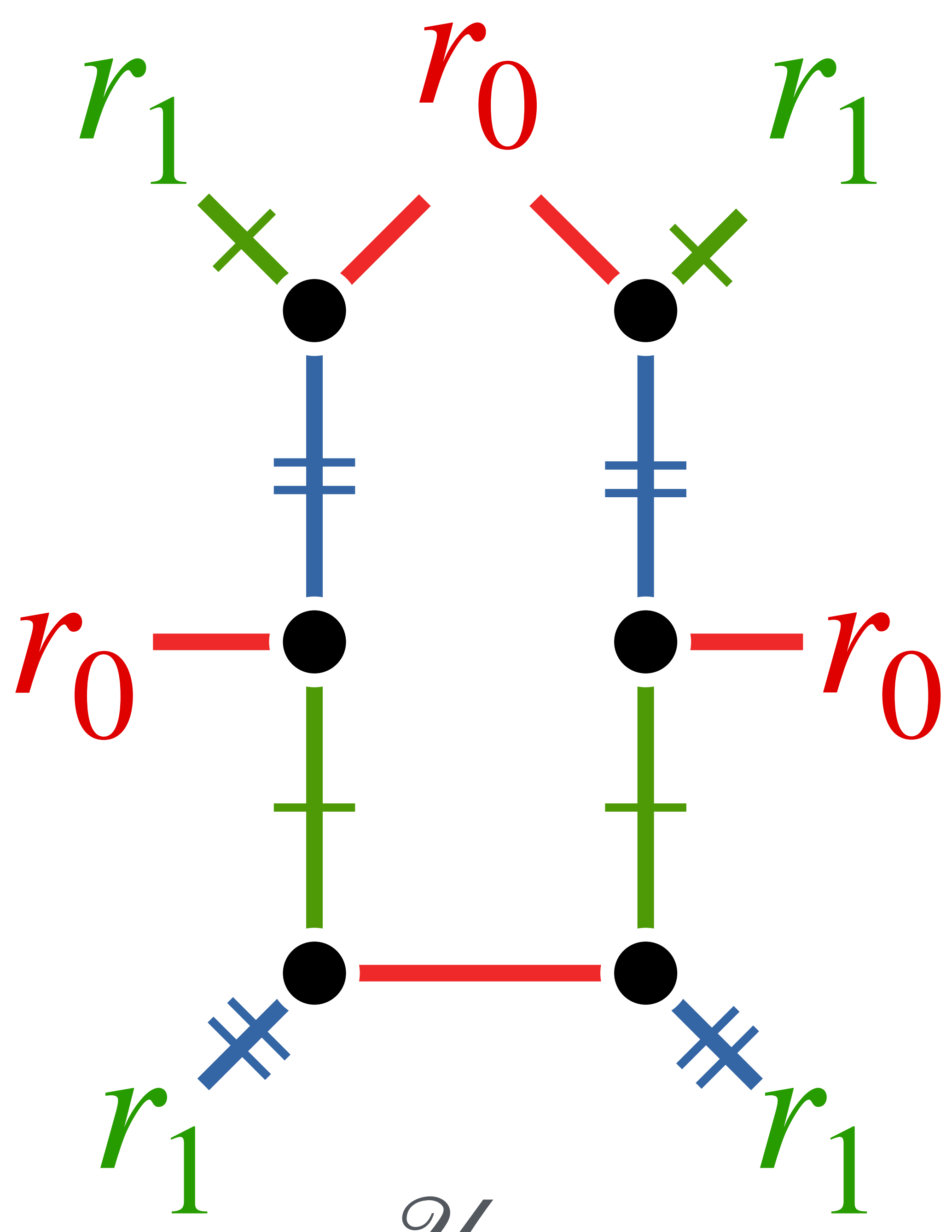

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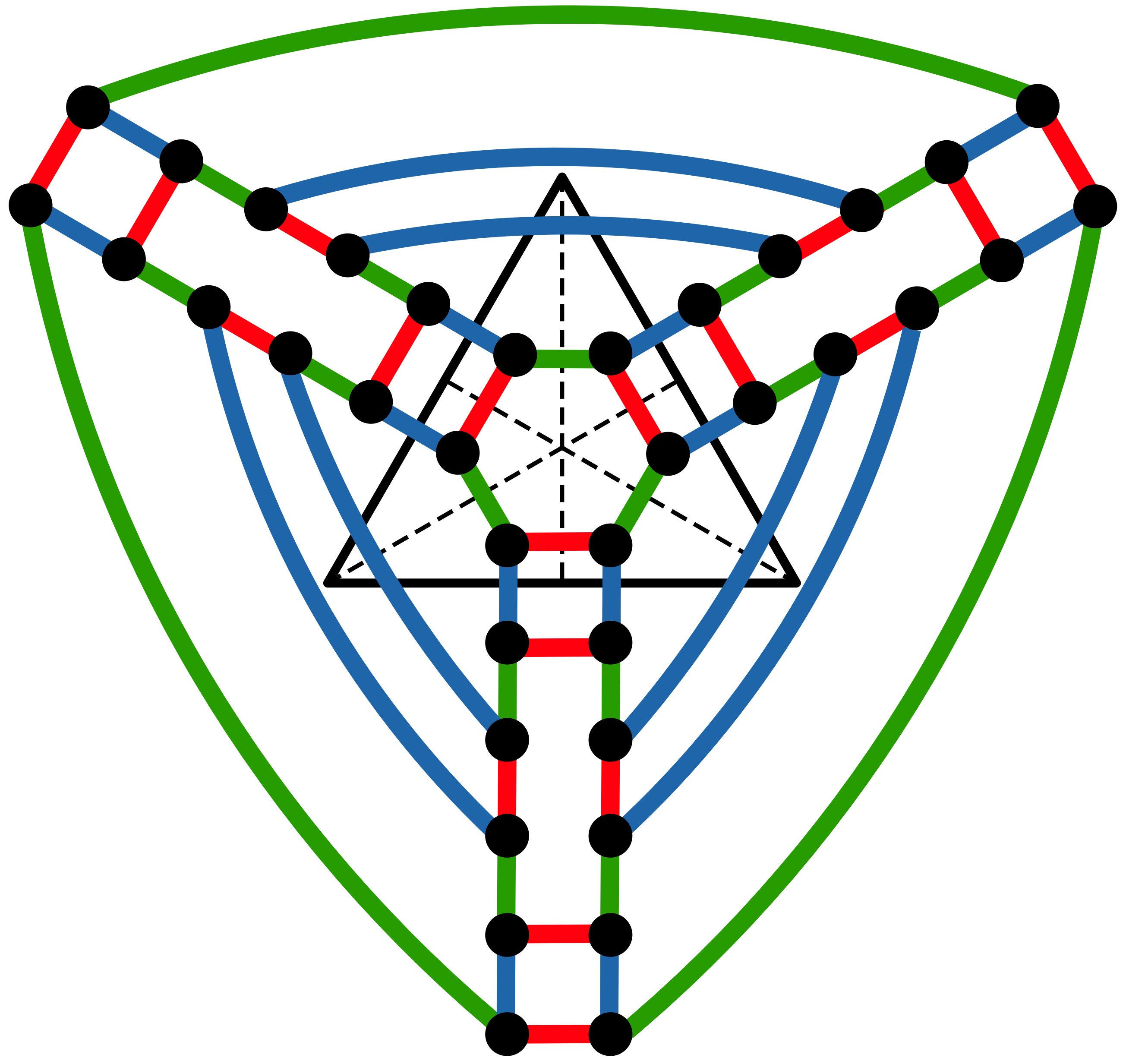

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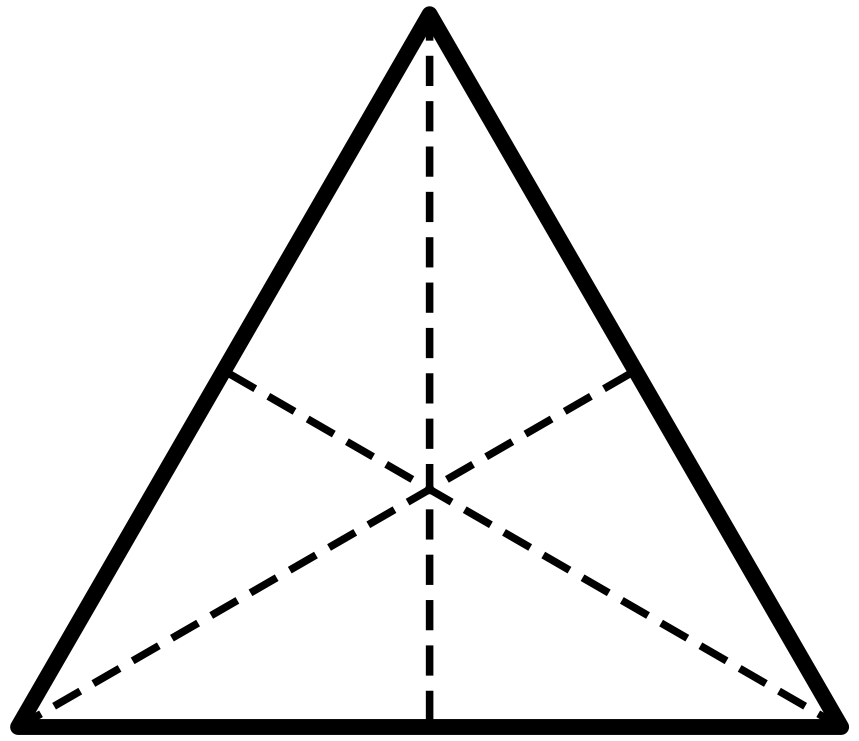

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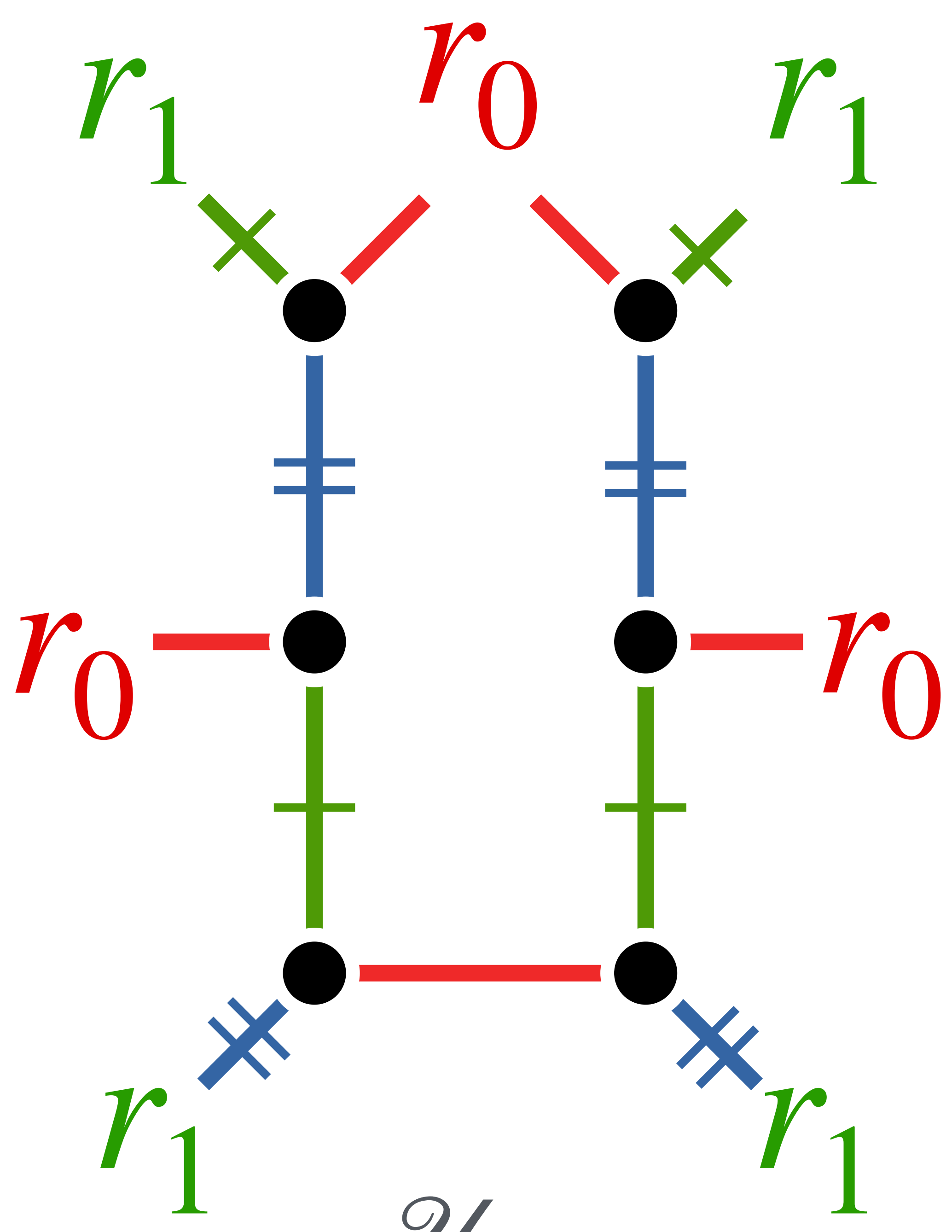

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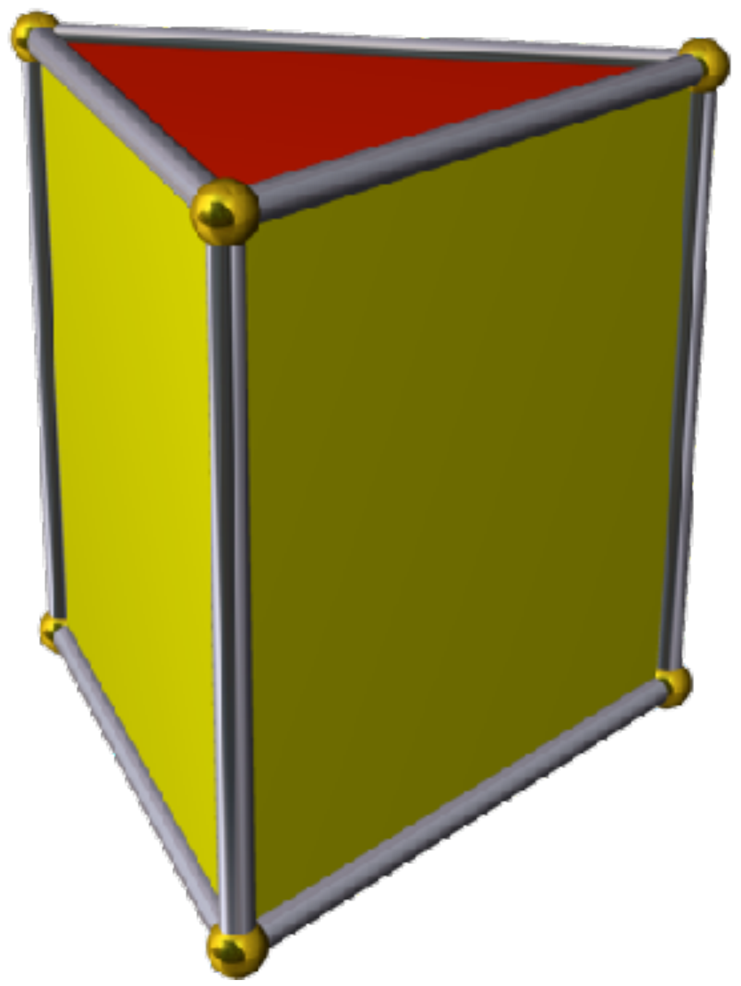

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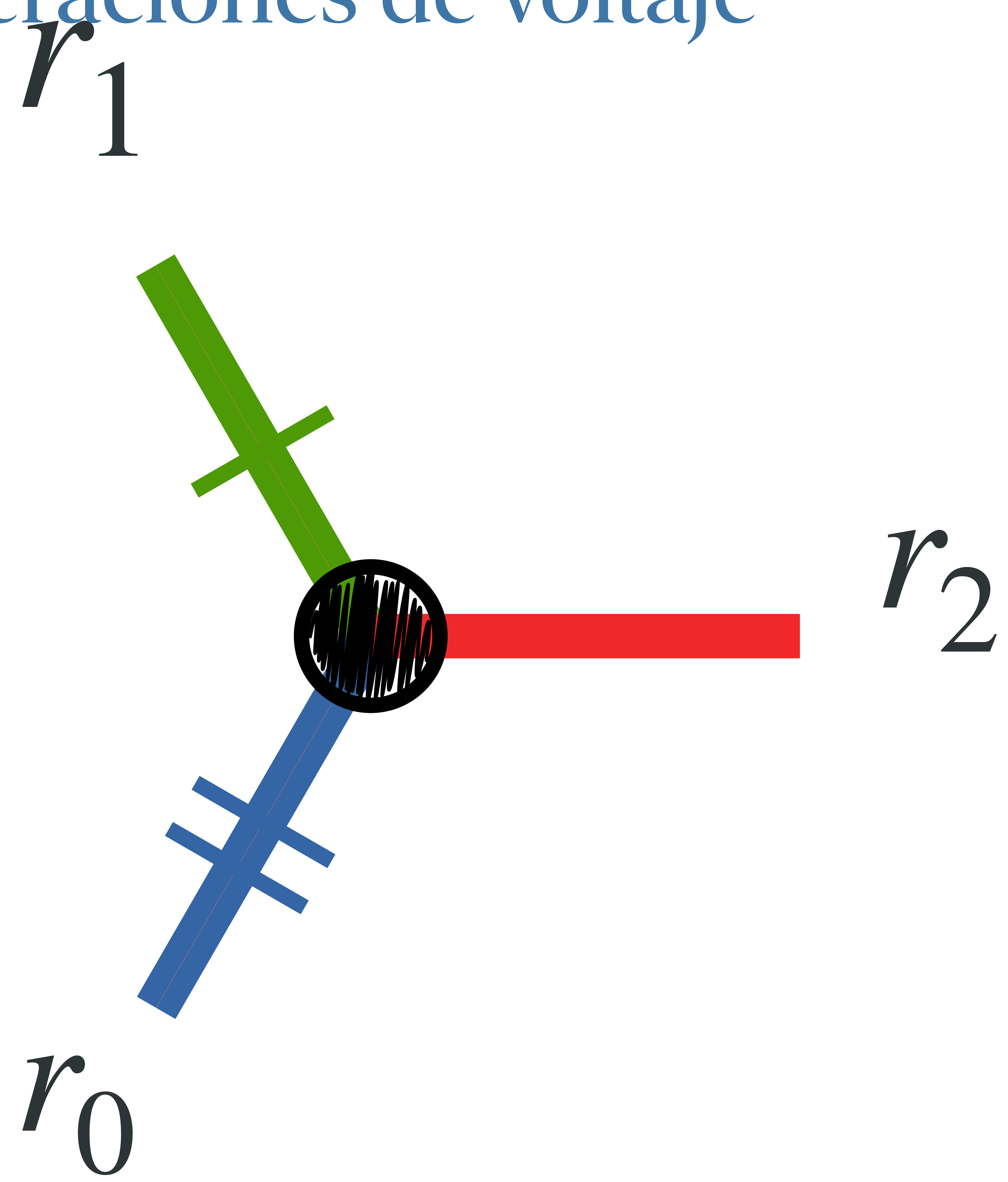
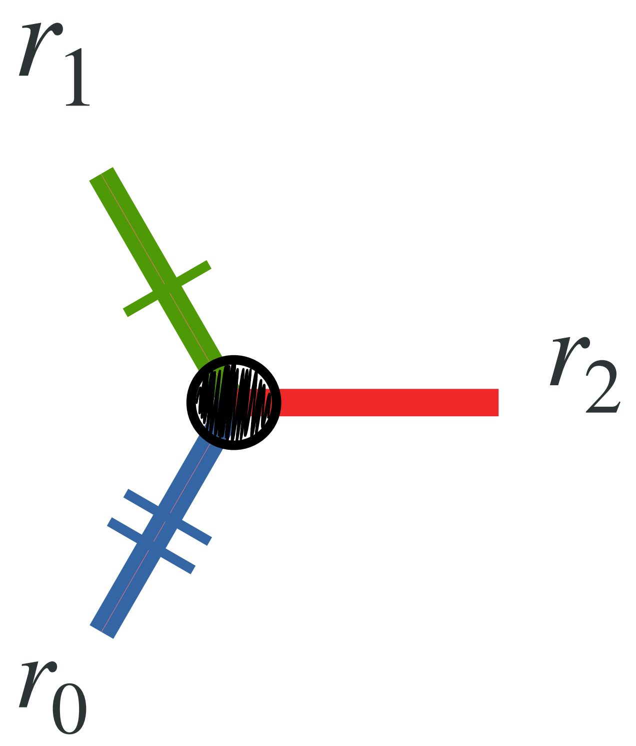
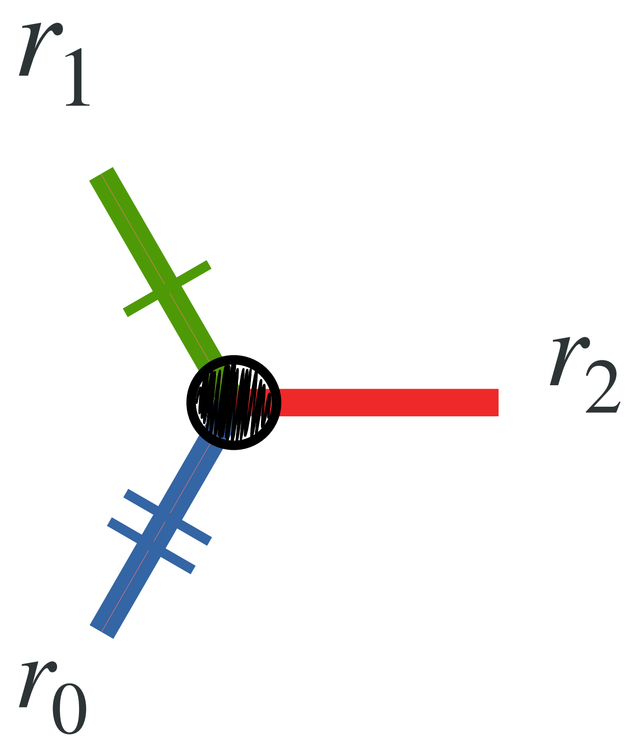
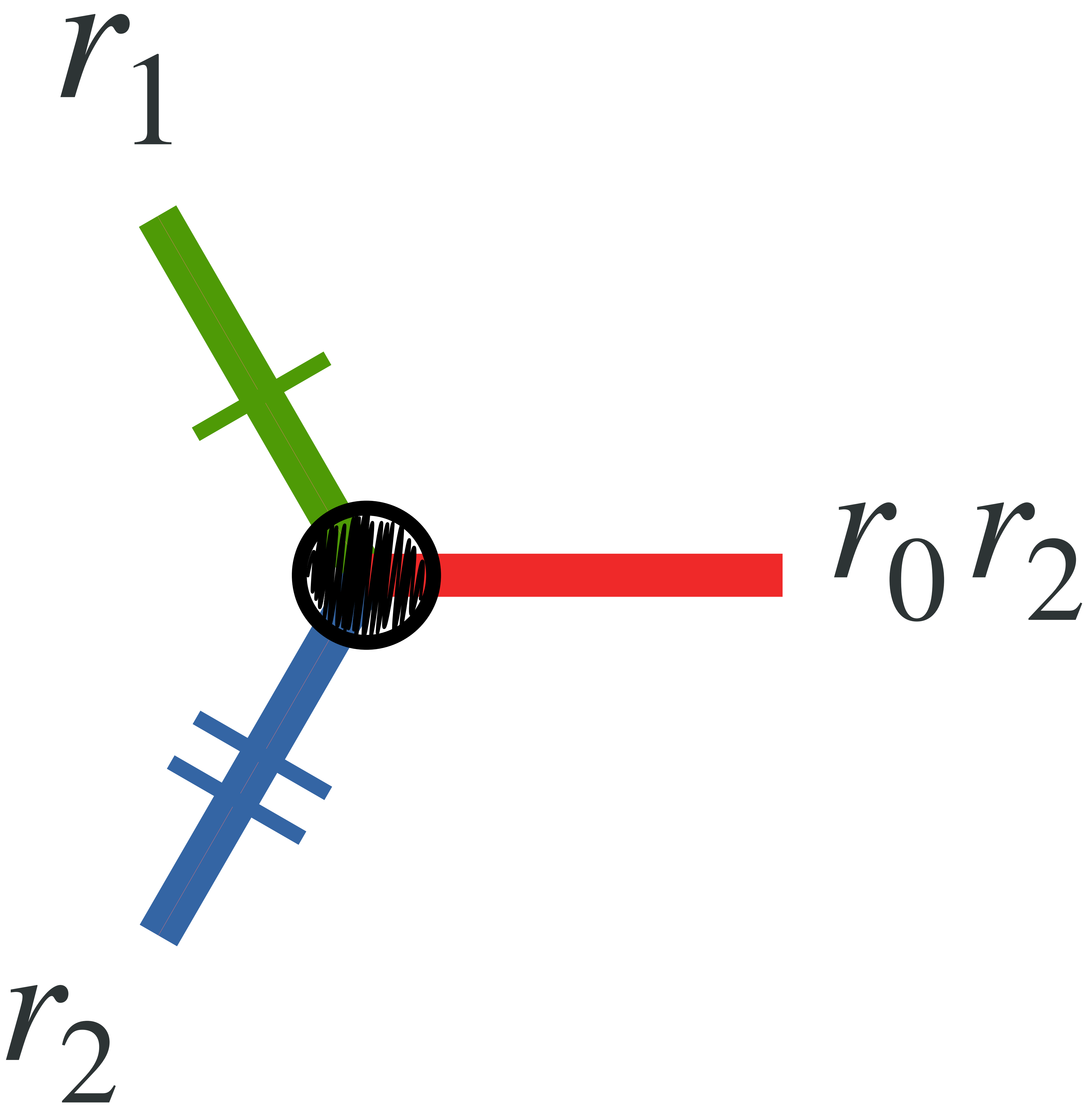


Operaciones de voltaje

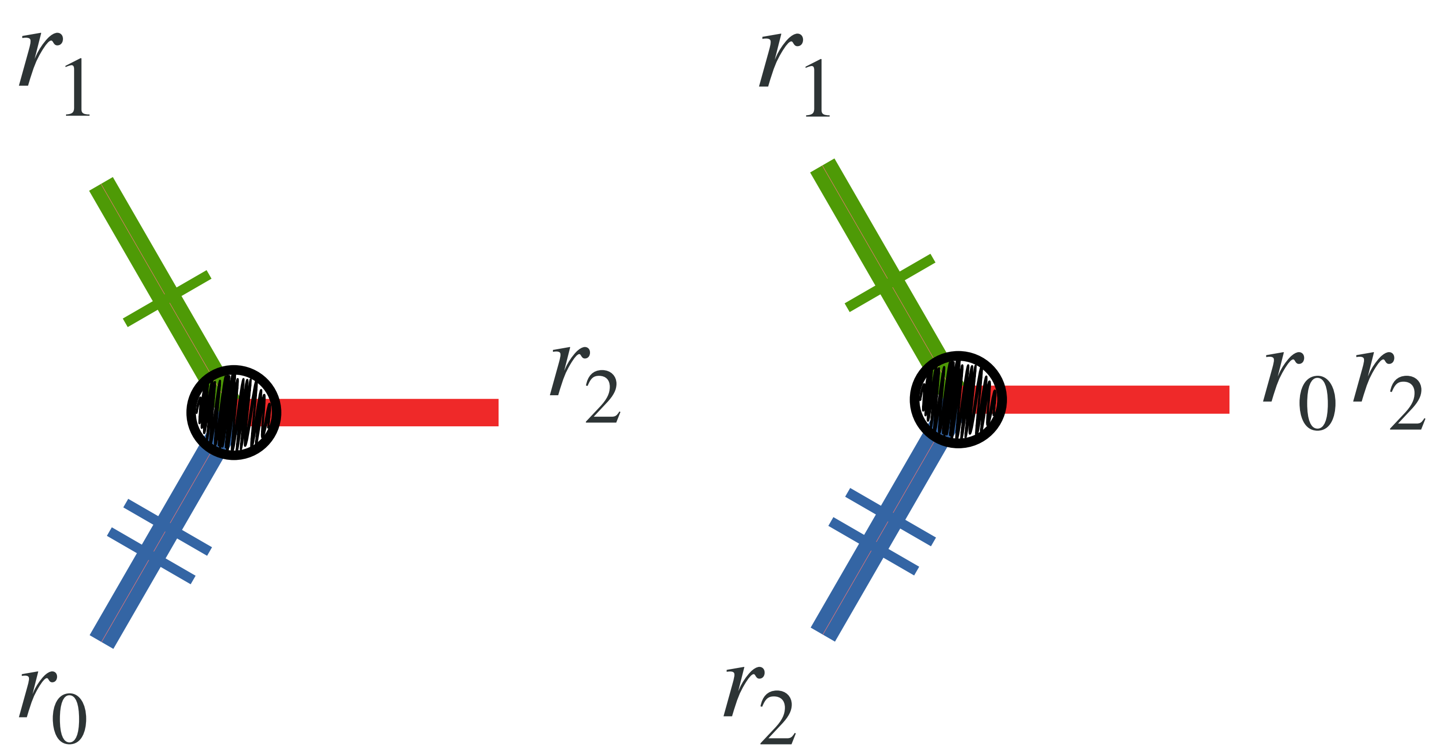


Operaciones de voltaje



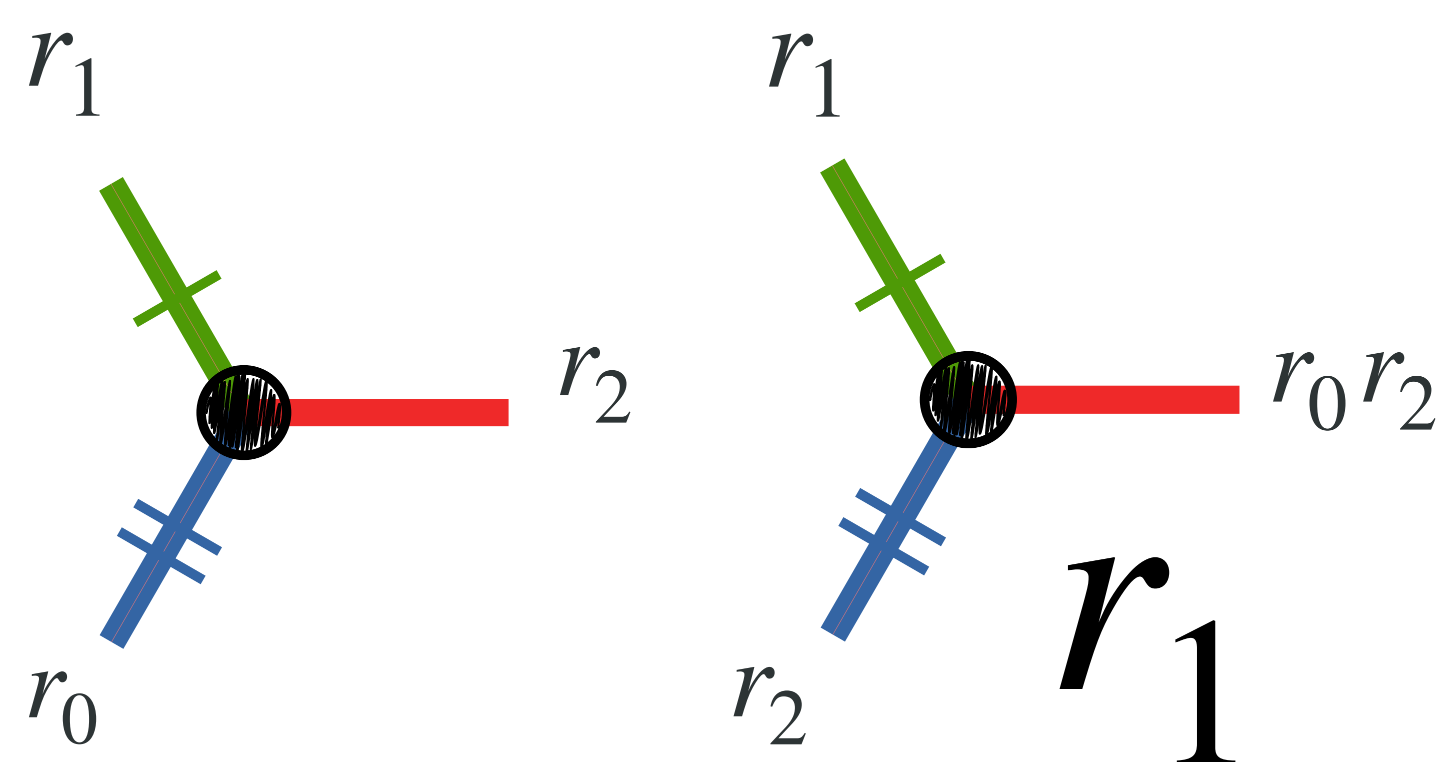


Operaciones de voltaje



D-morfismos
Operadores de Wilson

Operaciones de voltaje



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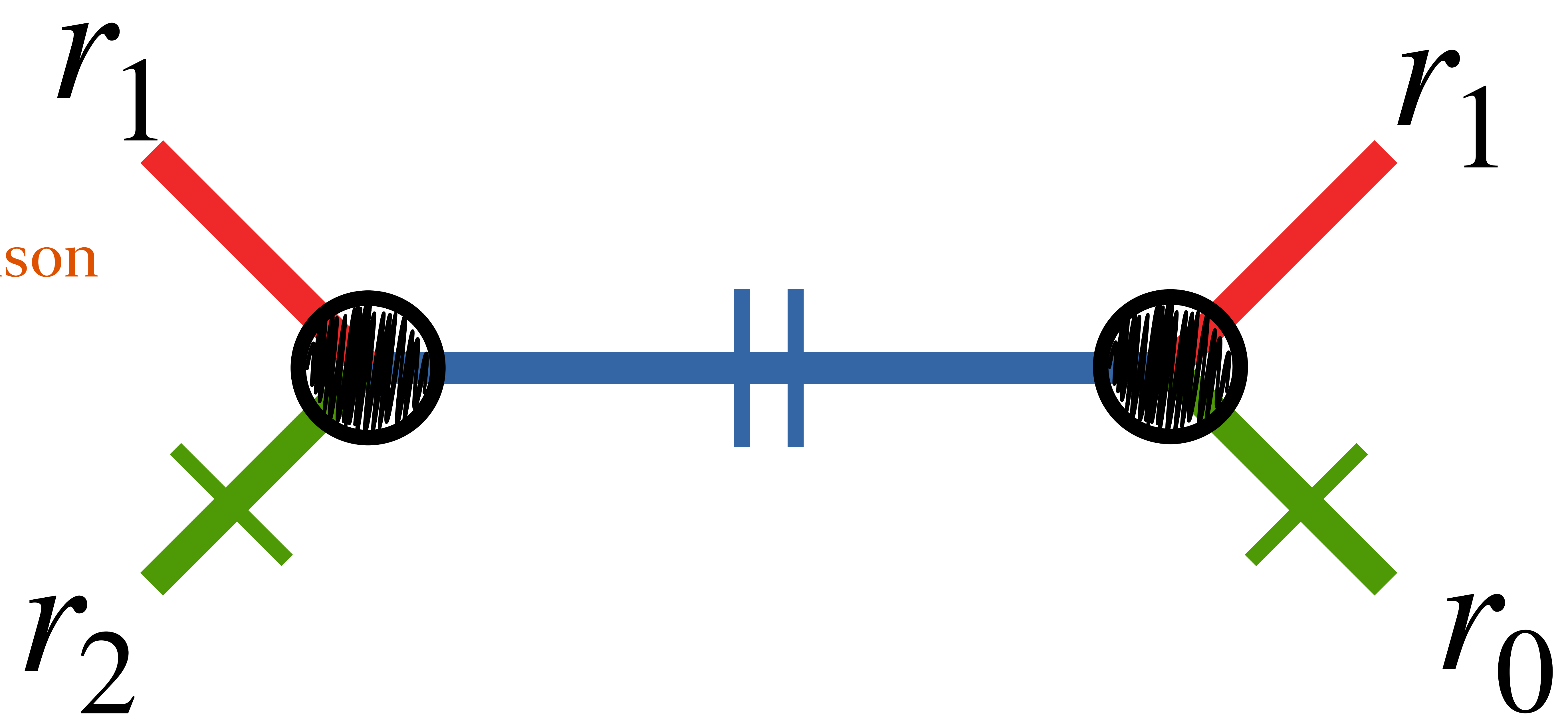
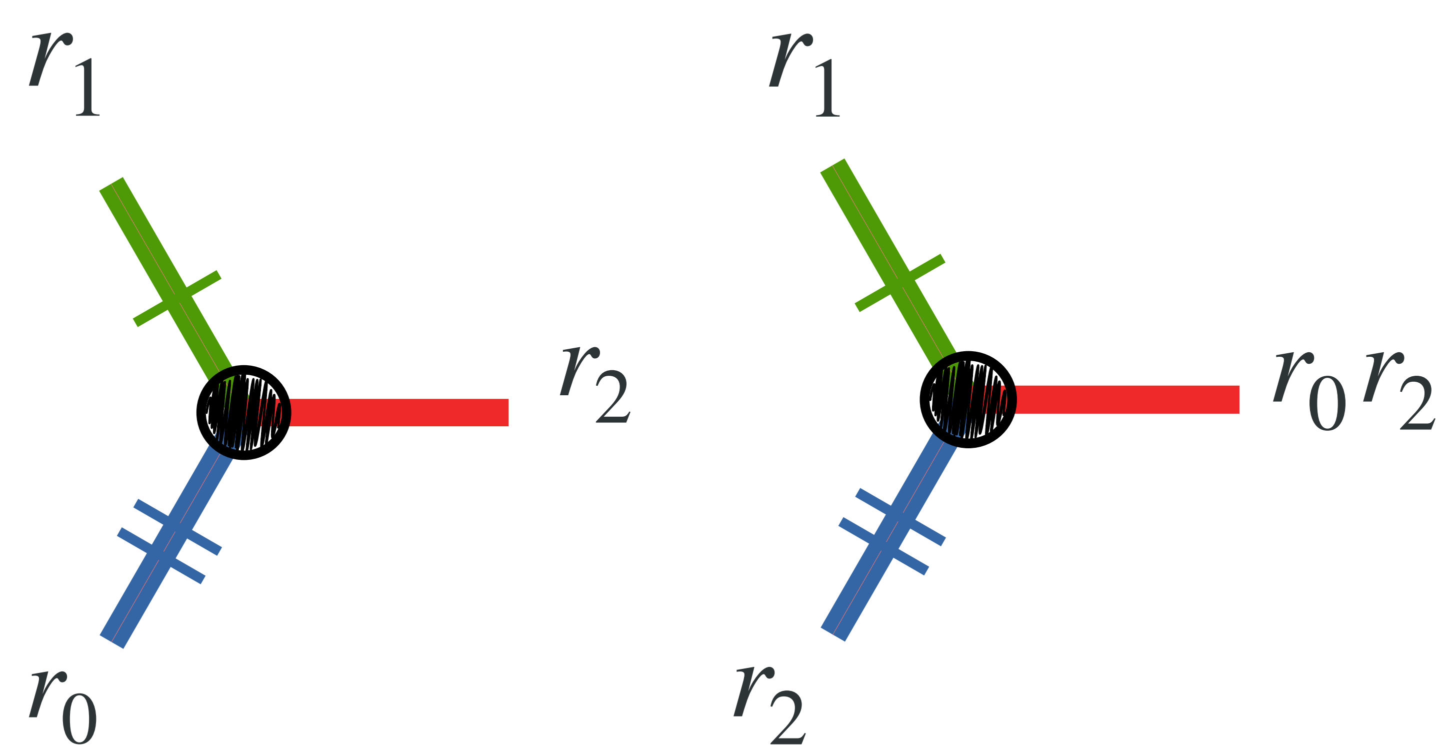
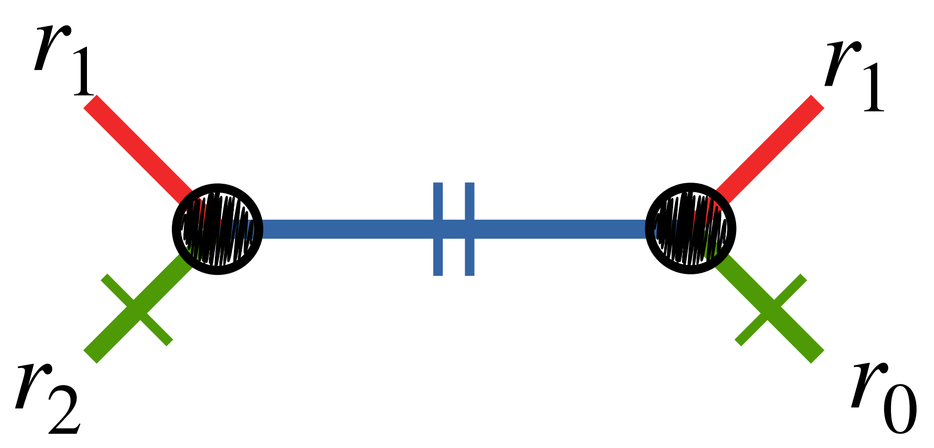
Operadores de Wilson



Operaciones de voltaje

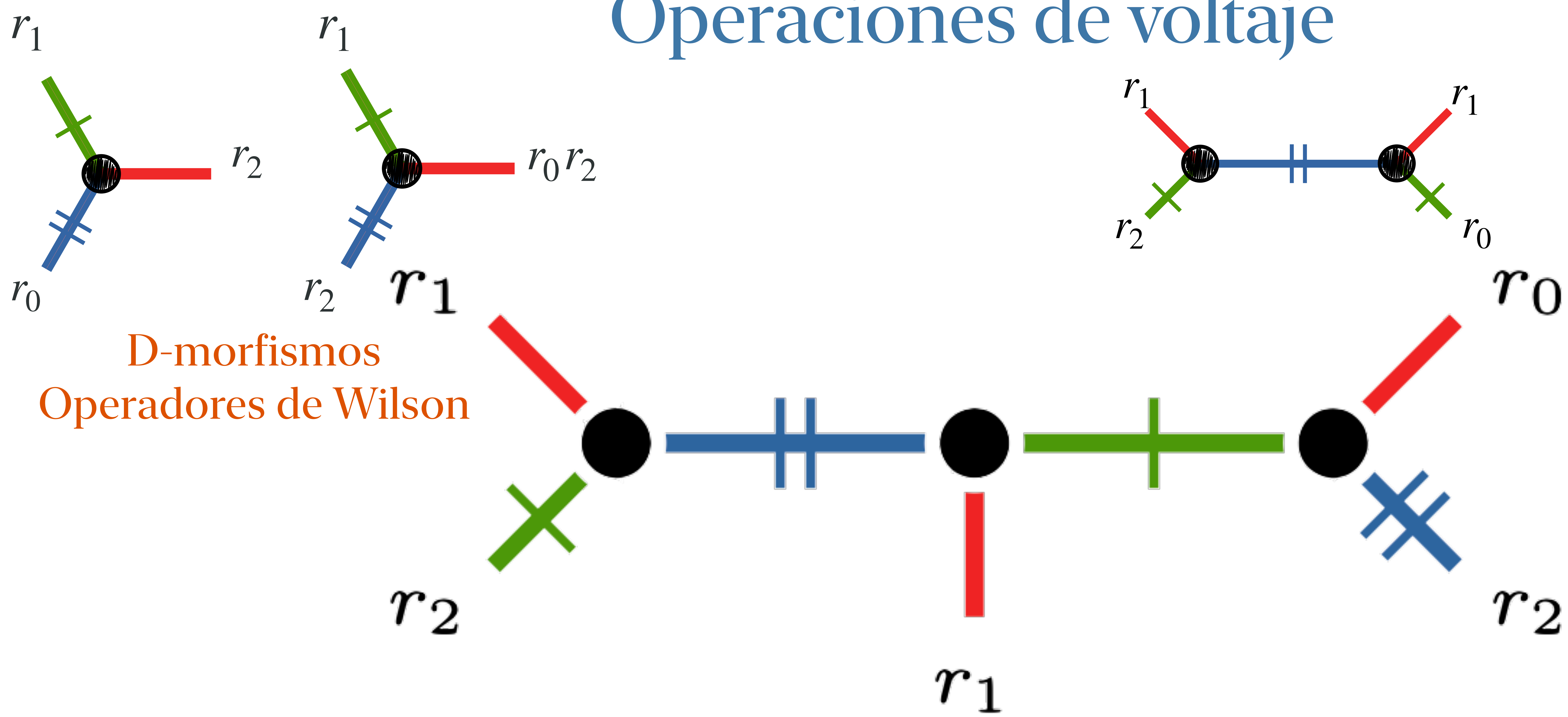


D-morfismos
Operadores de Wilson



Operaciones de voltaje

D-morfismos
Operadores de Wilson



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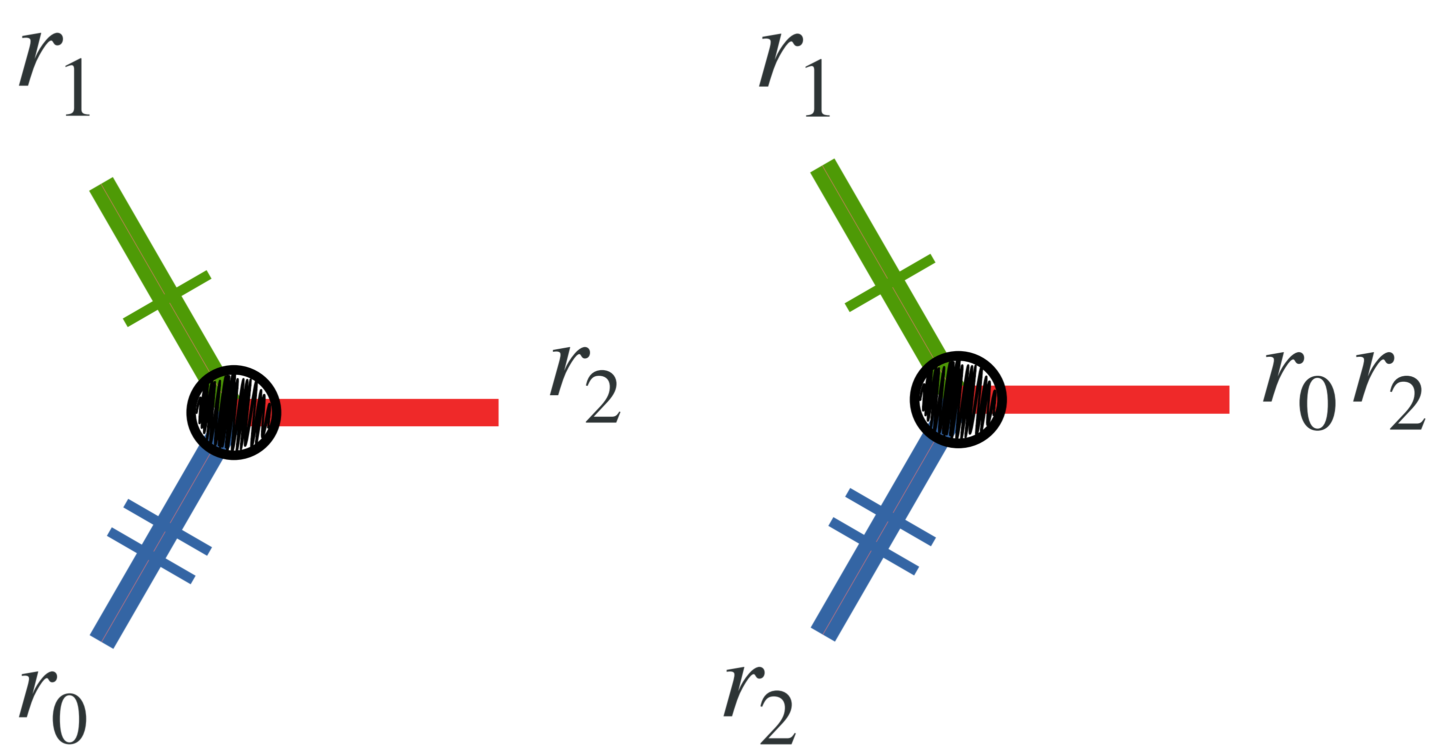

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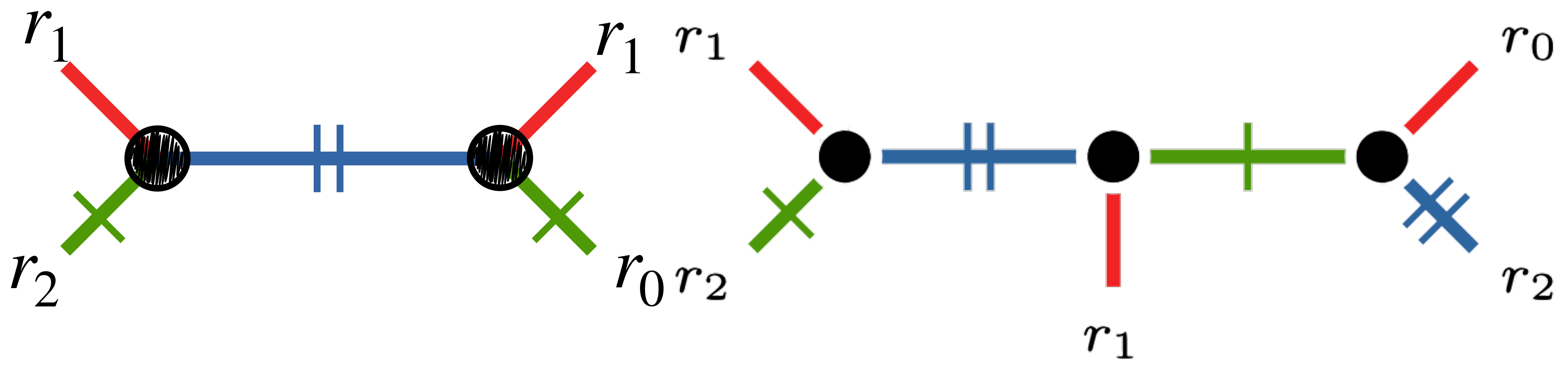
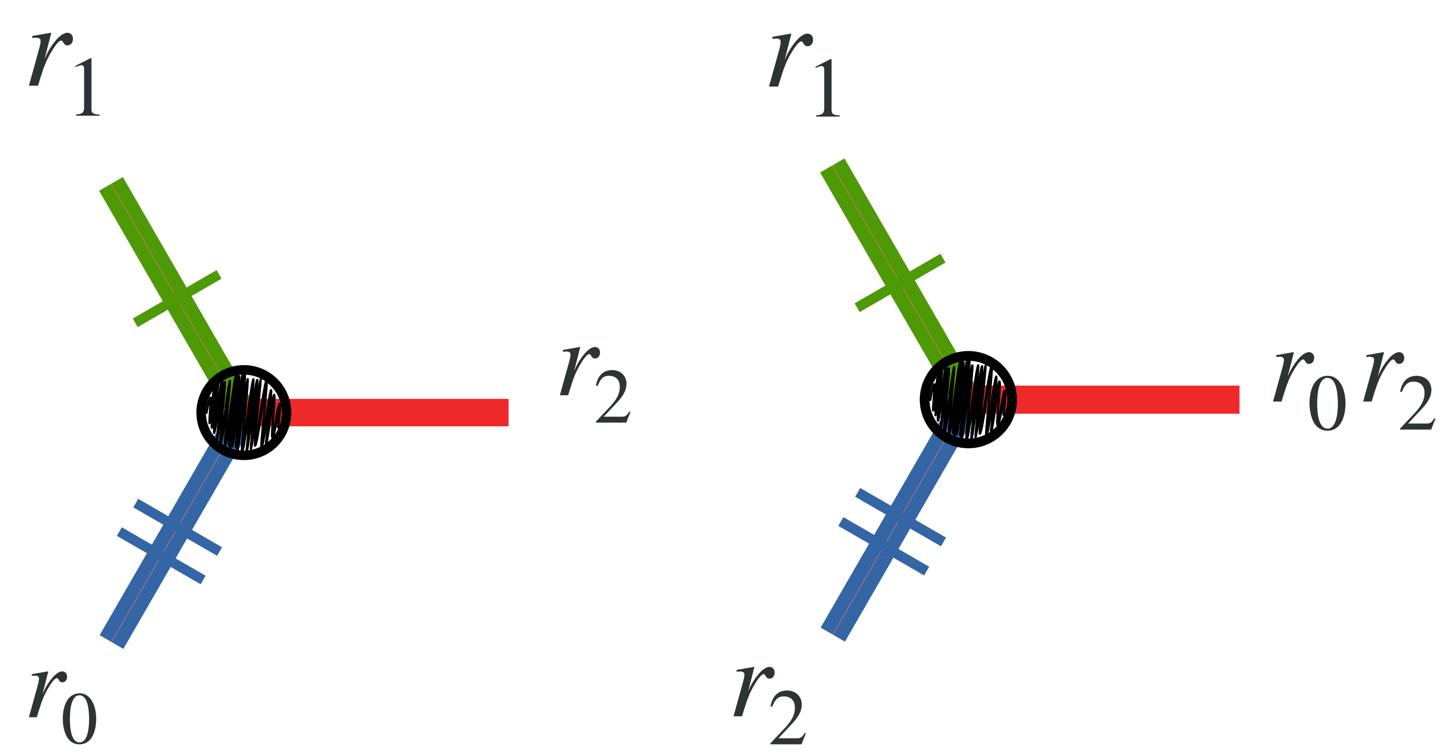


Ops. de Wythoff

Operaciones de voltaje



D-morfismos
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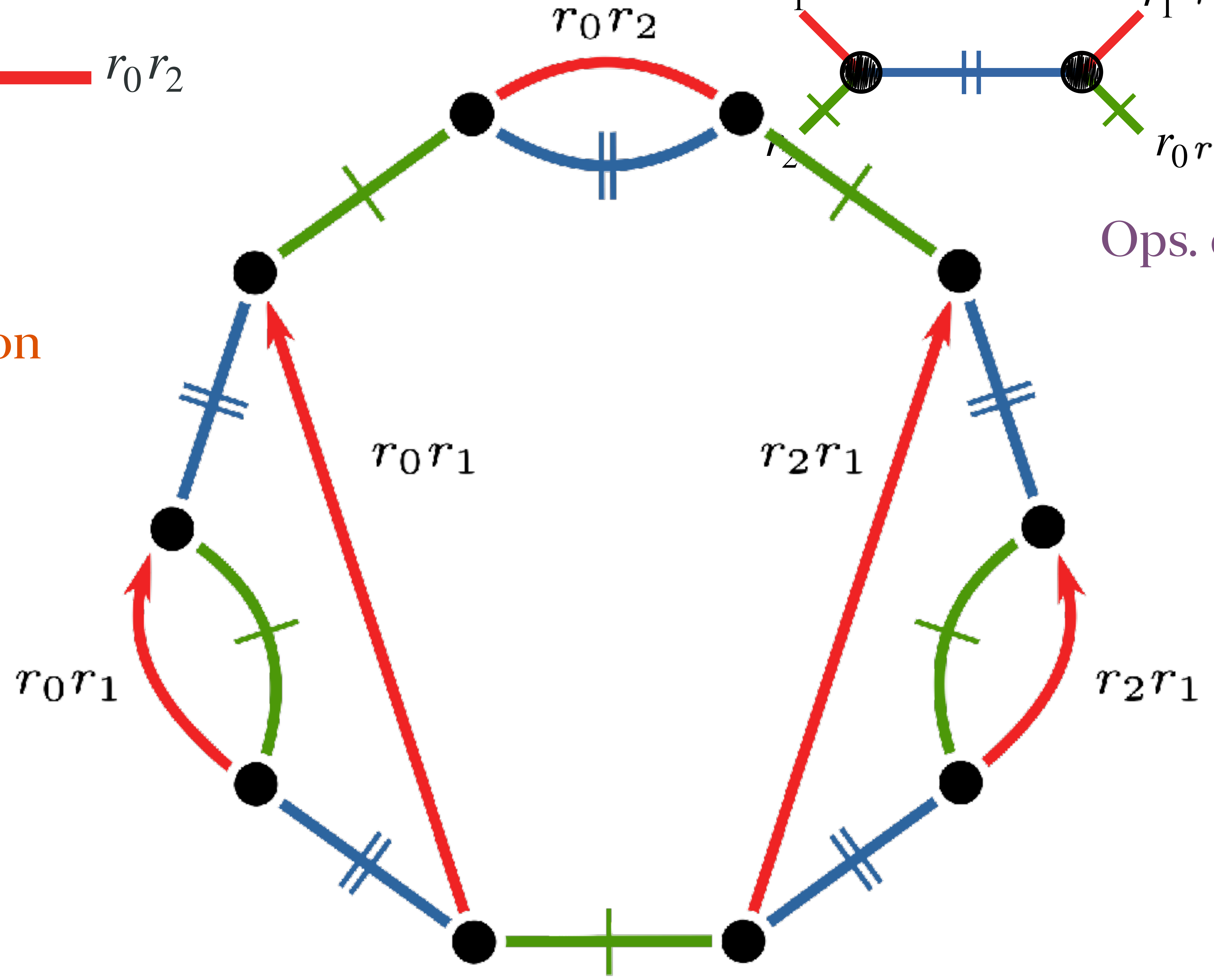

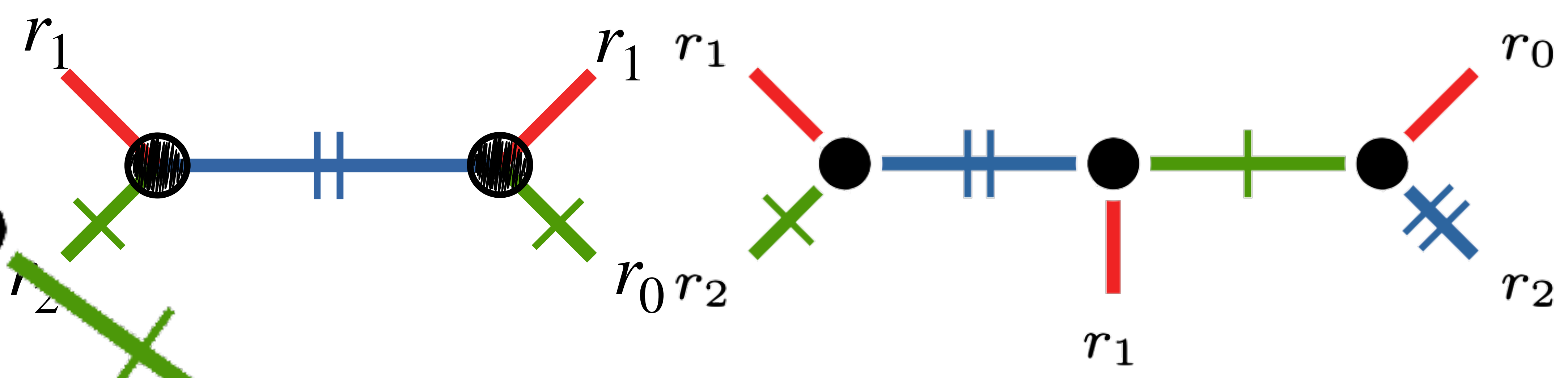

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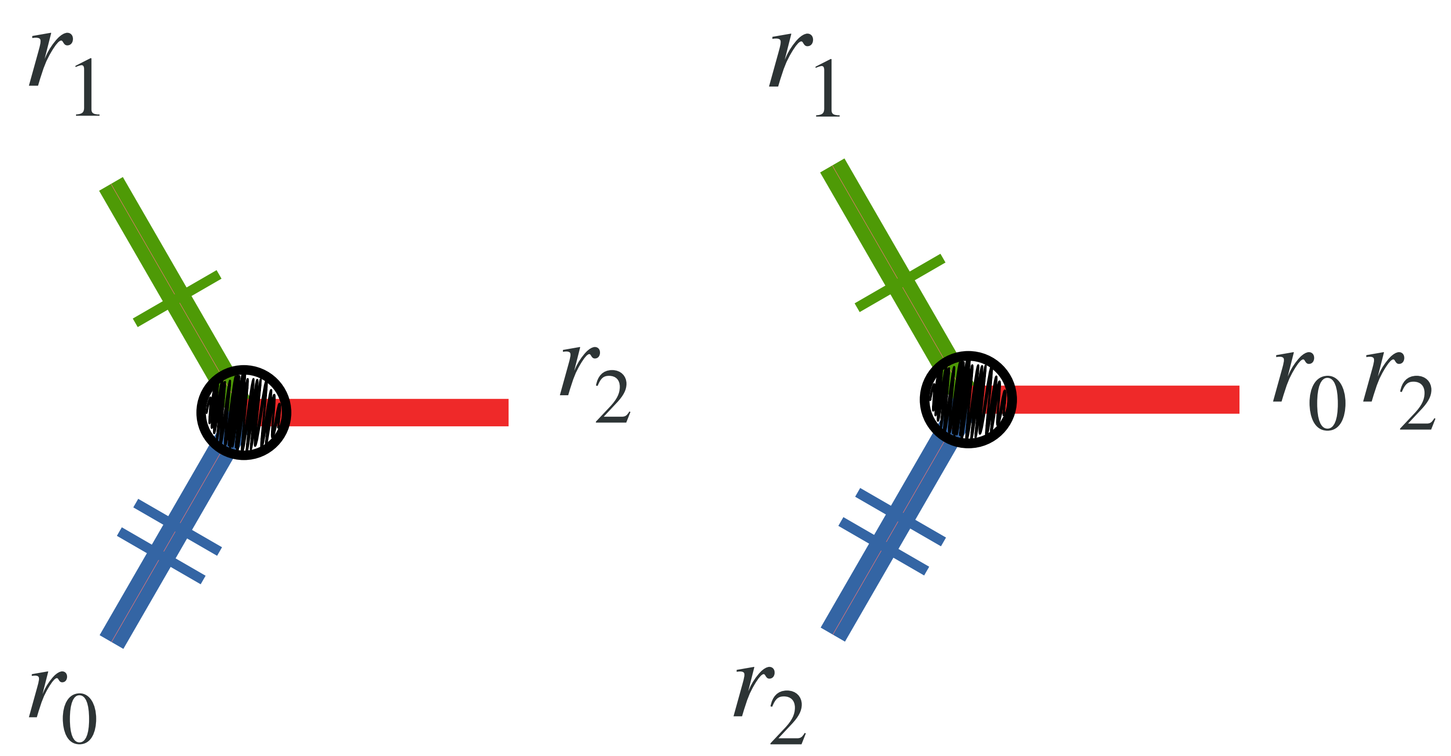

D-morfismos
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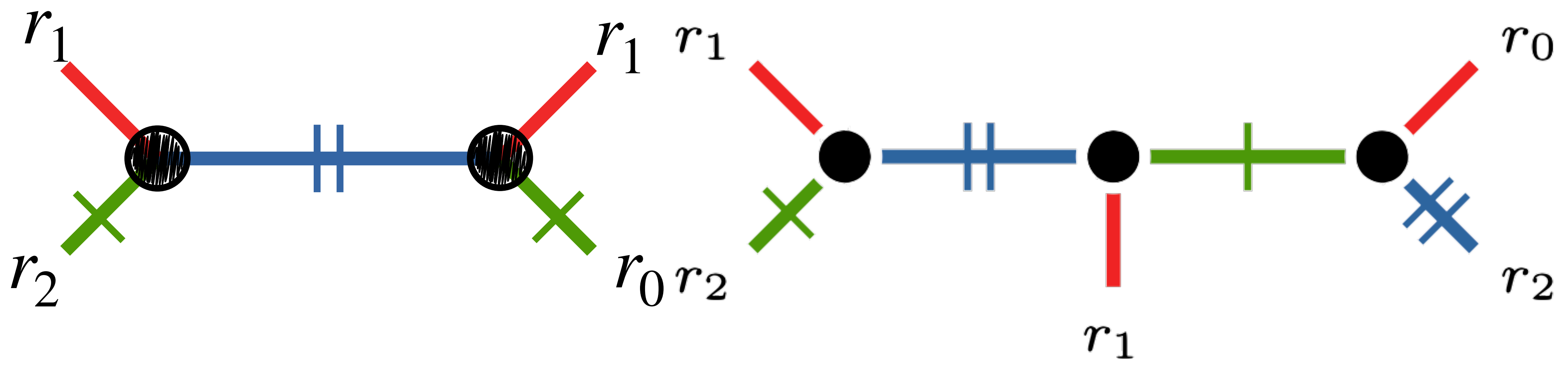

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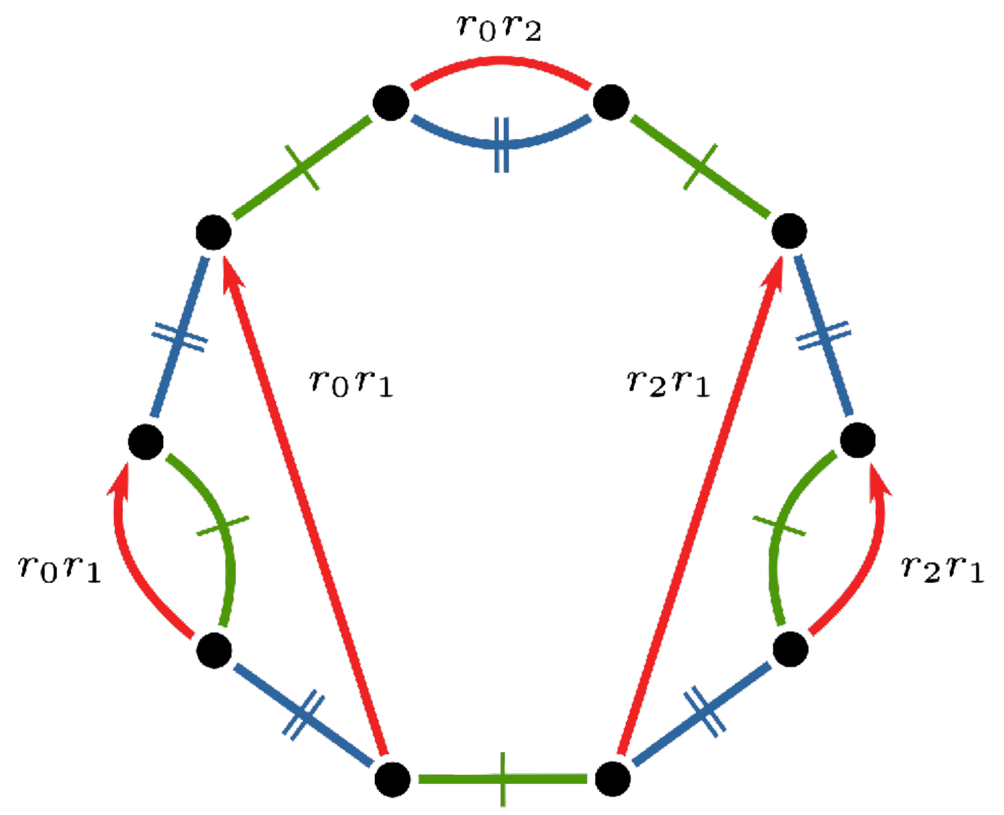
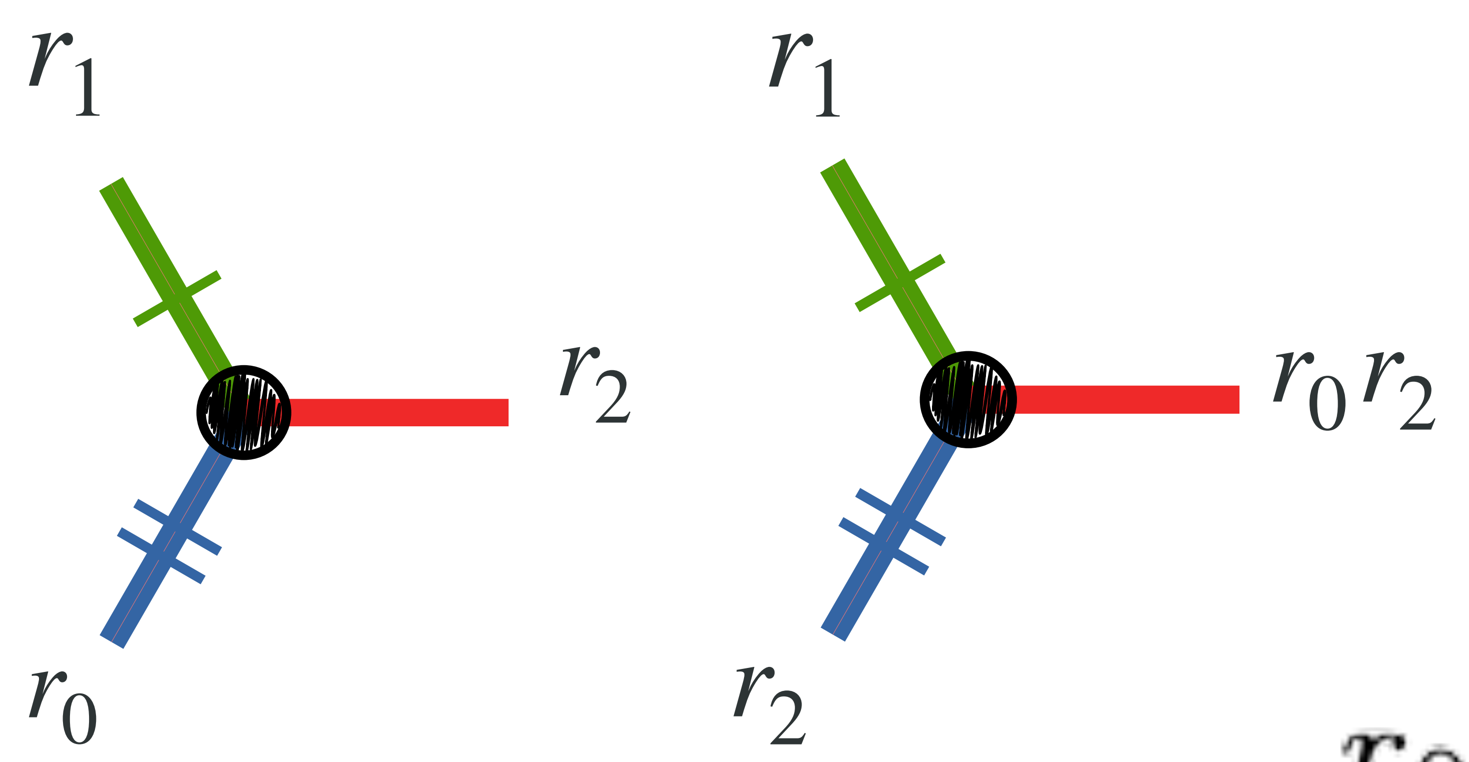


Snub

Operaciones de voltaje



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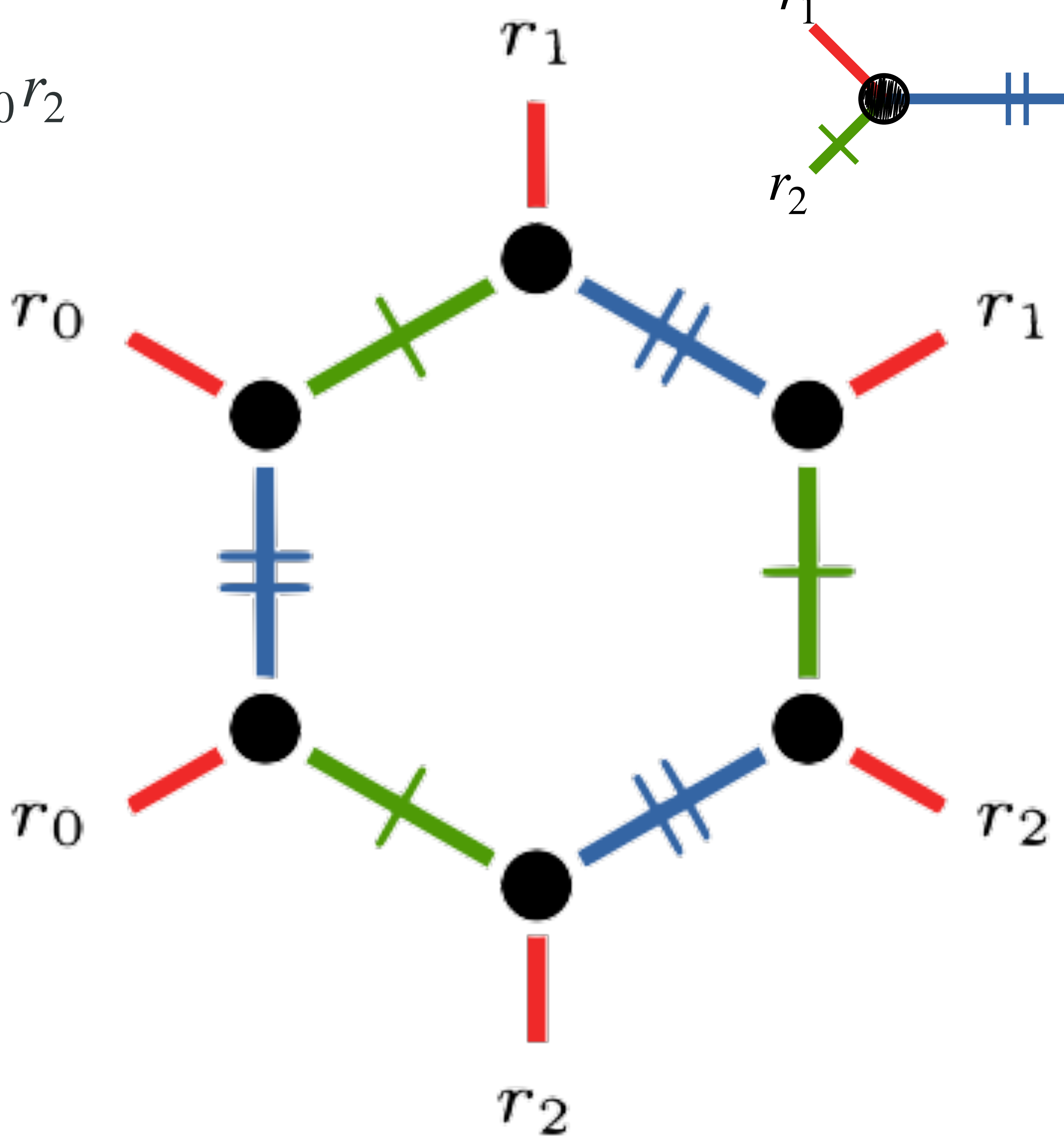
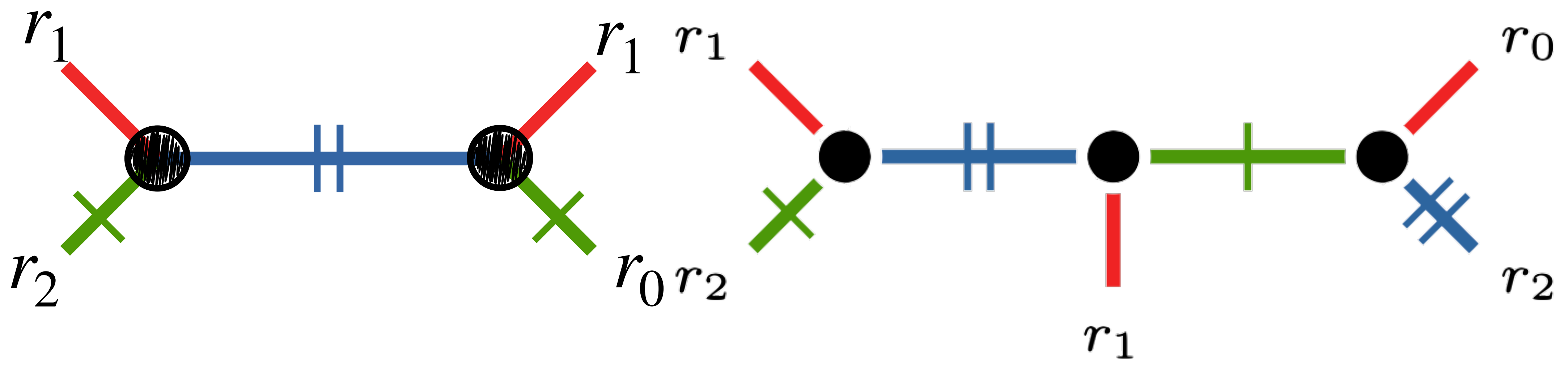
Operadores de Wilson





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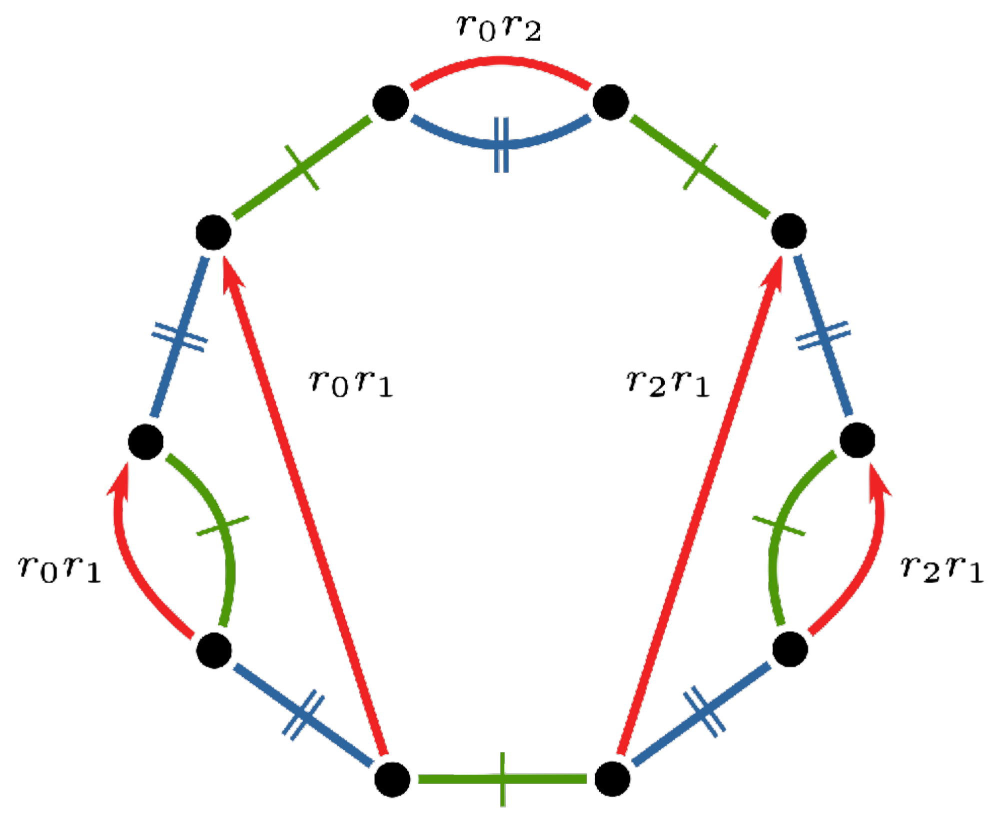
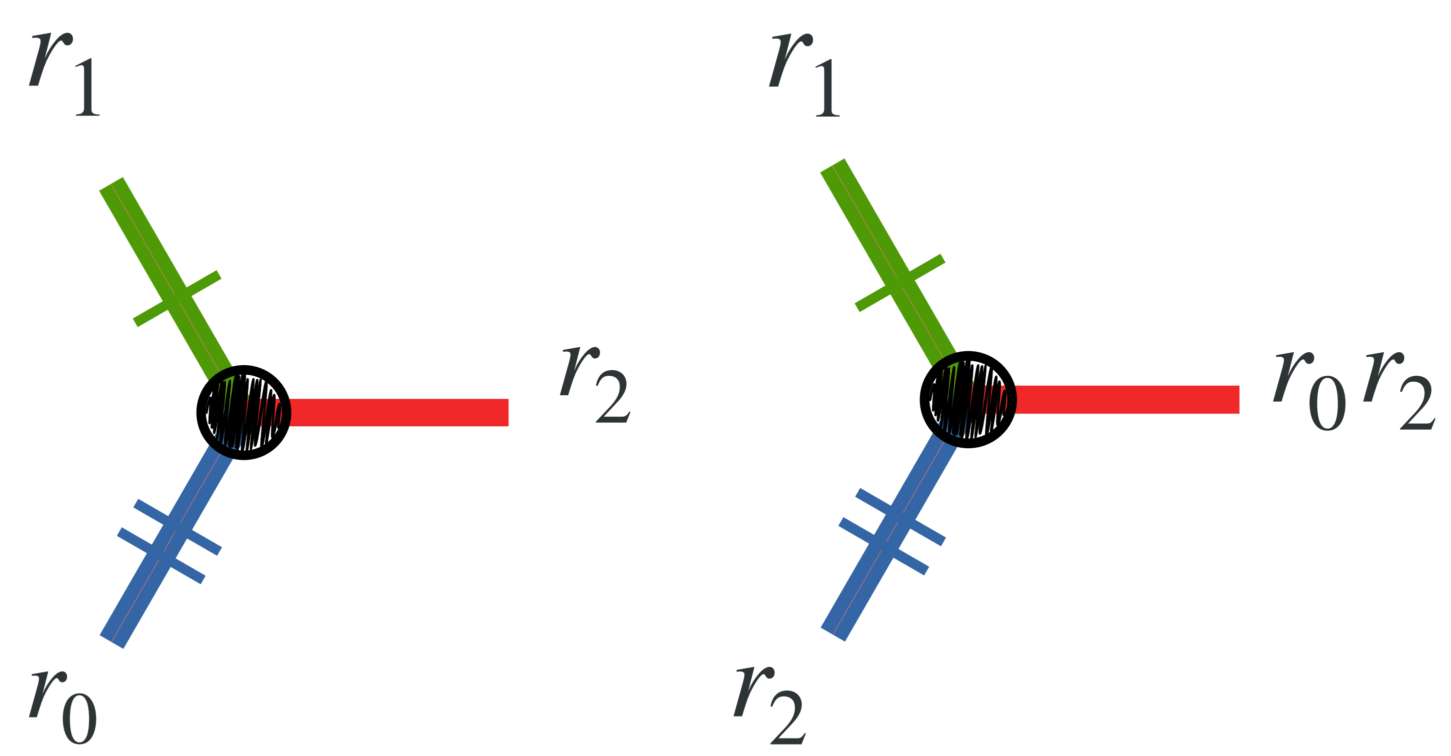
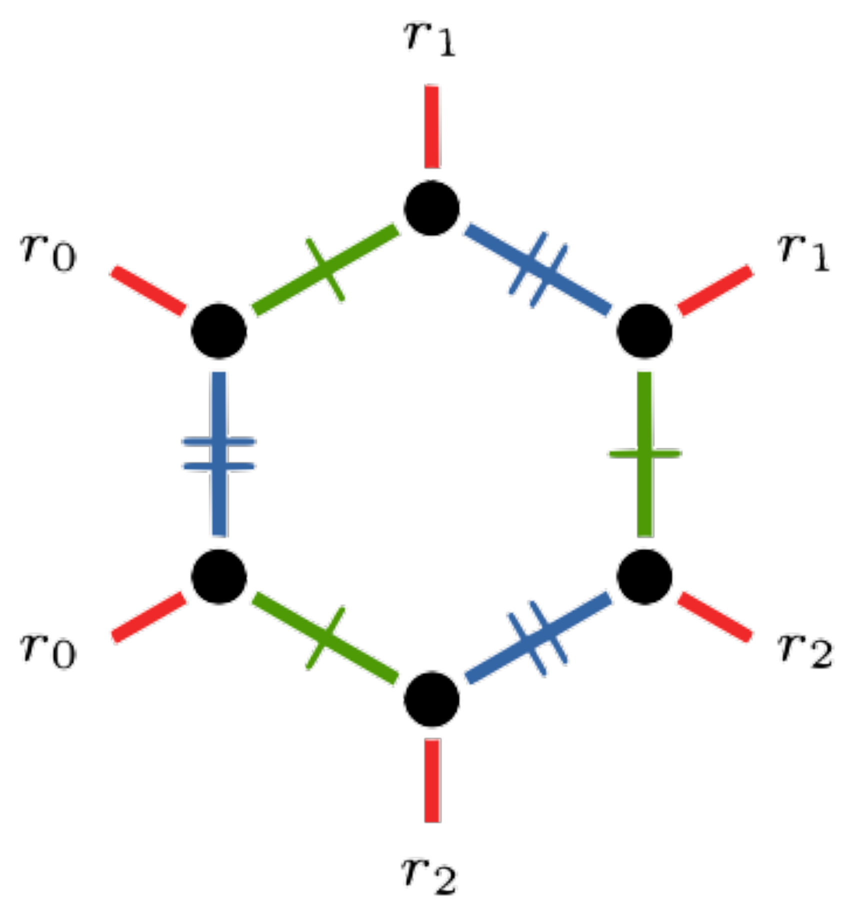


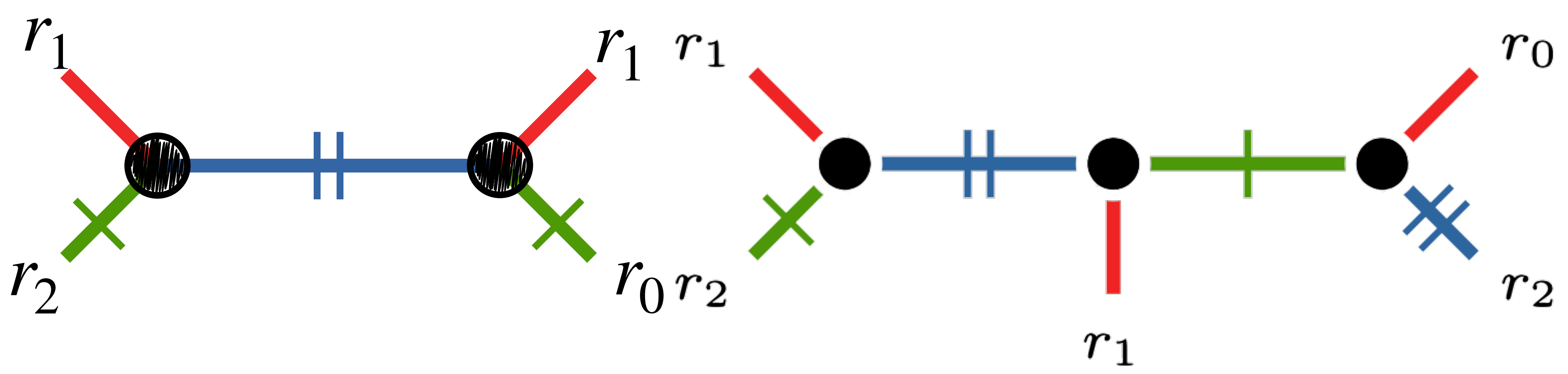
Snub

Operaciones de voltaje



D-morfismos
Operadores de Wilson





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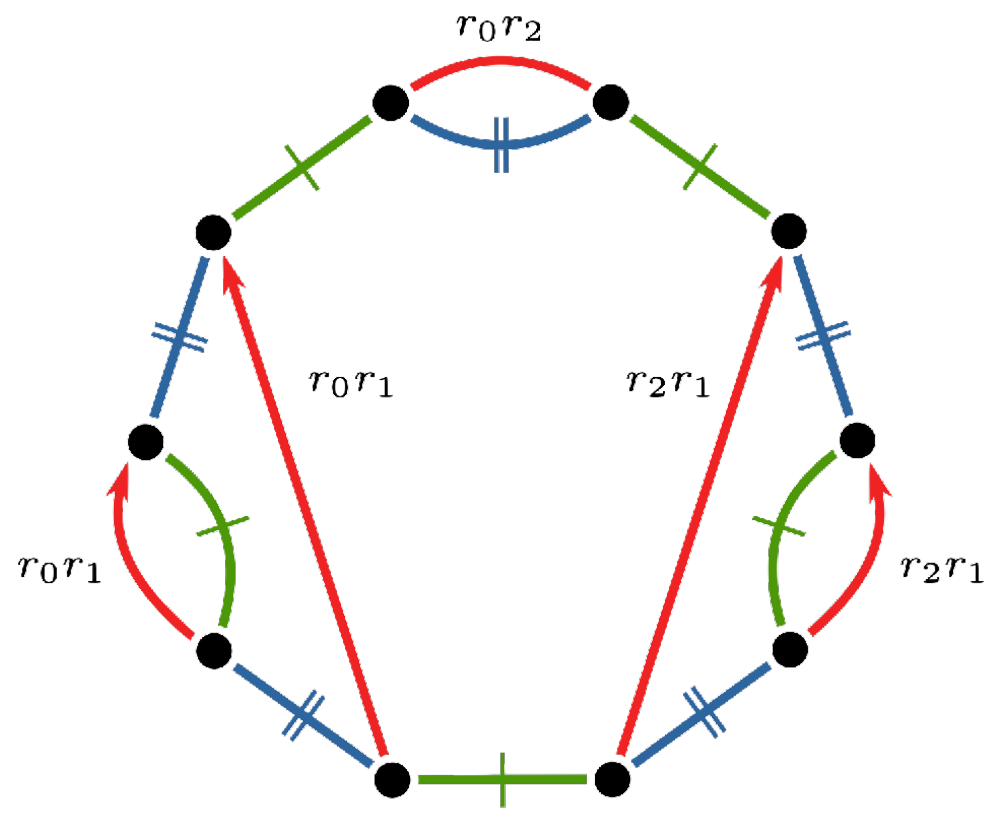

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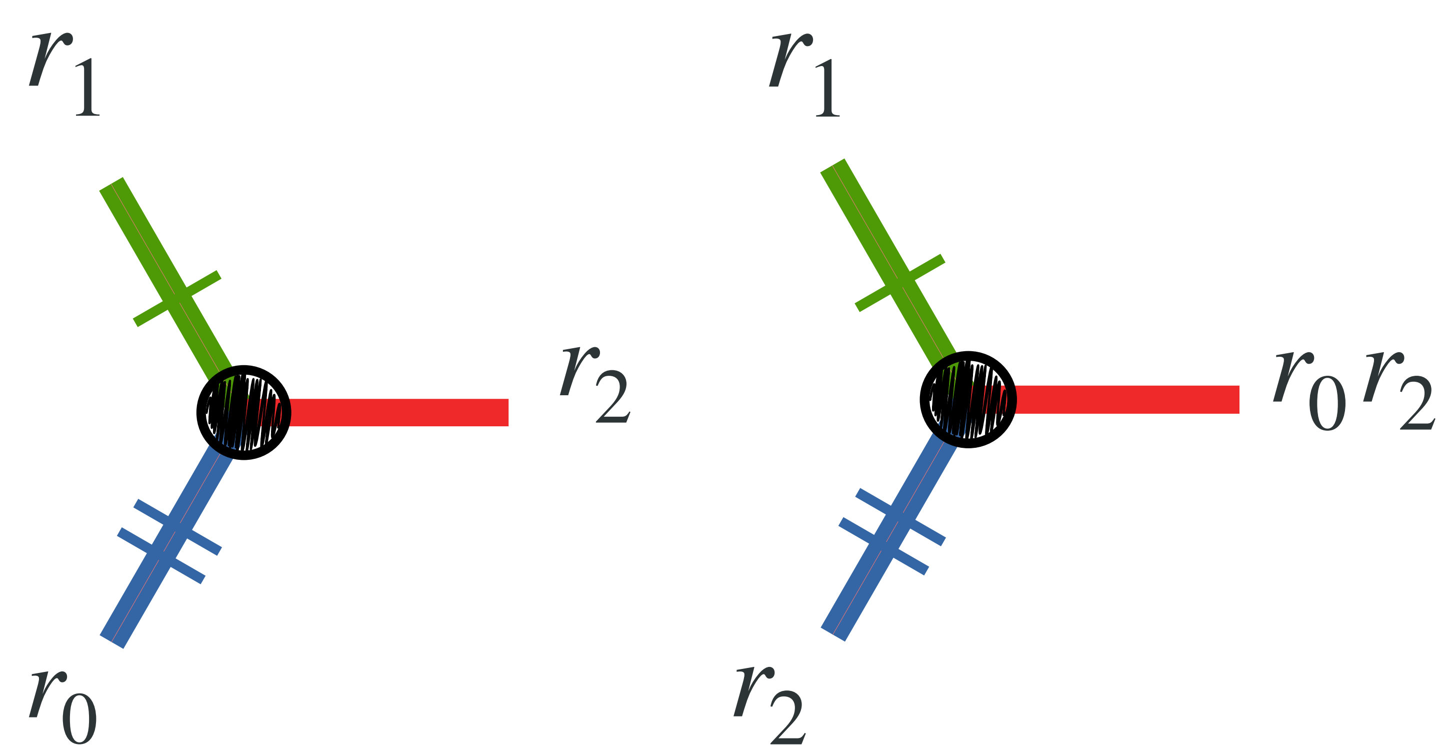

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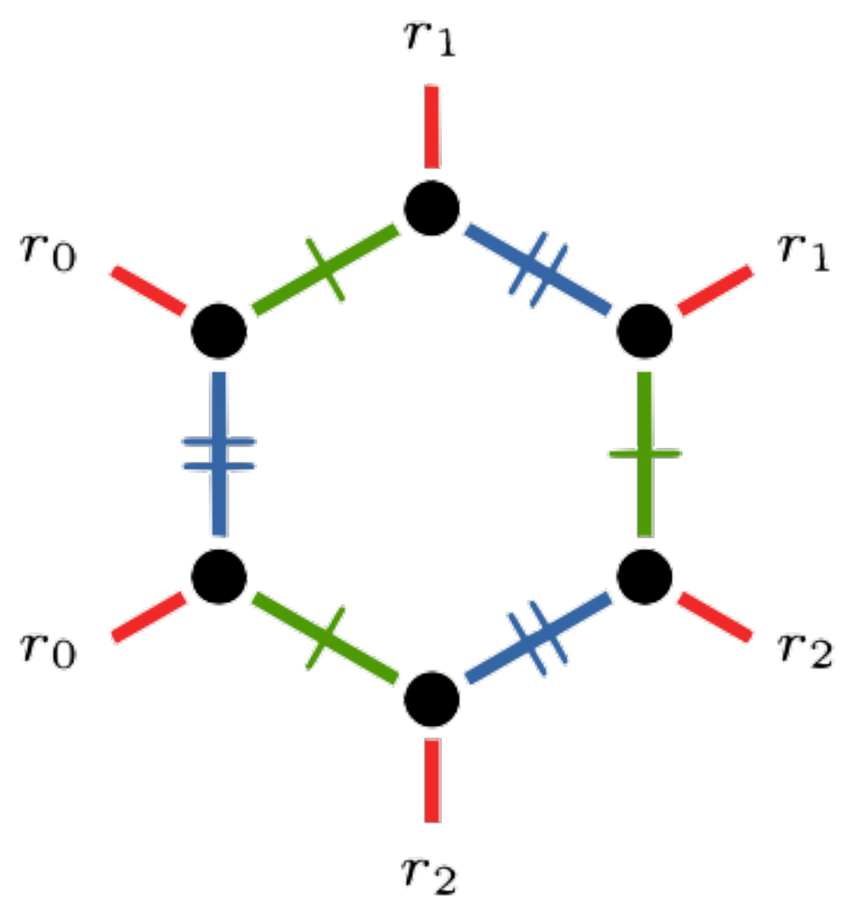

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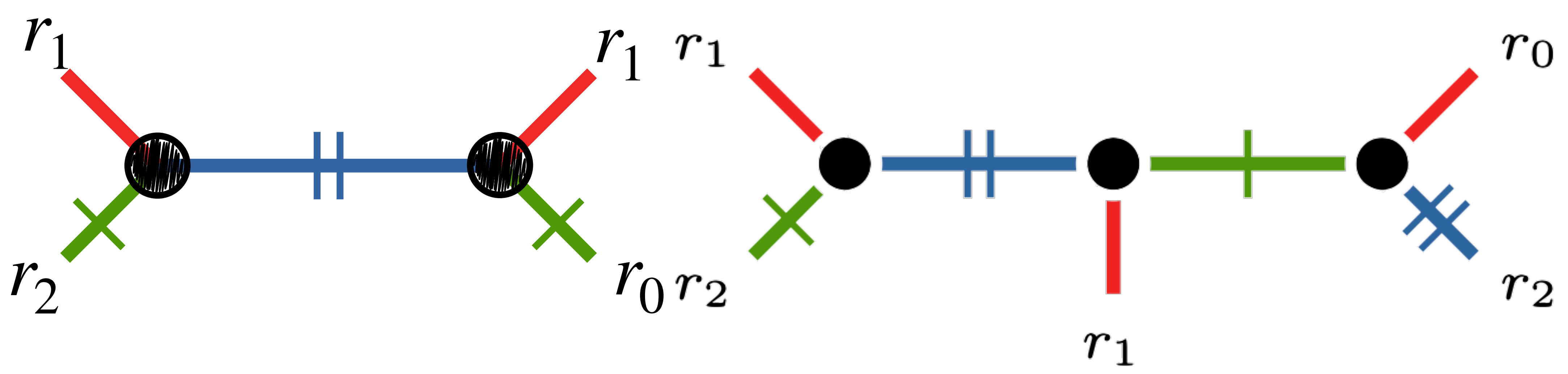

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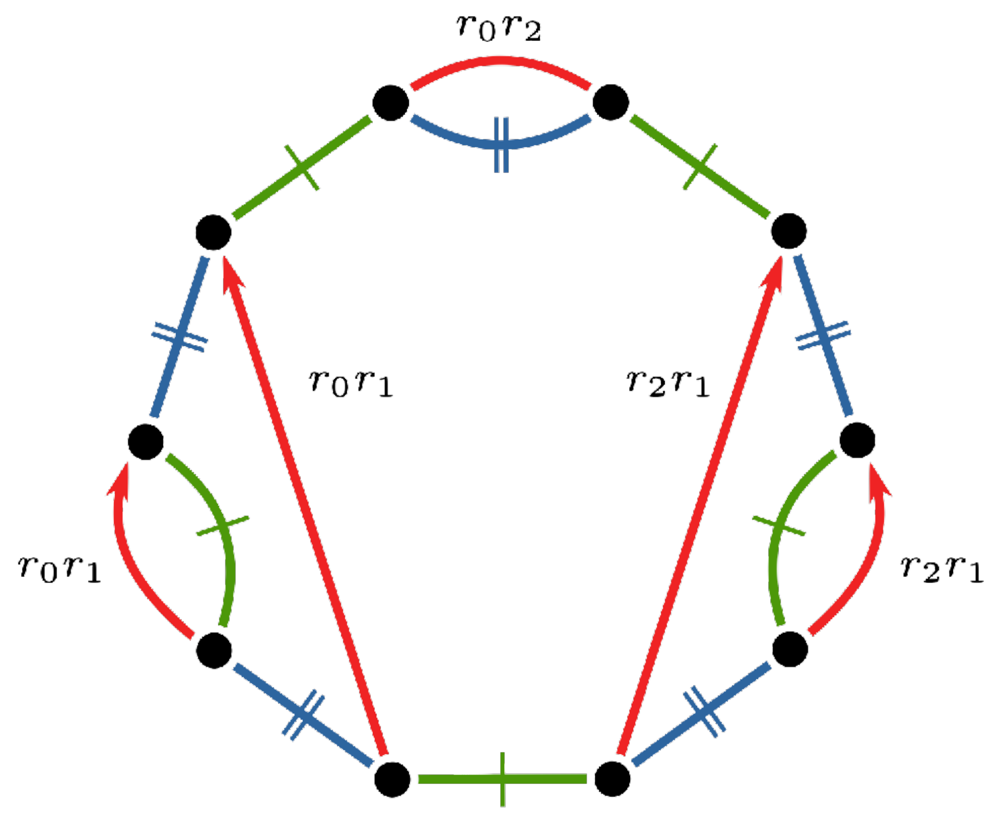

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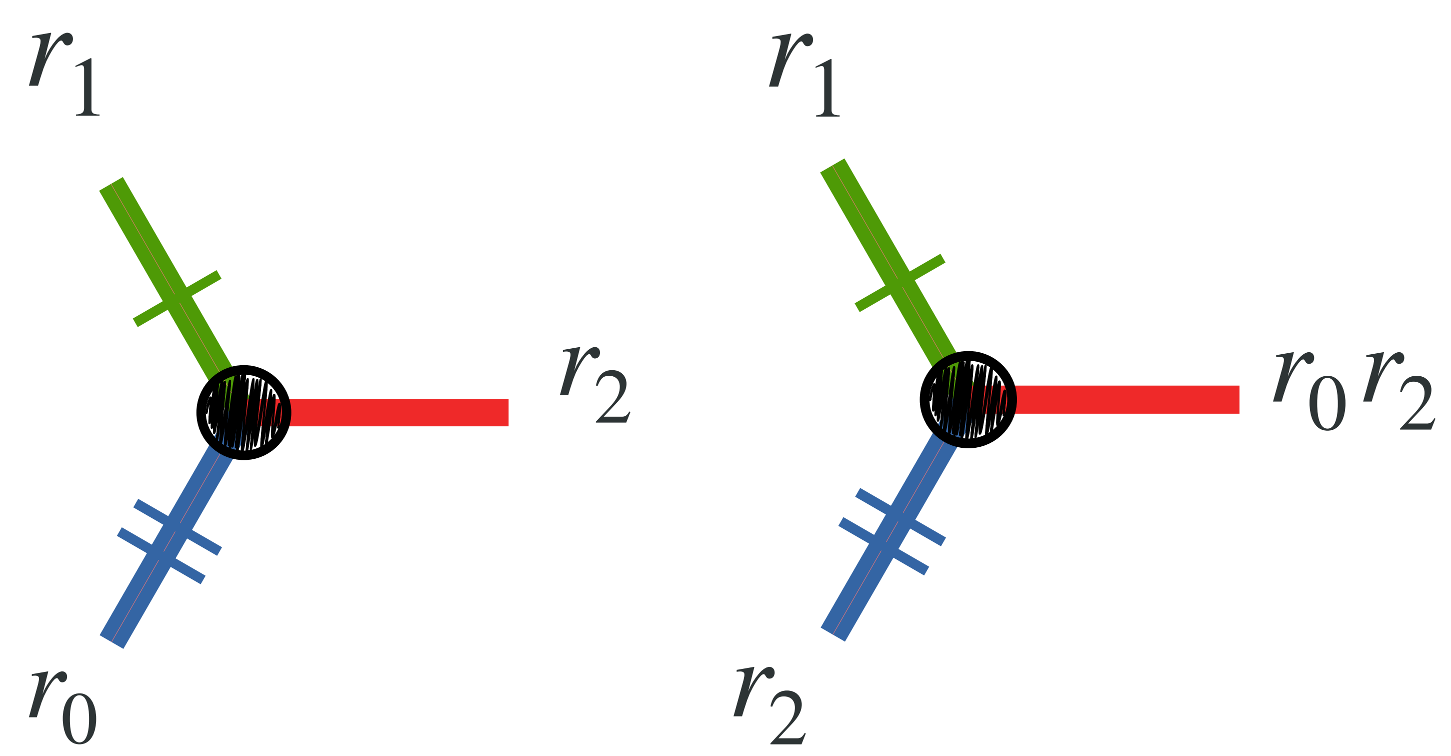

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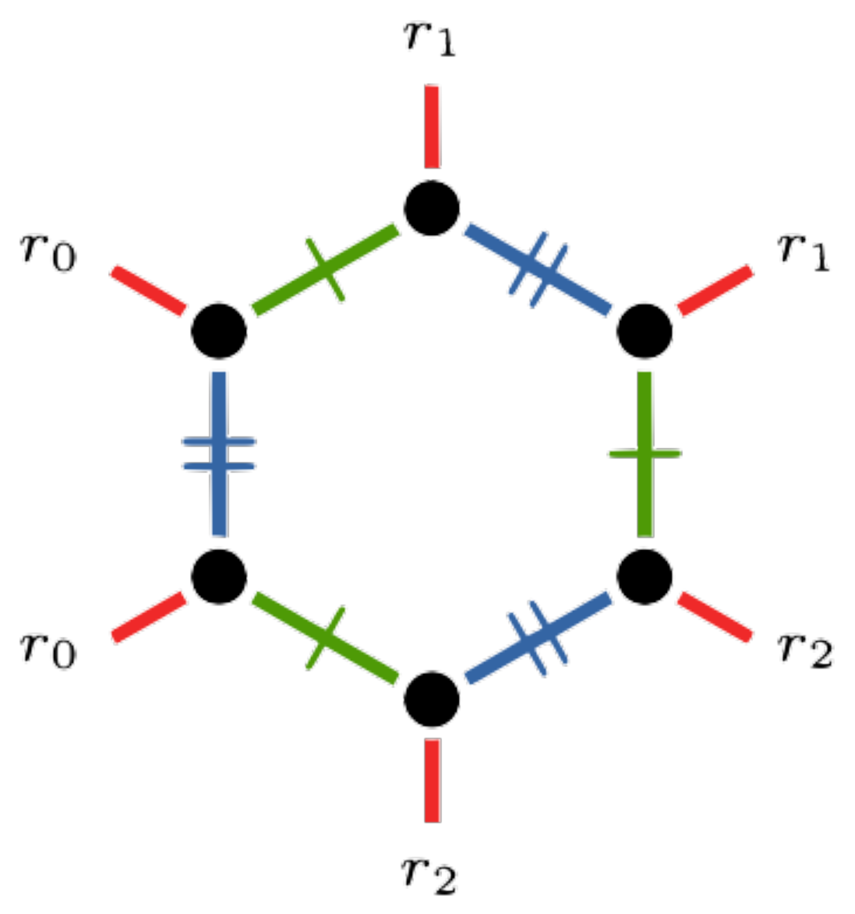

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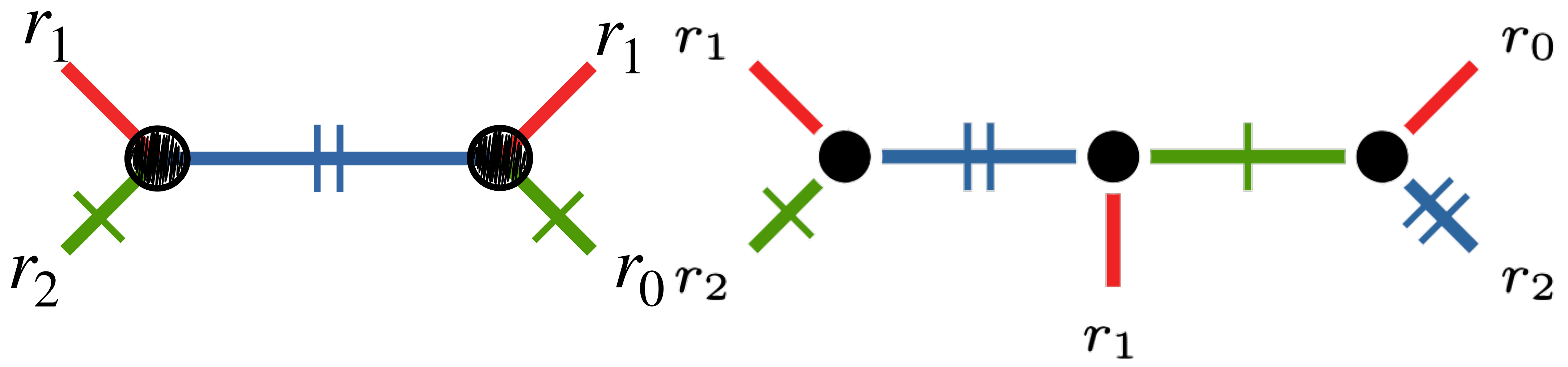

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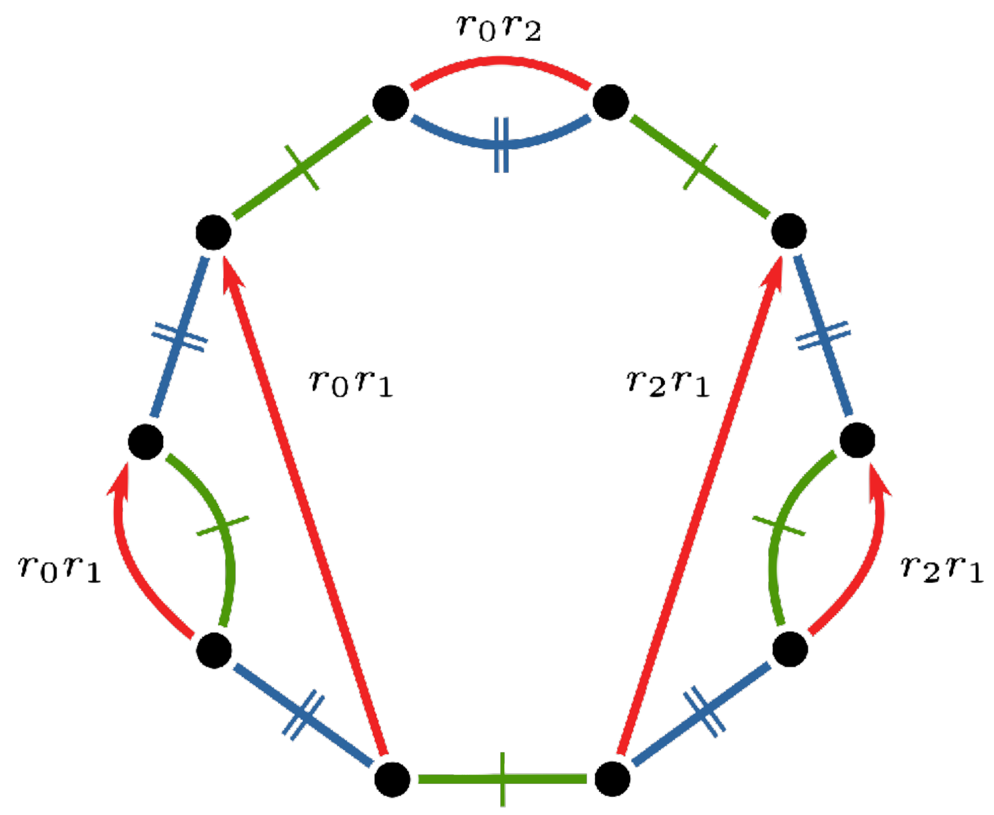

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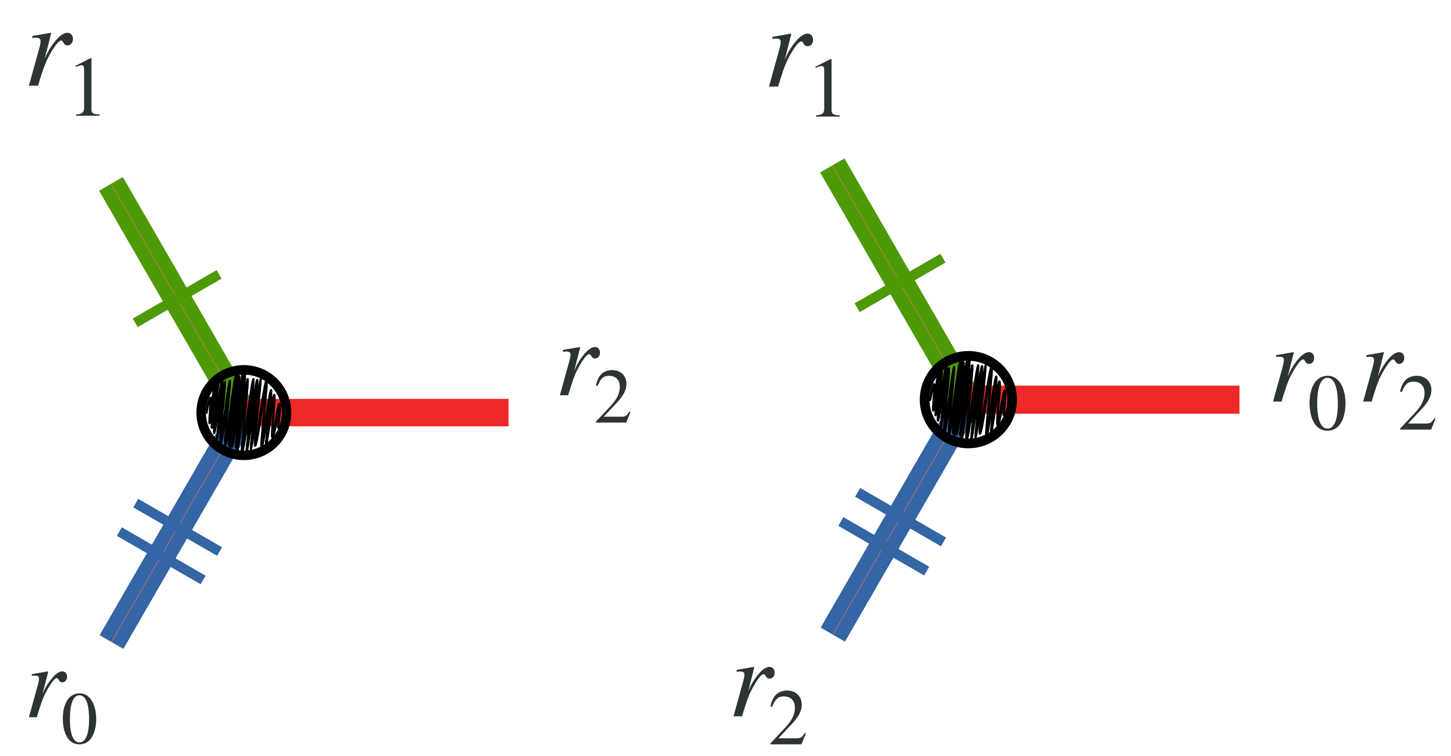

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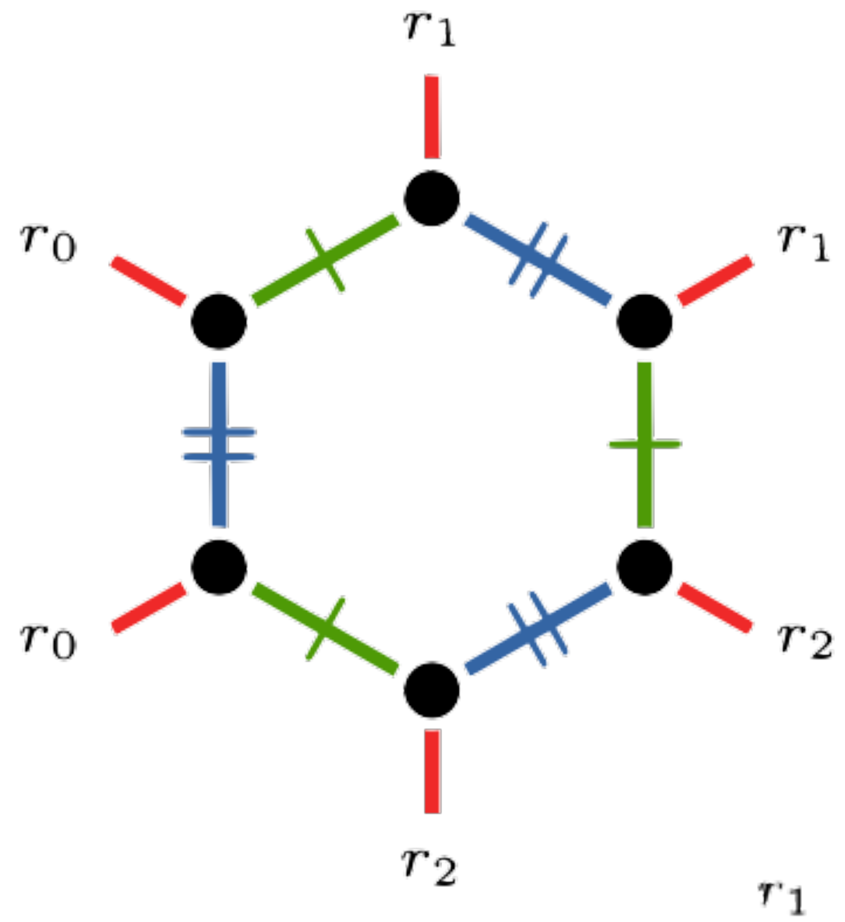

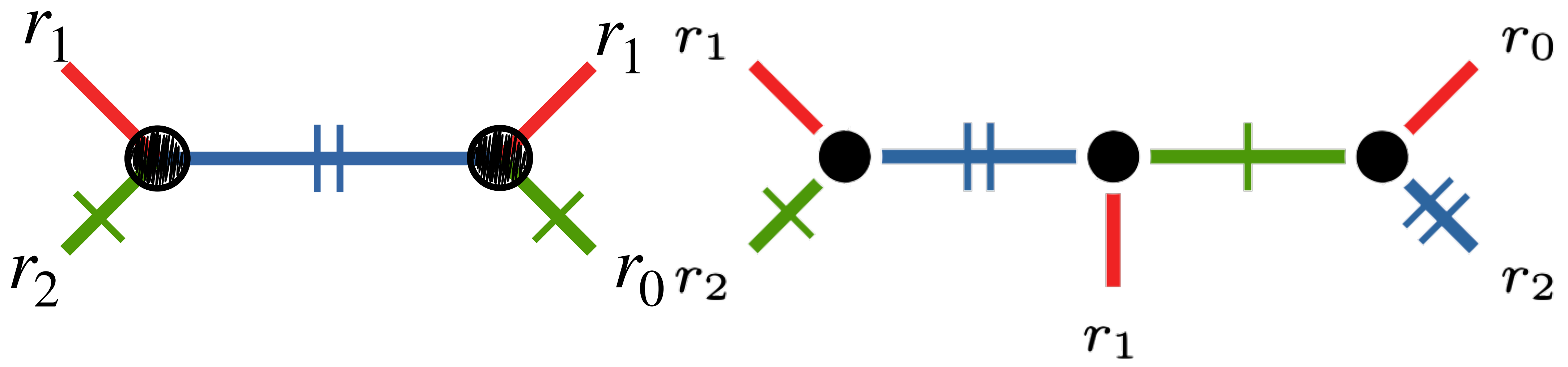

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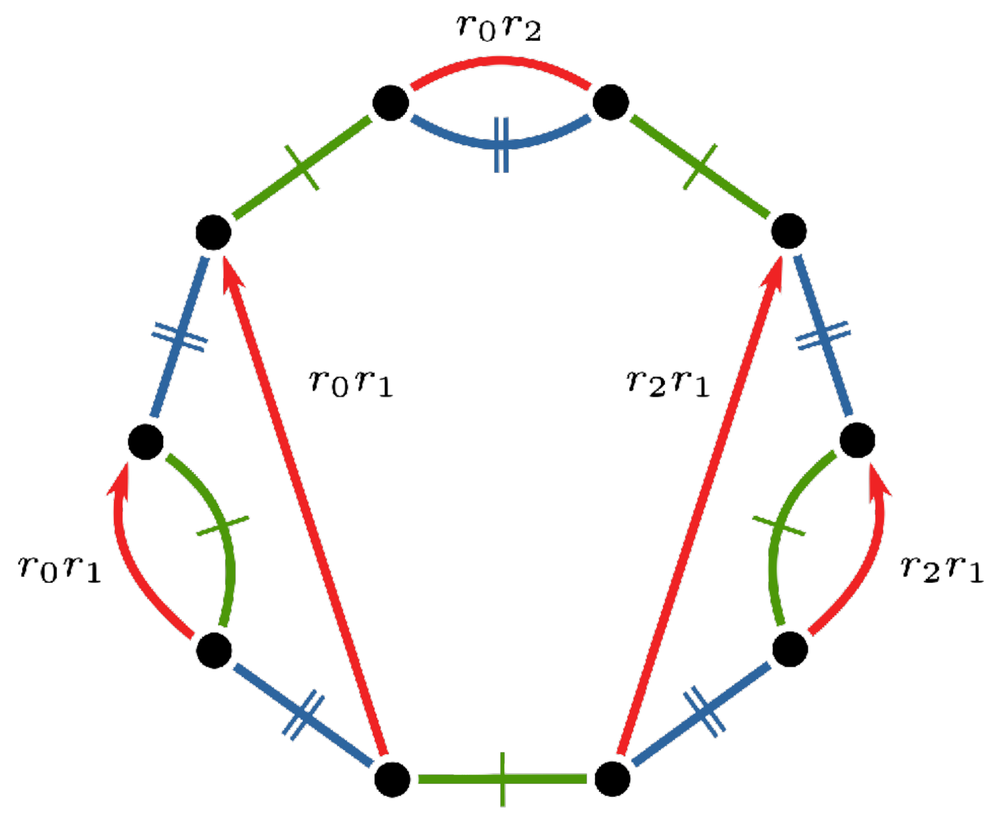

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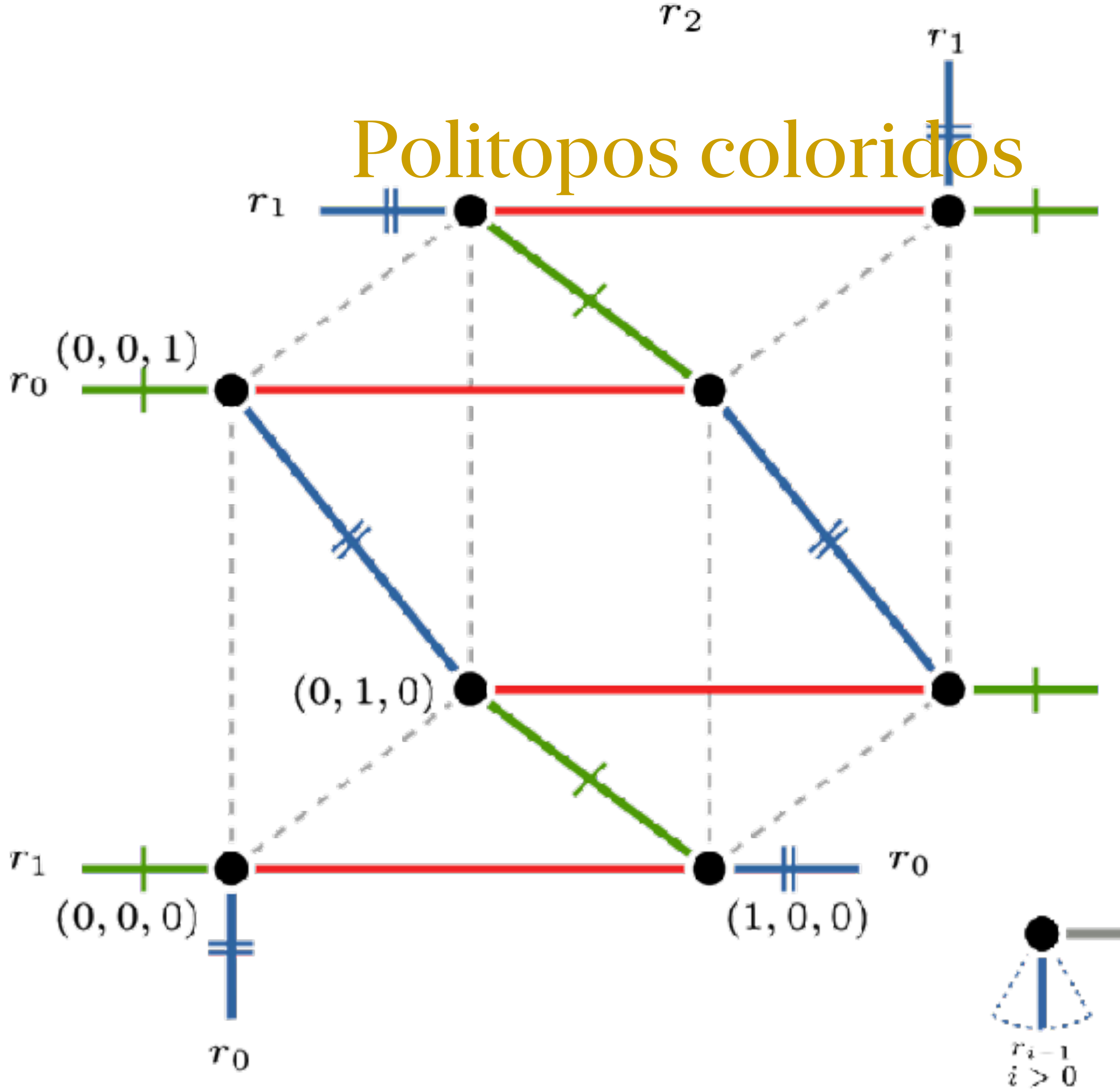

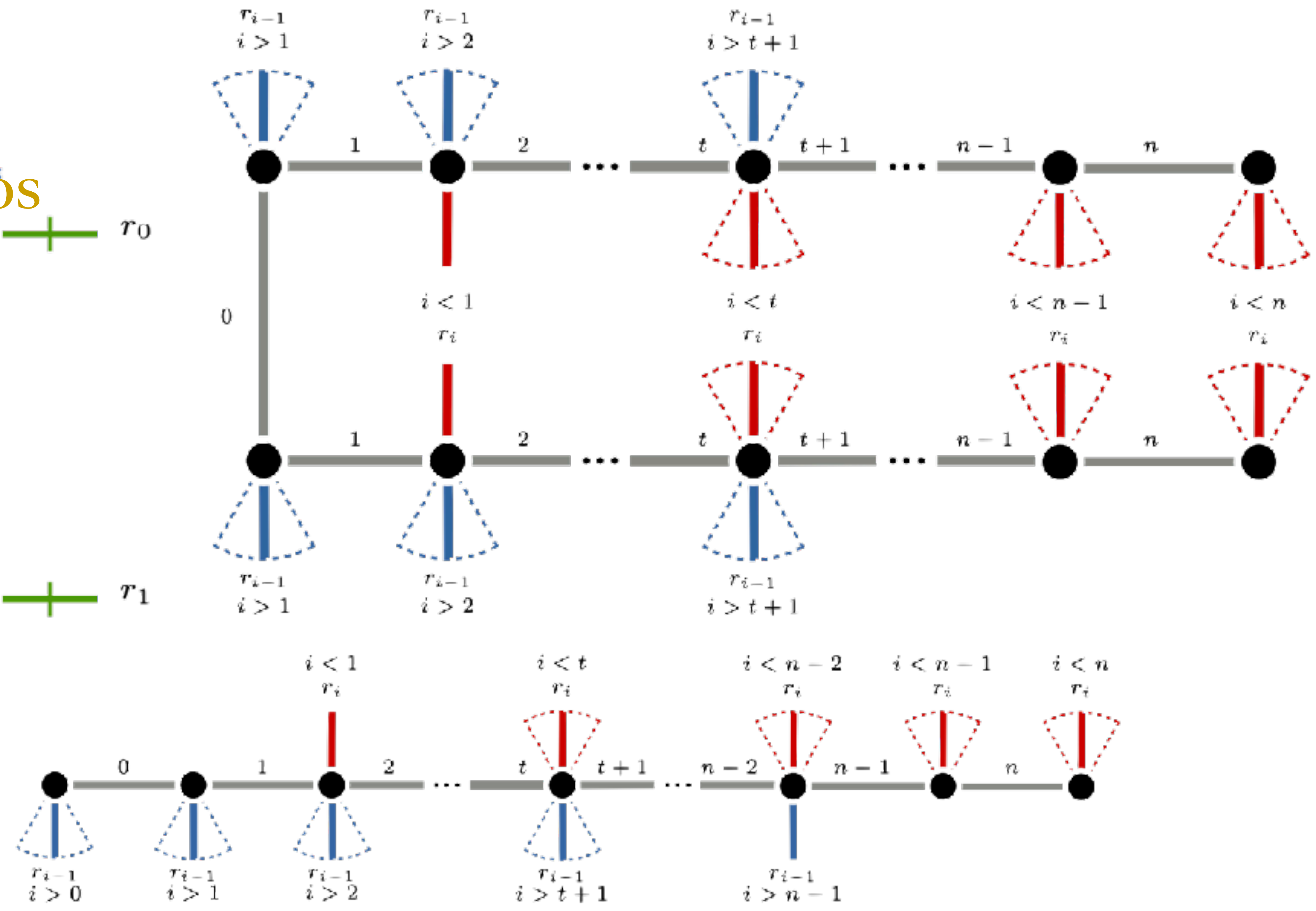

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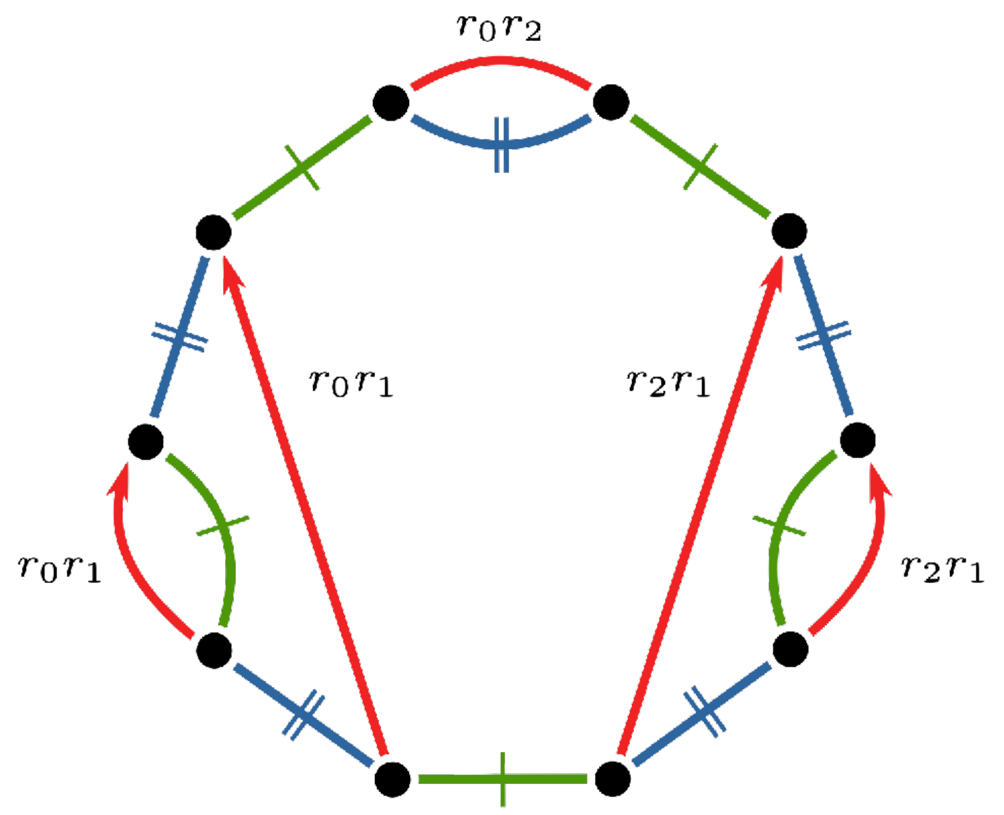

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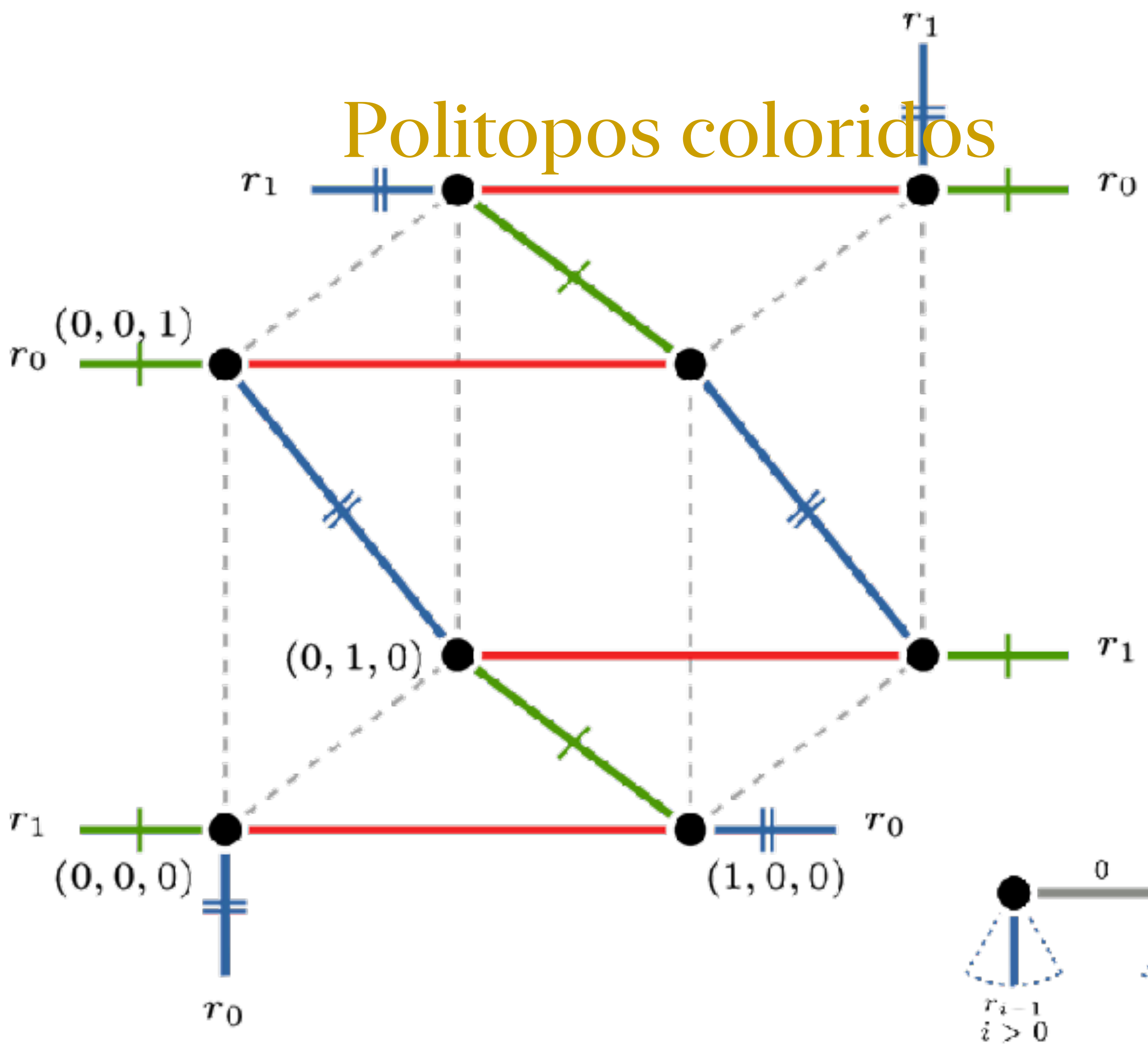
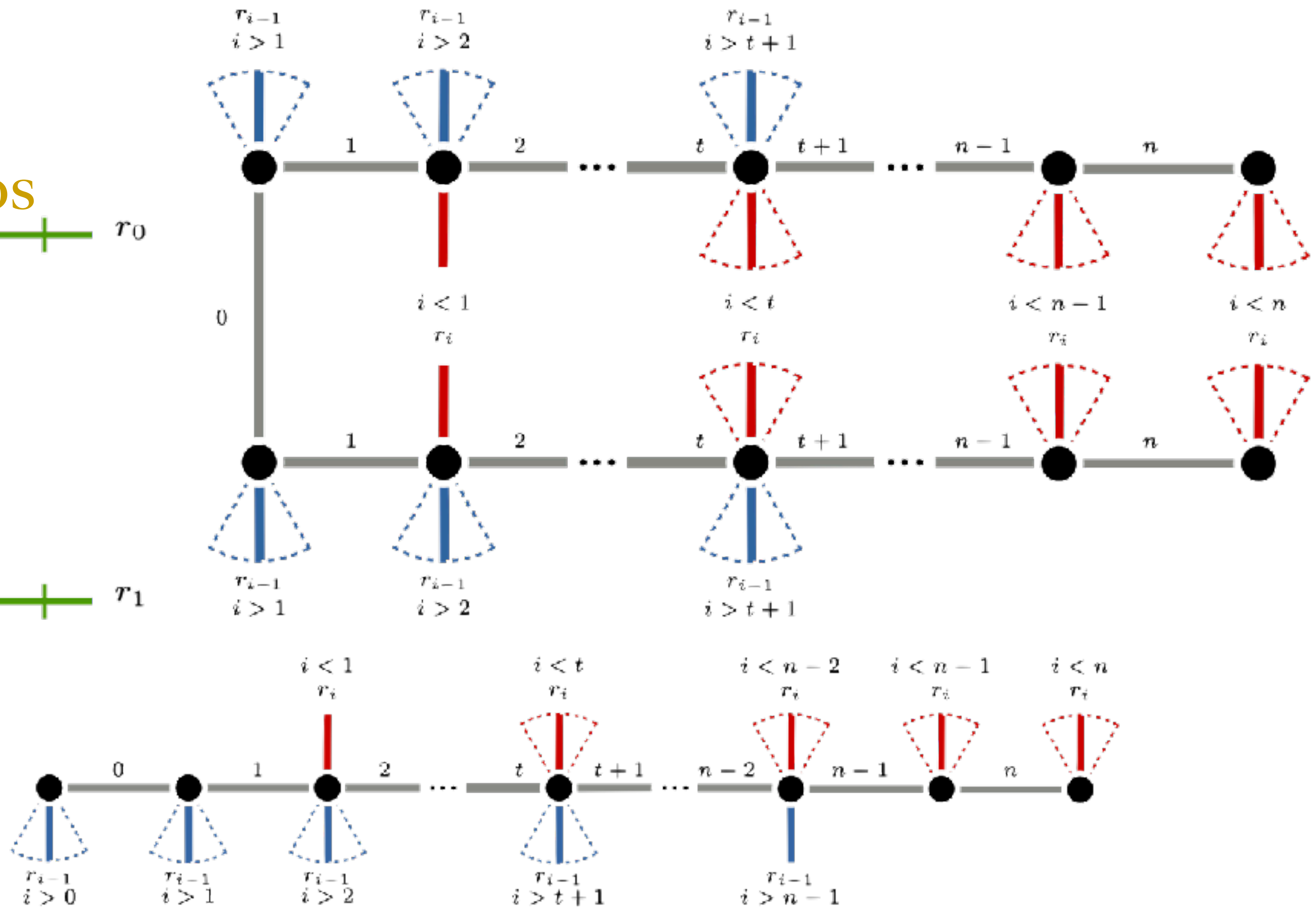
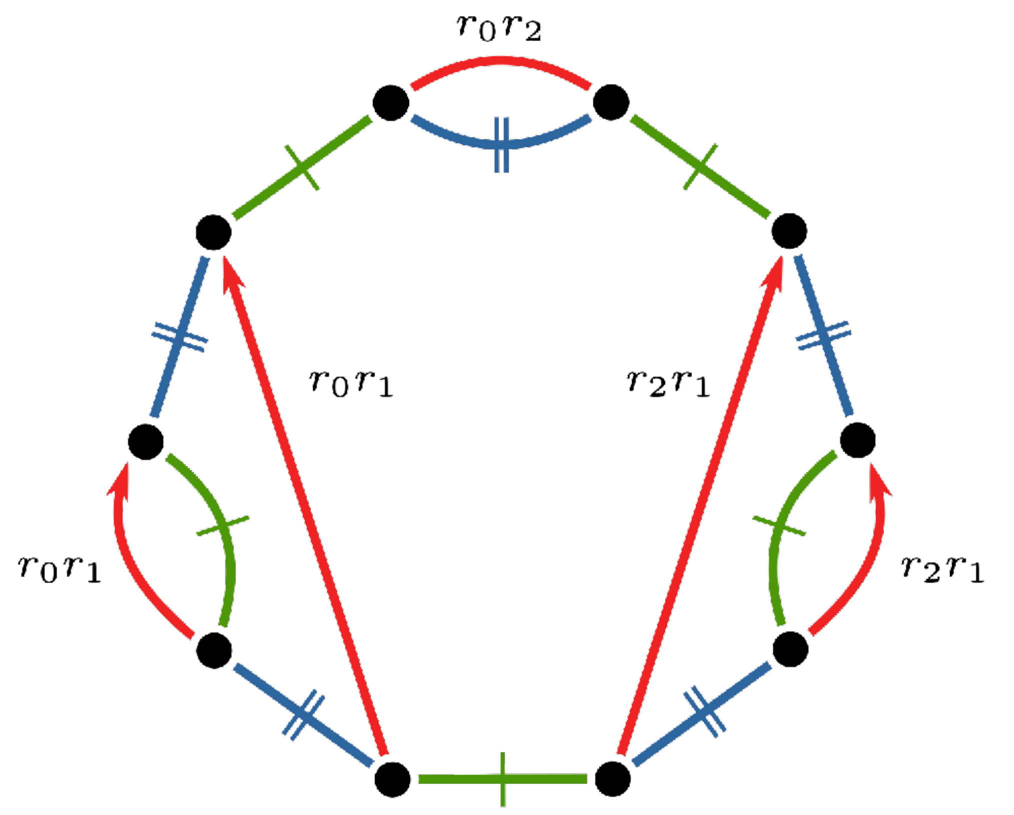




Operaciones de voltaje

$\mathcal{M} \diamond \mathcal{N}$



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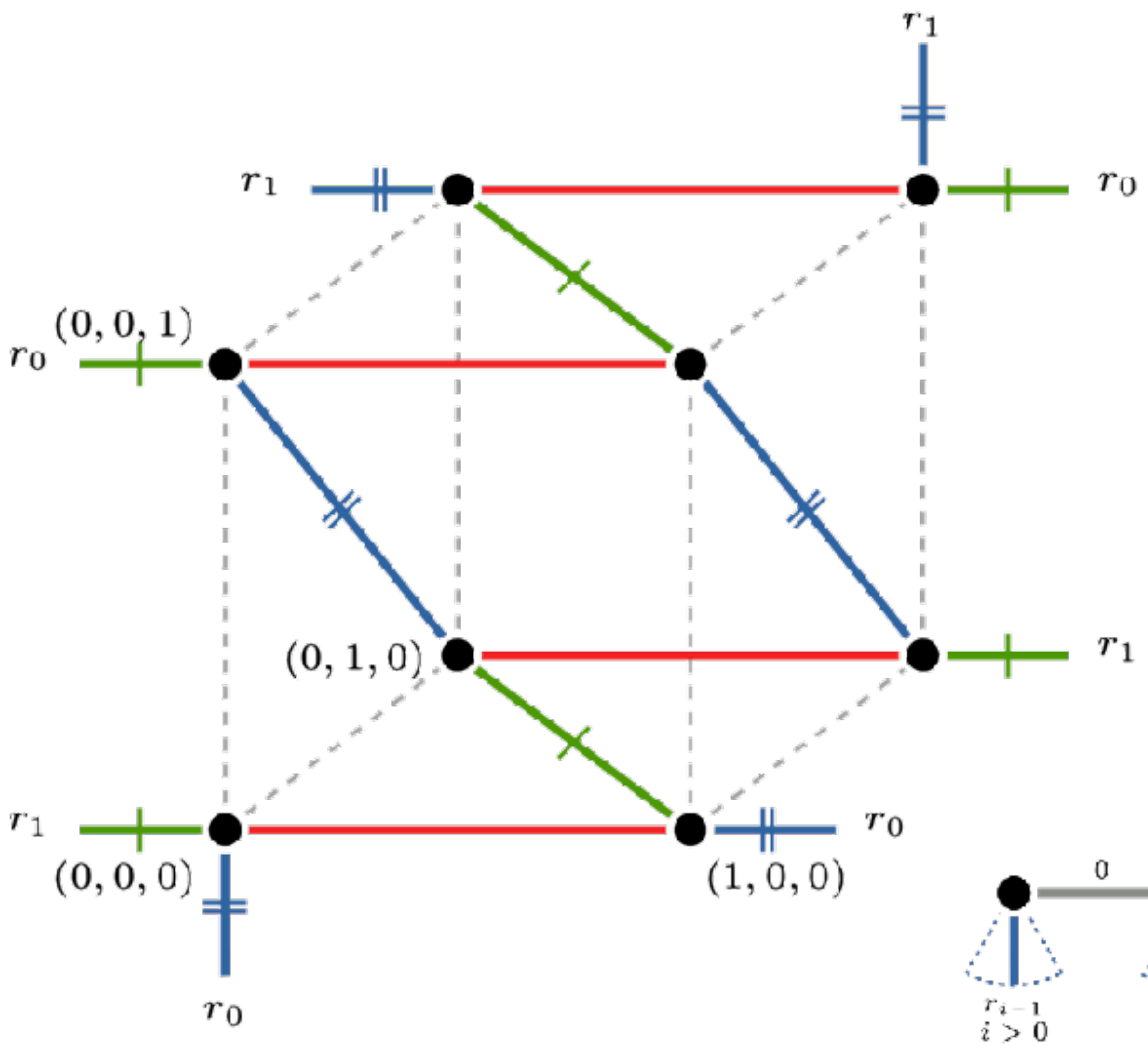

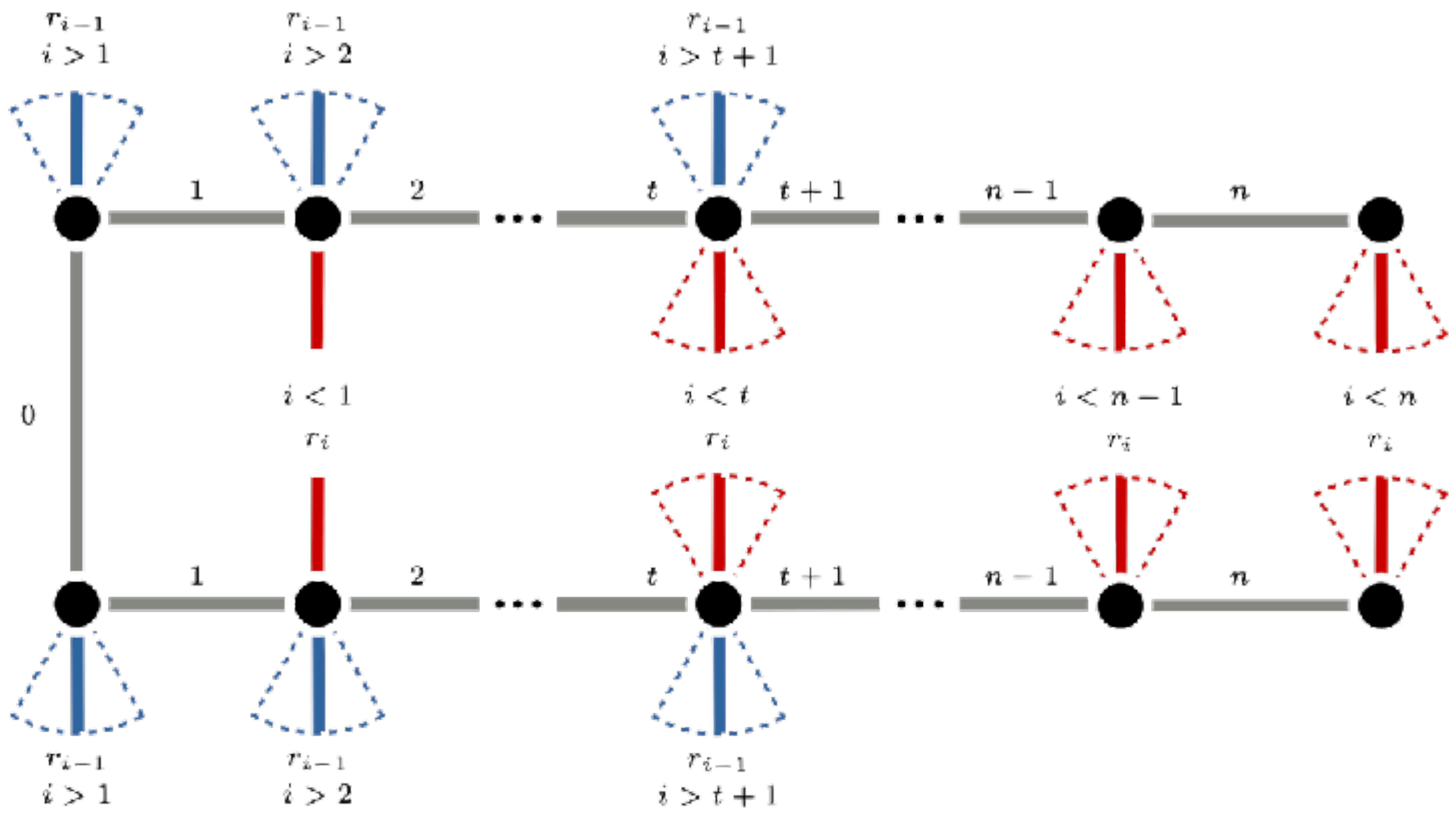

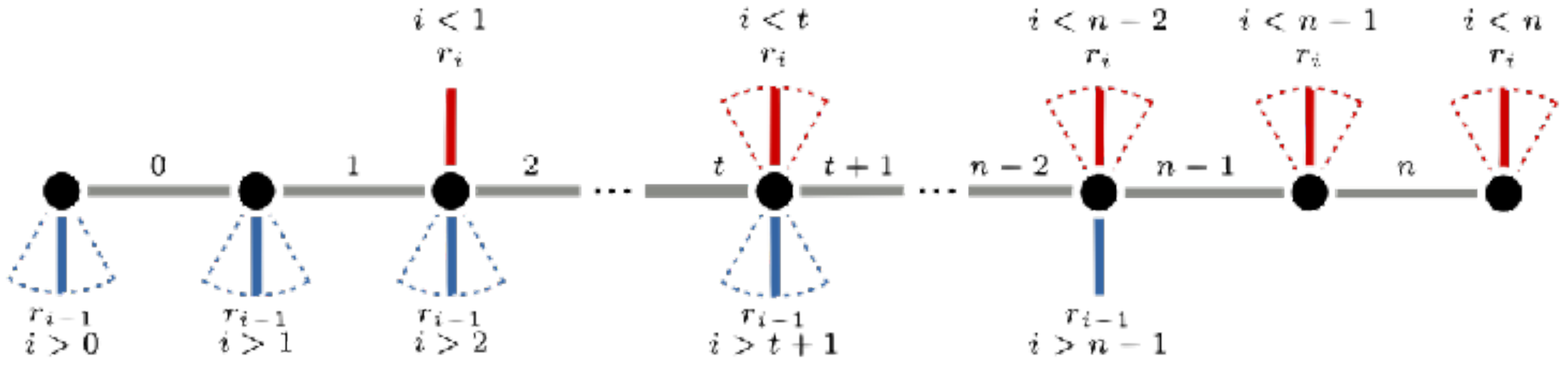

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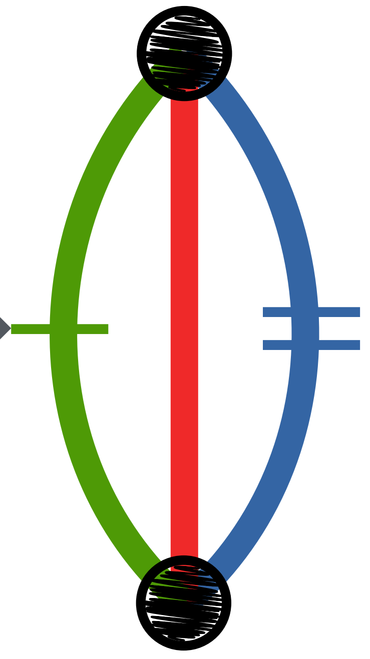
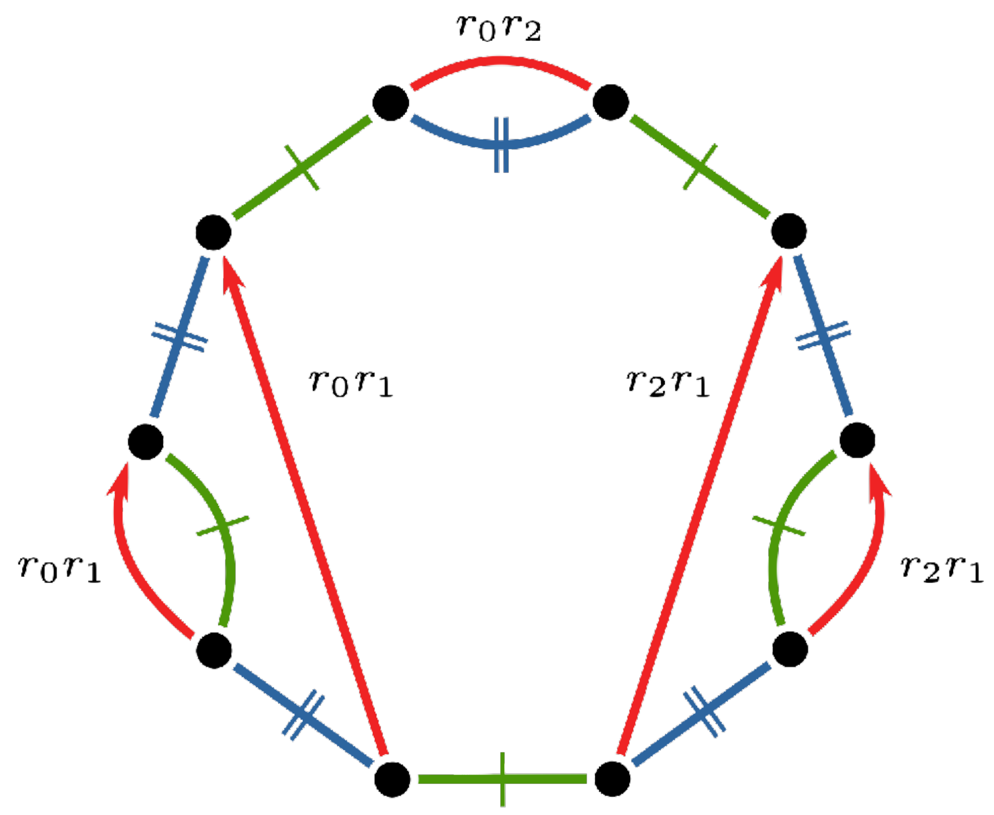
$\mathscr{M} \diamond$ 



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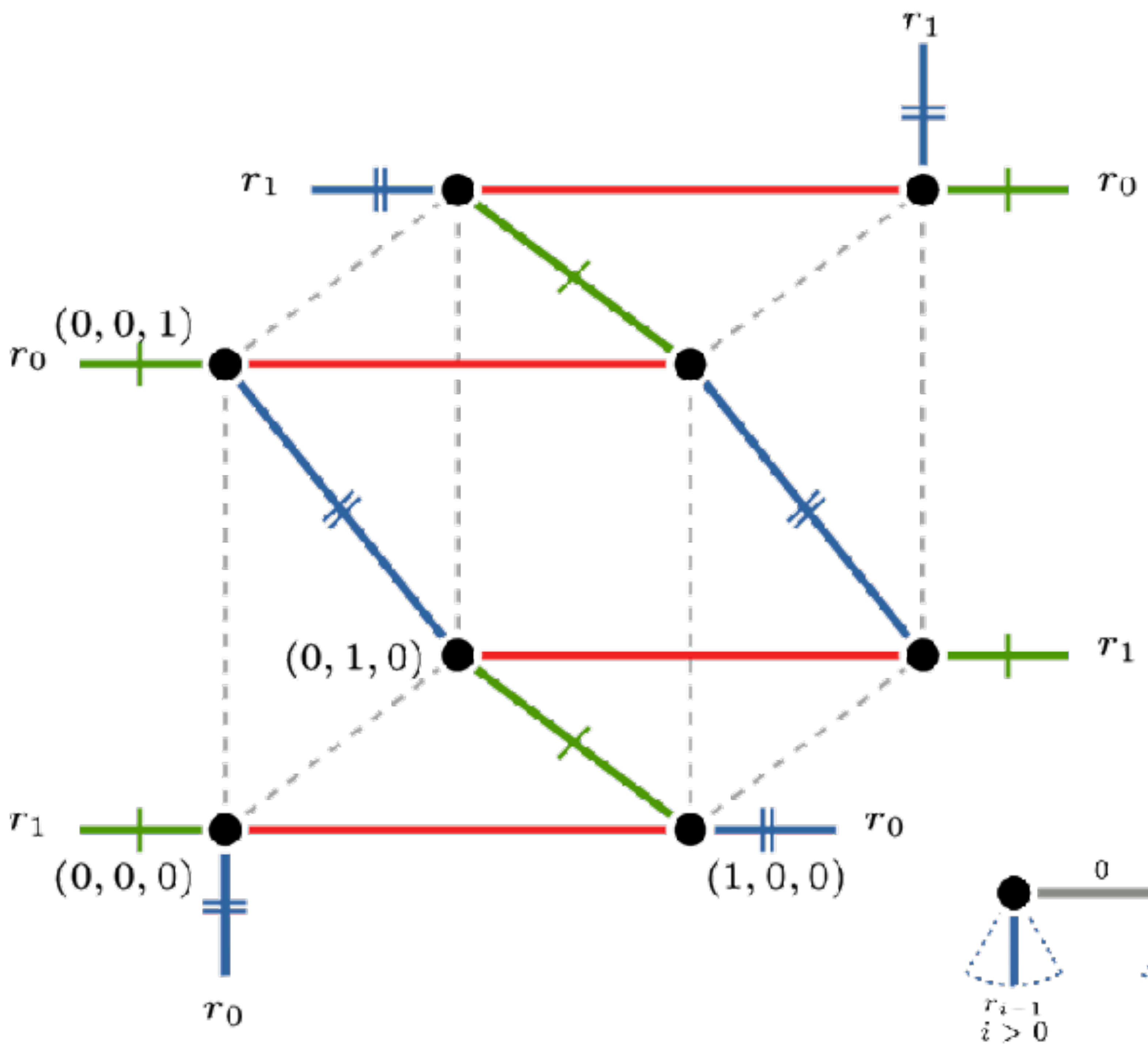

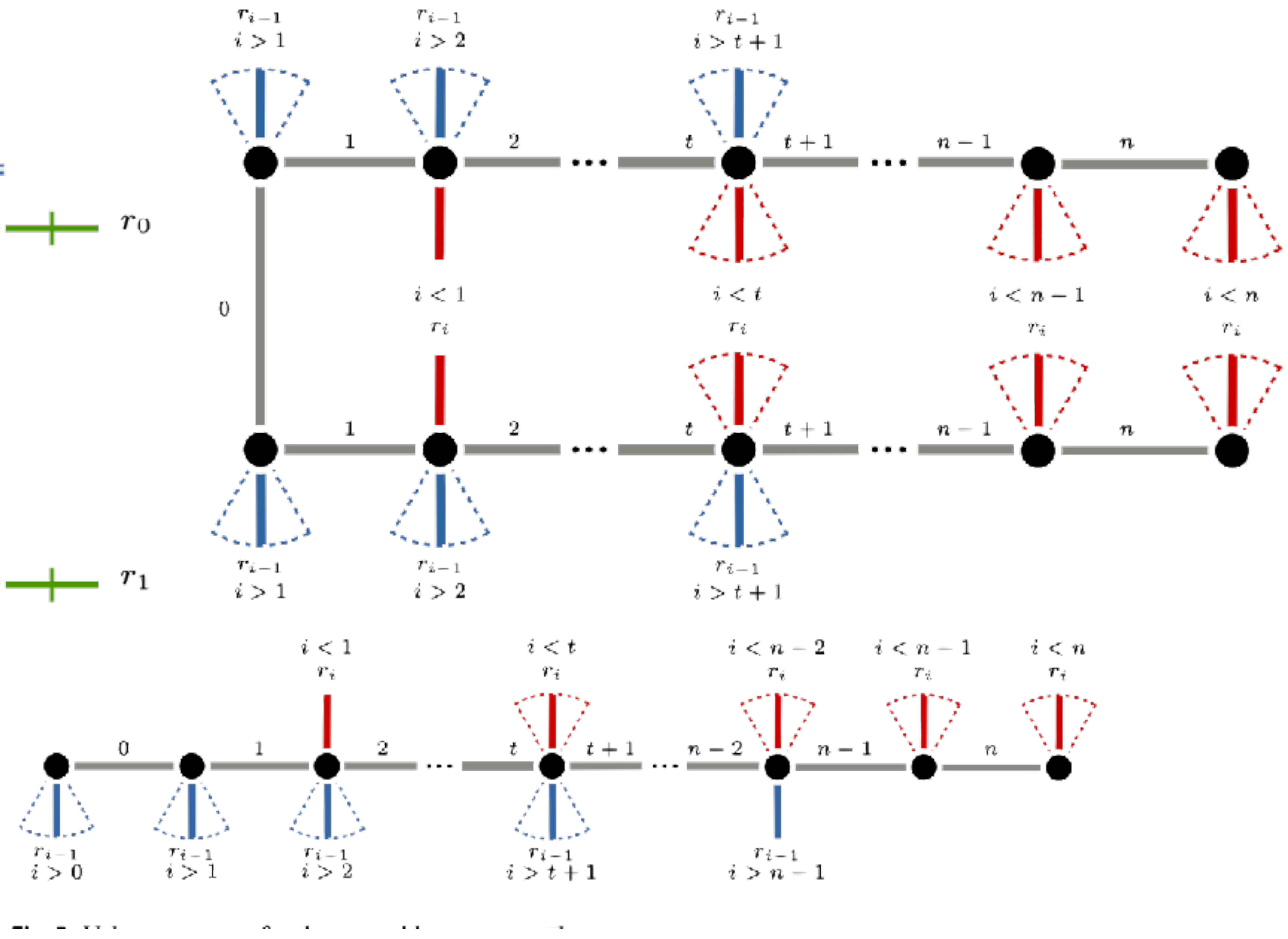

Operaciones de voltaje

Operaciones de voltaje

Hubard, Mochán, M. (2023)

$(\mathscr{Y}, \eta)$ un operador de voltajes
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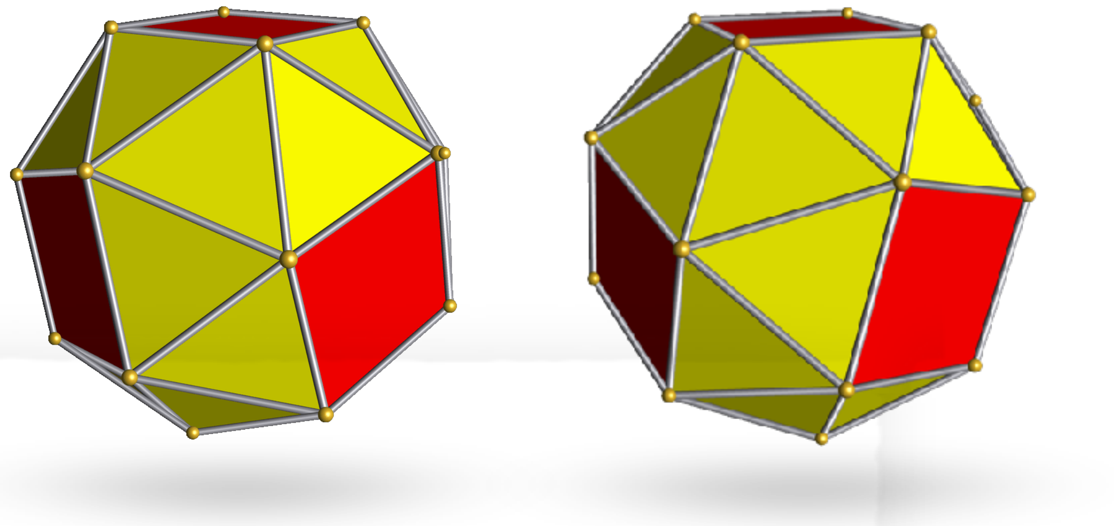

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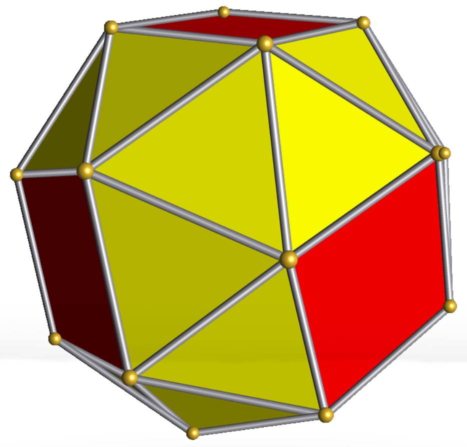

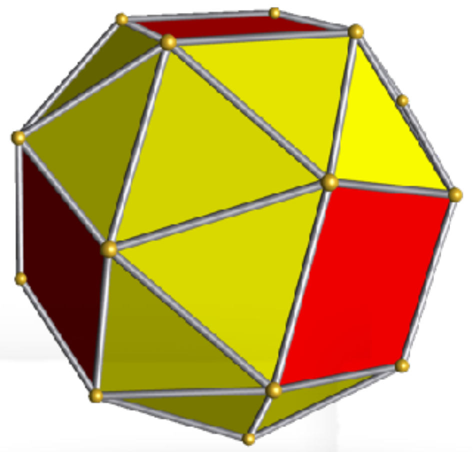

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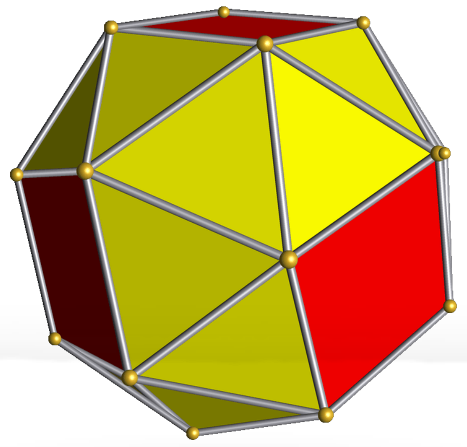

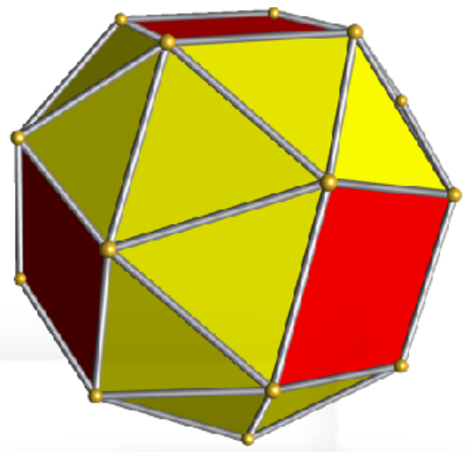

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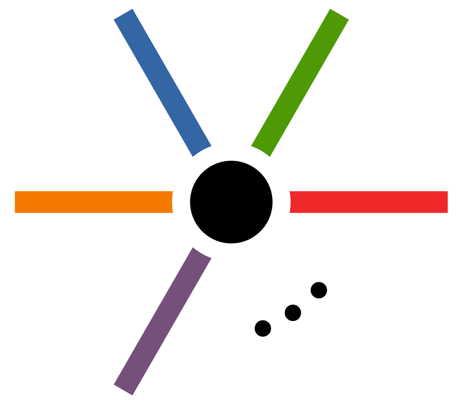
$\mathrm{STG}(\mathscr{X}) =$ 

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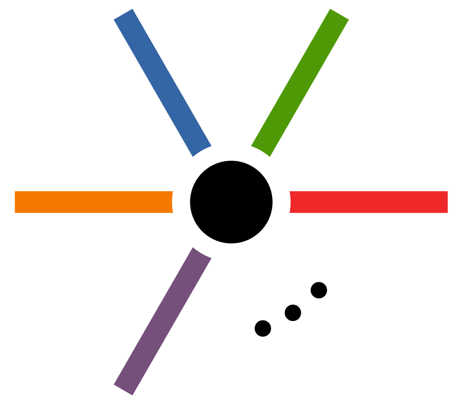
$\mathrm{STG}(\mathscr{X}) =$ 

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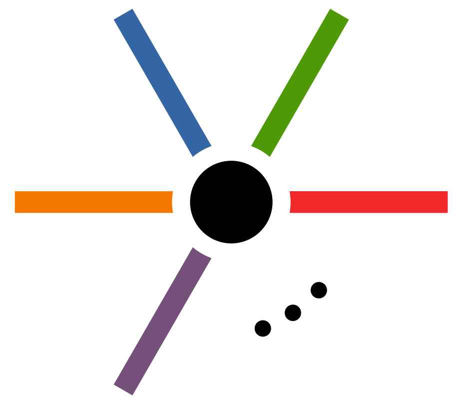
$\mathrm{STG}(\mathscr{X}) =$ 

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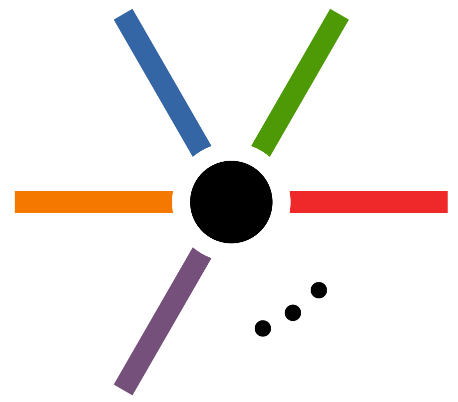
$\mathrm{STG}(\mathscr{X}) =$ 

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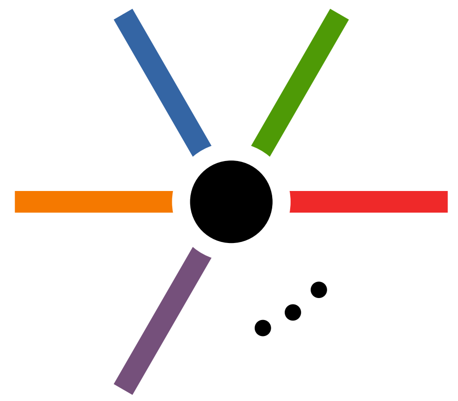
$\mathrm{STG}(\mathscr{X}) =$ 

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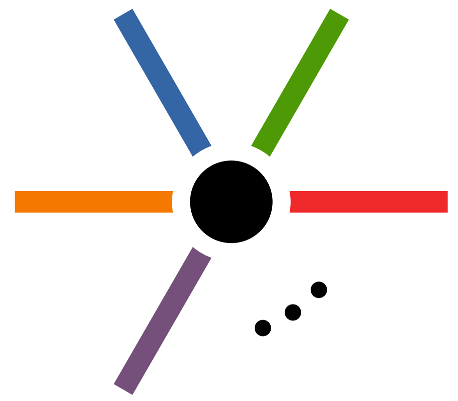
$\mathrm{STG}(\mathscr{X}) =$ 

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$$\mathrm{Aut}(\mathscr{X}) \lneq \mathrm{Aut}(\mathscr{X} \rtimes_\eta \mathscr{Y})$$

Simetrías de operaciones de voltajes

Map operations and k-orbit maps

Alen Orbanić[a,1], Daniel Pellicer[b], Asia Ivić Weiss[b,2]

[a] *University of Ljubljana, Faculty of Mathematics and Physics, Jadranska 21, 1000 Ljubljana, Slovenia*
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A B S T R A C T

A k-orbit map is a map with k flag-orbits under the action of its automorphism group. We give a basic theory of k-orbit maps and classify them up to $k \leqslant 4$. “Hurwitz-like” upper bounds for the cardinality of the automorphism groups of 2-orbit and 3-orbit maps on surfaces are given. Furthermore, we consider effects of

Simetrías de operaciones de voltajes

Proposition 4.3. *Let $M = (\mathcal{C}/N, \mathcal{C})$ be a k-orbit map. Then* $\mathrm{Tr}(M)$ *is either a k-orbit map, a $\frac{3k}{2}$-orbit map or a 3k-orbit map.*

Proof. There are three cosets in $\mathcal{C}_3/3_3^0$, namely $3_3^0 a$, $a \in A = \{id, s_1, s_{121}\}$. Therefore, every element $x \in \mathcal{C}_3$ is of the form $x = ta$, $t \in 3_3^0$, $a \in A$.

Let $N' = \varphi(N)$ (see diagram in Fig. 5). For any $a \in A$ let $x, y \in 3_3^0 a$, with $x = ta$, $y = sa$, for some $t, s \in 3_3^0$. If both x and y normalize N', it follows that $t^{-1}N't = aN'a^{-1} = s^{-1}N's$. Hence t and s must be in the same coset in $3_3^0/\mathcal{N}$, where $\mathcal{N} = \mathrm{Norm}_{3_3^0}(N') \leqslant 3_3^0$. This implies that $\mathrm{Norm}_{\mathcal{C}_3}(N')$ consists of cosets in $\mathcal{C}_3/\mathcal{N}$, where at most one such coset can be contained in any of the cosets $3_3^0 a$, $a \in A$. Since

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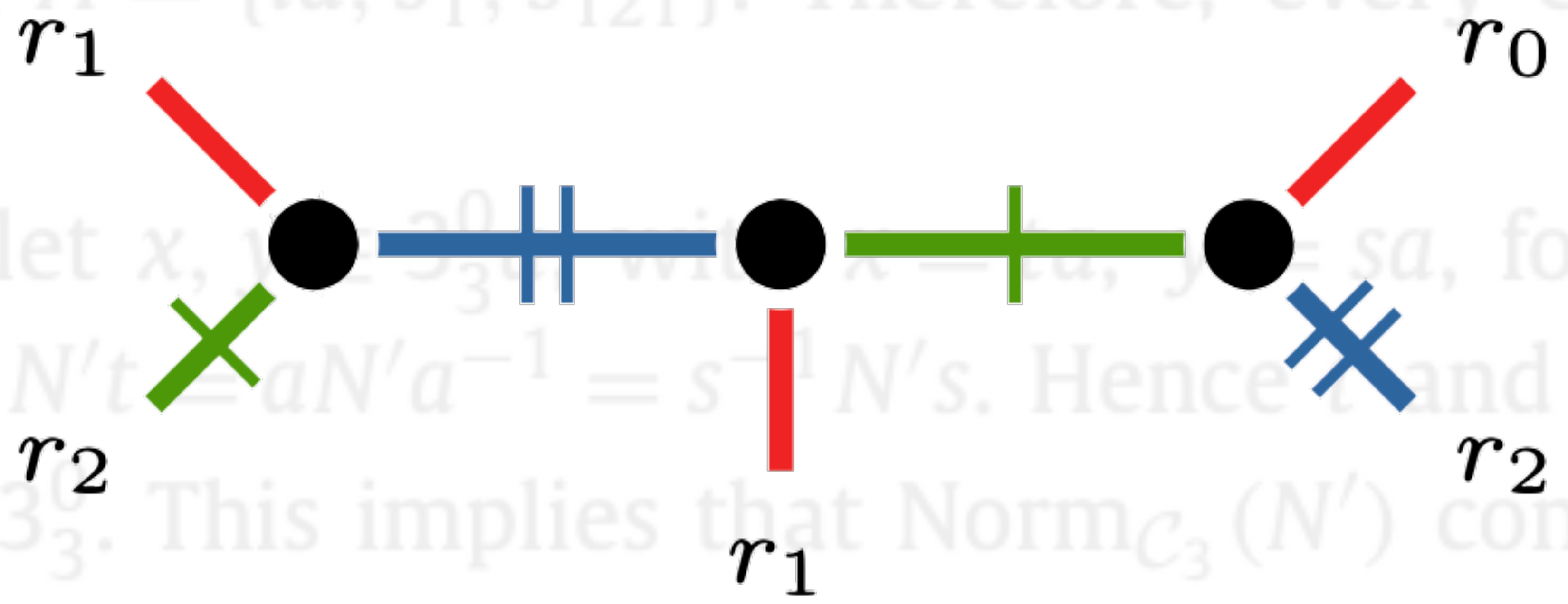

from the fact that $[3_3^0 : \mathcal{N}] = k$. $\square$

We illustrate the proposition with the following examples.

The truncations of the regular maps of Schläfli type $\{4,4\}$ and $\{6,3\}$ are have faces of two different sizes and therefore are 3-orbit maps. The truncation of the regular map $\{3,6\}_{(t,0)}$ is the regular map $\{6,3\}_{(t,t)}$, whereas the truncation of the regular map $\{3,6\}_{(t,t)}$ is the regular map $\{6,3\}_{(3t,0)}$. Finally, the map given in dotted lines in Fig. 6 is a 3-orbit map on an orientable surface of genus 2. As we can see, this map can be obtained by truncating the regular map $\{3,8|\cdot,\cdot,2\}$ (see Fig. 6(a)) or by truncating the 2-orbit map given in Fig. 6(b) (belonging to class 2_{01}).

The core of 3_3^0 is the index two subgroup $K = \langle s_0, s_{101}, s_{21012}\rangle$ of $\mathcal{C}_3$. The group K plays an important role for the truncation operation, as is shown by the following two results.

Proposition 4.4. *Let $M = (\mathcal{C}/N, \mathcal{C})$ be a k-orbit map such that* $\mathrm{Tr}(M)$ *is $\frac{3k}{2}$-orbit or k-orbit. Then M must necessarily be 2_{01}-compatible.*

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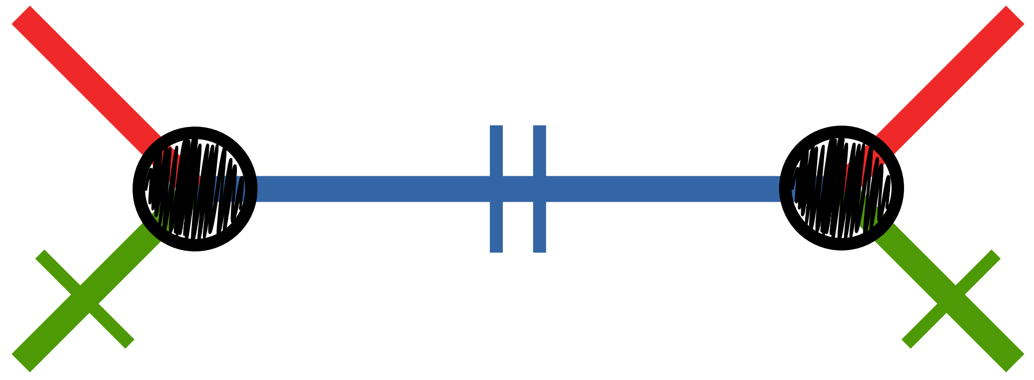
$\mathscr{M}$ cubre a 

Simetrías de operaciones de voltajes

Simetrías de operaciones de voltajes

Hubard, Mochán, M. (2025)

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$(\mathcal{Y}, \eta)$ un operador de voltajes

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Simetrías de operaciones de voltajes

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Para un operador de voltajes $(\mathscr{Y}, \eta)$ existe una familia $\mathscr{F}$ de premaniplexes que cumple:

- $|\mathscr{F}| \leq |\mathscr{Y}/\mathrm{Aut}(\mathscr{Y})|$
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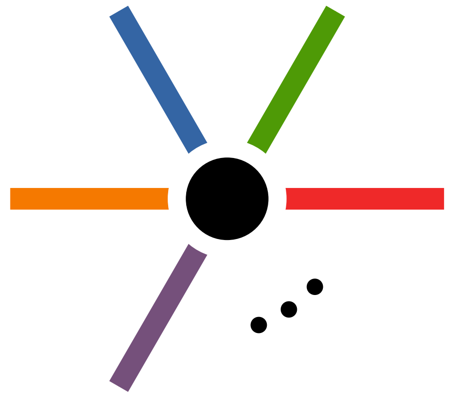
No es poco común que  $\in \mathscr{F}$

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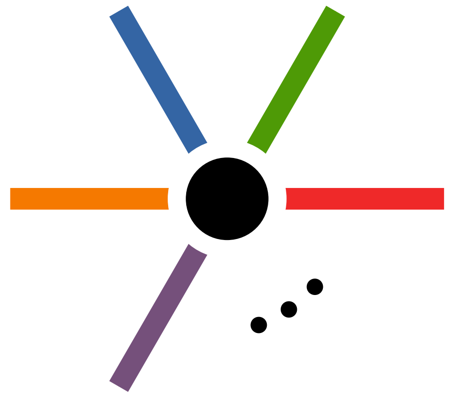
No es poco común que  $\in \mathscr{F}$

En particular, cada automorfismo no trivial de $\mathscr{Y}$ induce este caso.

Podemos entender esos?

Simetrías de operaciones de voltajes

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\begin{tikzcd}
\mathscr{X} \rtimes_\eta \mathscr{Y} \arrow[r, "\tau^*"] \arrow[d] & \mathscr{X} \rtimes_\eta \mathscr{Y} \arrow[d] \\
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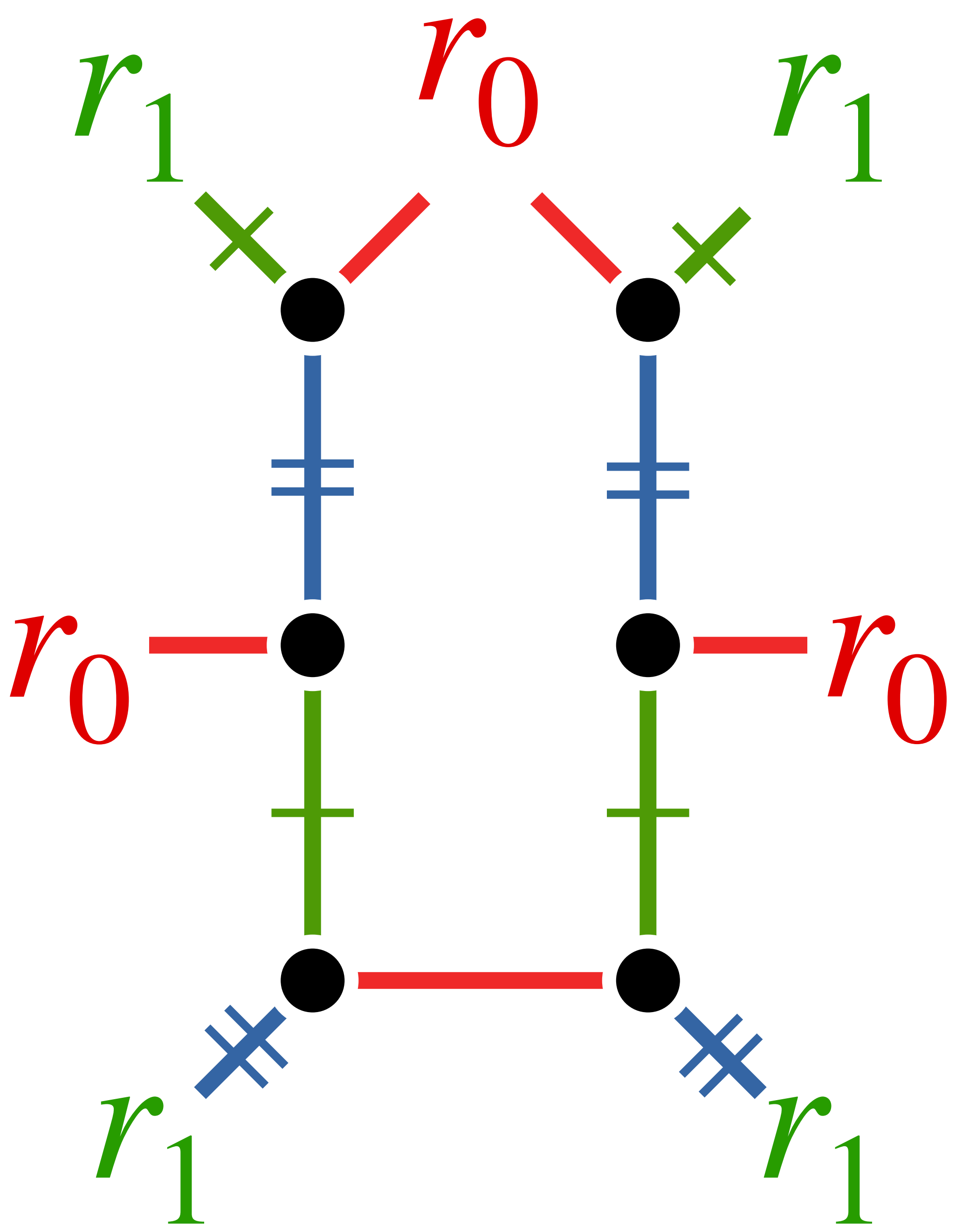
Los automorfismos de $\mathscr{X} \rtimes_\eta \mathscr{Y}$ que se proyectan en *id* son exactamente los inducidos porAut($\mathscr{X}$).

Simetrías de operaciones de voltajes



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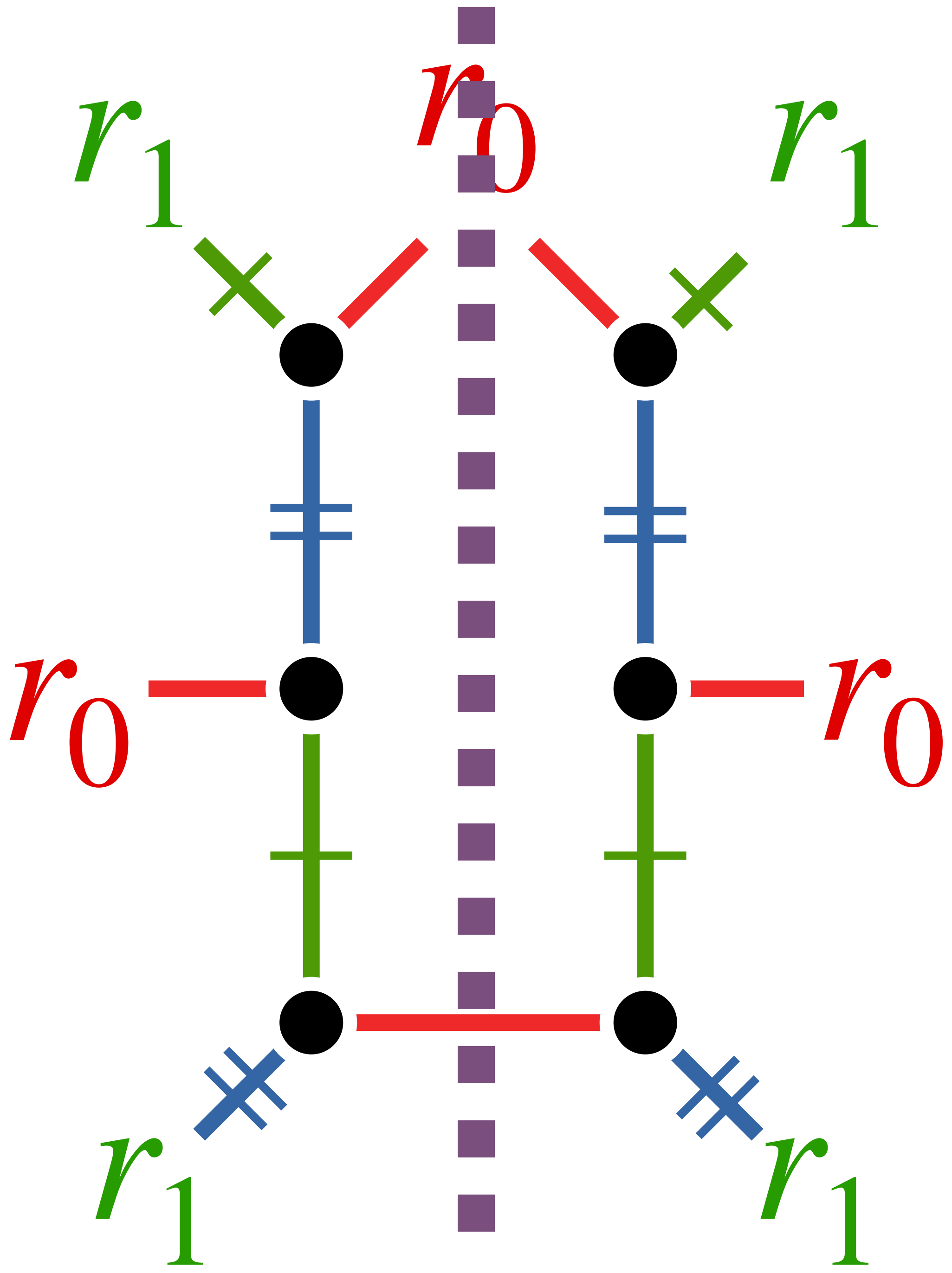

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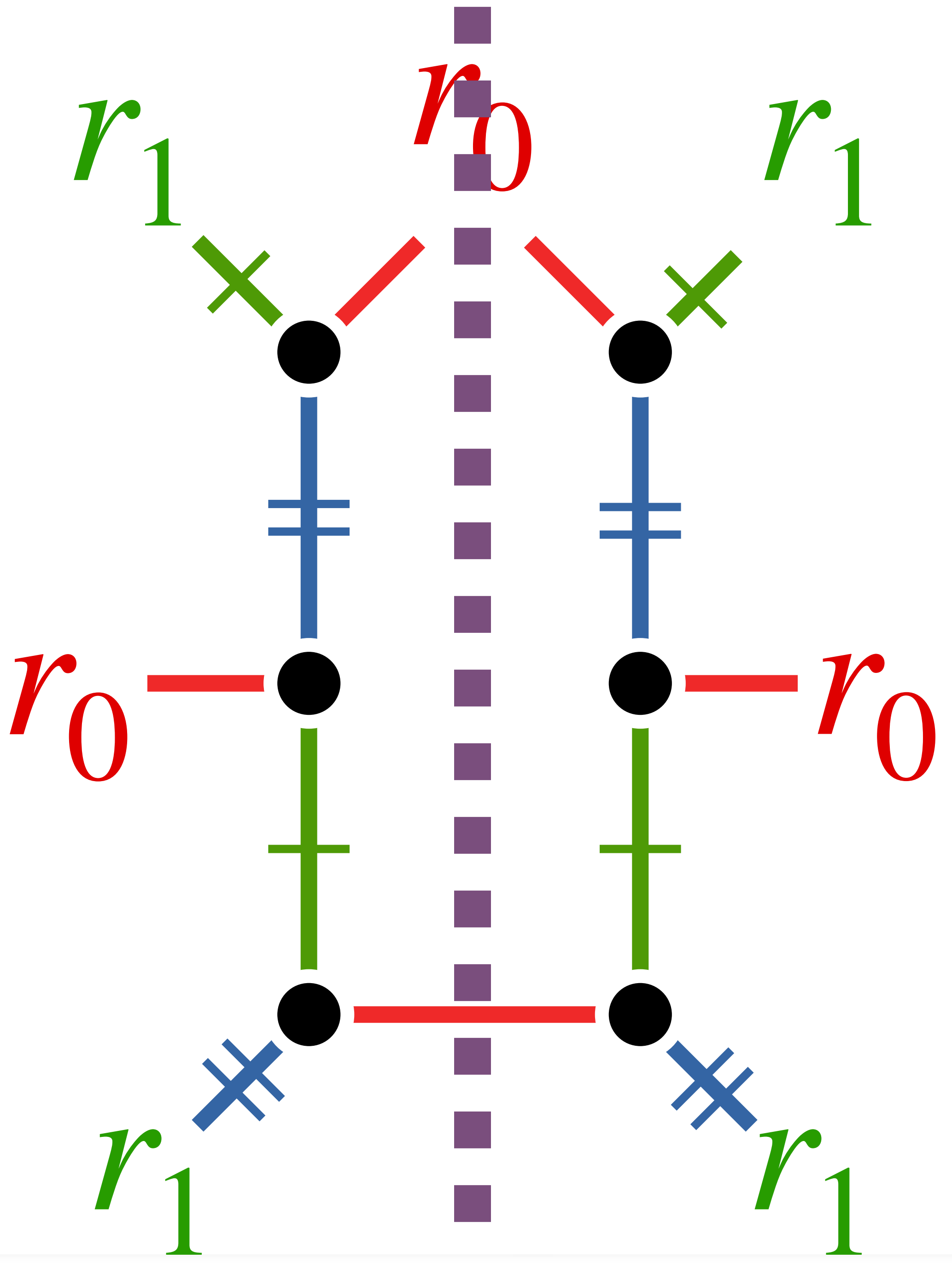

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El grupo $\mathrm{Aut}(\mathscr{Y}, \eta) \leq \mathrm{Aut}(\mathscr{Y})$ de automorfismos de $\mathscr{Y}$ que preservan η siempre se levanta a $\mathscr{X} \rtimes_\eta \mathscr{Y}$ e induce un grupo isomorfo a $\mathrm{Aut}(\mathscr{X}) \times \mathrm{Aut}(\mathscr{Y}, \eta)$.

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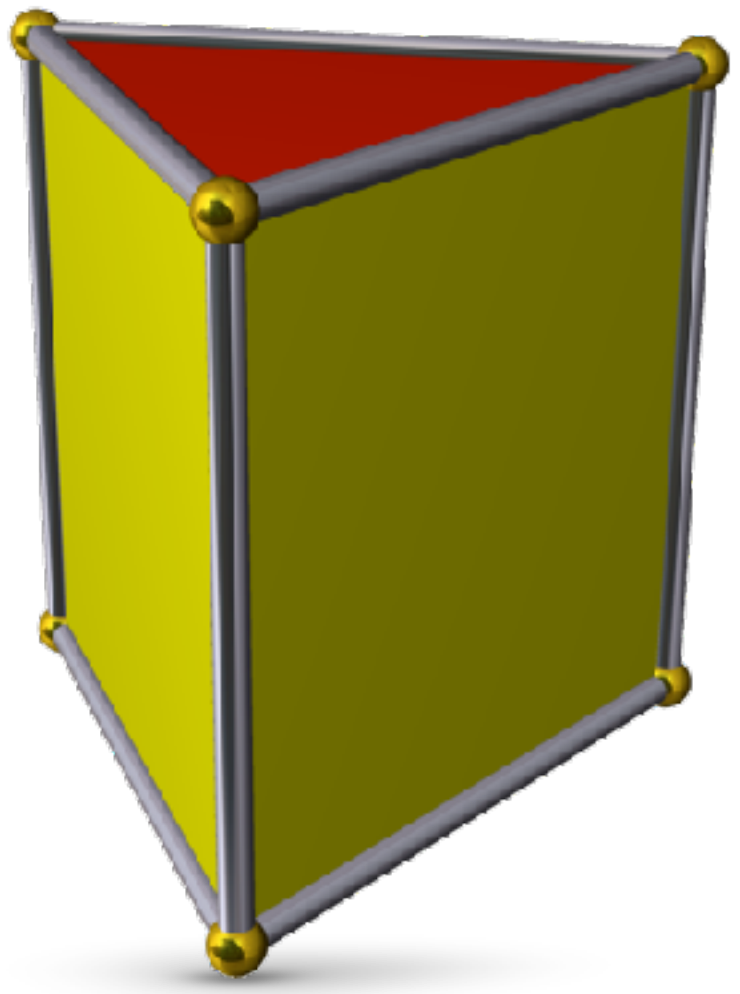

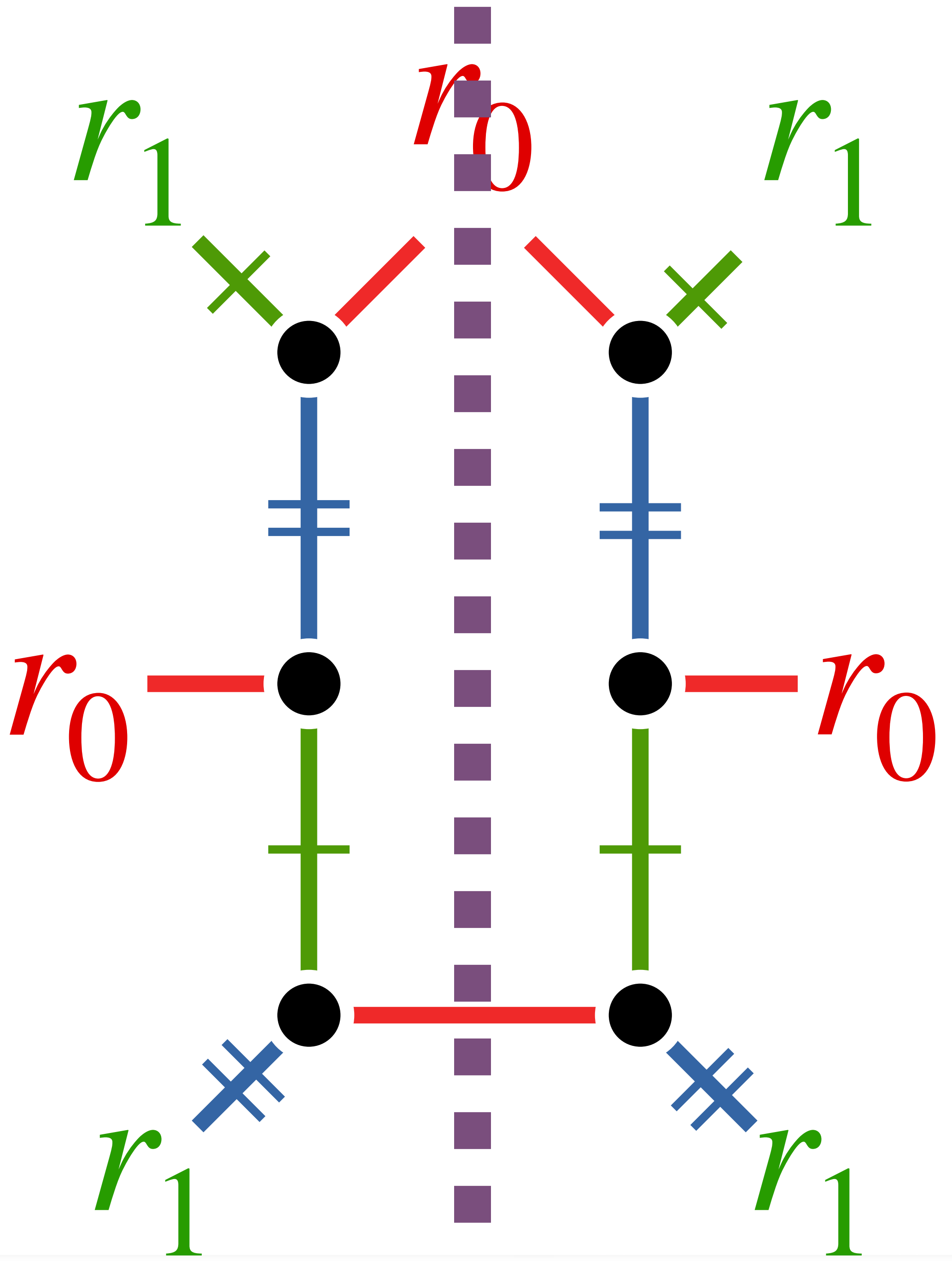

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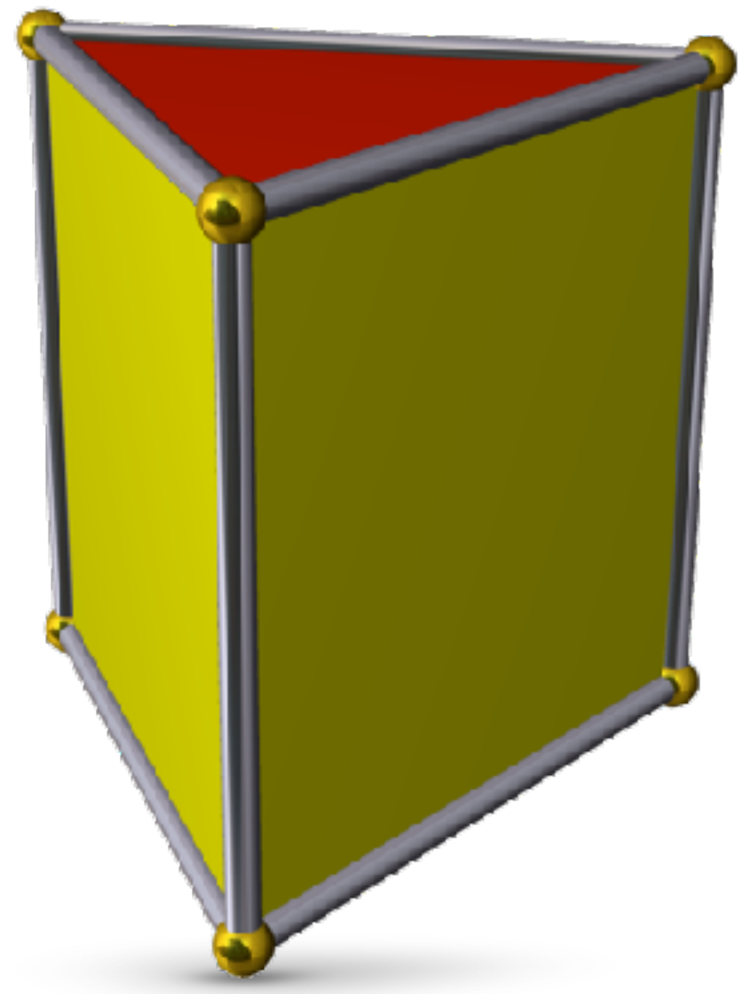

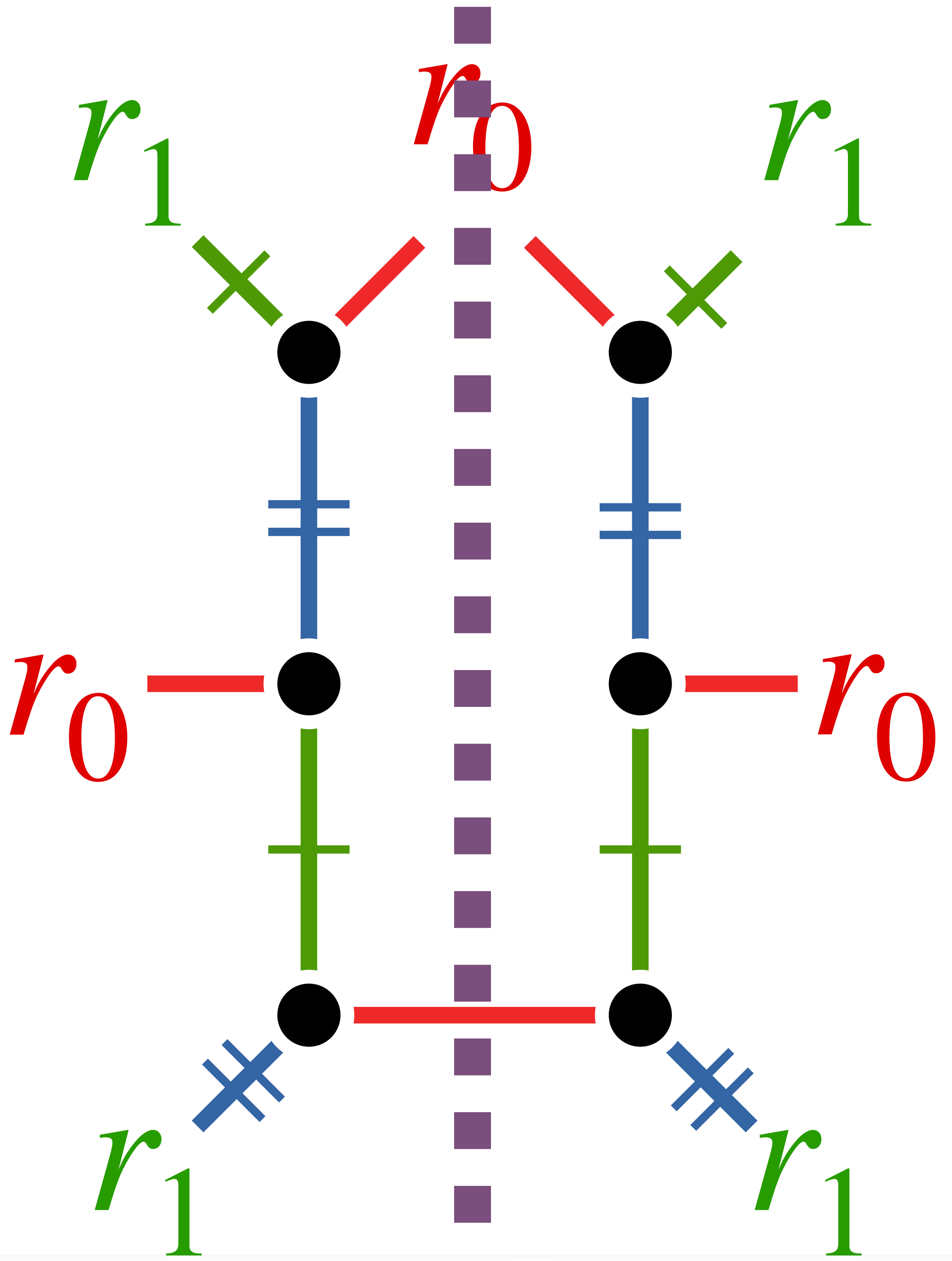

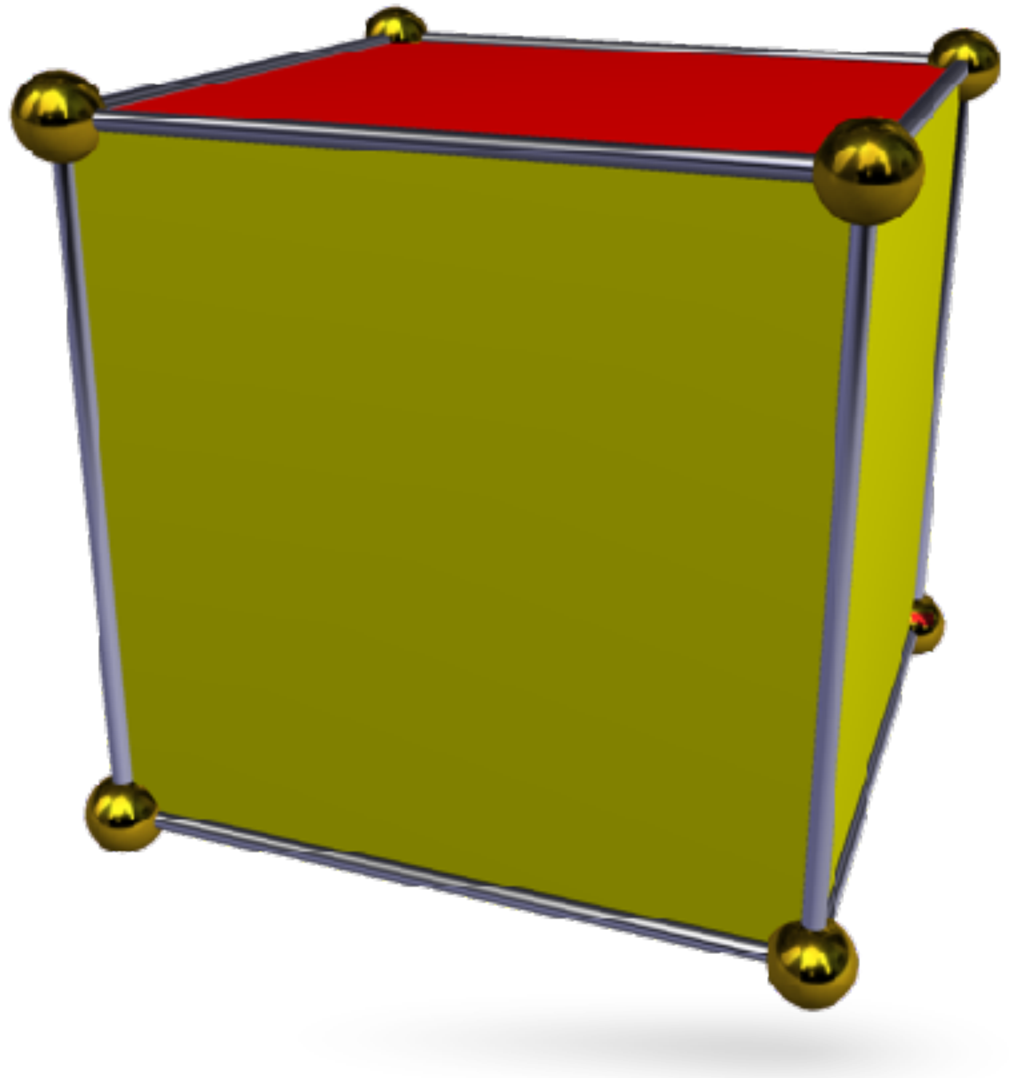

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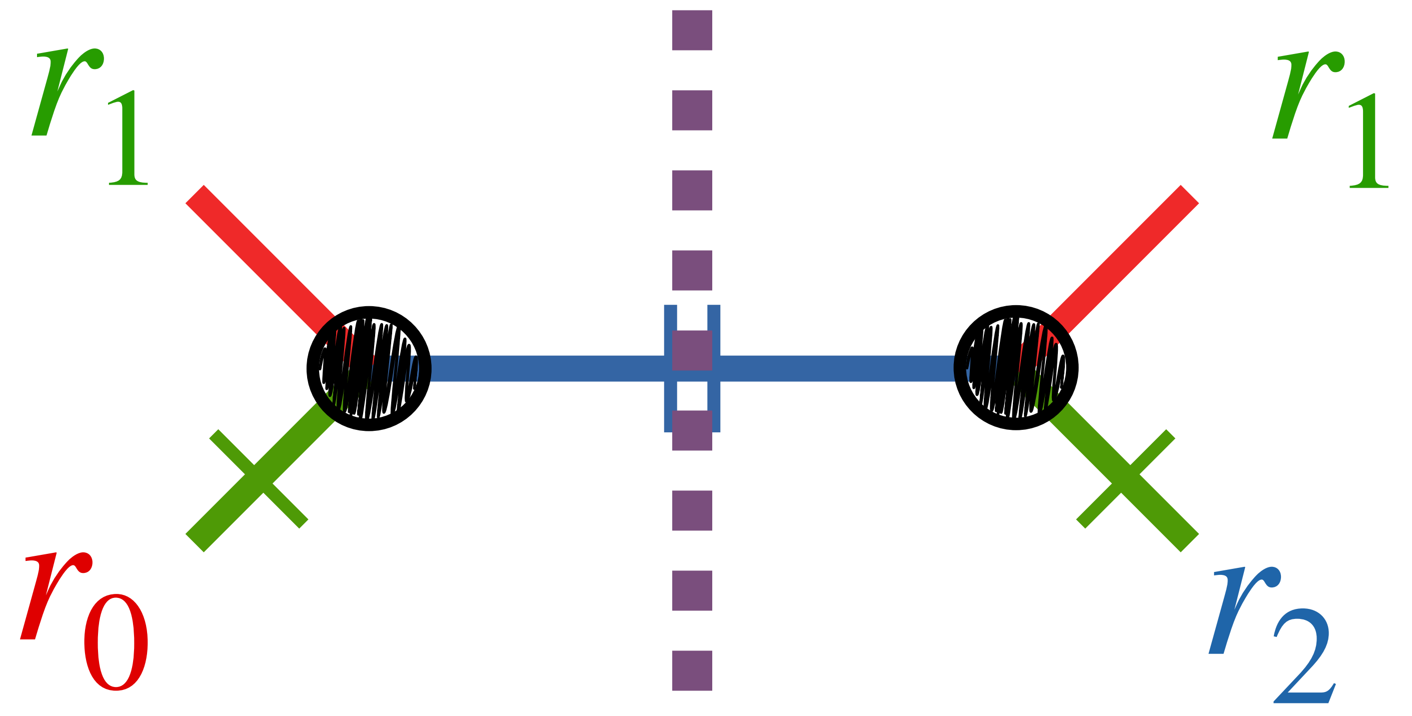

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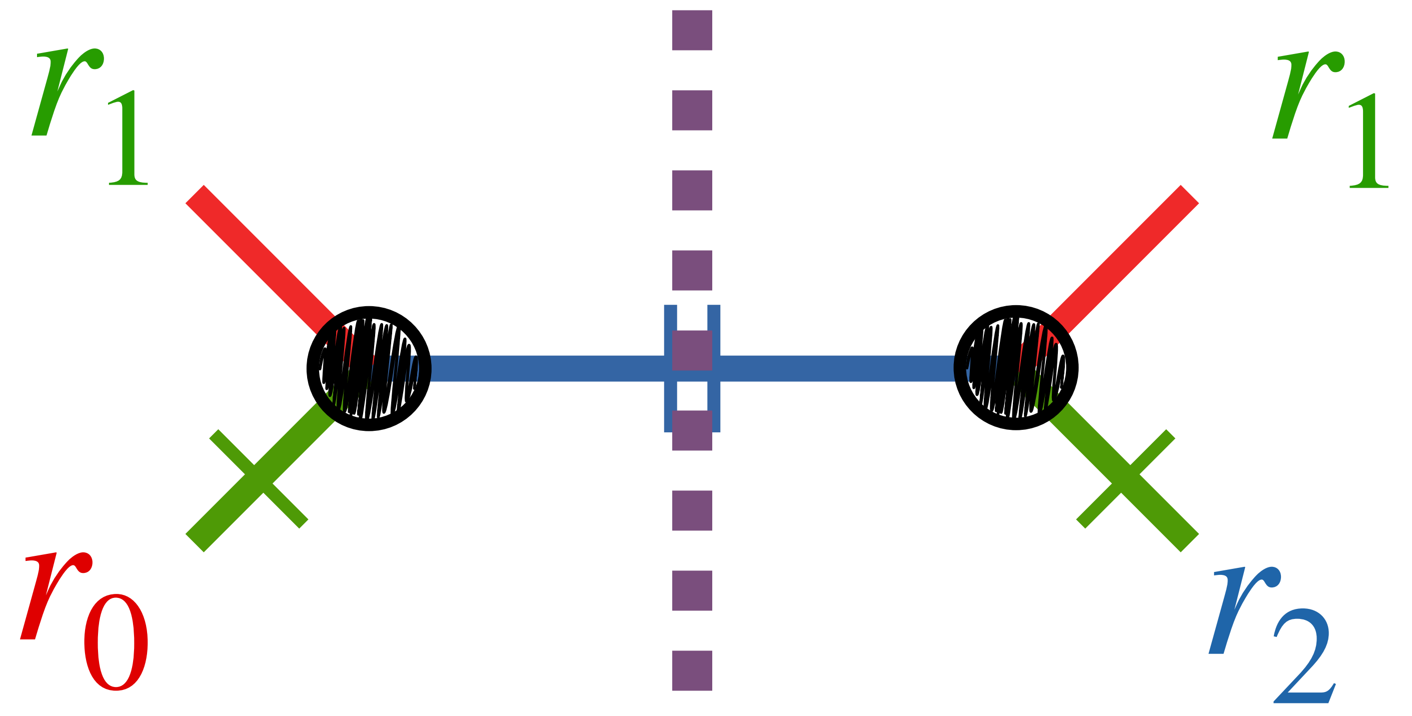

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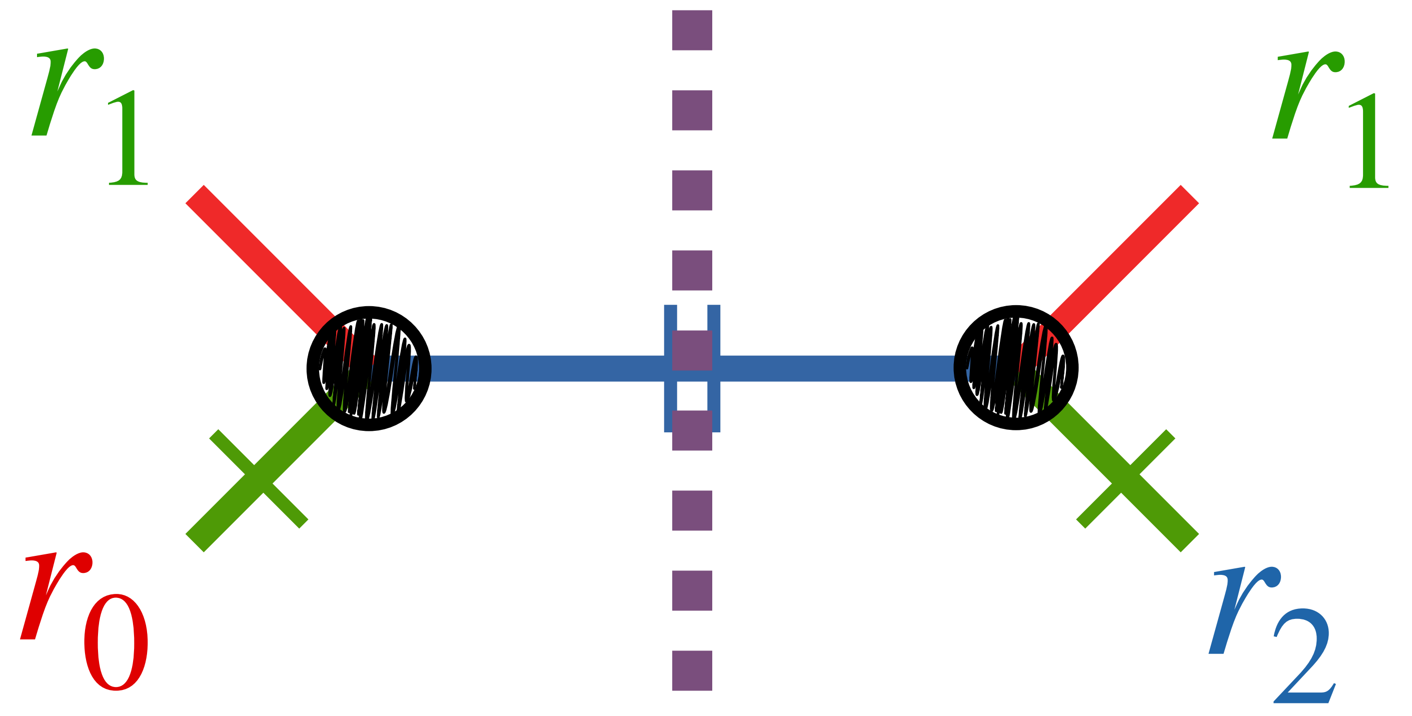

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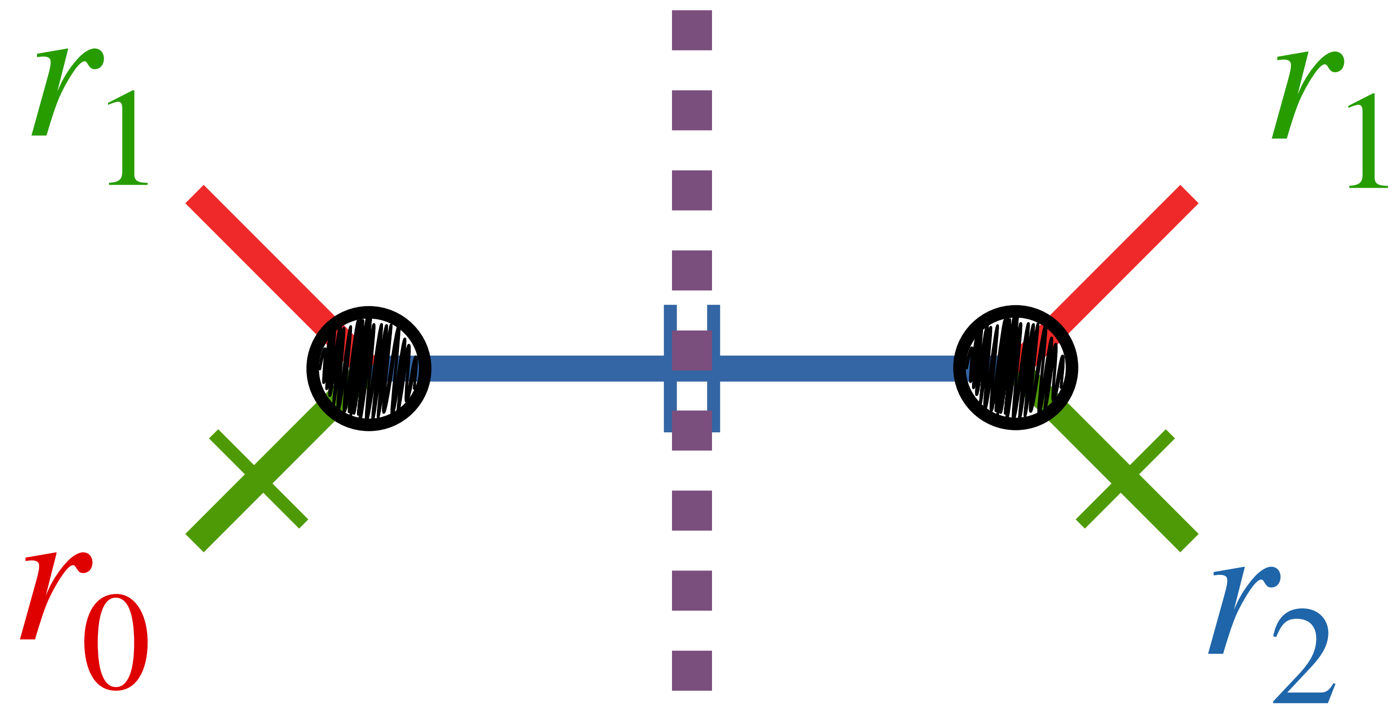

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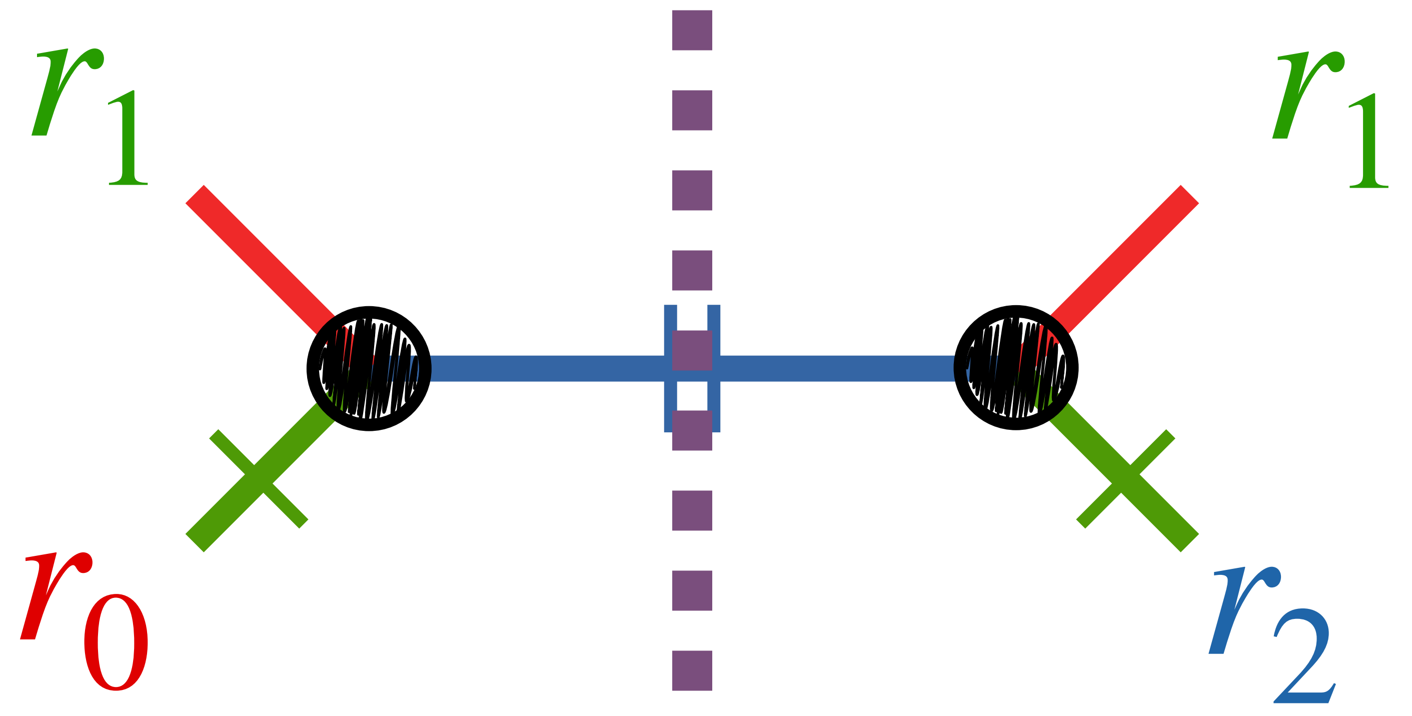

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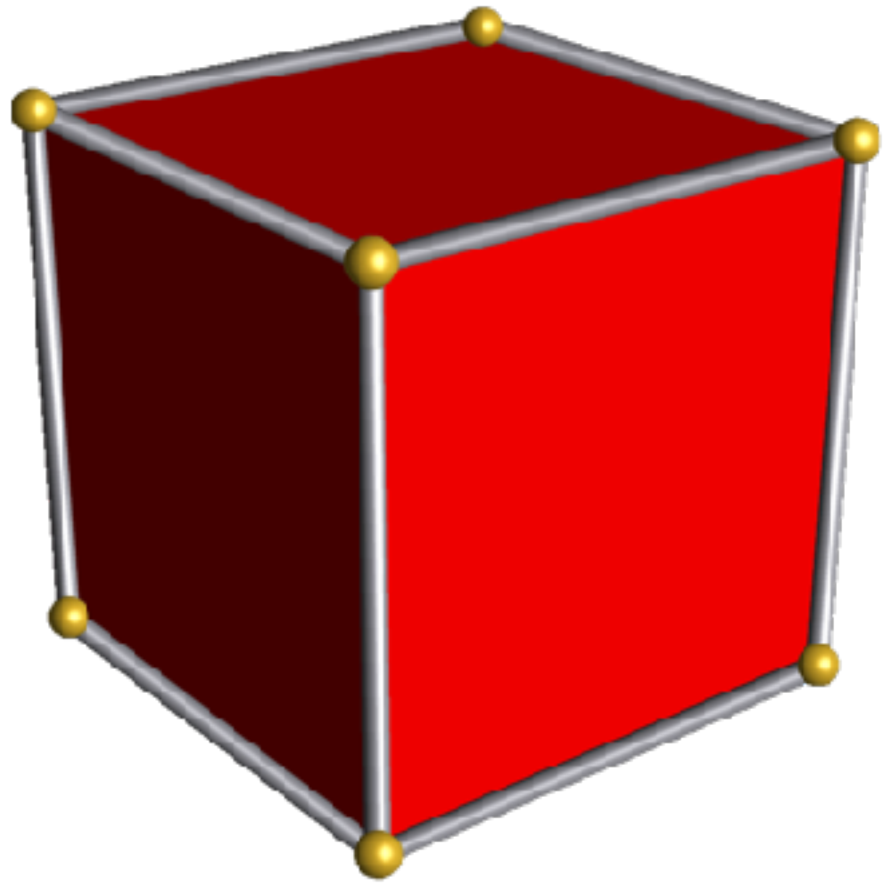

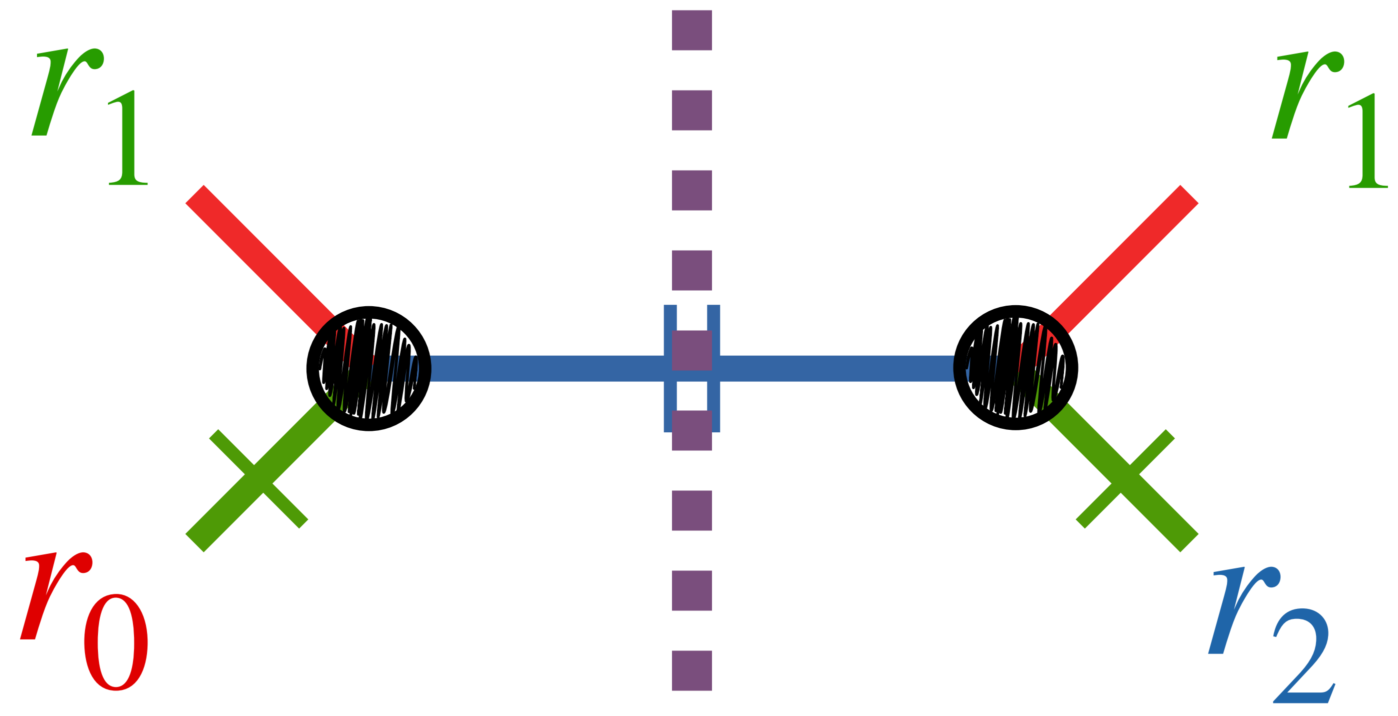

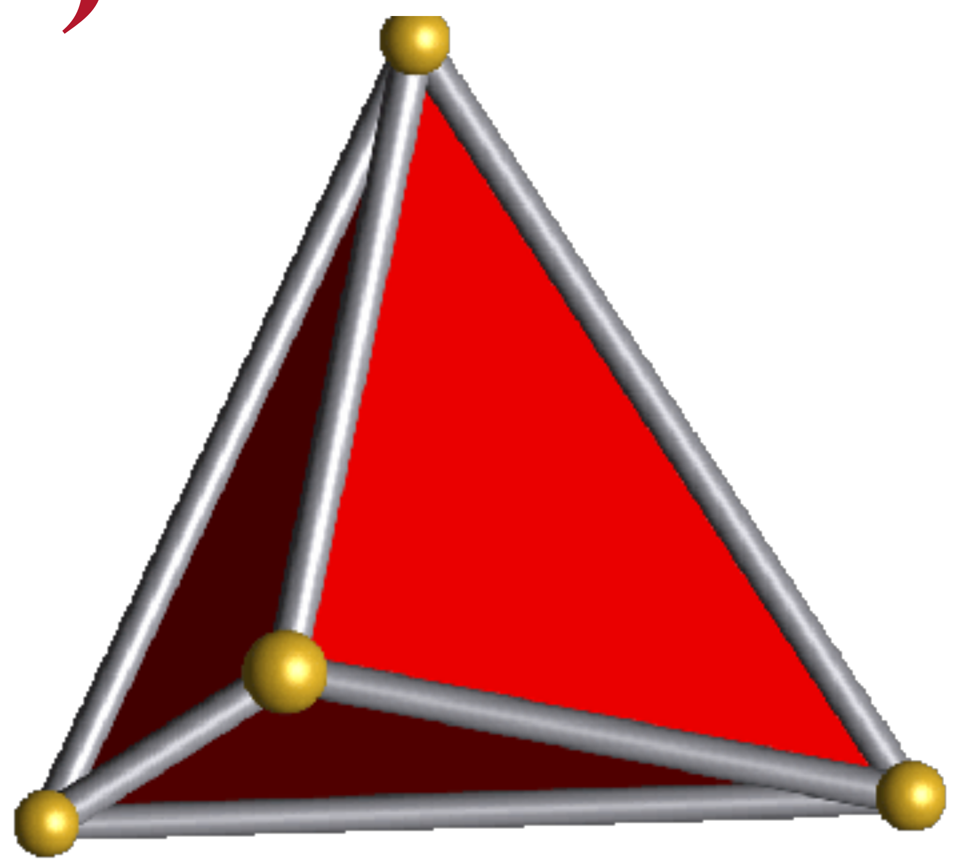

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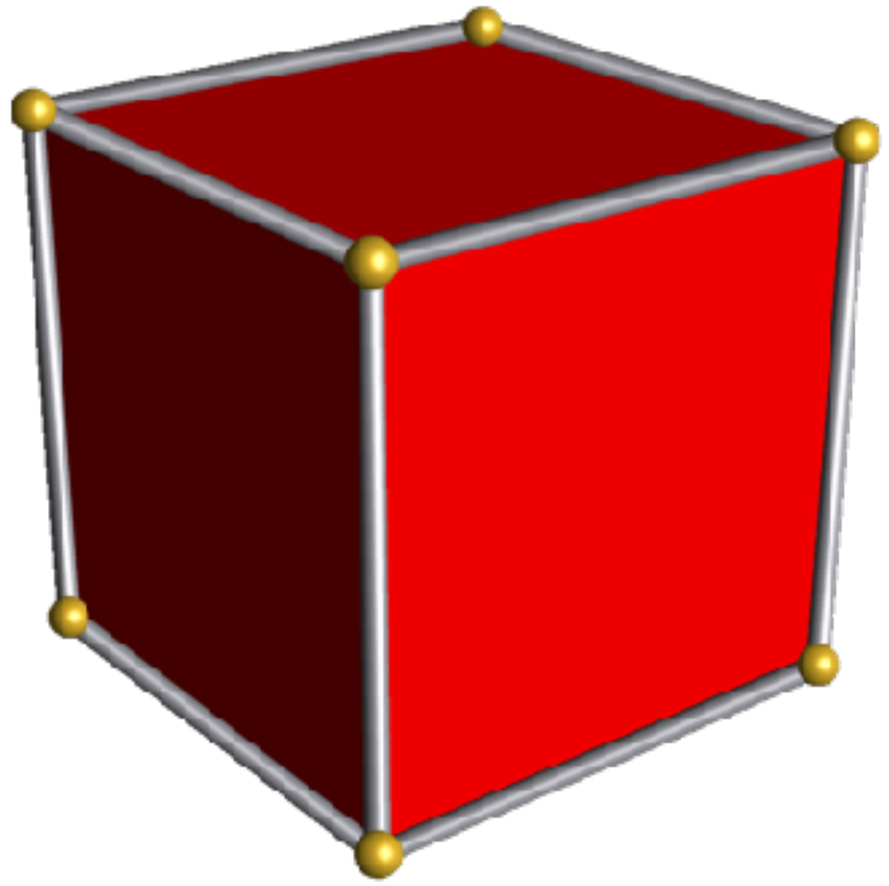

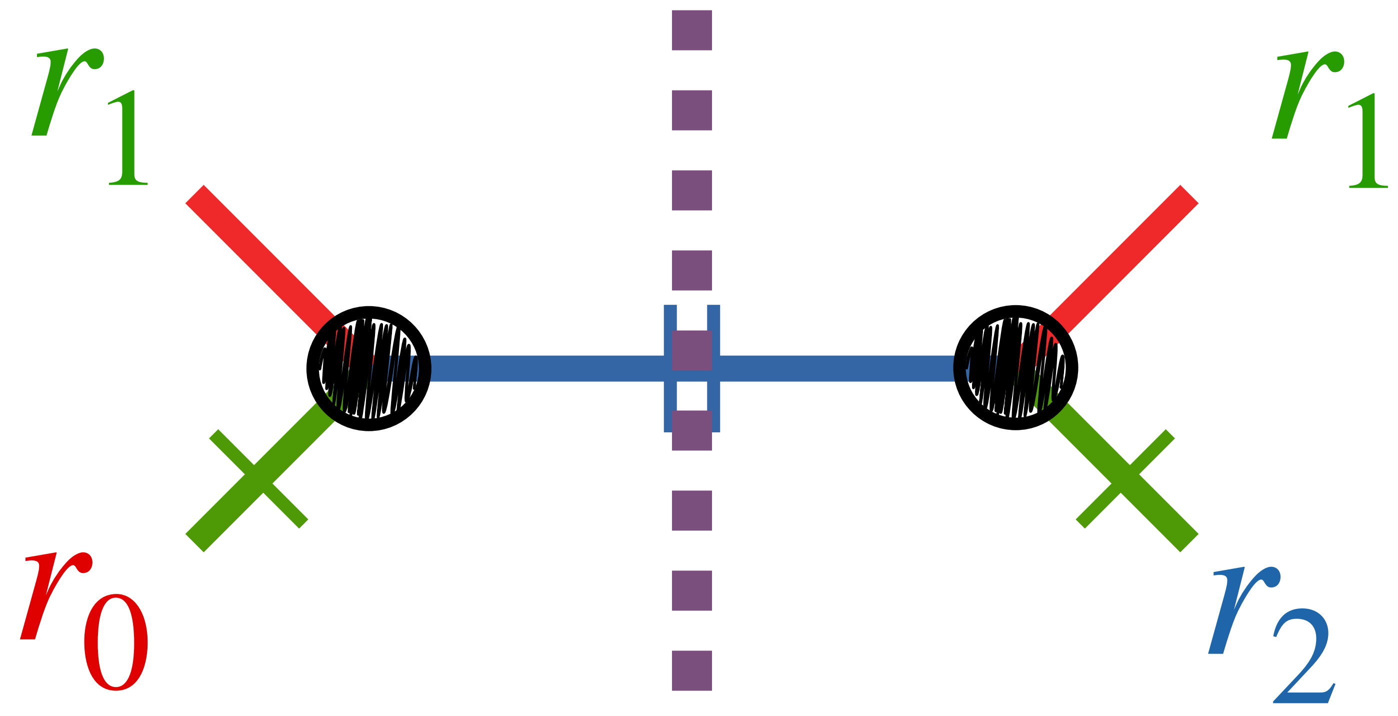

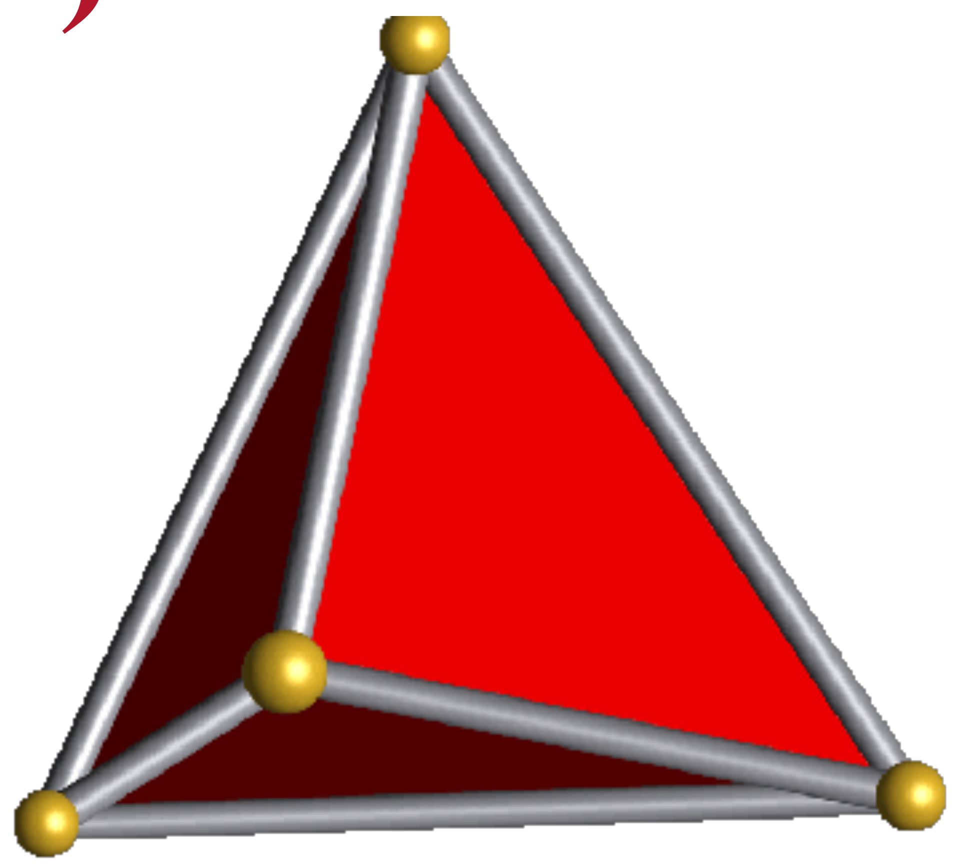

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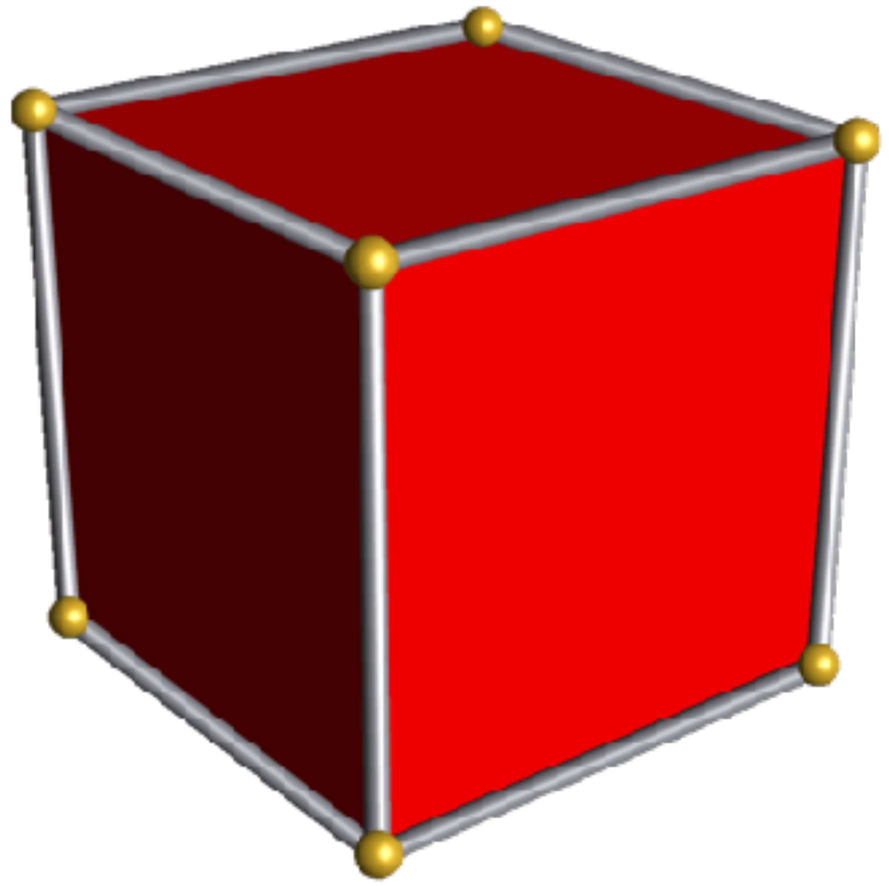

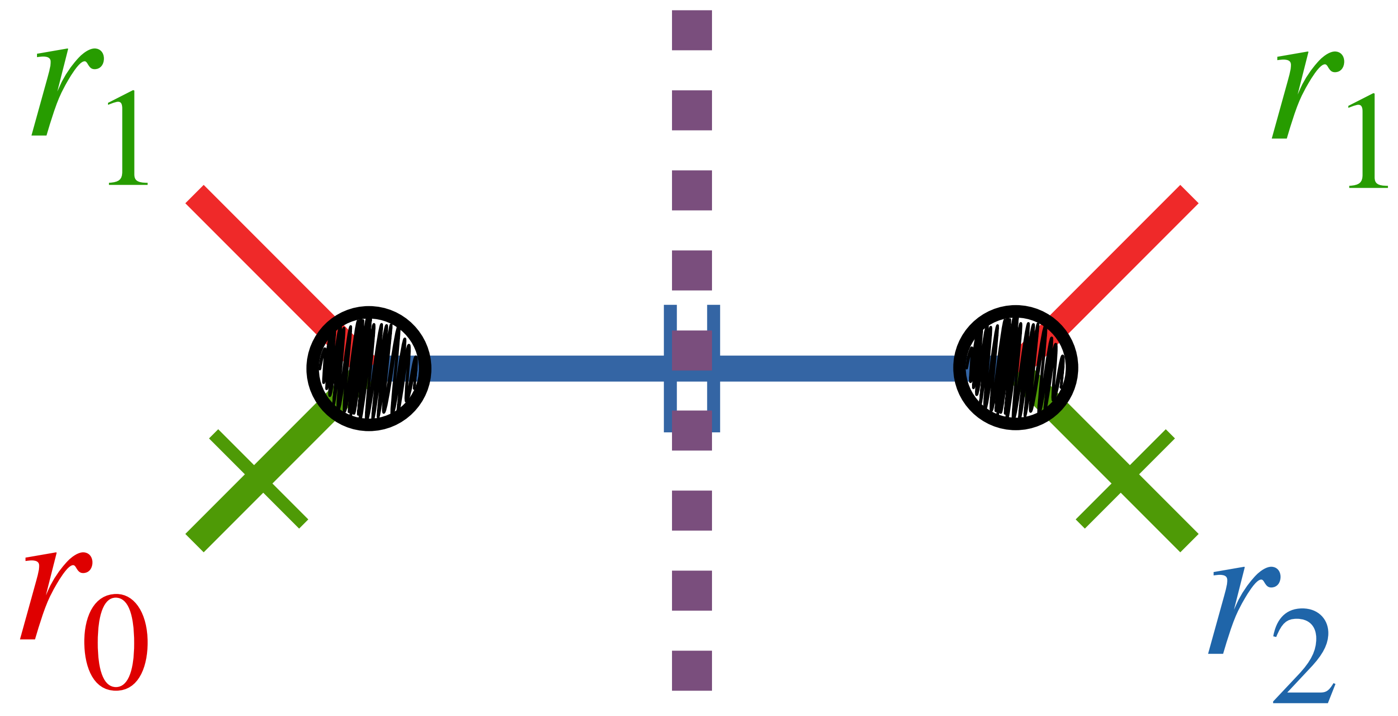

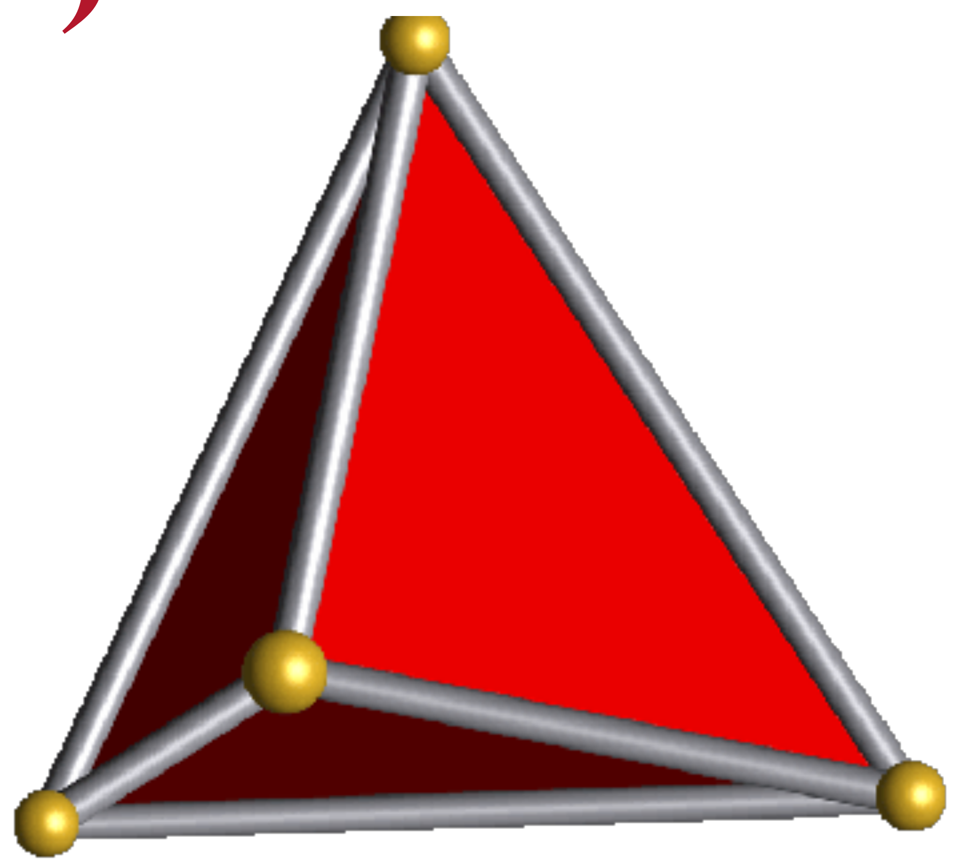

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Hay algo más?

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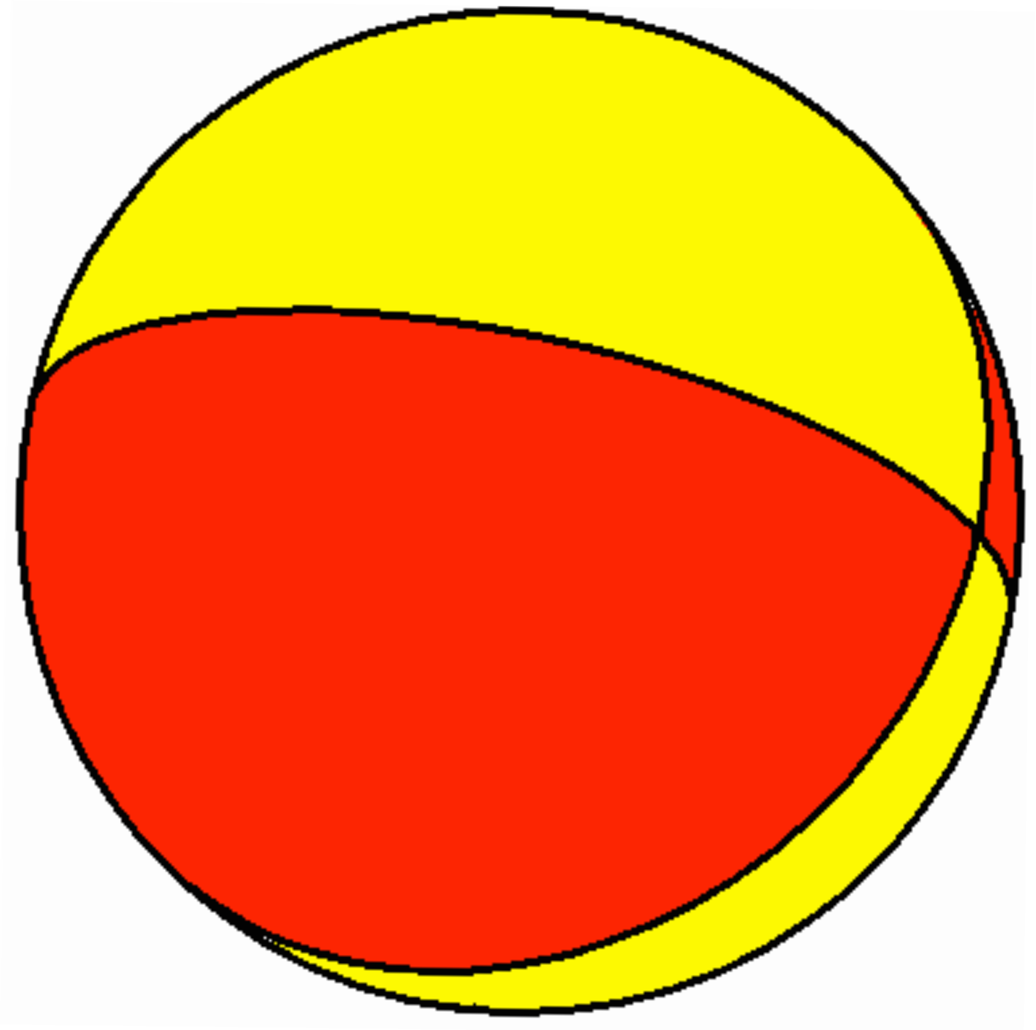
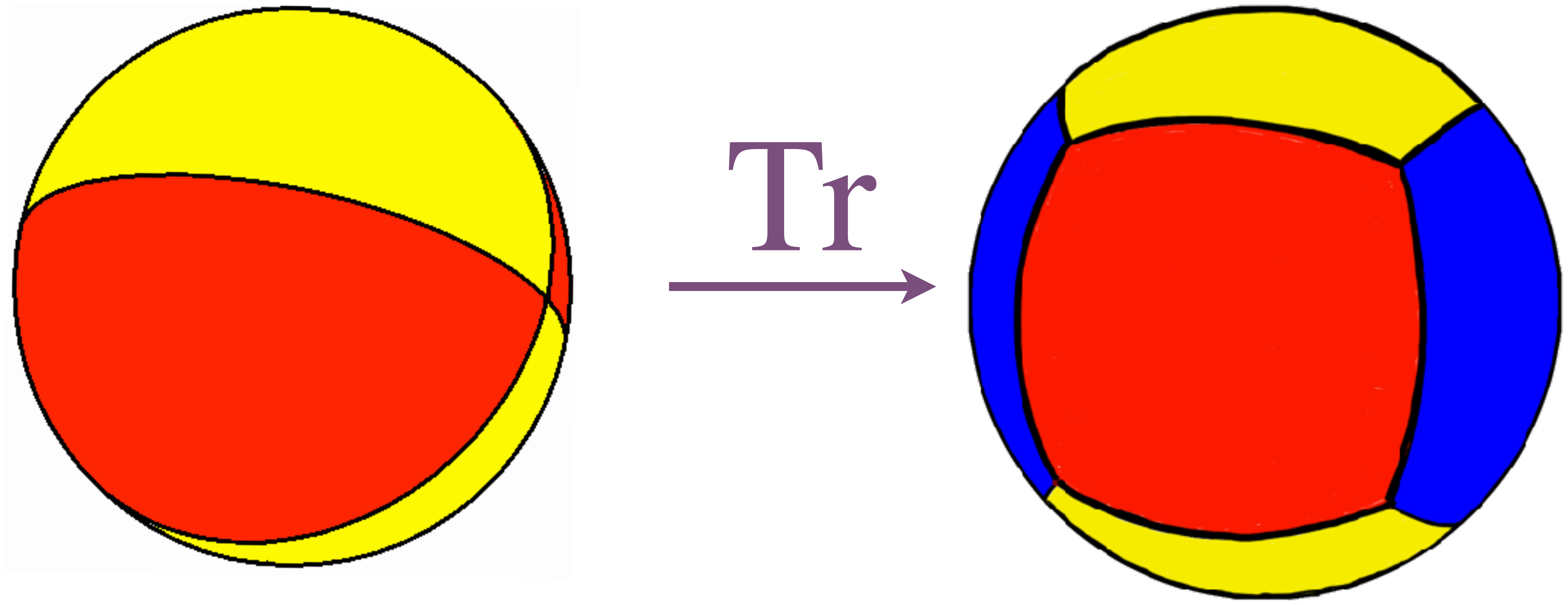


Simetrías de operaciones de voltajes

Hay algo más?



Simetrías de operaciones de voltajes

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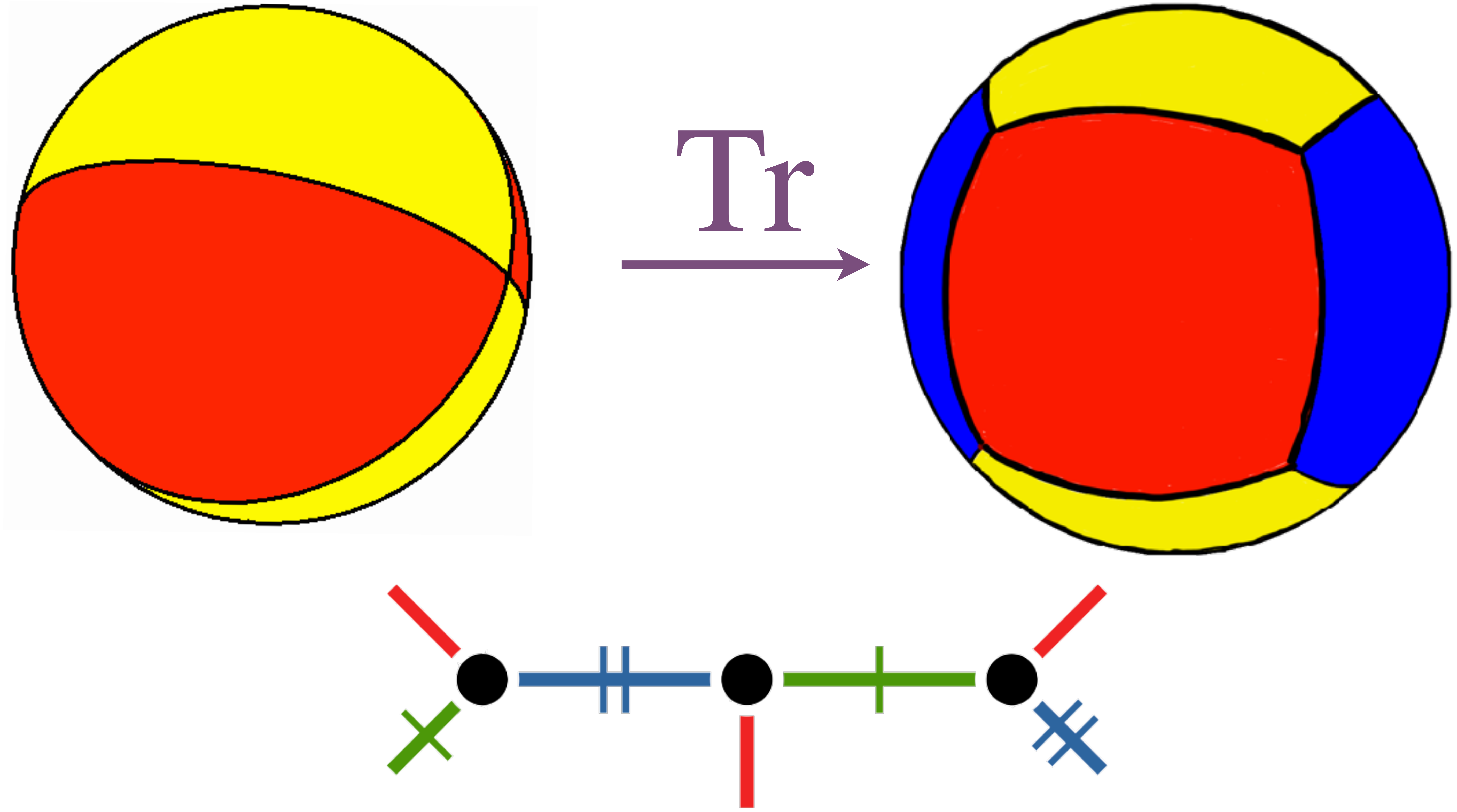

Simetrías de operaciones de voltajes

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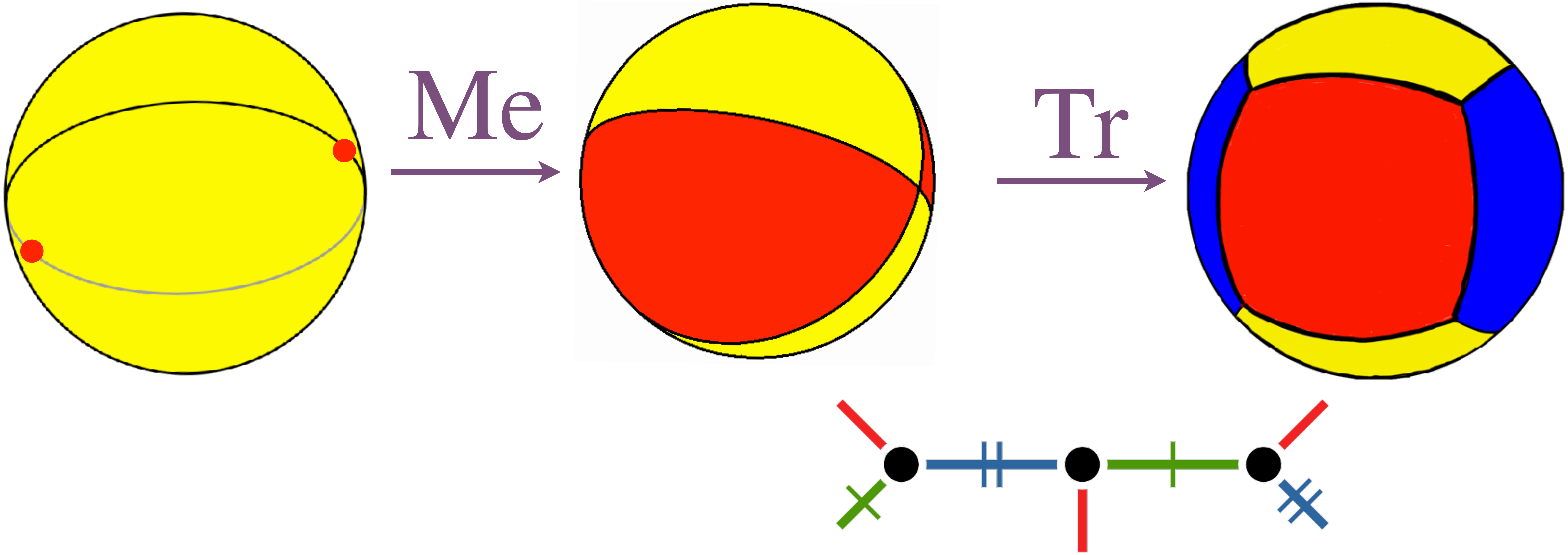
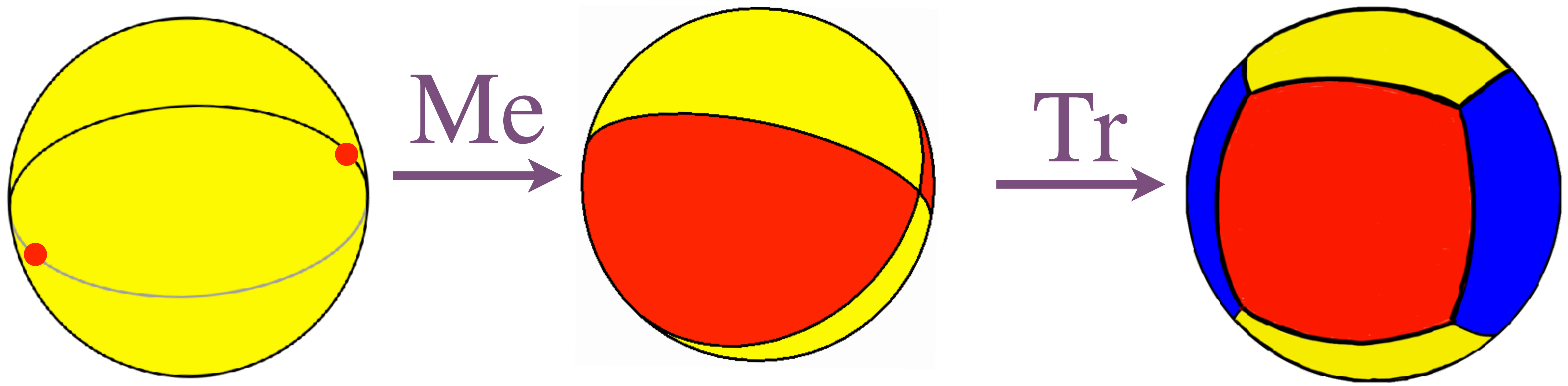
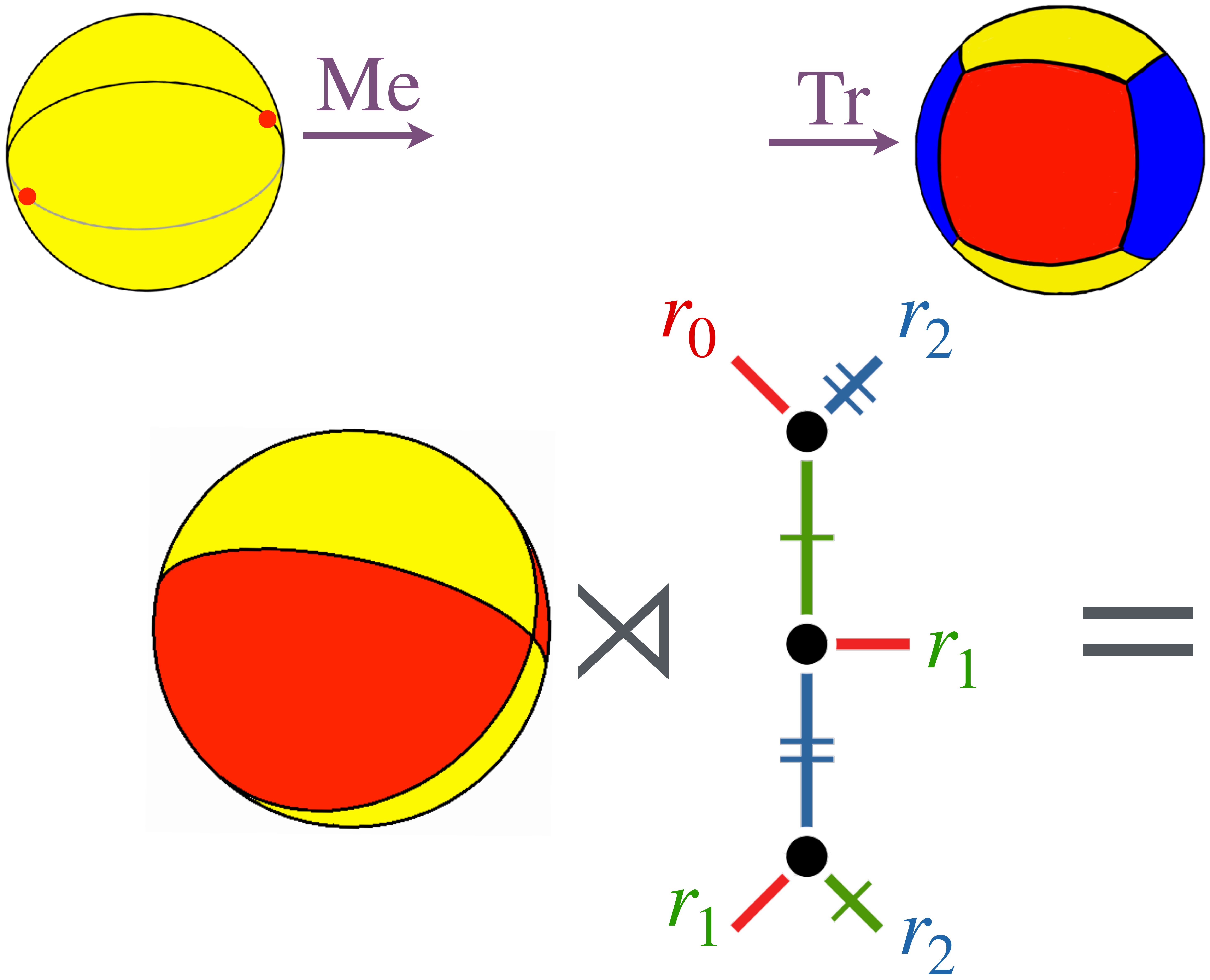


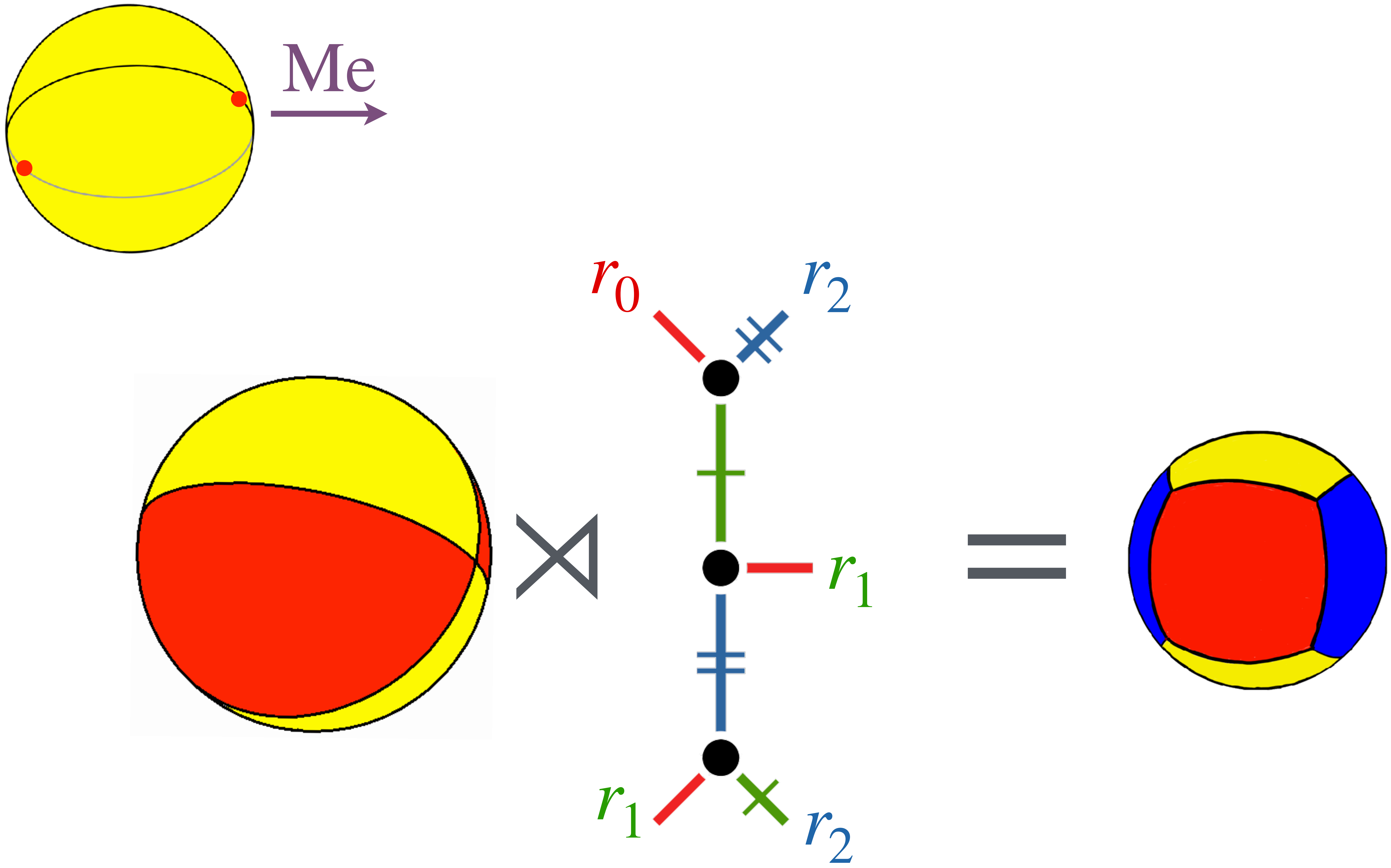
Simetrías de operaciones de voltajes



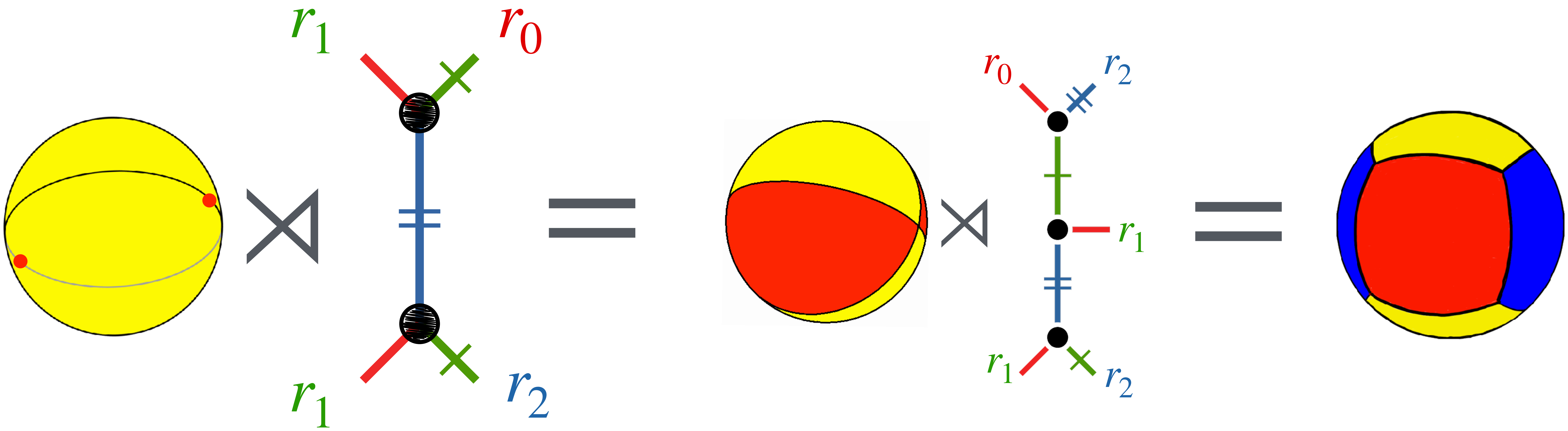
Simetrías de operaciones de voltajes



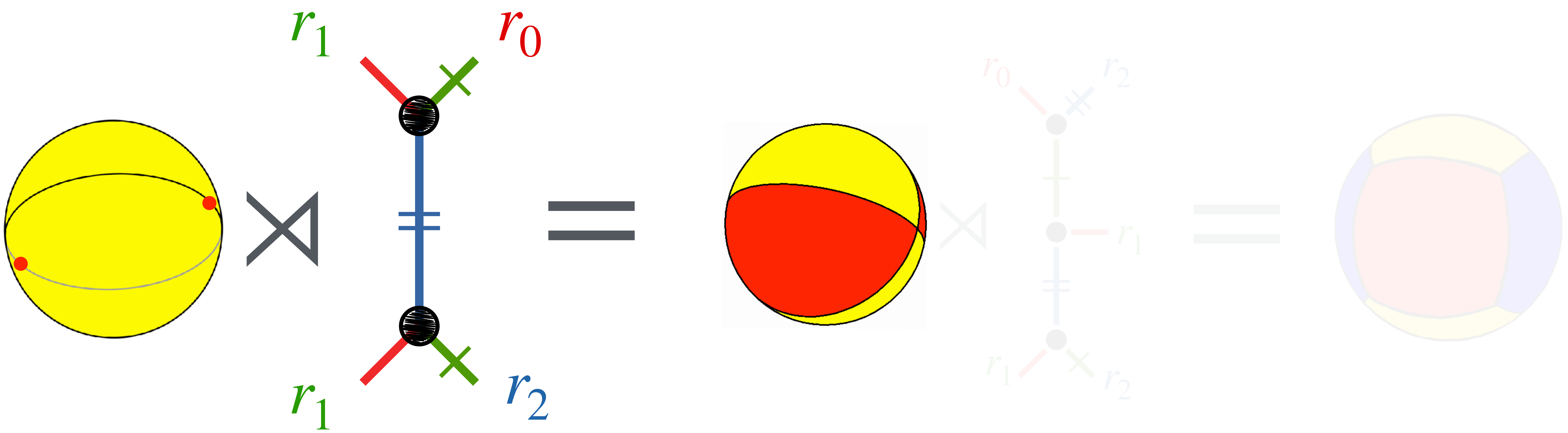
Simetrías de operaciones de voltajes



Simetrías de operaciones de voltajes



Simetrías de operaciones de voltajes



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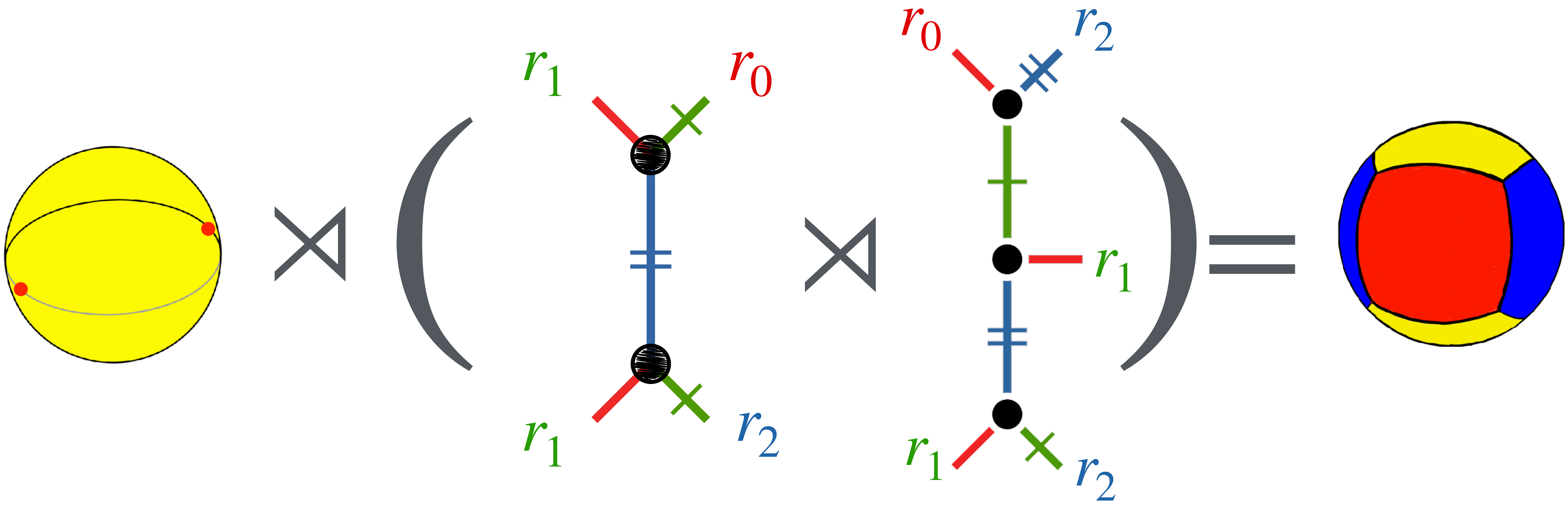

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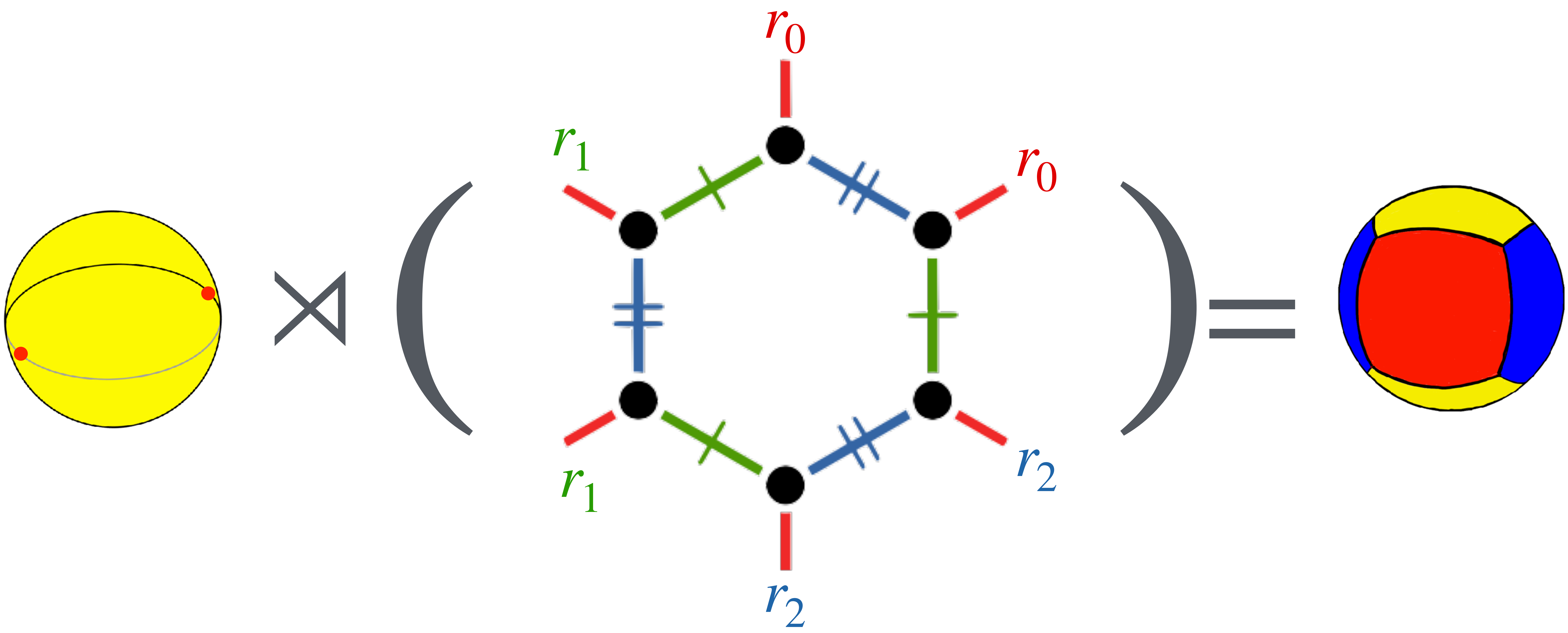

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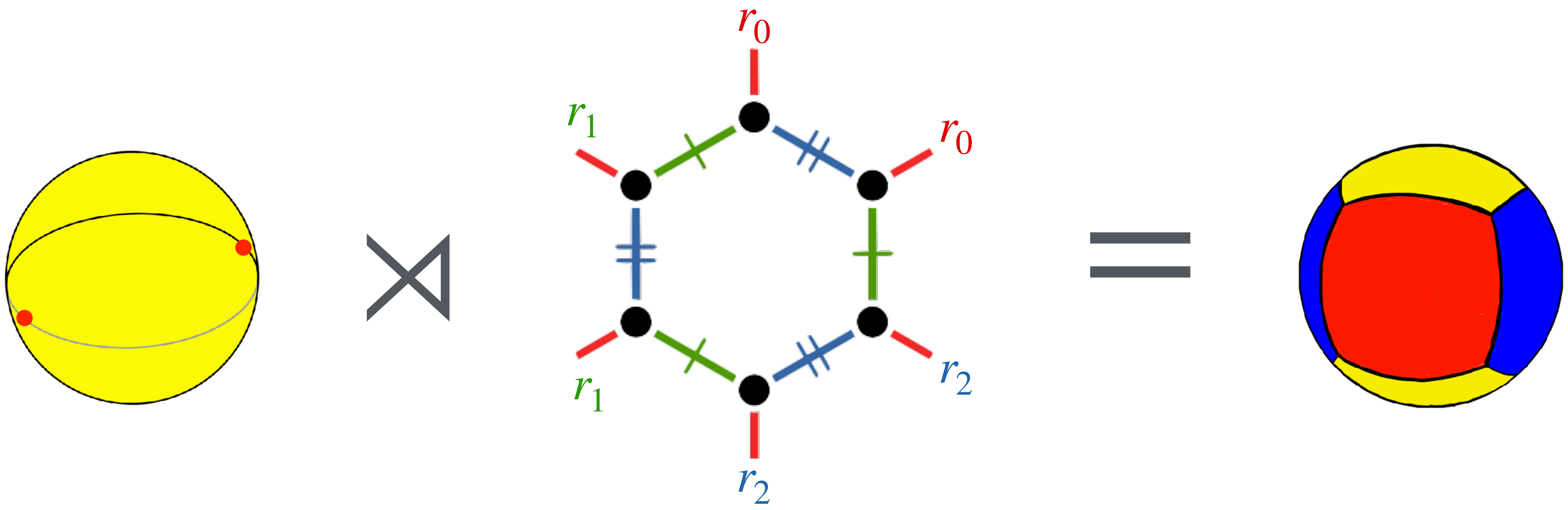
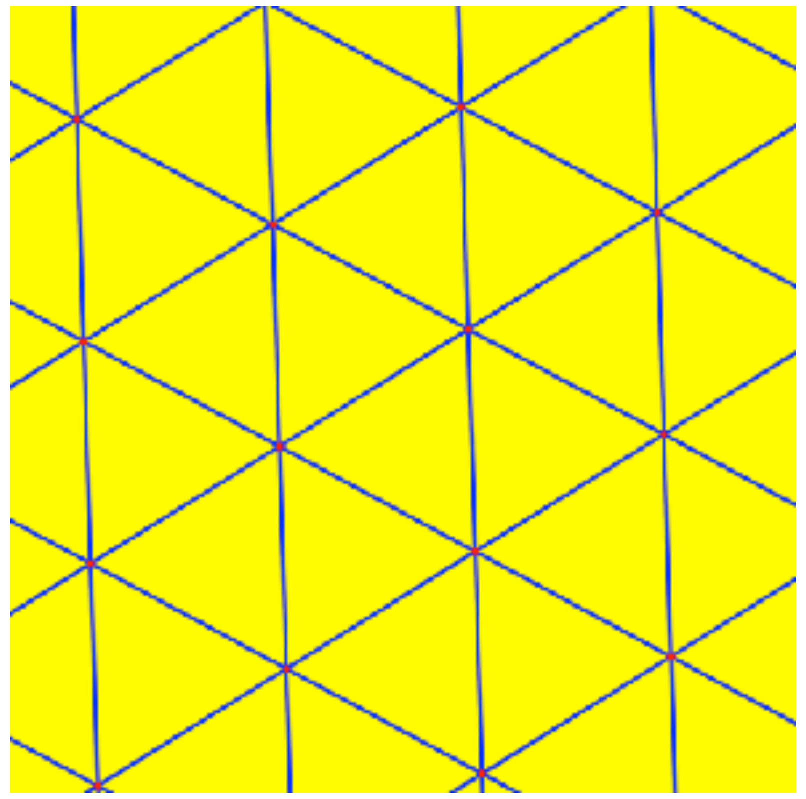


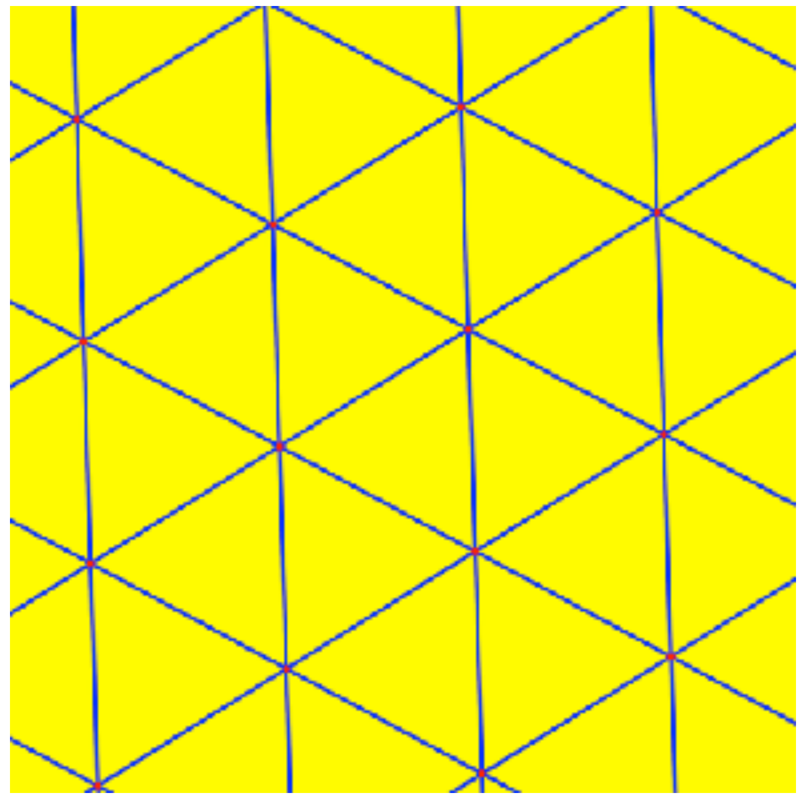
$\mathrm{Aut}(\mathscr{X}) = \mathbb{Z}_2^3$ $\mathrm{Aut}(\mathscr{Y}) = S_3$ $\mathrm{Aut}(\mathscr{X} \rtimes_\eta \mathscr{Y}) = \mathbb{Z}_2^3 \rtimes S_3$

Simetrías de operaciones de voltajes

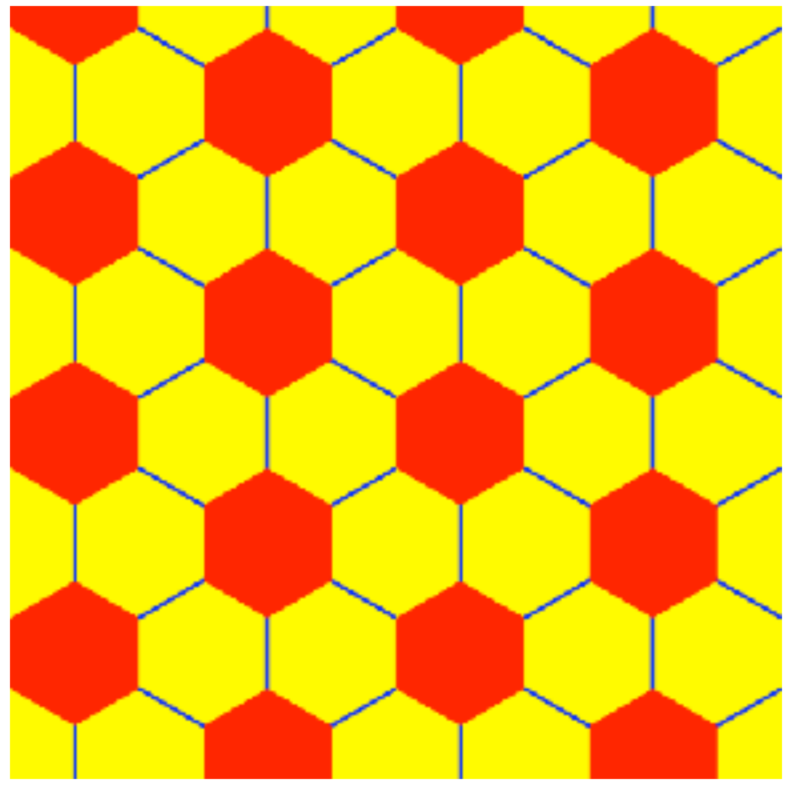


{3,6}

Simetrías de operaciones de voltajes



$\xrightarrow{\text{Tr}}$

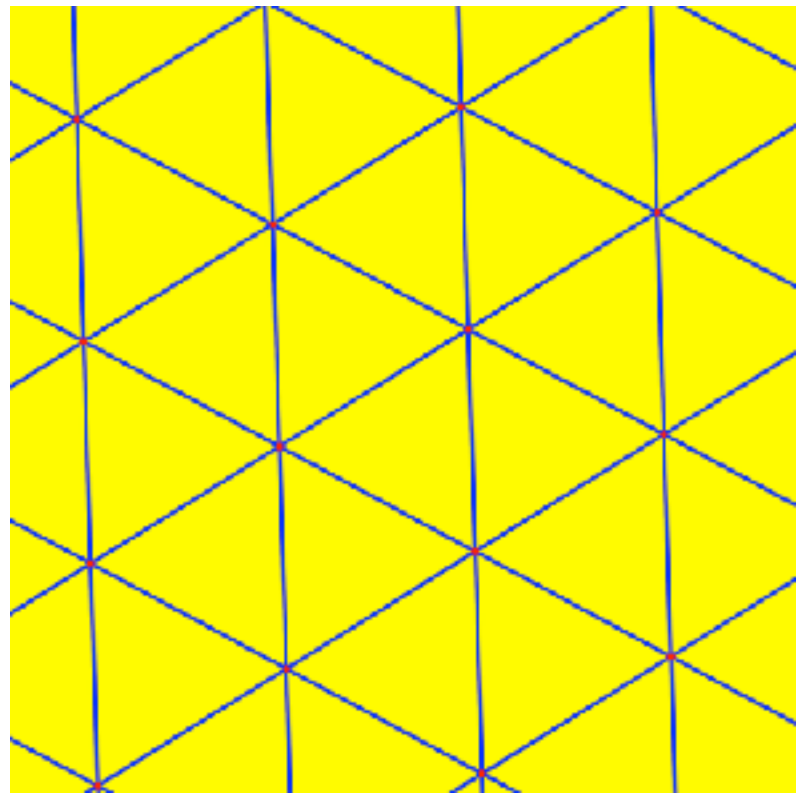


$\{3,6\}$

$\text{Tr}\{3,6\} = \{6,3\}$

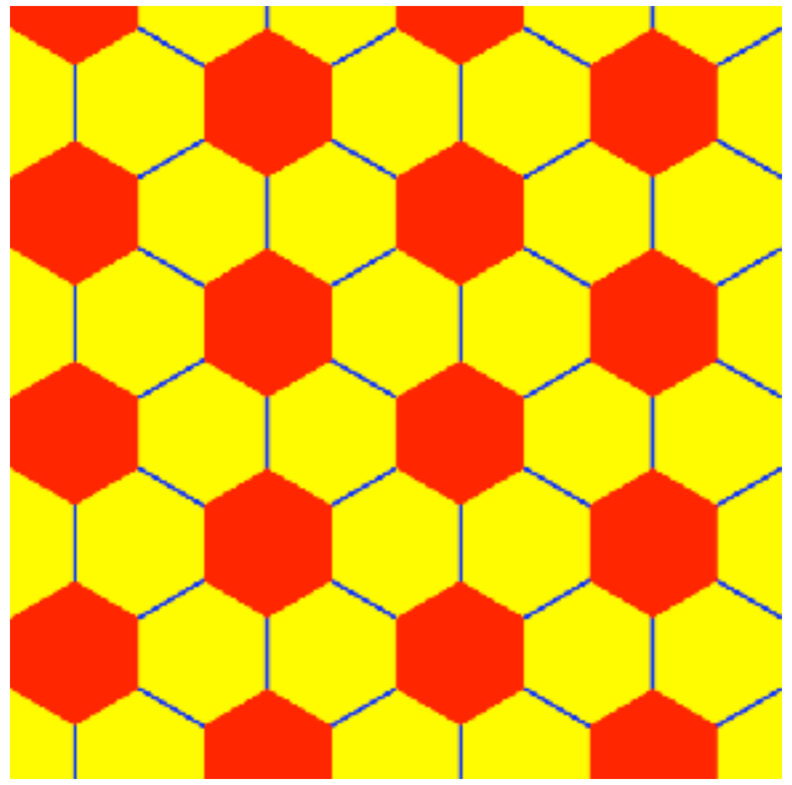
Simetrías de operaciones de voltajes

$\xrightarrow{\text{Me}}$



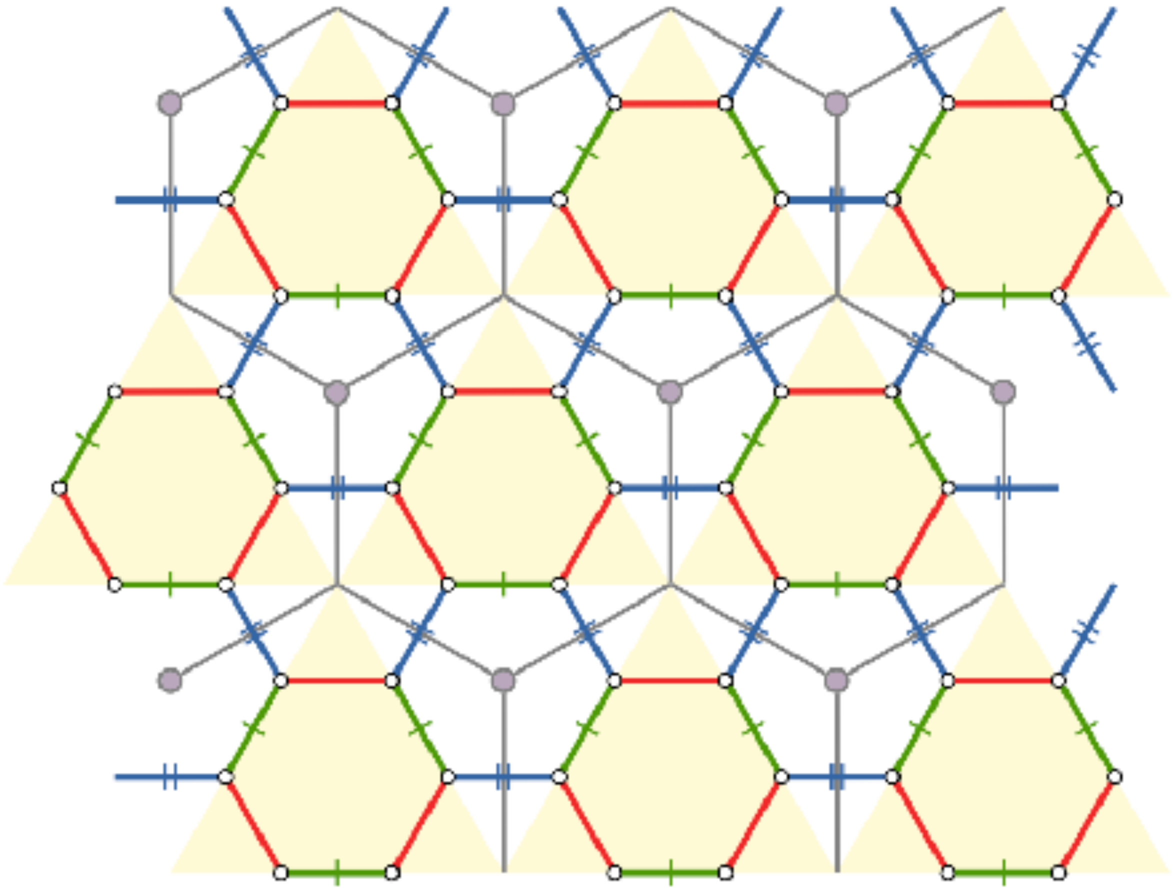
{3,6}

$\xrightarrow{\text{Tr}}$

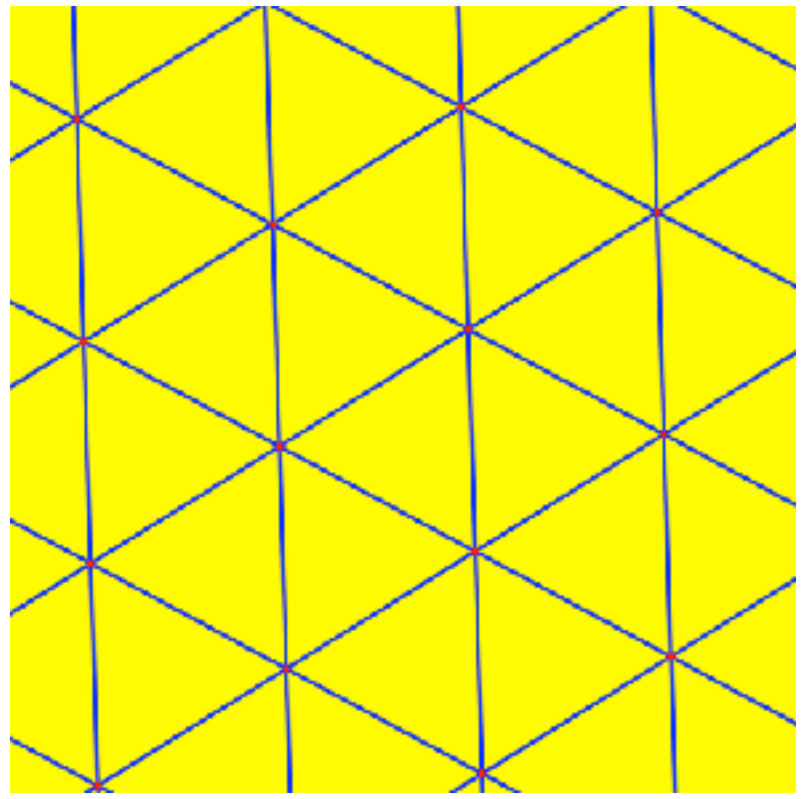


Tr{3,6} = {6,3}

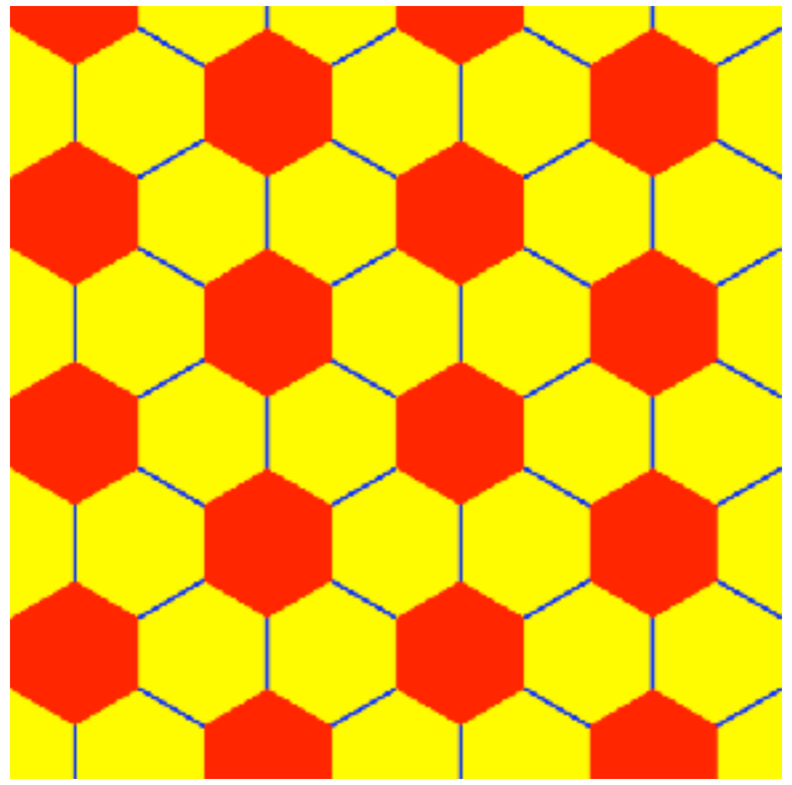
Simetrías de operaciones de voltajes



$\xrightarrow{\mathrm{Me}}$



$\xrightarrow{\mathrm{Tr}}$

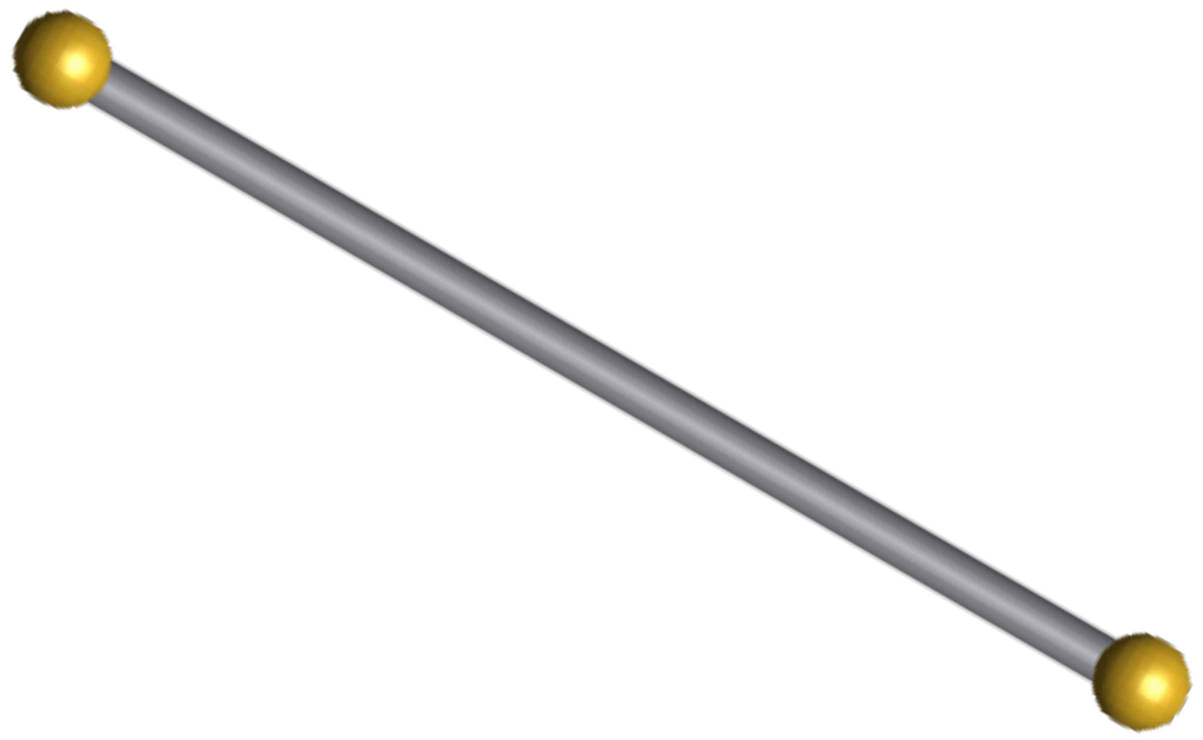


(3,3,3)

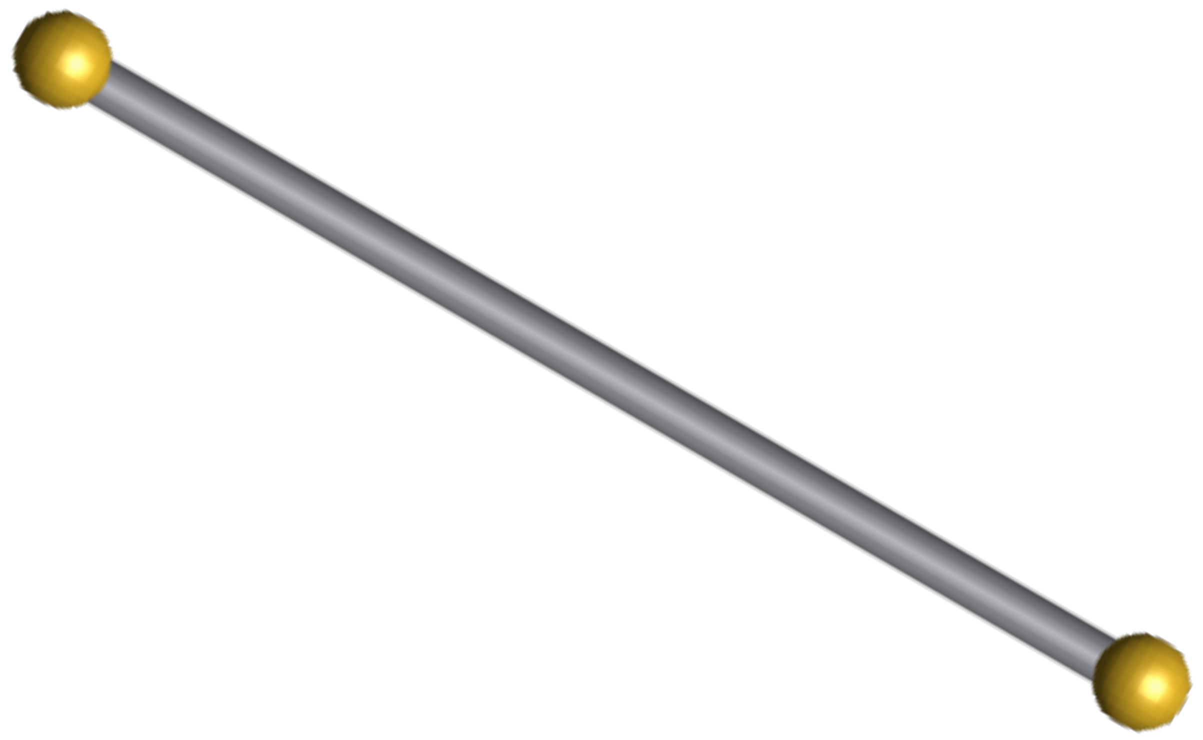
{3,6}

Tr{3,6} = {6,3}

Simetrías de operaciones de voltajes

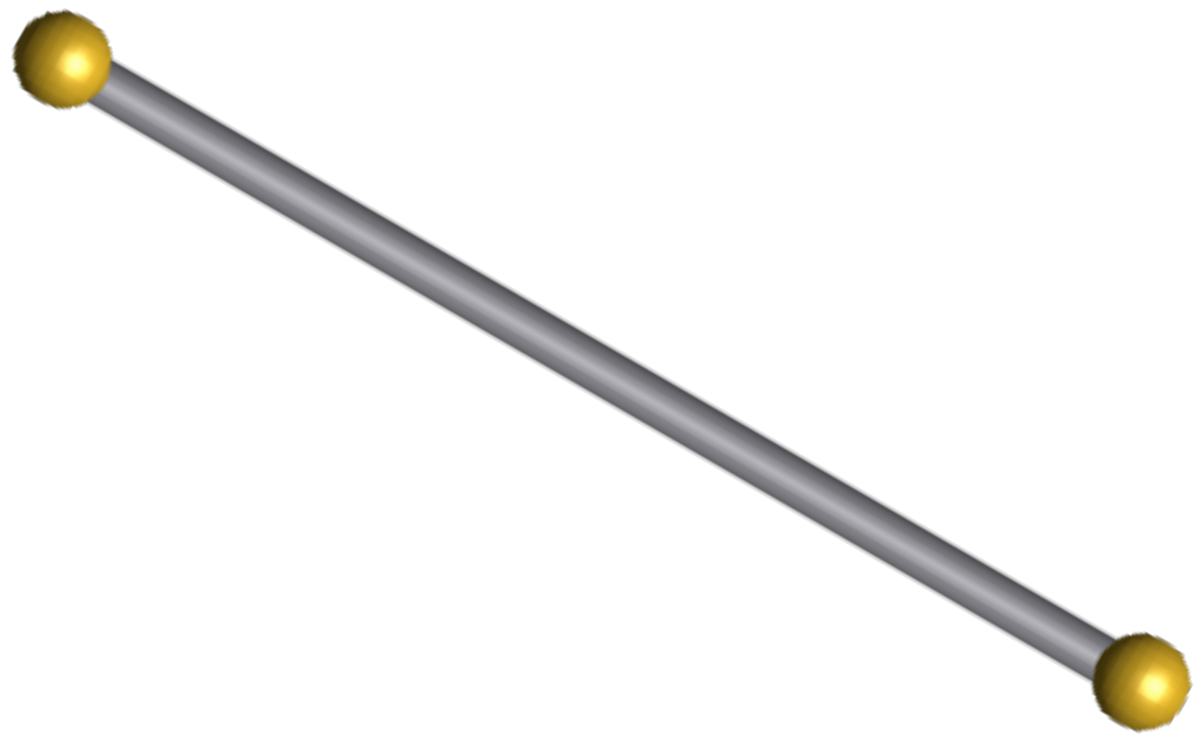


Simetrías de operaciones de voltajes



$\xrightarrow{\text{Pri}}$

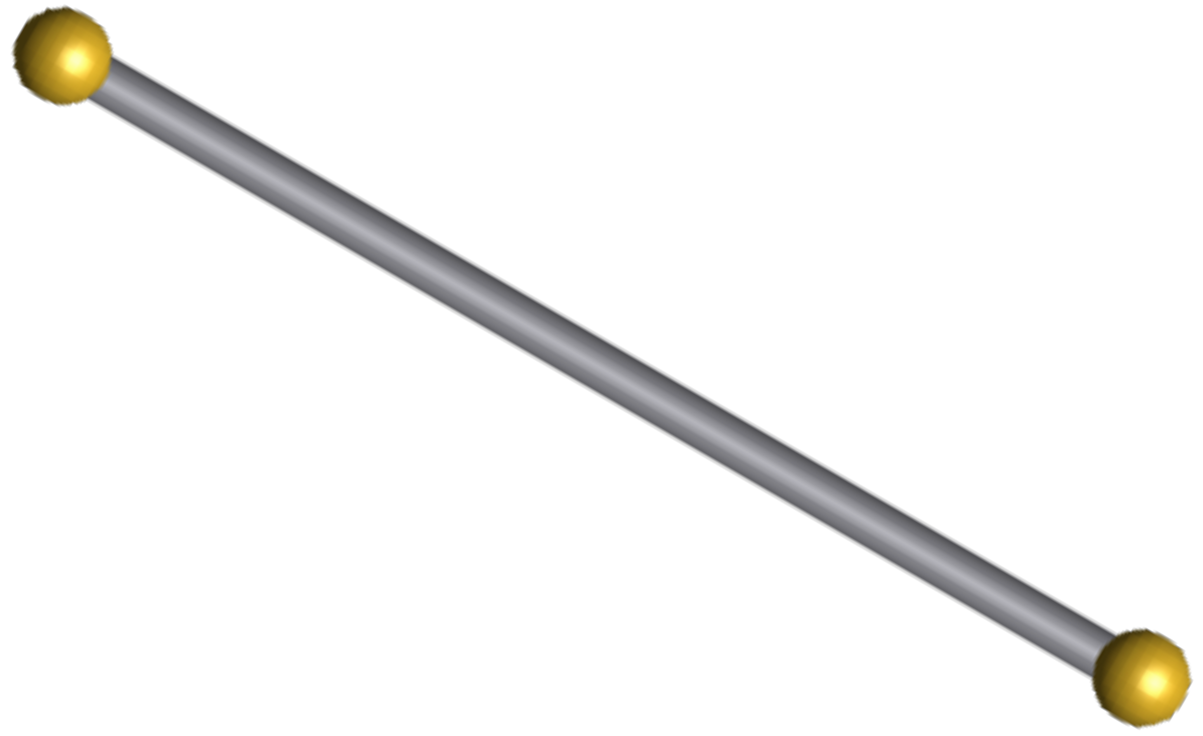
Simetrías de operaciones de voltajes



$\xrightarrow{\text{Pri}}$

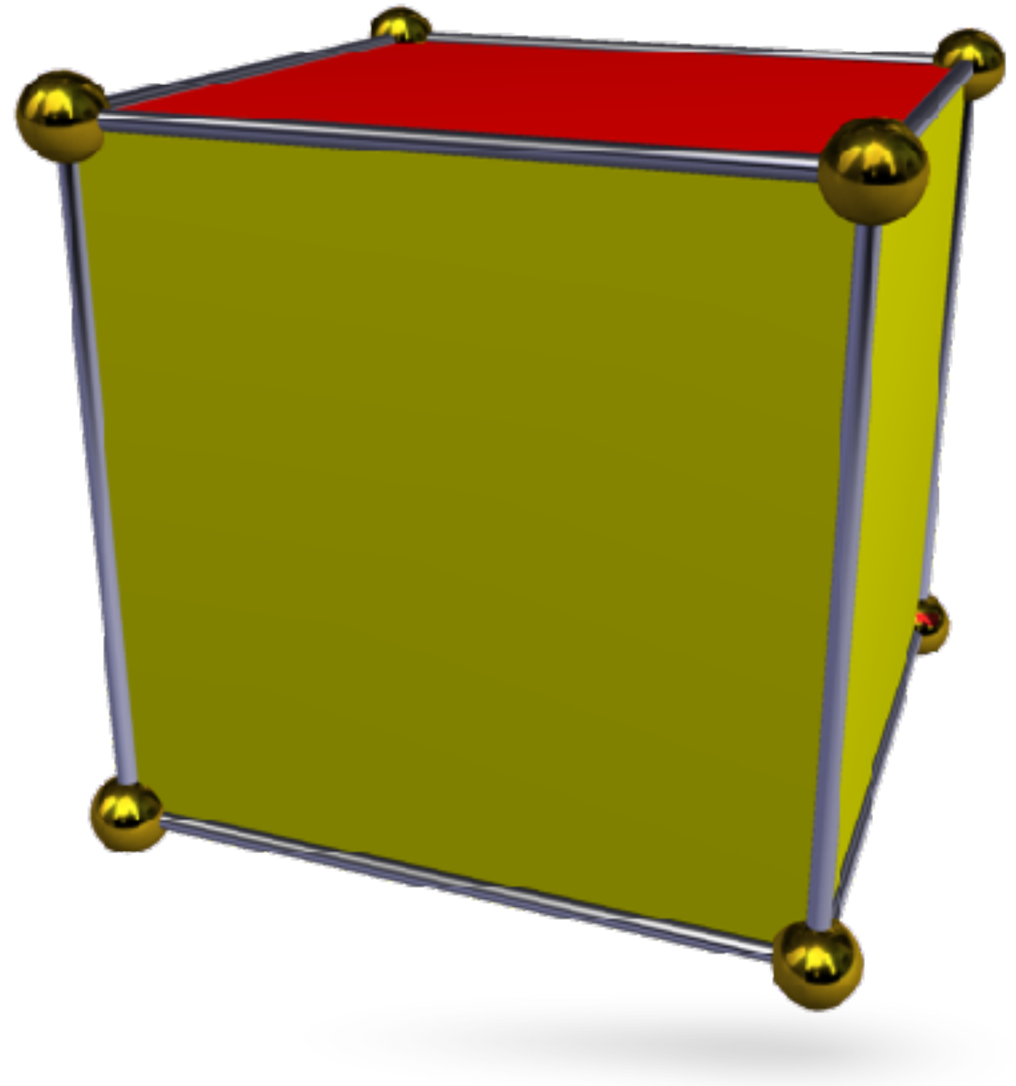
$\xrightarrow{\text{Pri}}$

Simetrías de operaciones de voltajes

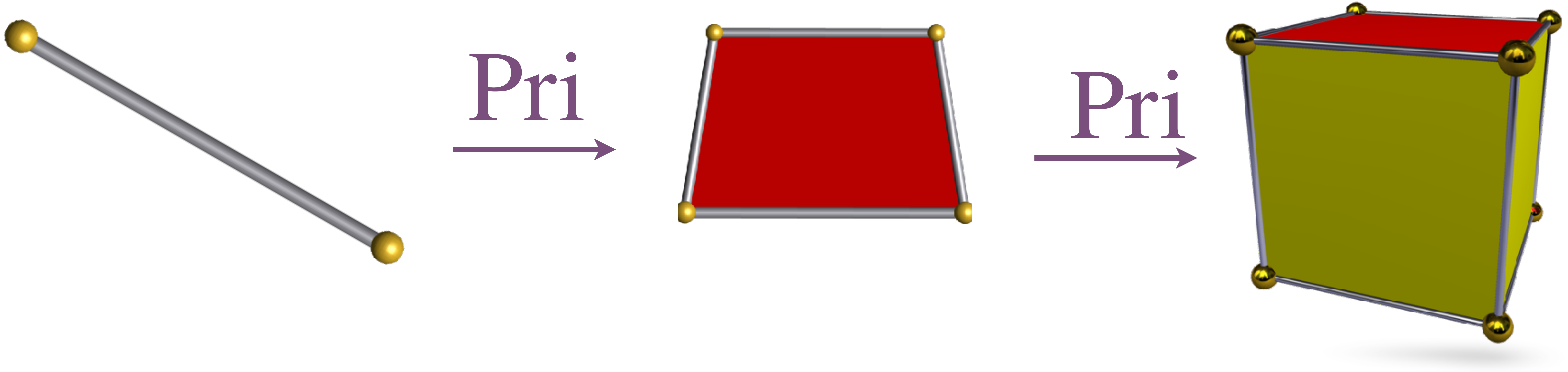


$\xrightarrow{\text{Pri}}$

$\xrightarrow{\text{Pri}}$



Simetrías de operaciones de voltajes

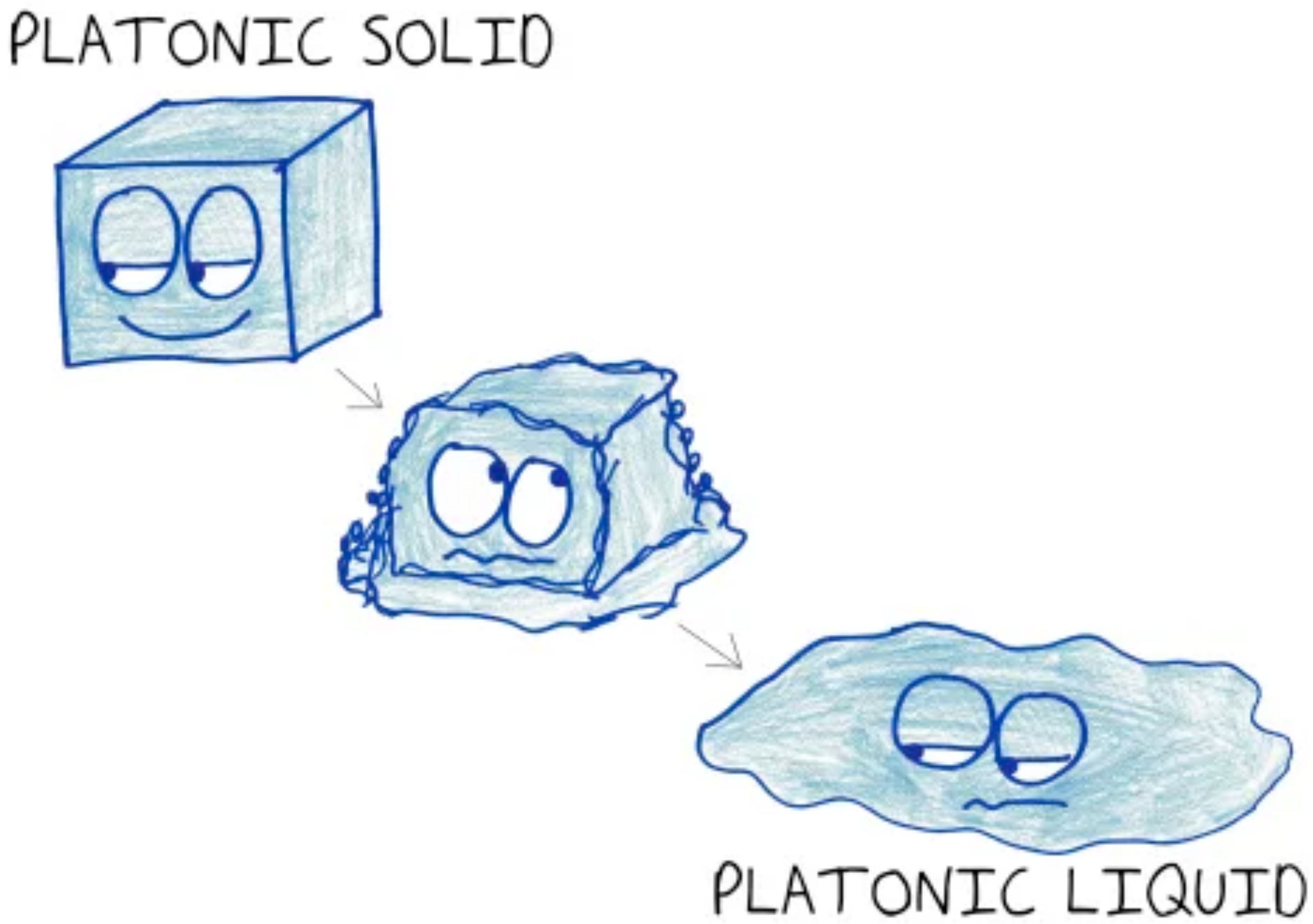


Simetrías de operaciones de voltajes

Conjectura

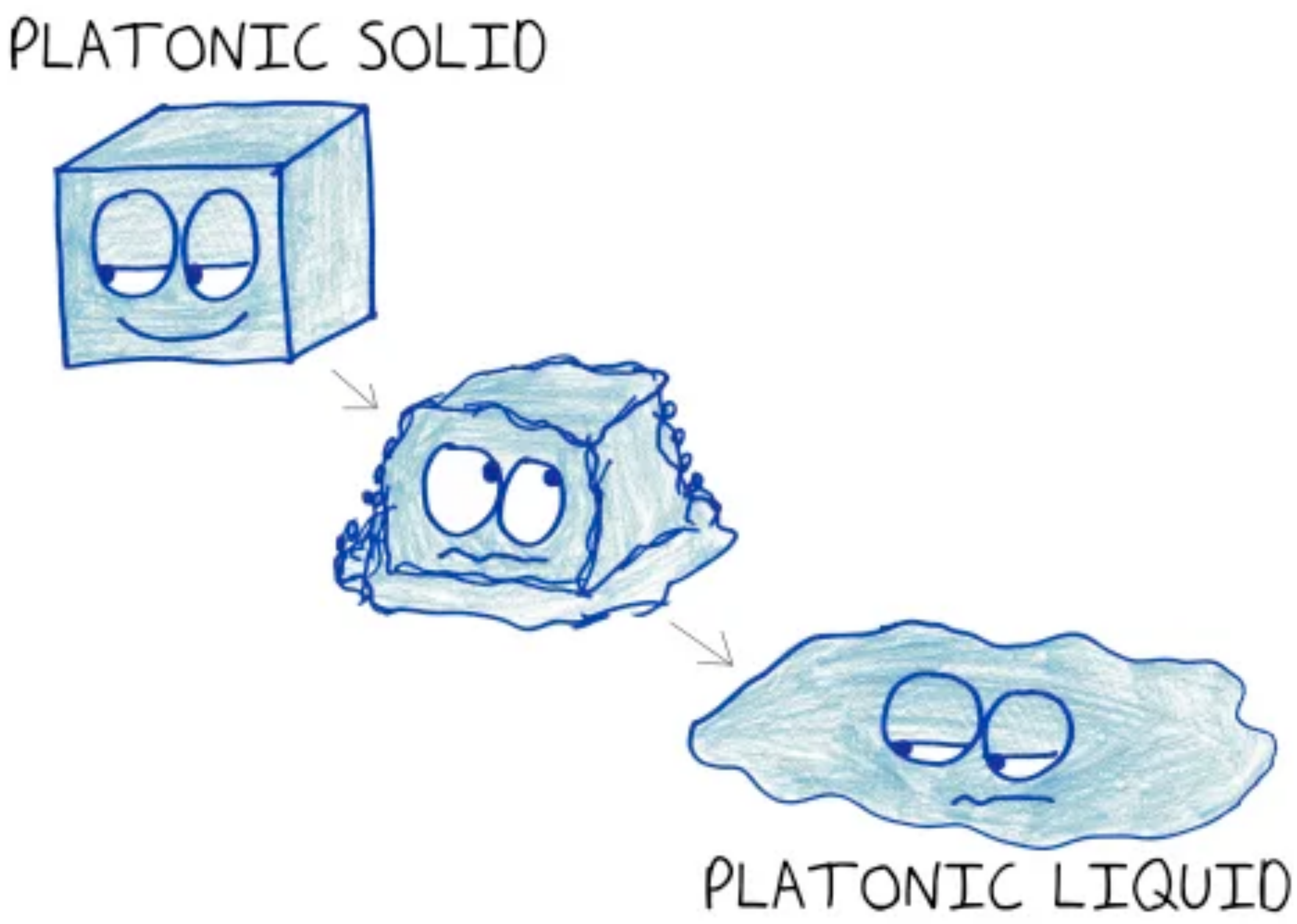
Si $\mathscr{X}$ no se puede escribir como $\mathscr{X} \cong \mathscr{W} \rtimes_\theta \mathscr{Z}$ para algún operador de voltajes $(\mathscr{Z}, \theta)$, entonces todo los automorfismos de $\mathscr{X} \rtimes_\eta \mathscr{Y}$ son levantamientos de automorfismos de $\mathscr{Y}$.

Esto no es una operación de voltajes:



Esto no es una operación de voltajes:

Slides



Referencias

¡GRACIAS!