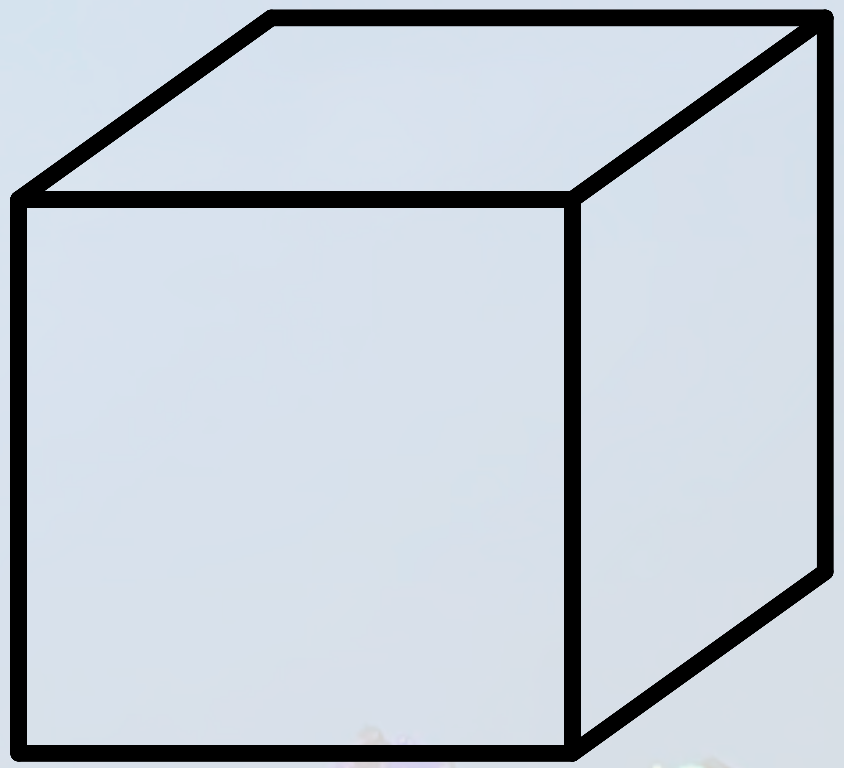
Faithfulness in maniplexes

Antonio Montero

University of Ljubljana

Based on joint work with
Gabe Cunningham, Elías Mochán, Primož Potočnik and Micael Toledo



Symmetries in Graphs, Maps and Polytopes 2026

Université Libre de Bruxelles, 13 - 17 July, 2026

Faithfulness in maniplexes

(How far should a maniplex be from being a polytope)

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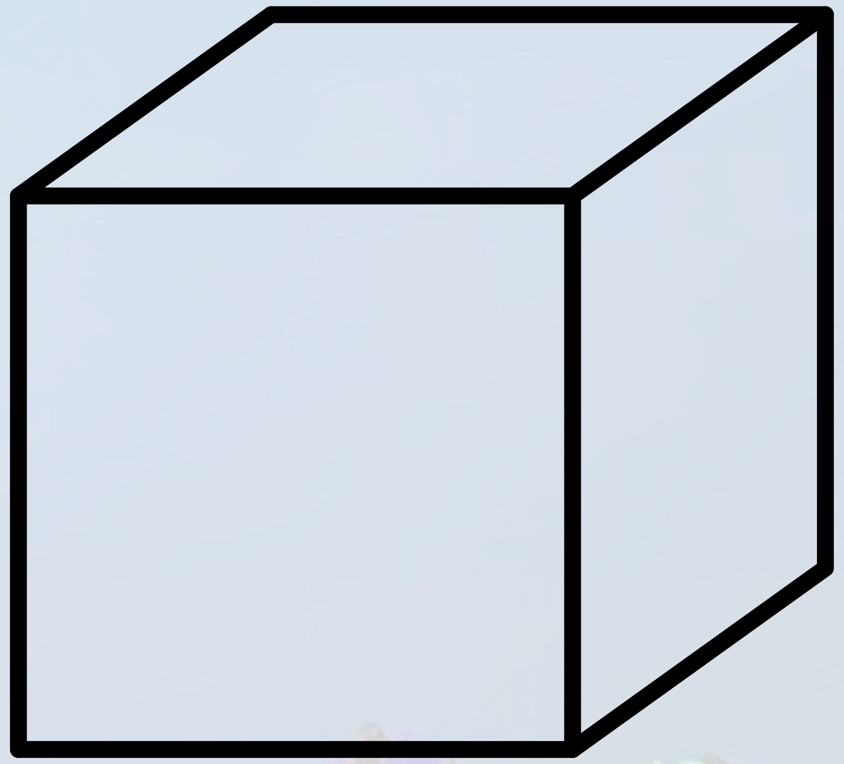

Symmetries in Graphs, Maps and Polytopes 2026

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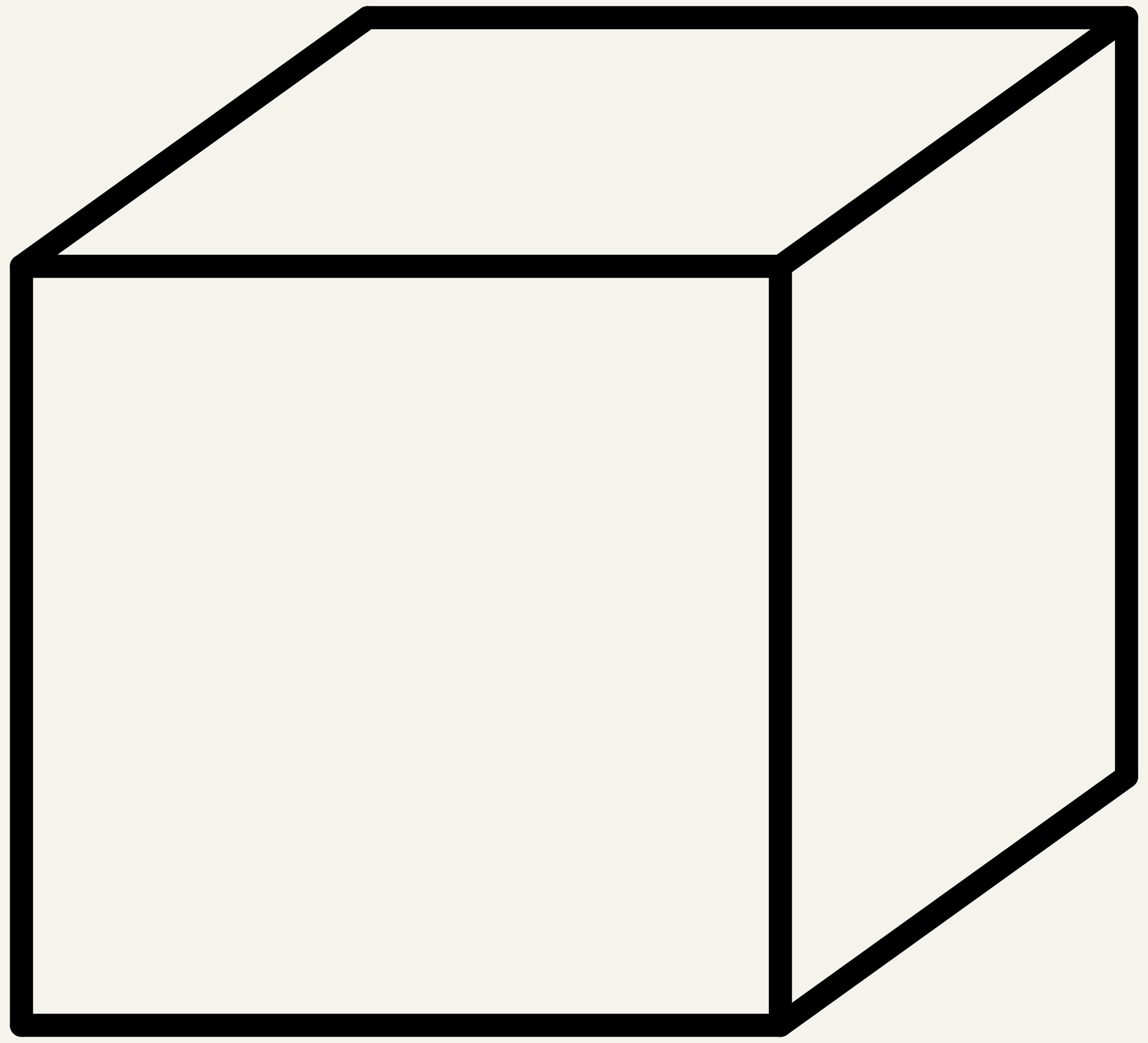
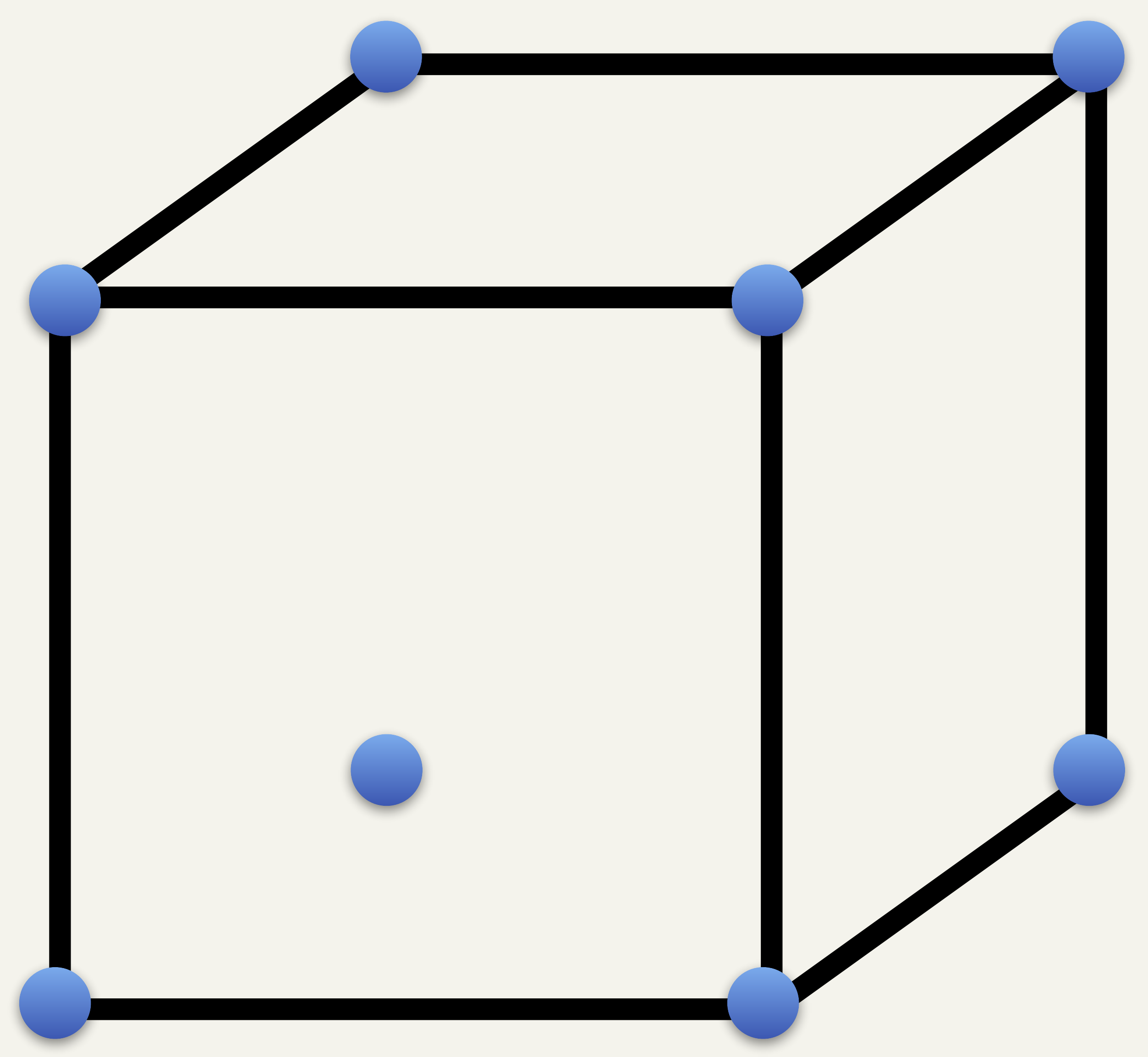
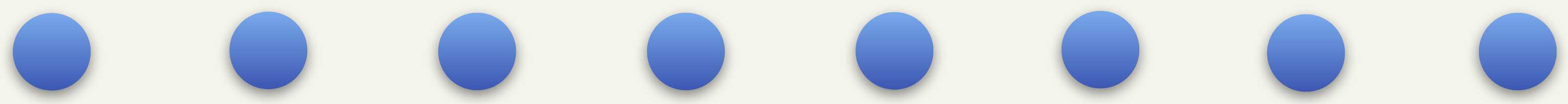
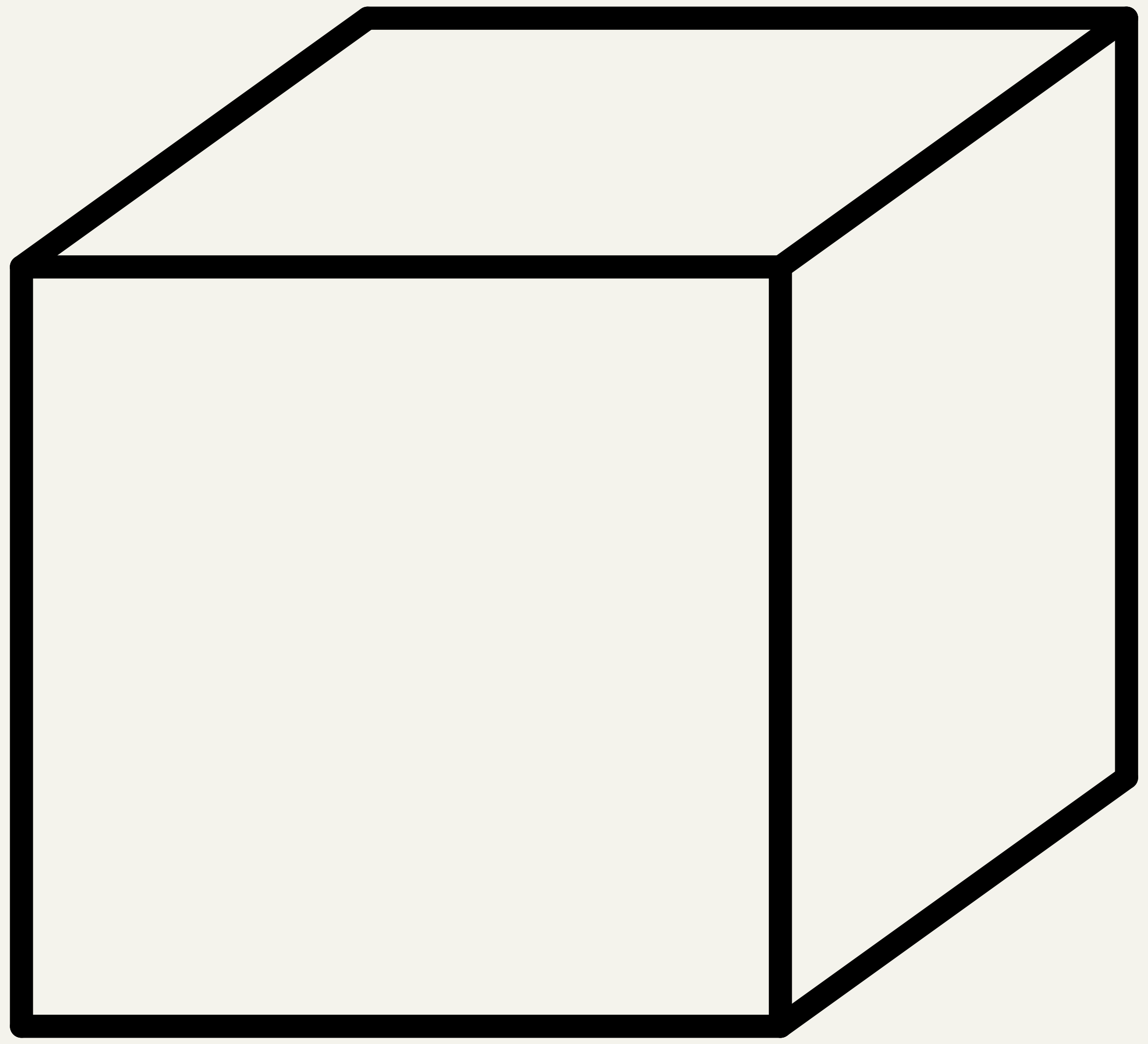
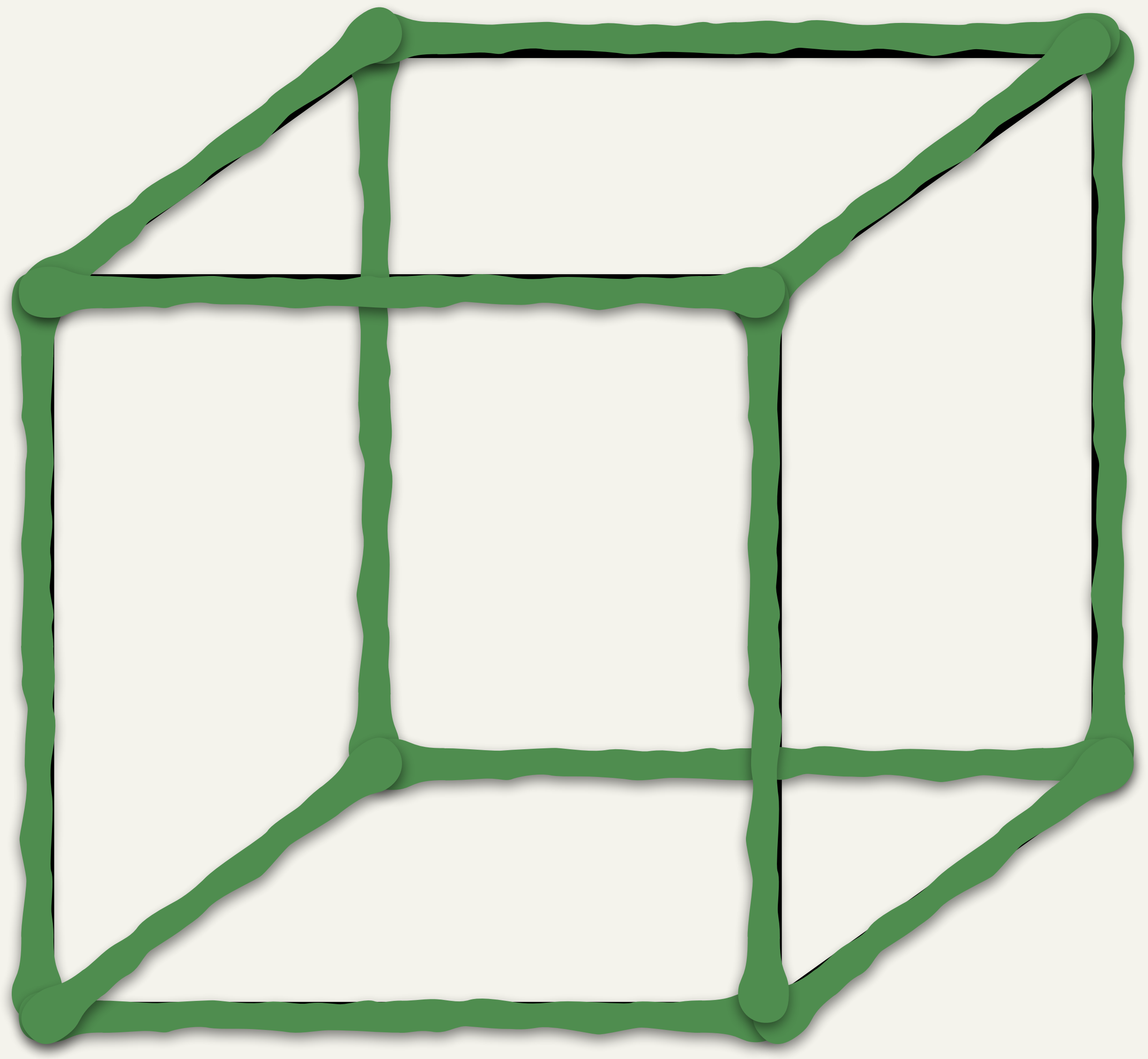
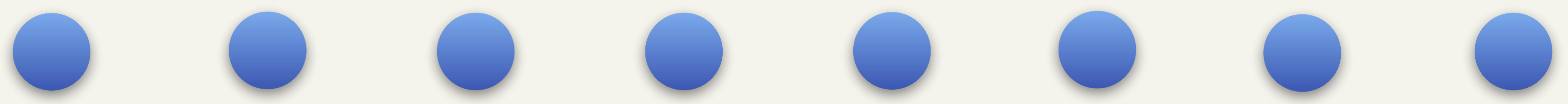


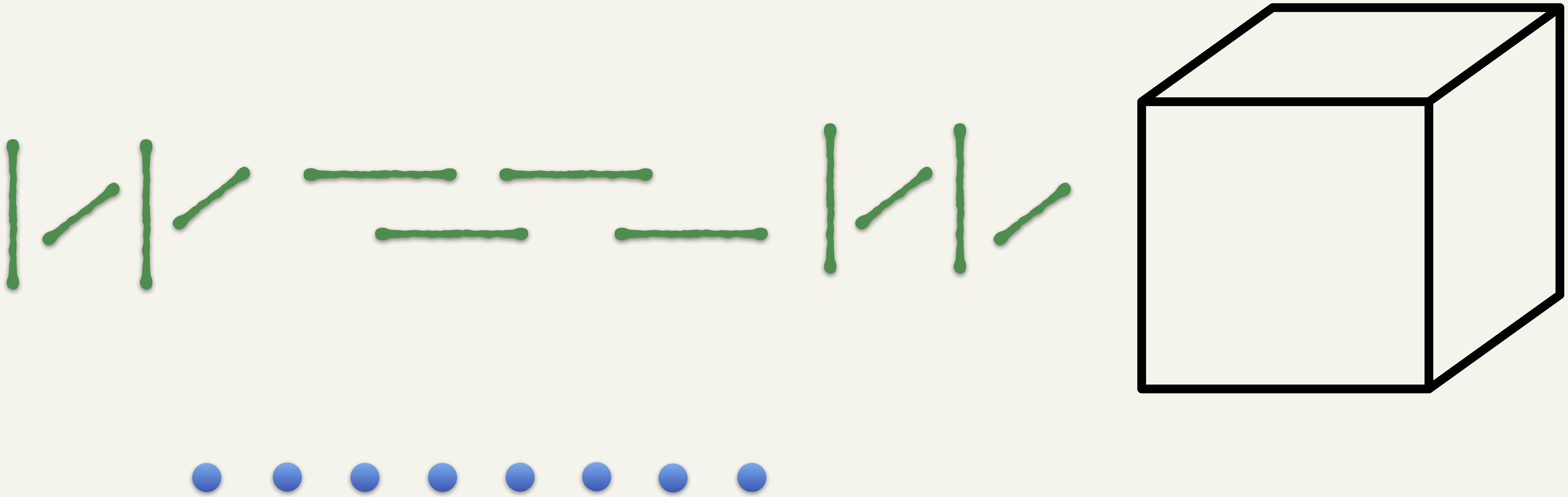


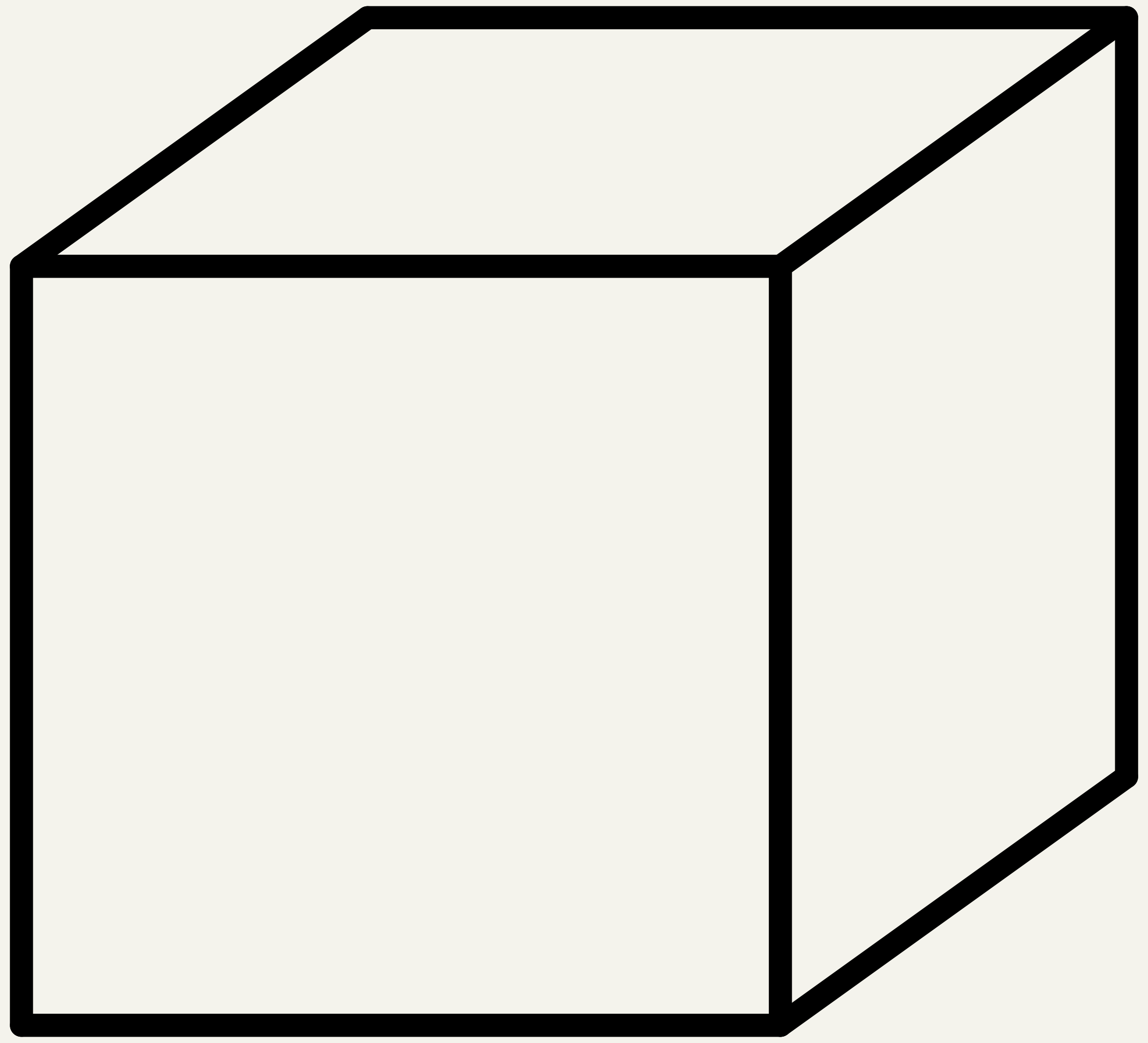








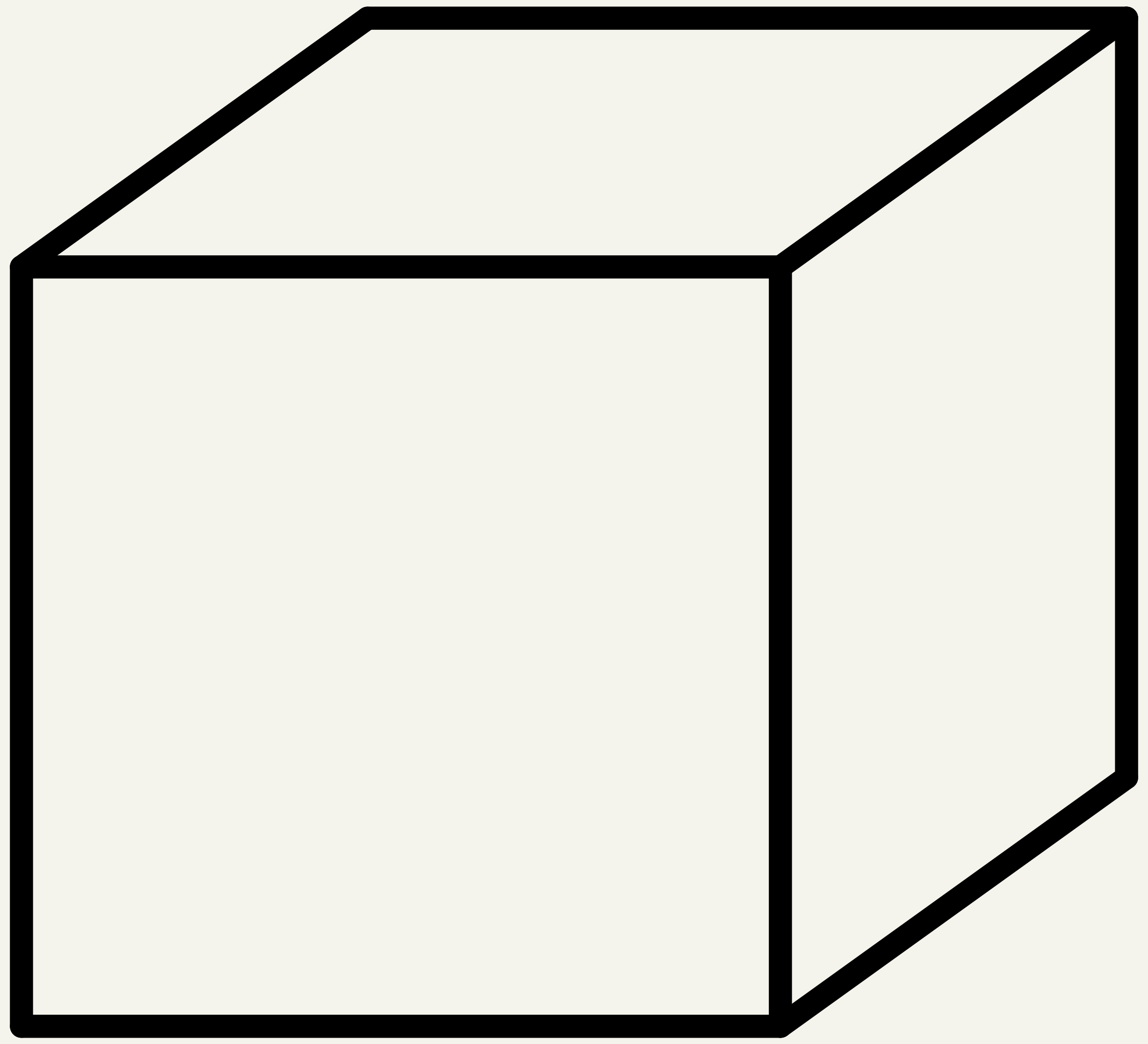




2-faces

1-faces

0-faces



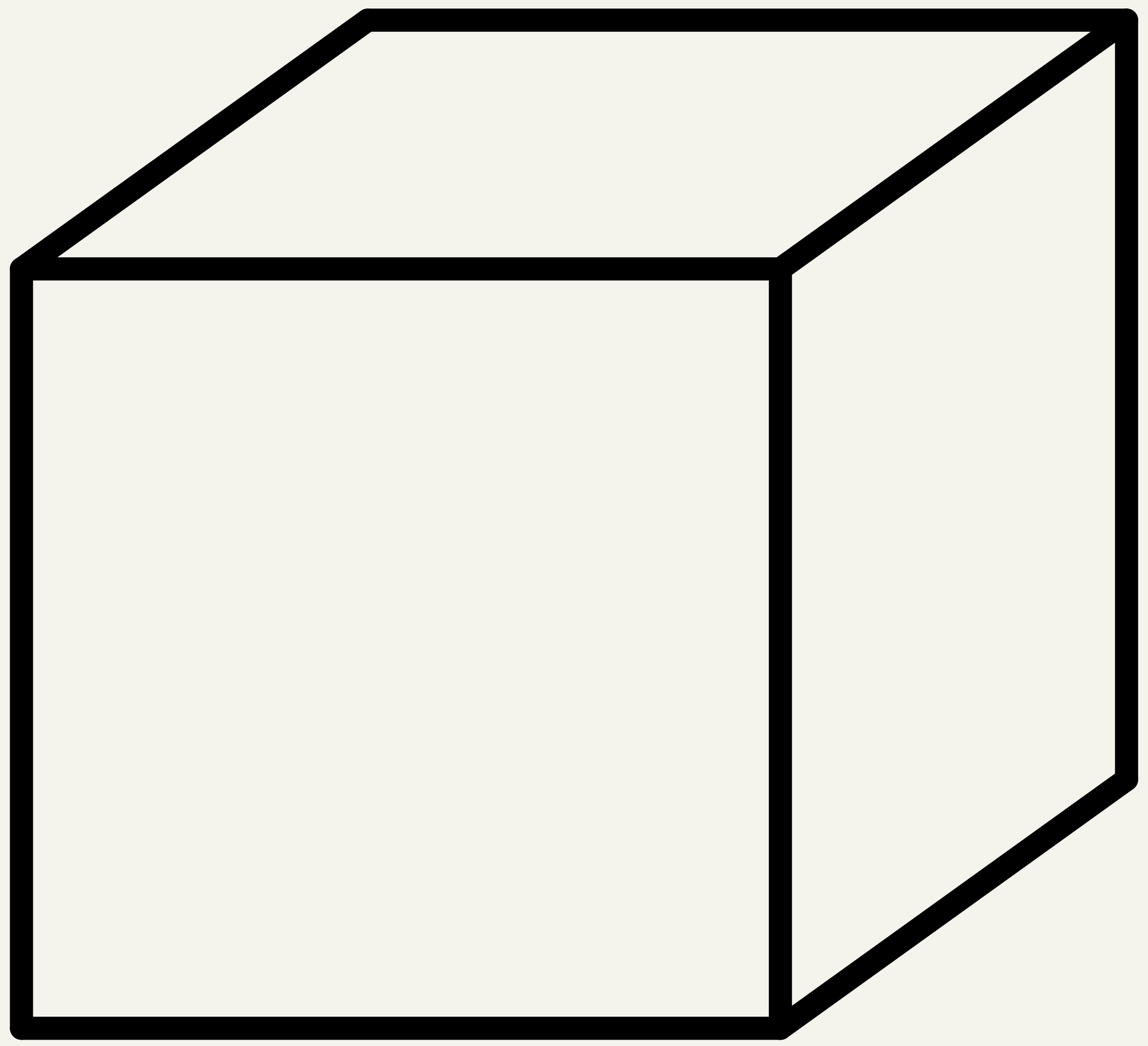
F_3

2-faces

1-faces

0-faces

F_{-1}



Abstract polytopes

F_3

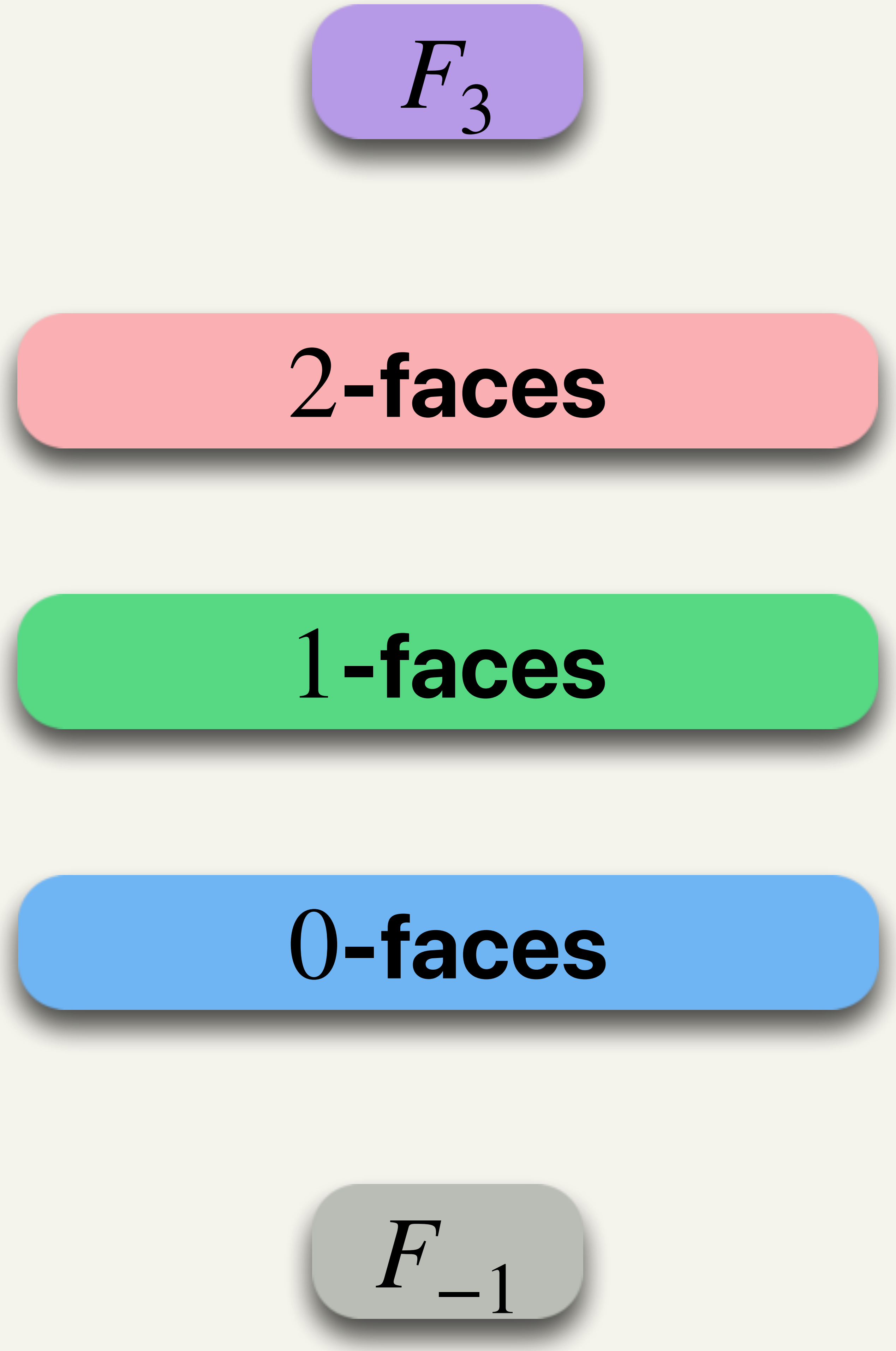
2-**faces**

1-**faces**

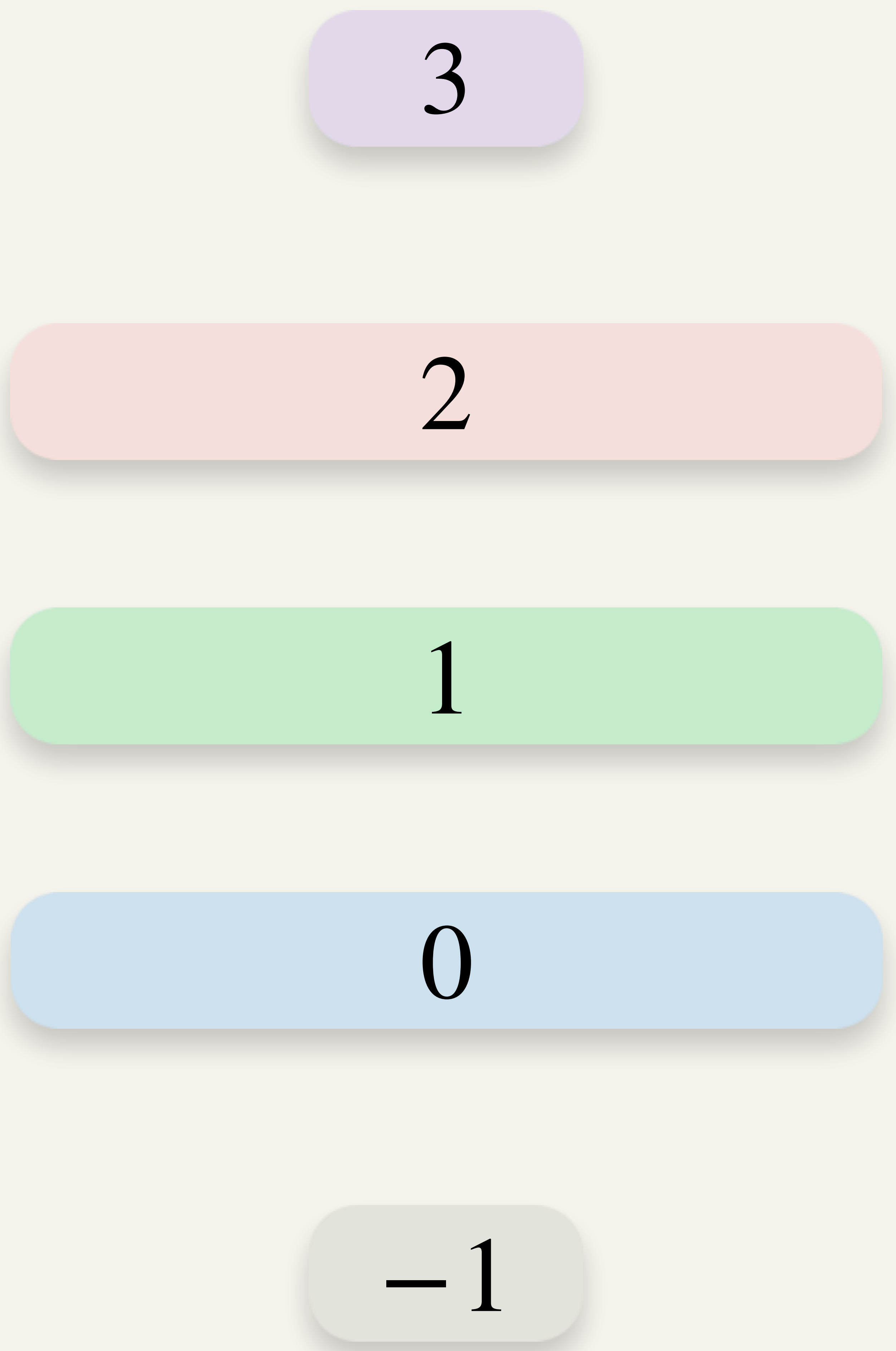
0-**faces**

F_{-1}

Abstract polytopes



Abstract polytopes



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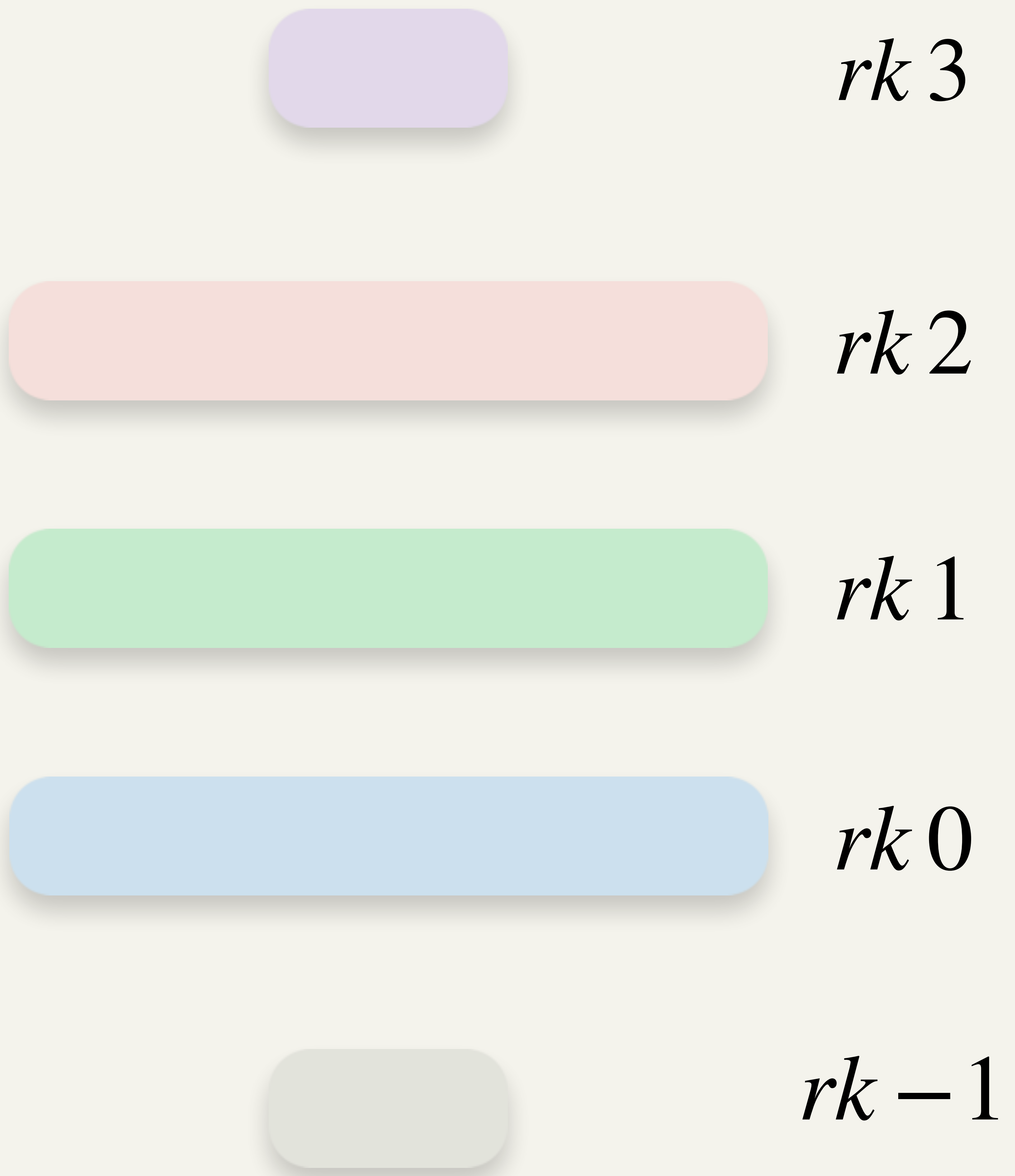
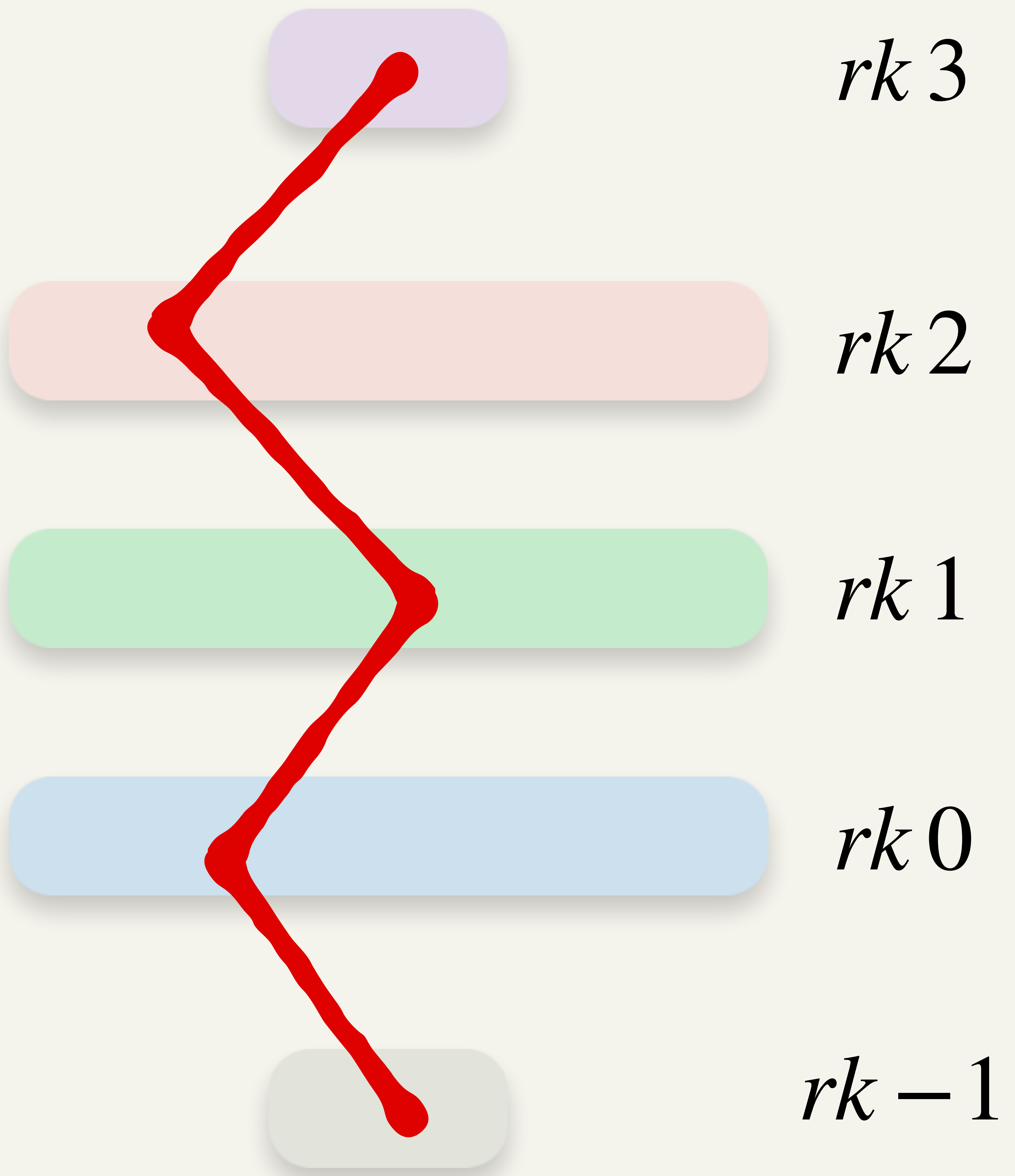


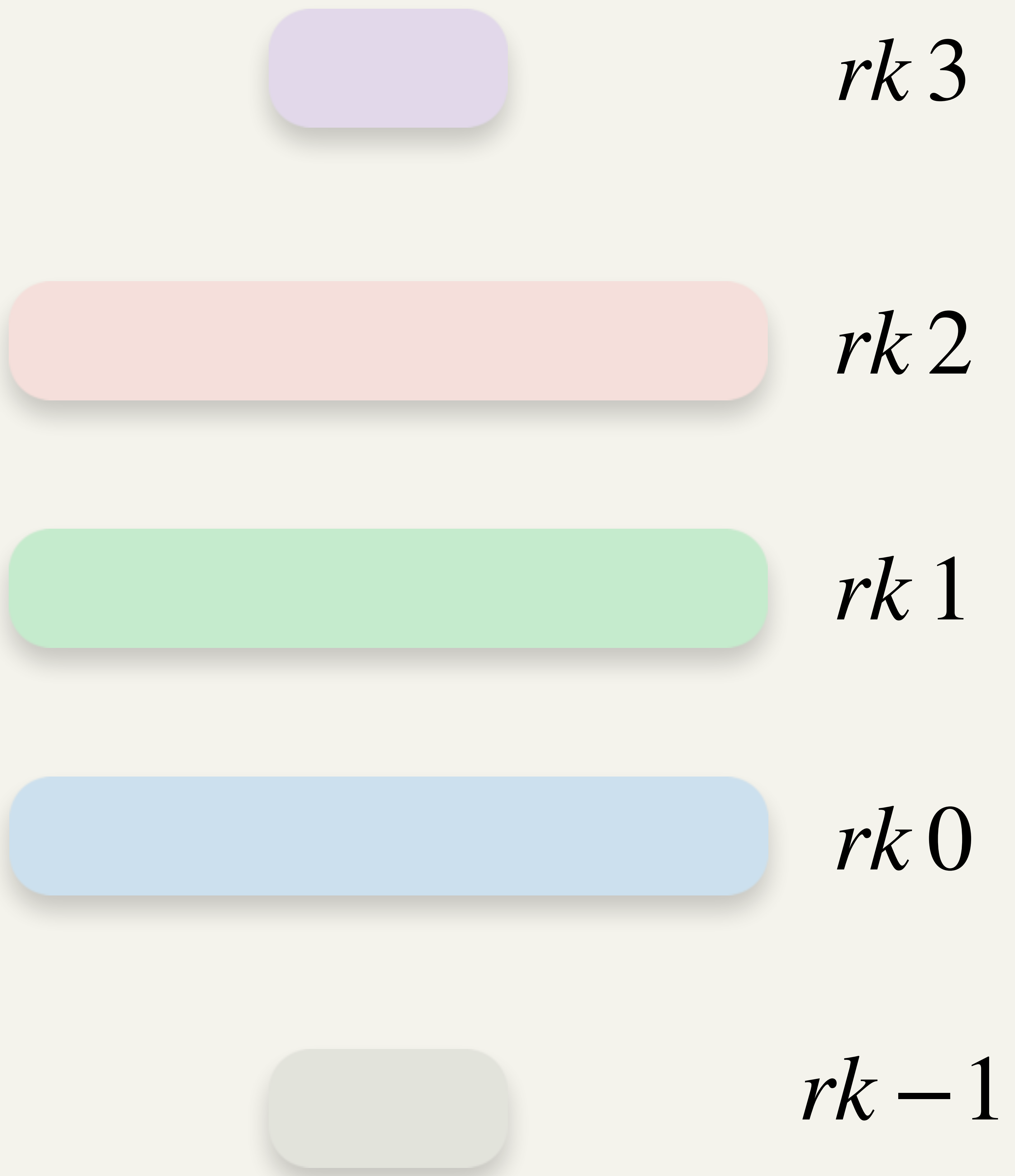
Abstract polytopes

- Ranked poset (each maximal chain contains $(n+2)$ elements);



Abstract polytopes

- Ranked poset (each maximal chain contains $(n+2)$ elements);
- Diamond condition;



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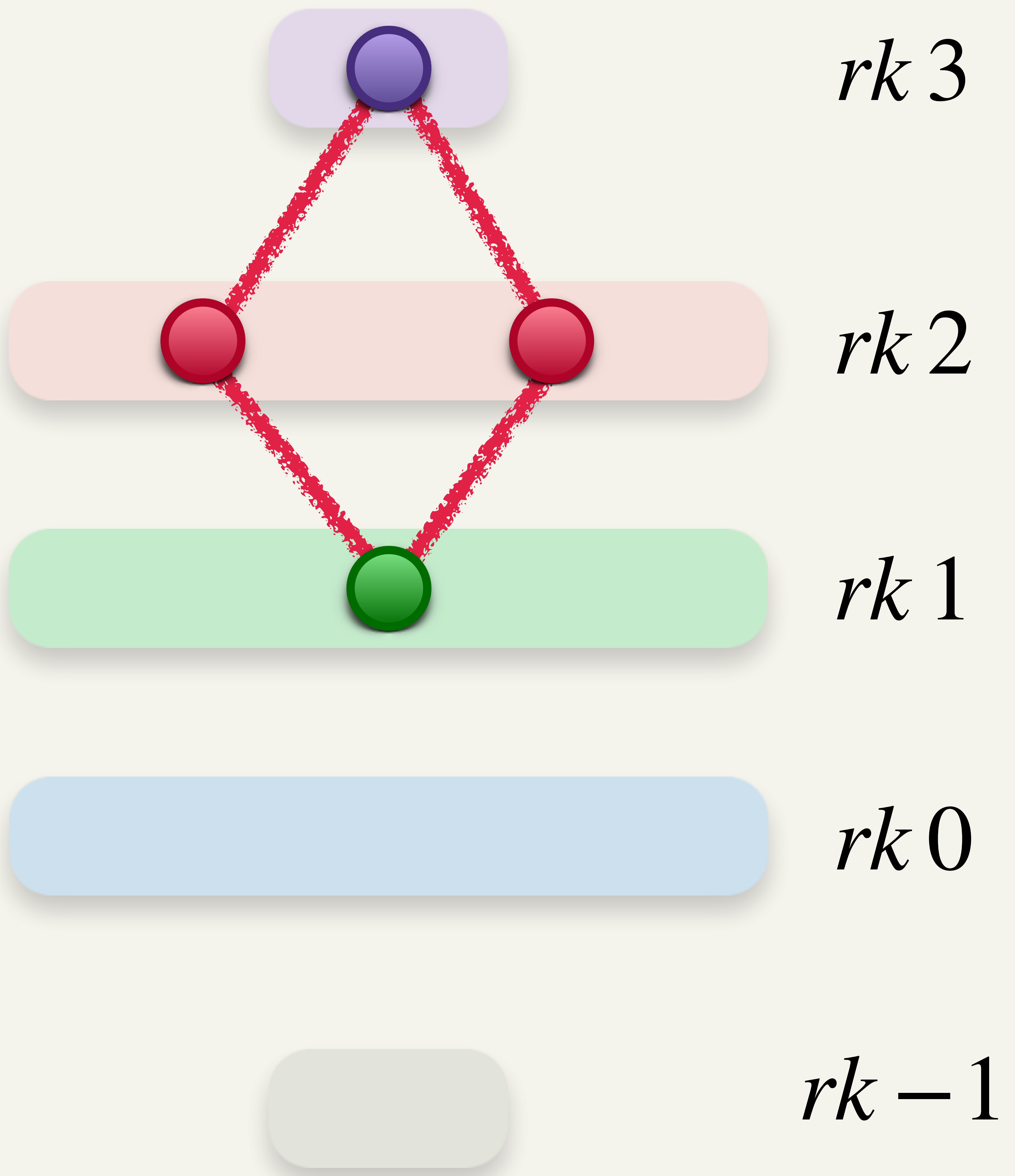

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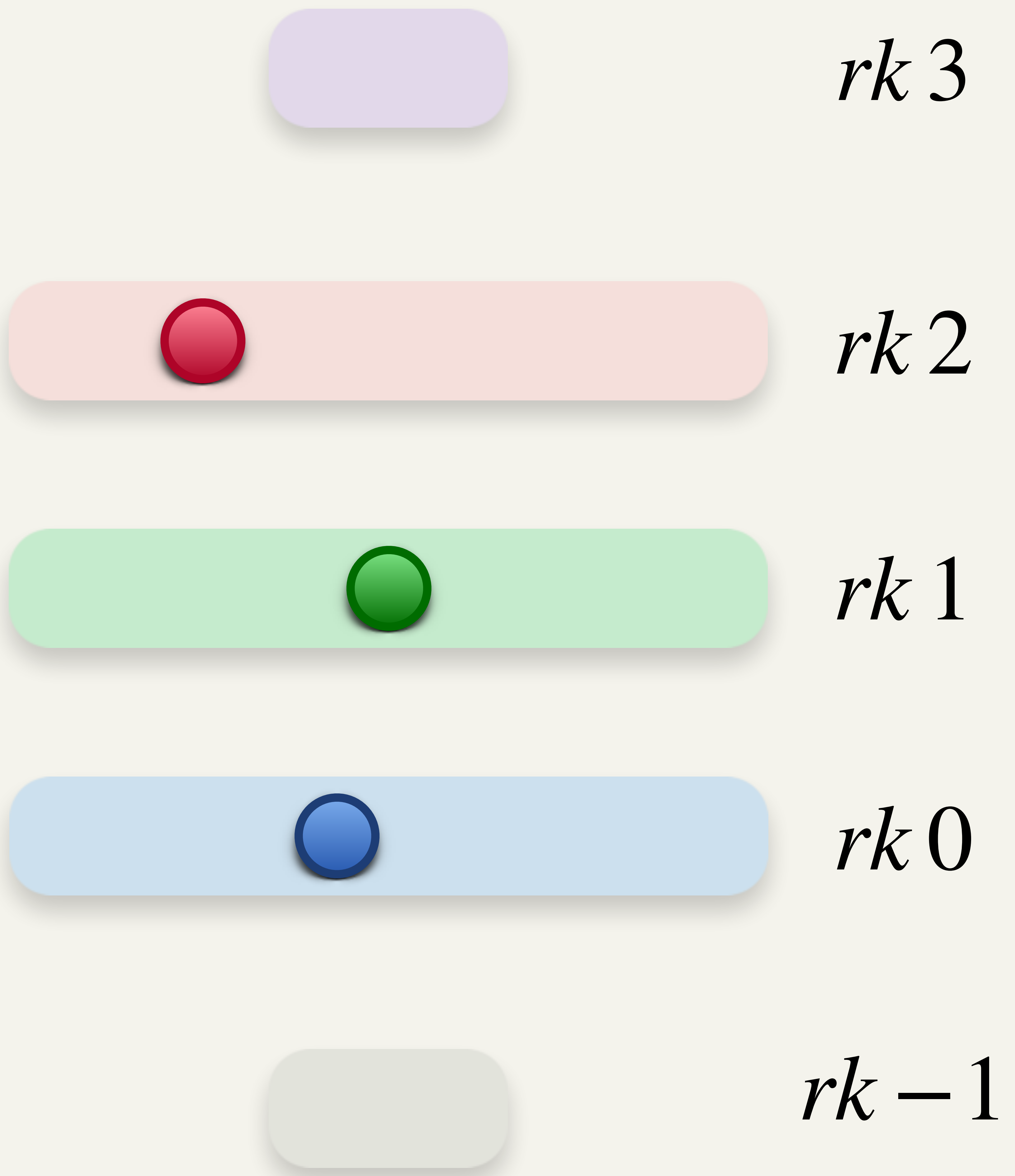

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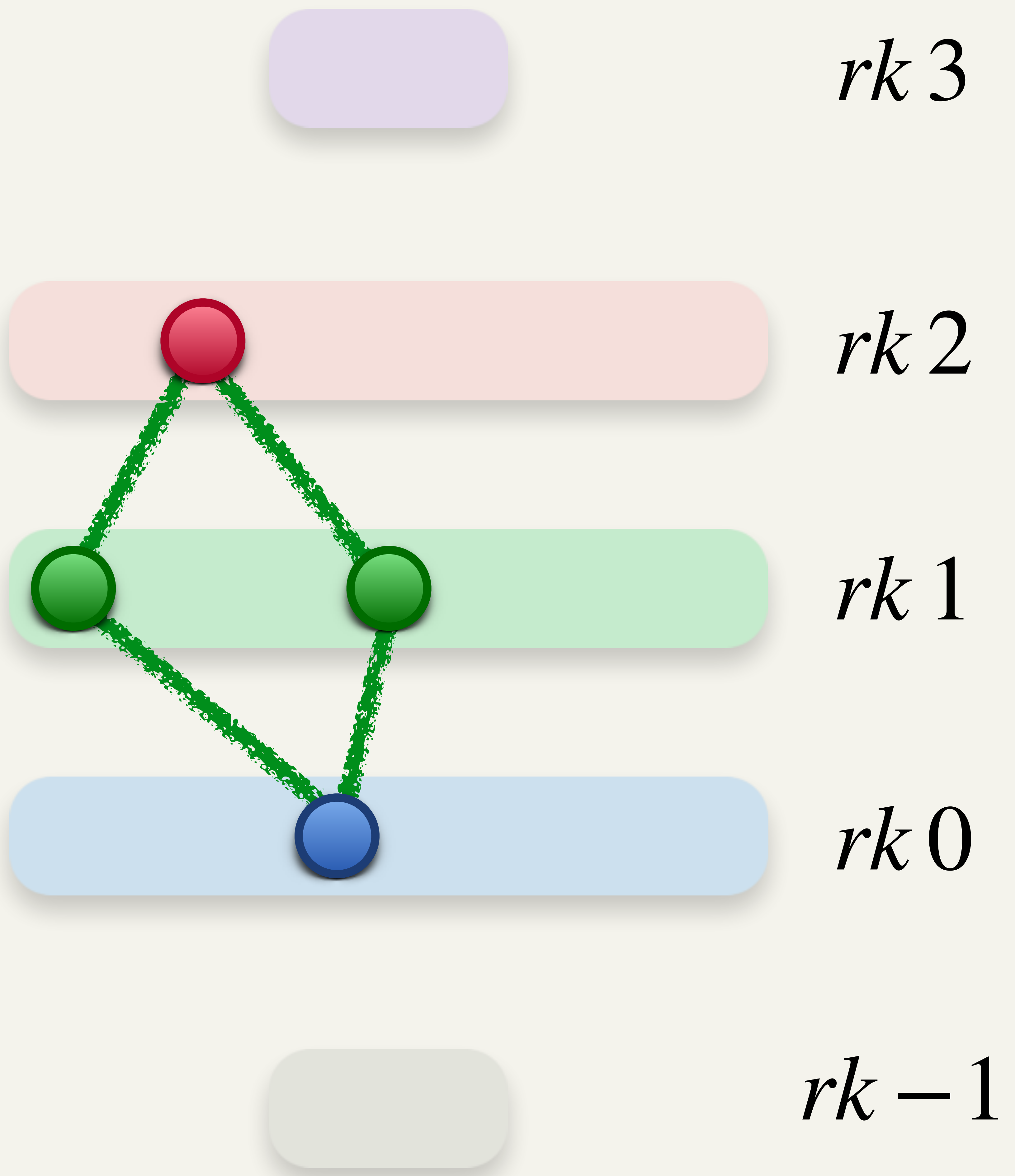

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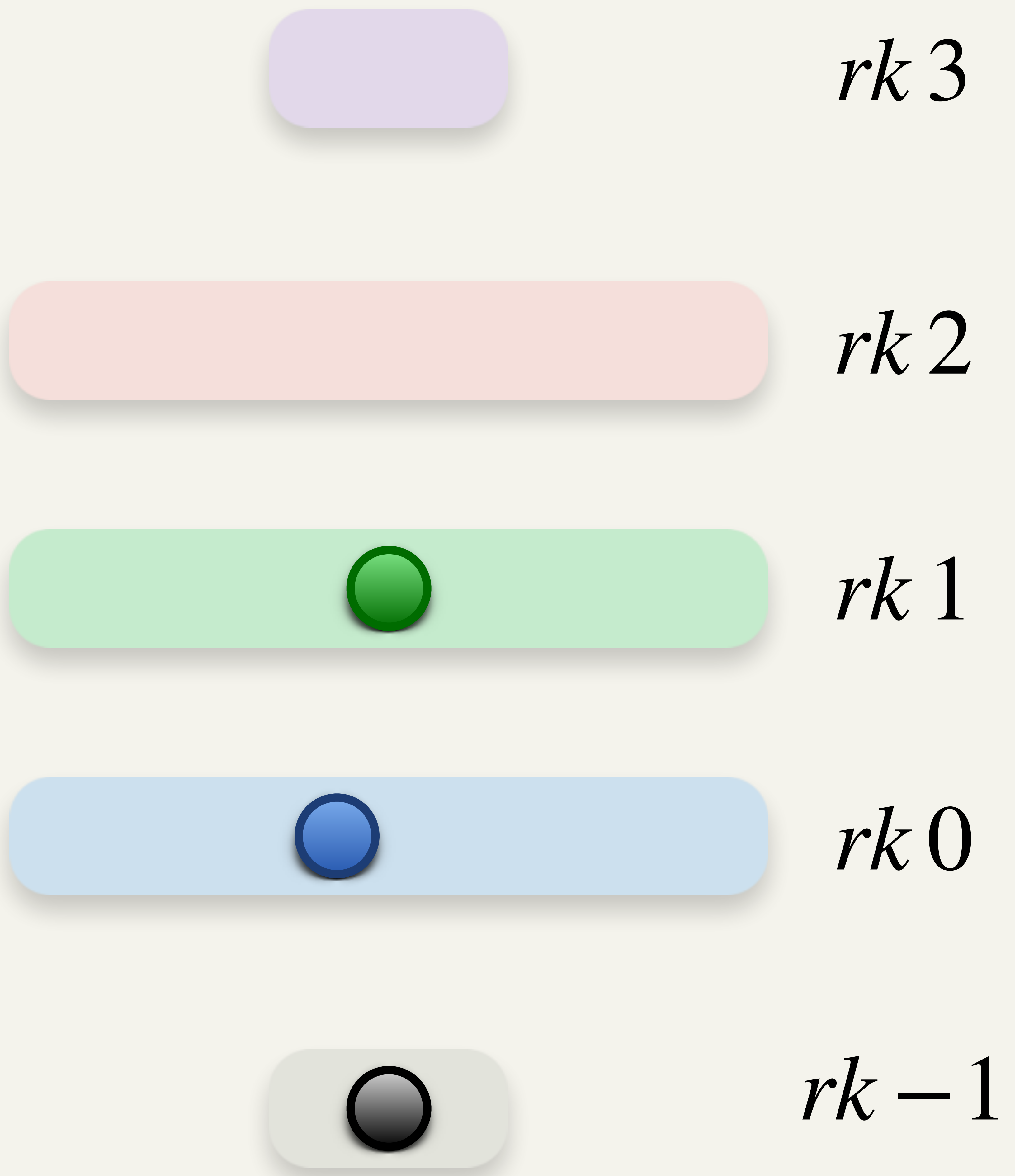

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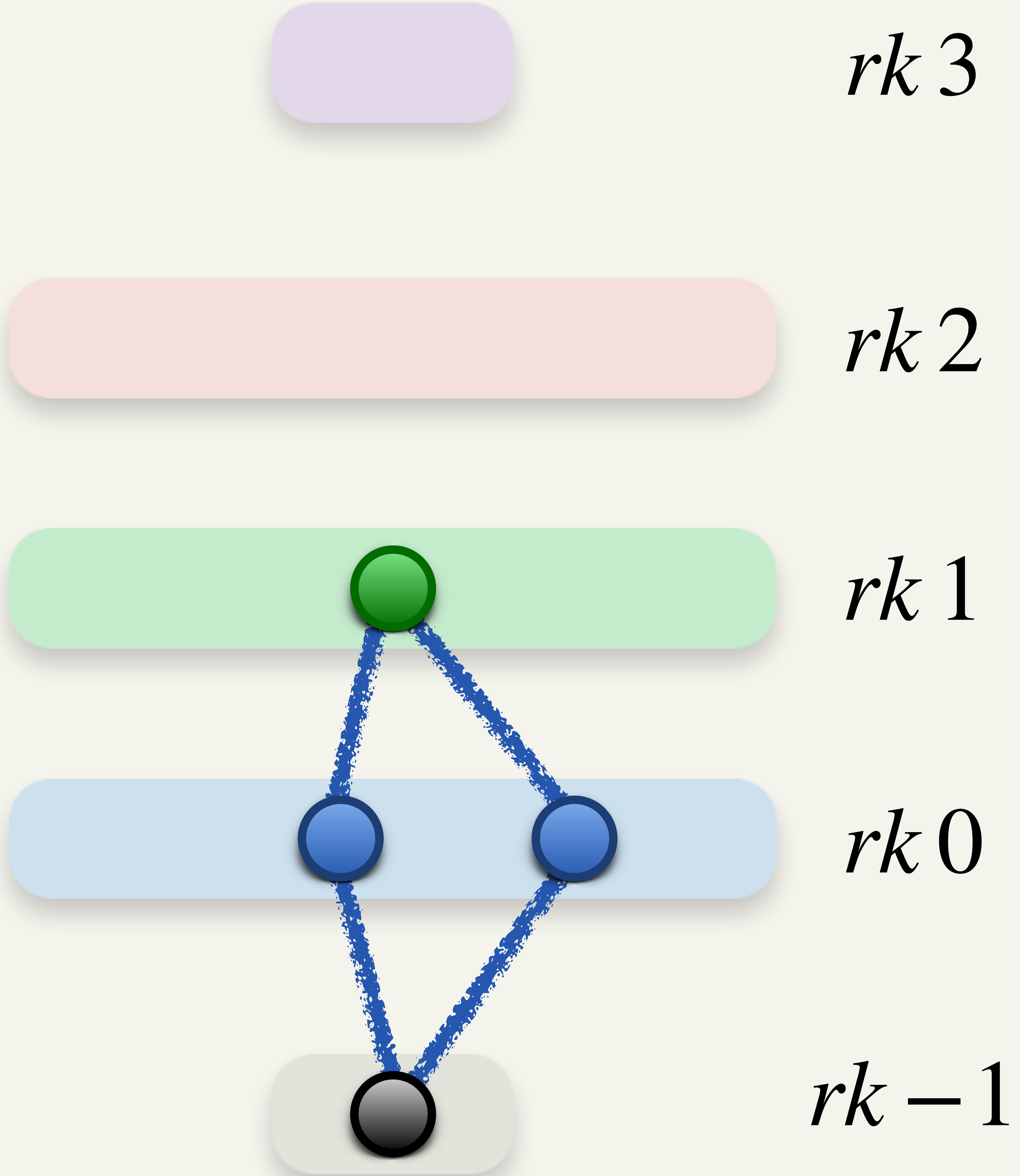
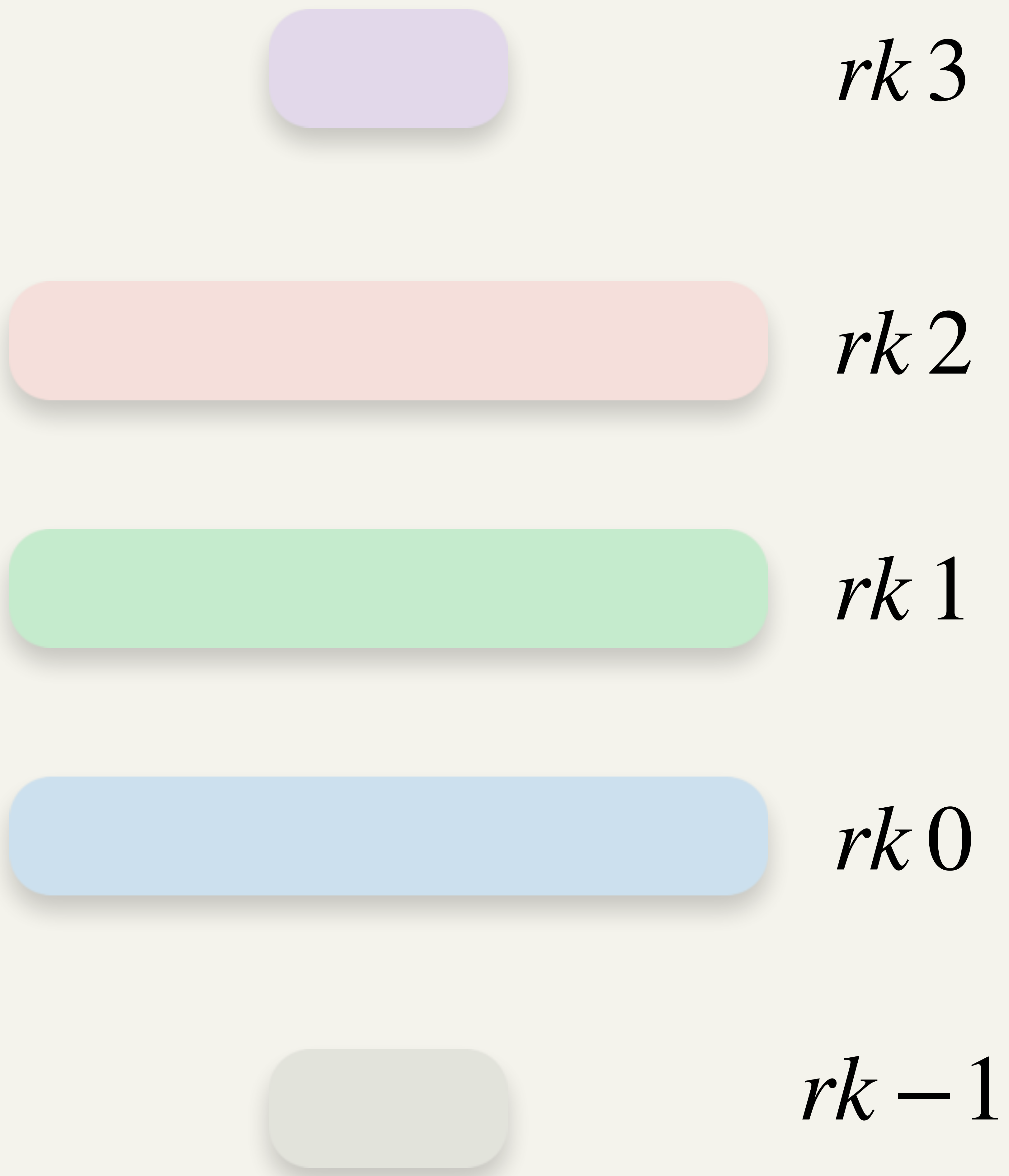
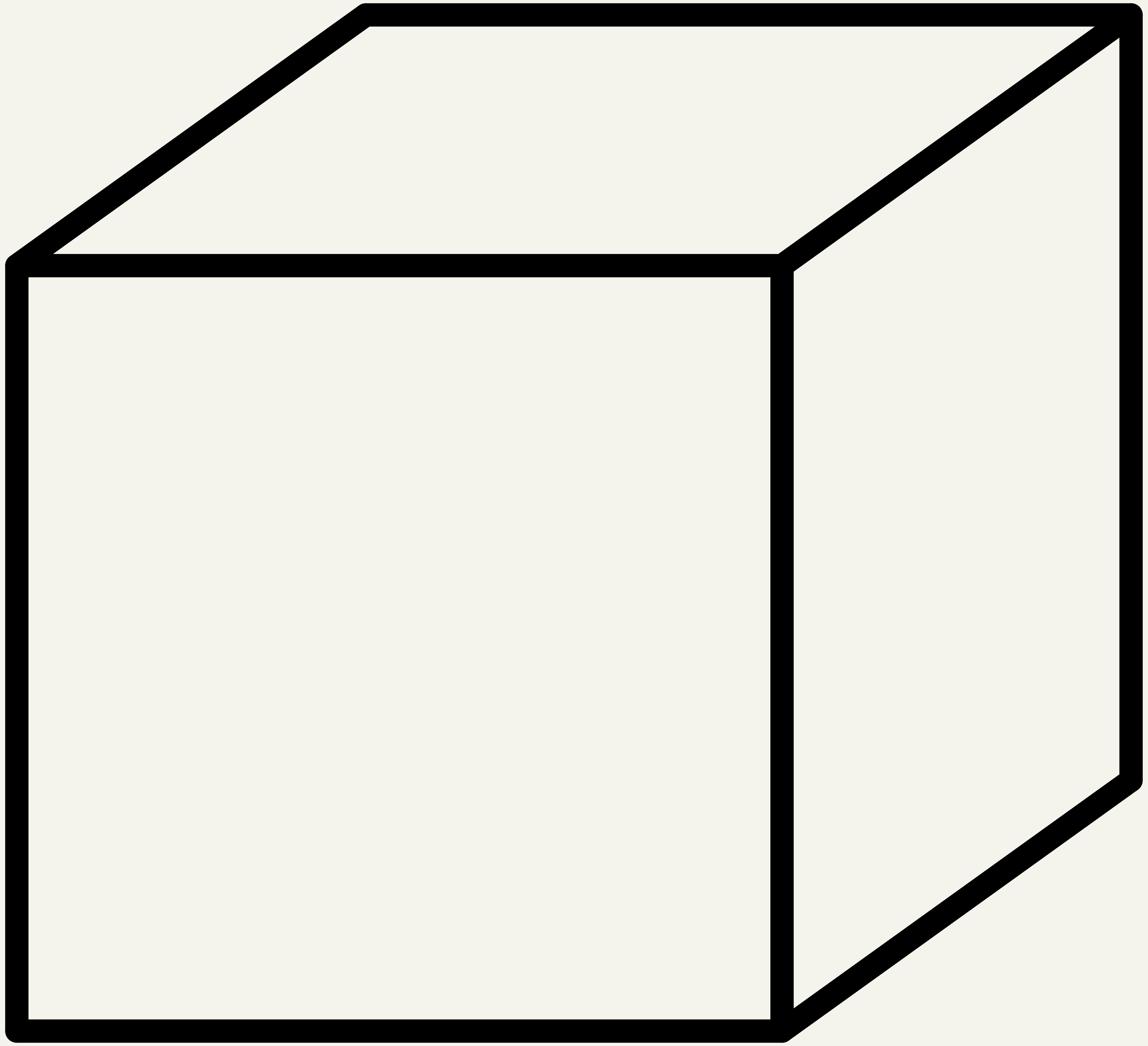


Abstract polytopes

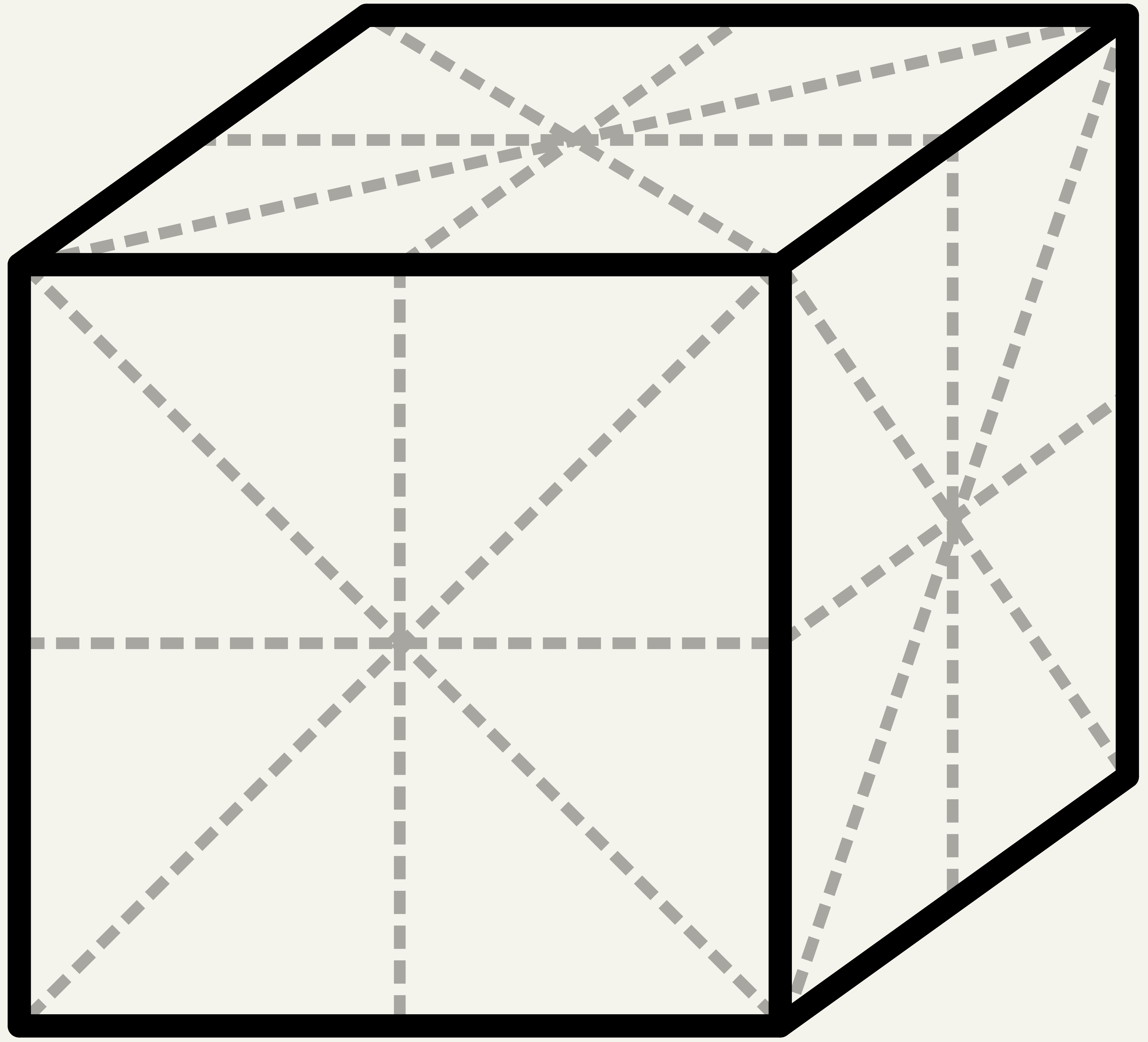
- Ranked poset (each maximal chain contains $(n+2)$ elements);
- Diamond condition;
- Strongly flag connected.



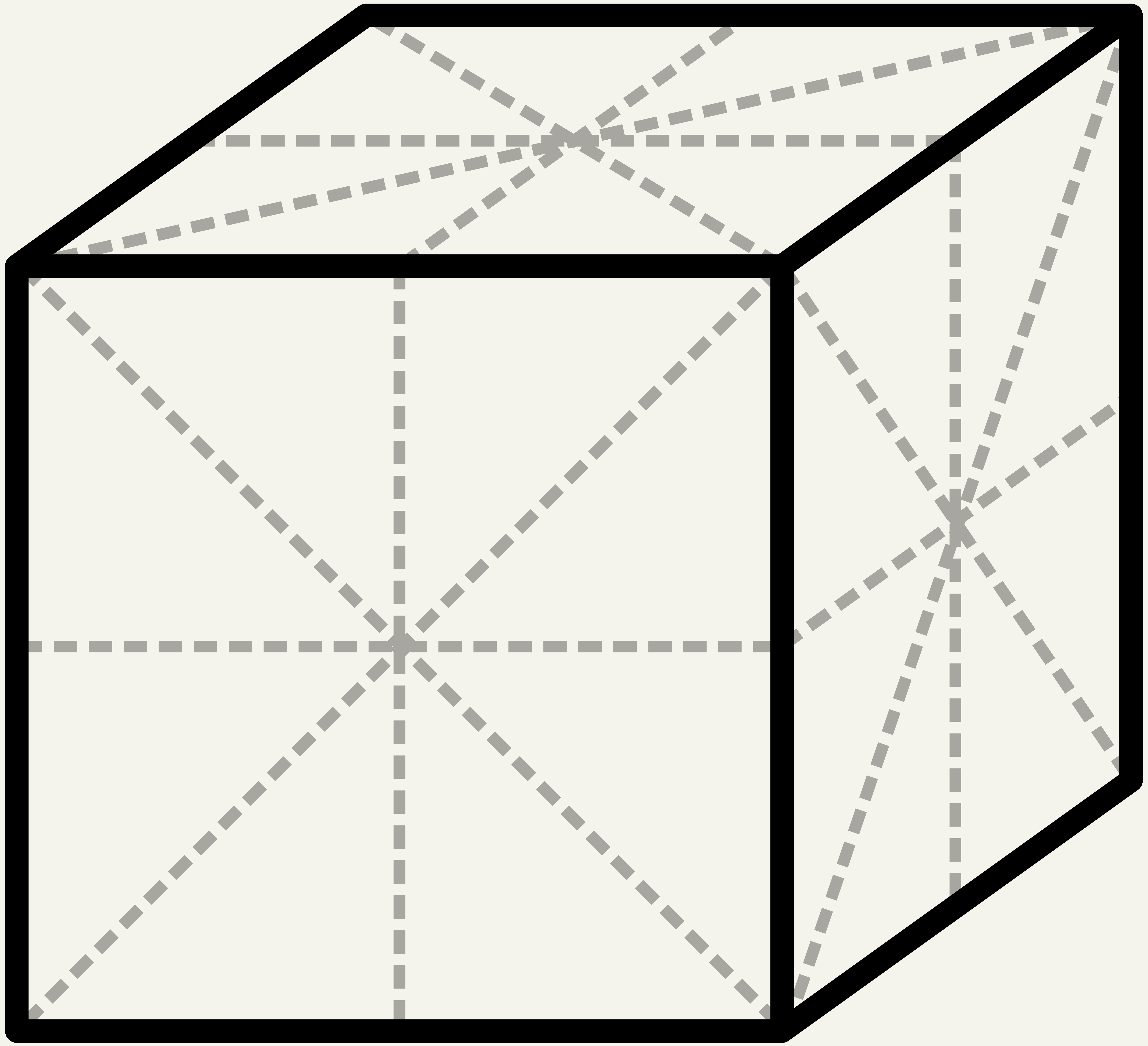
Flag graph



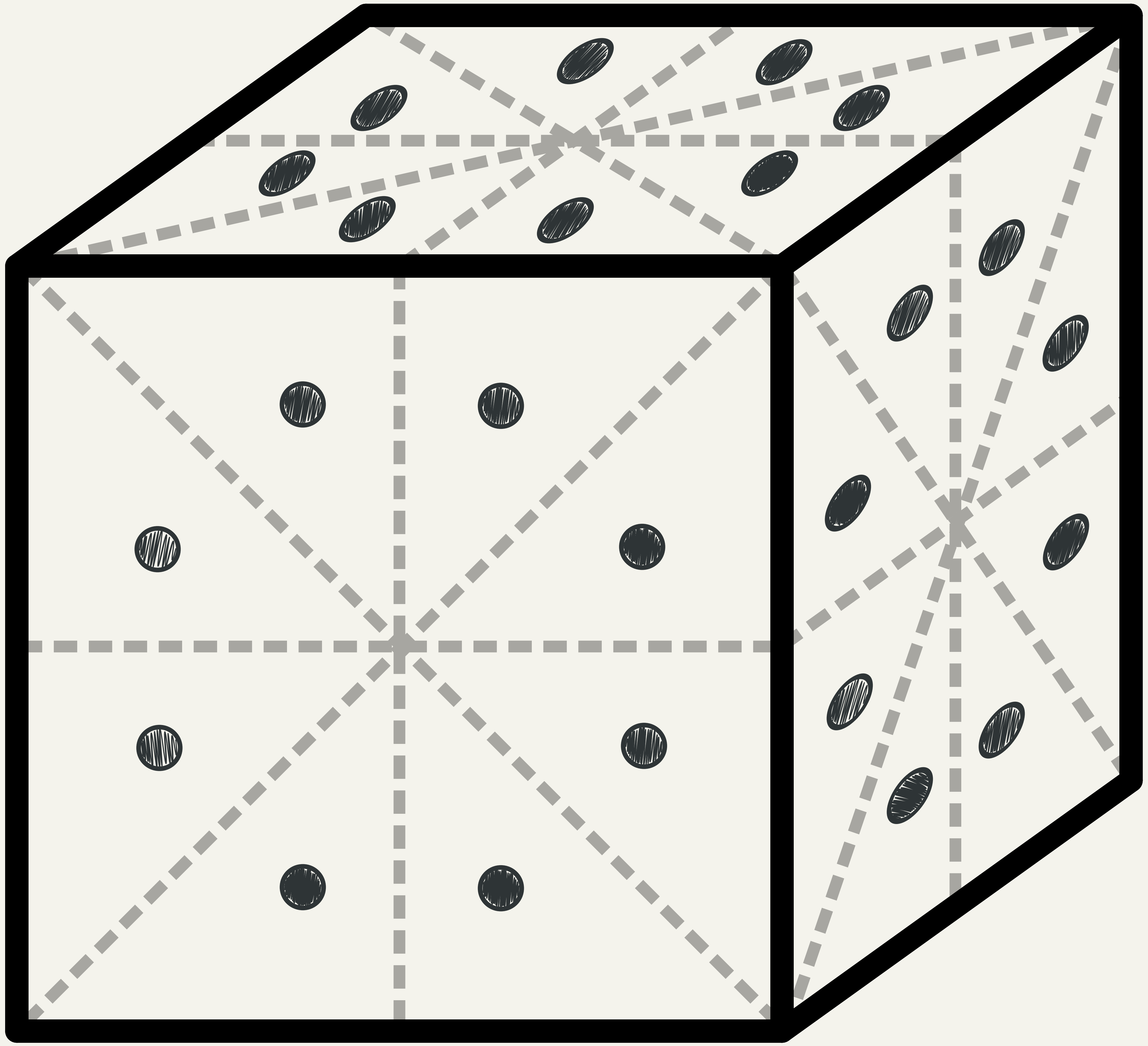
Flag graph



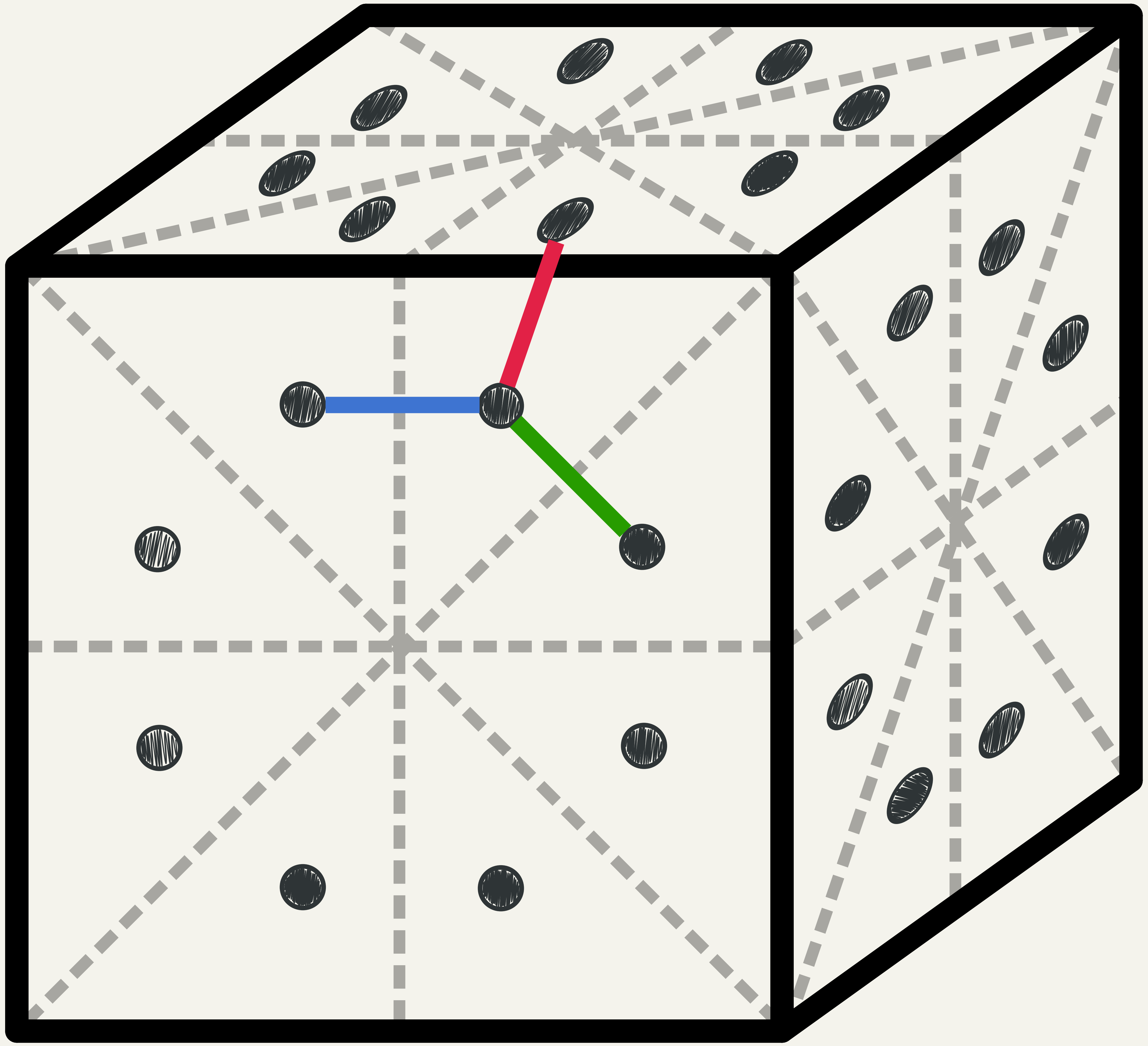
Flag graph



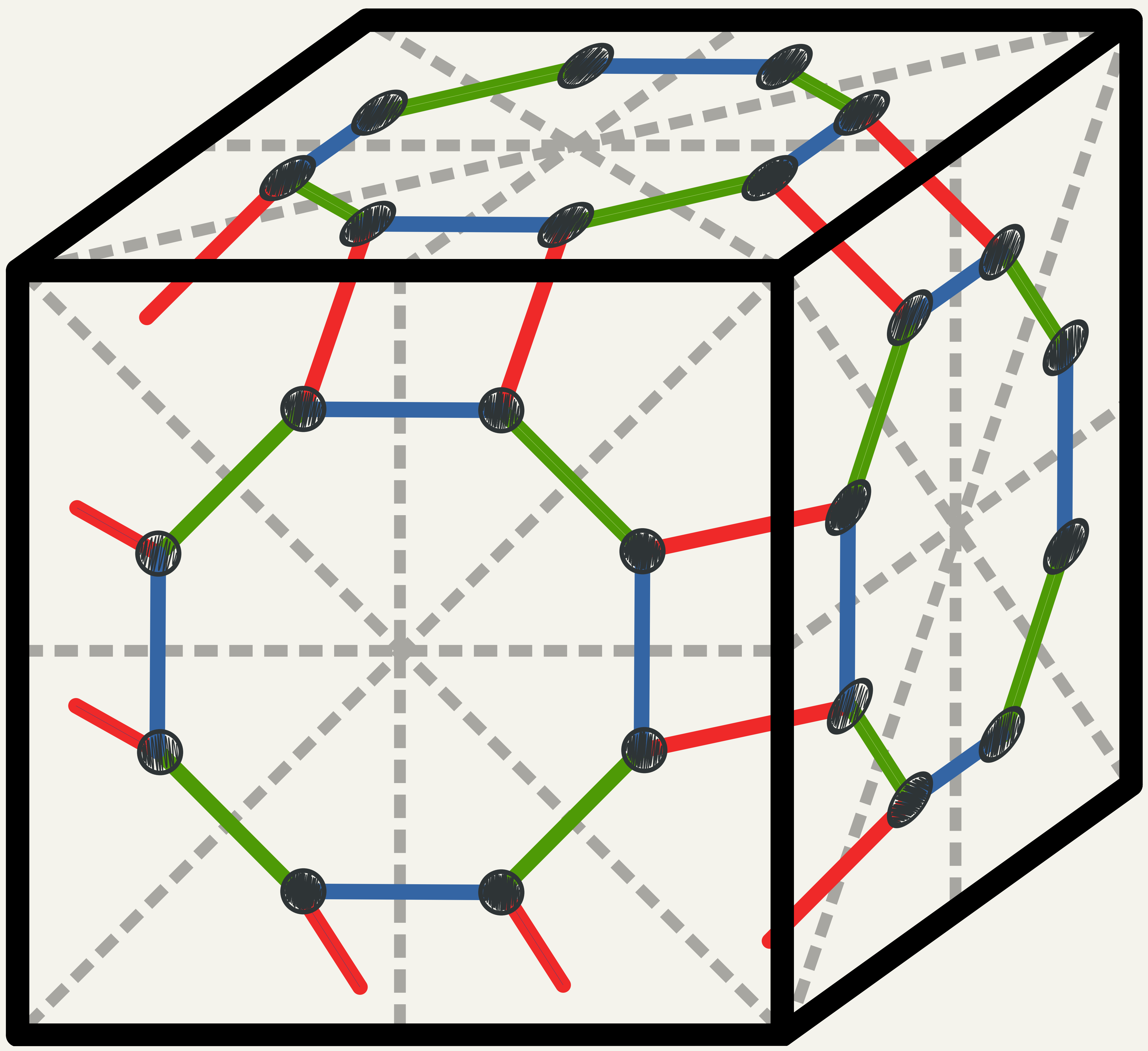
Flag graph



Flag graph



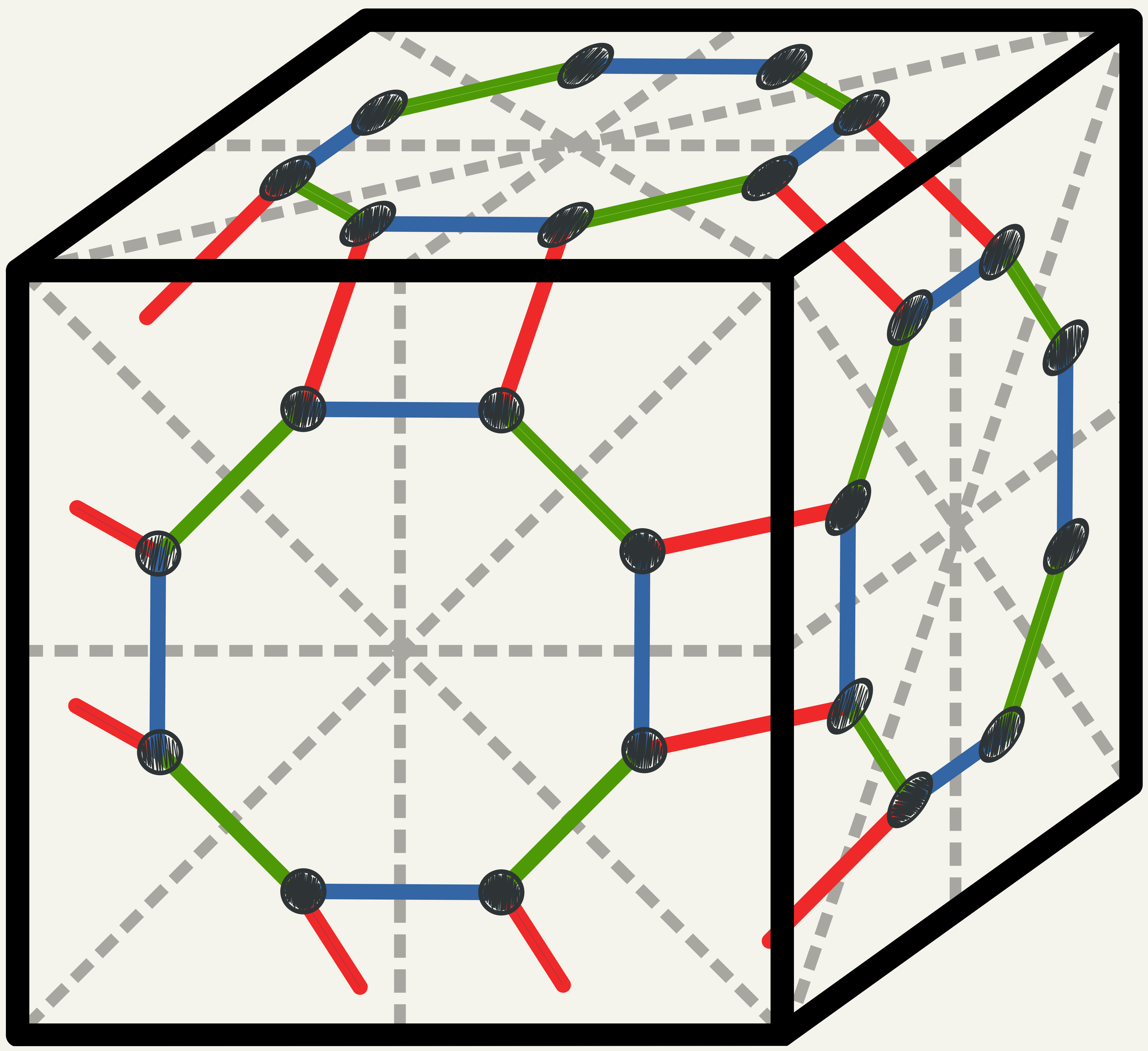
Flag graph



Flag graph

The flag graph $\mathrm{Fl}(\mathscr{P})$ of a polyhedron $\mathscr{P}$:

- Connected and simple,
- Valency 3,
- Properly-edge 3-coloured,
- The $(0,2,0,2)$-paths are alternating squares.



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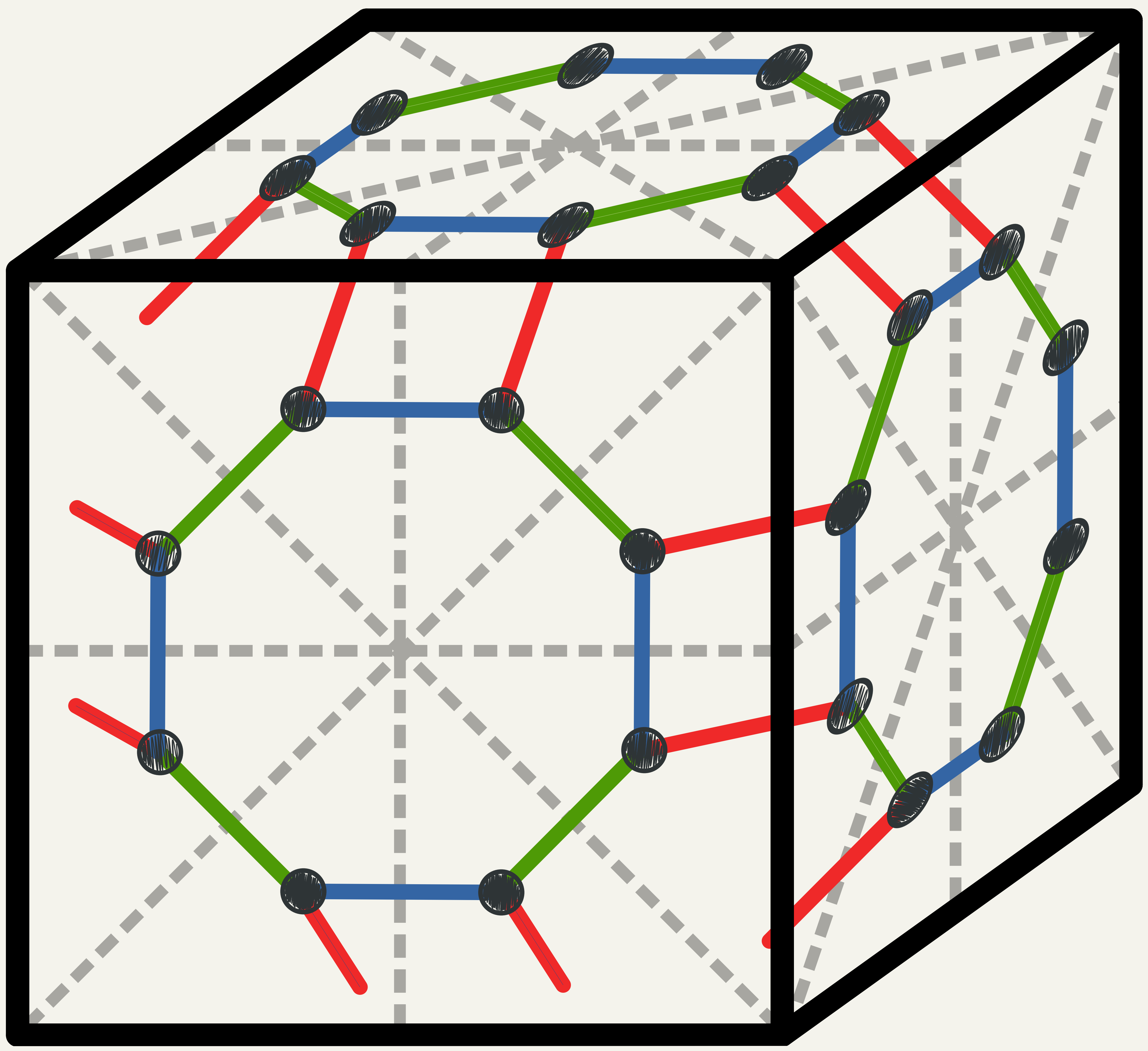
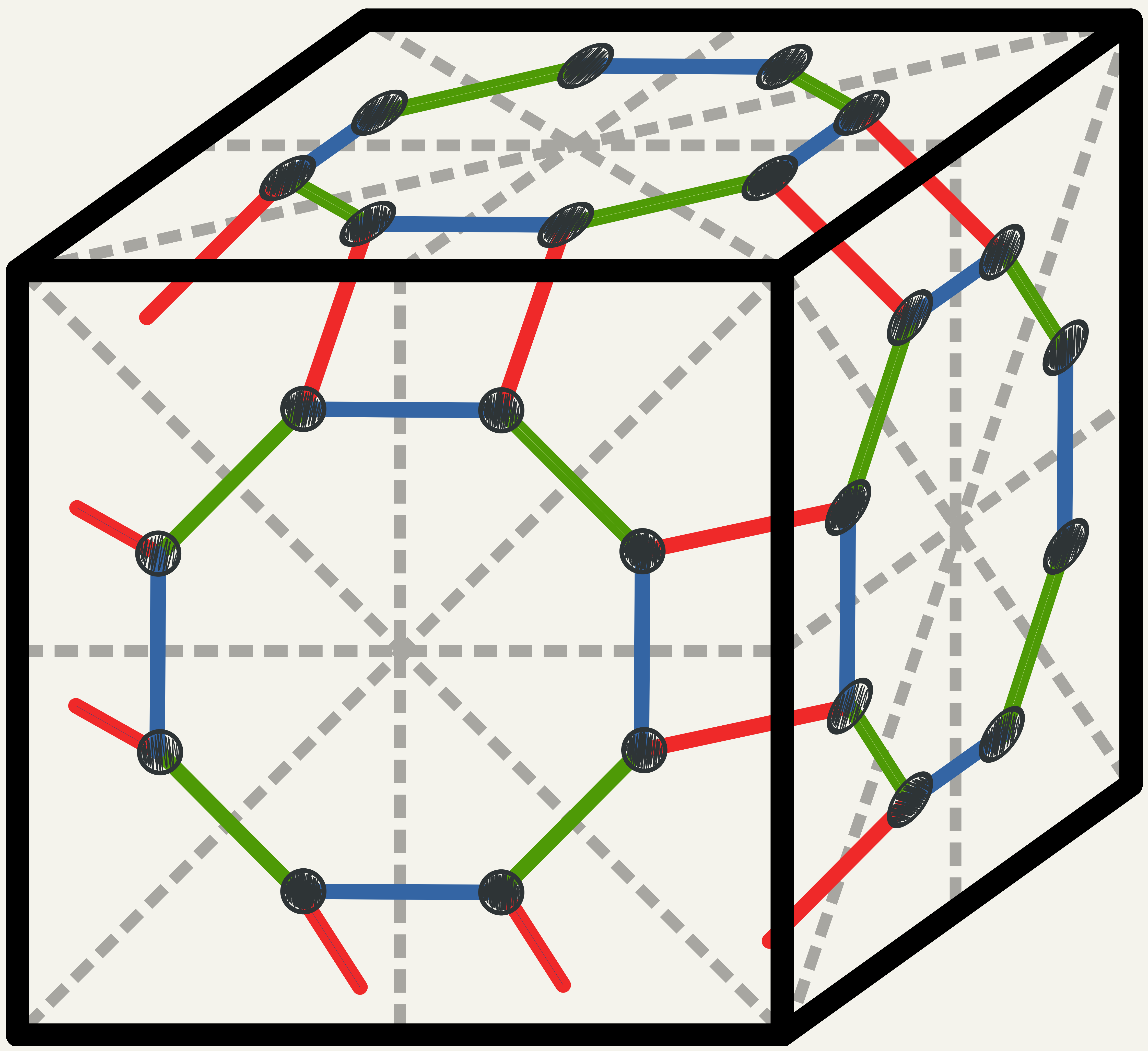


Maniplexes

A 3-maniplex is a graph:

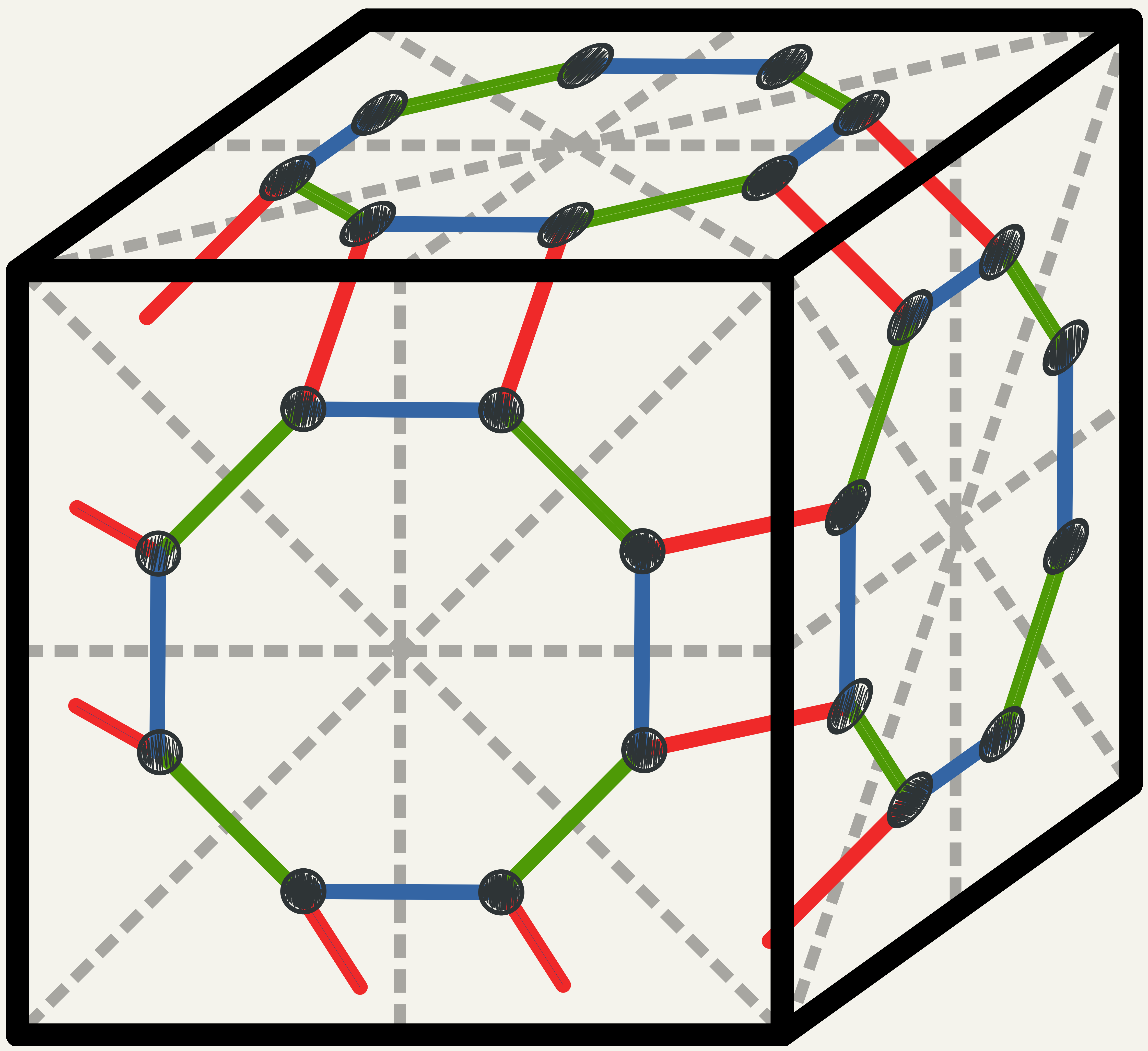
- Connected and simple,
- Valency 3,
- Properly-edge 3-coloured,
- The $(0,2,0,2)$-paths are alternating squares.



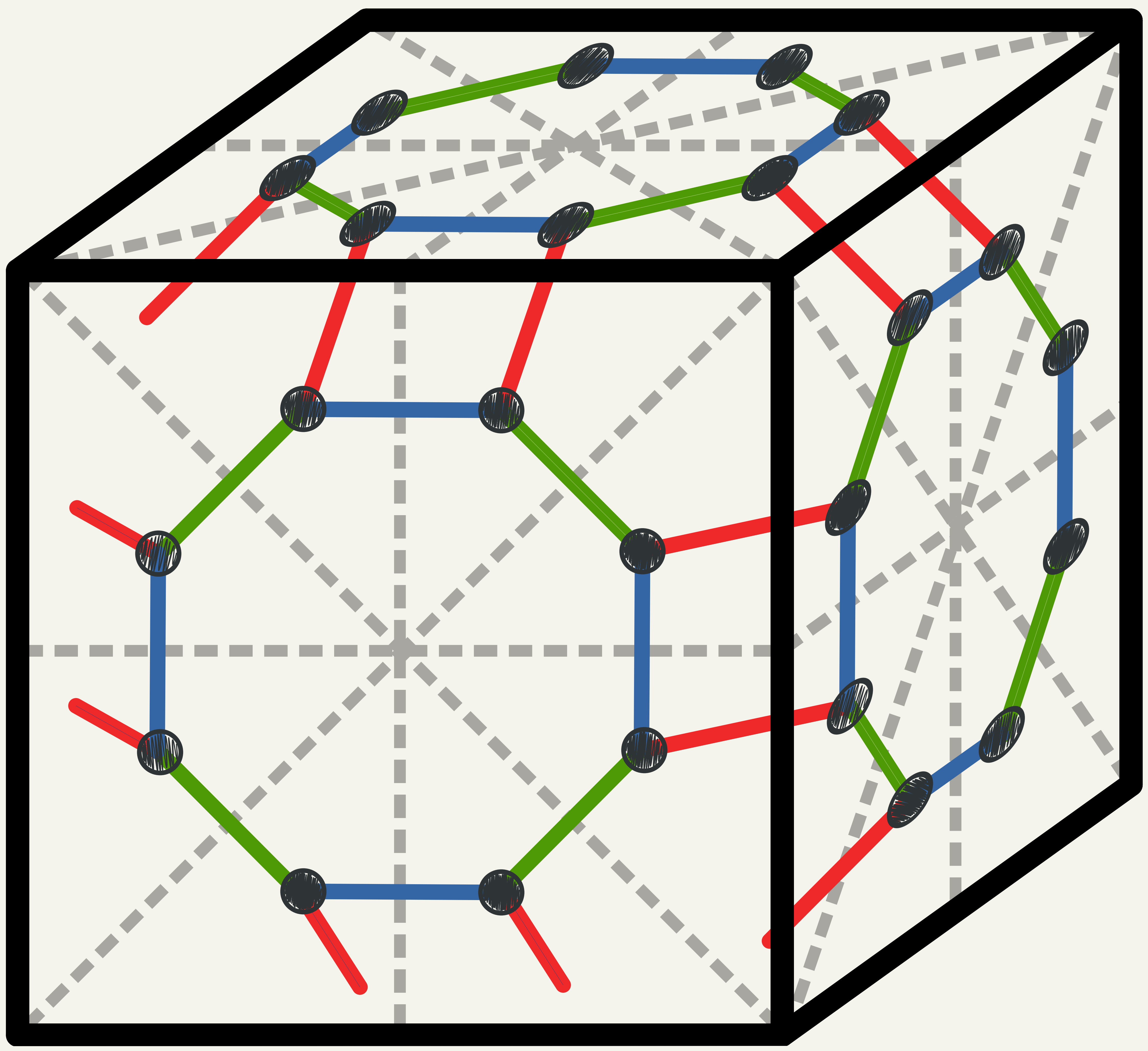
Maniplexes

A n-maniplex is a graph:

- Connected and simple,
- Valency n,
- Properly-edge n-coloured,
- The (i, j, i, j)-paths are alternating squares, whenever $|i - j| > 1$



Polytopes vs maniplexes



Polytopes vs maniplexes

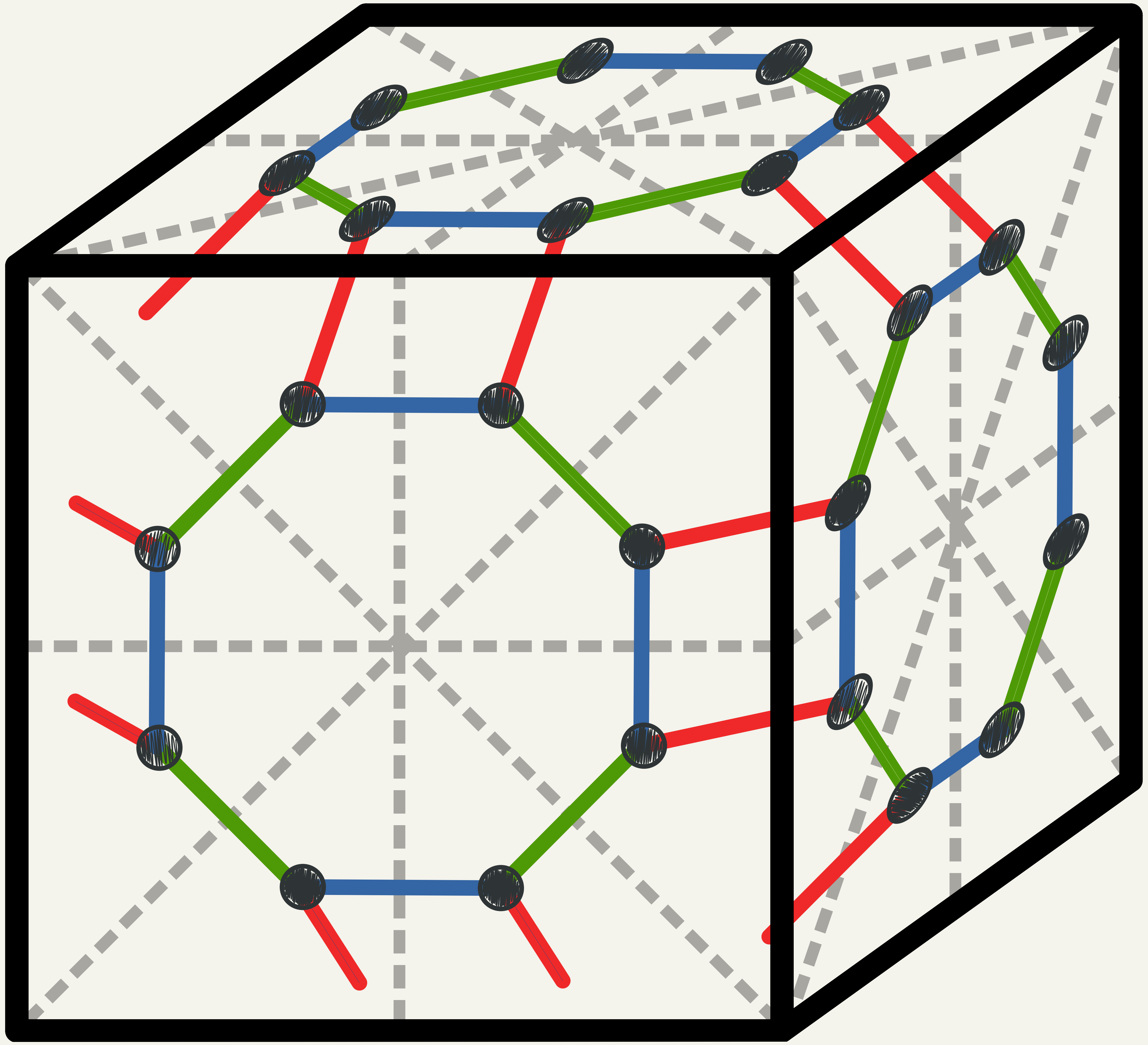
$rk\ 3$

$rk\ 2$

$rk\ 1$

$rk\ 0$

$rk\ -1$



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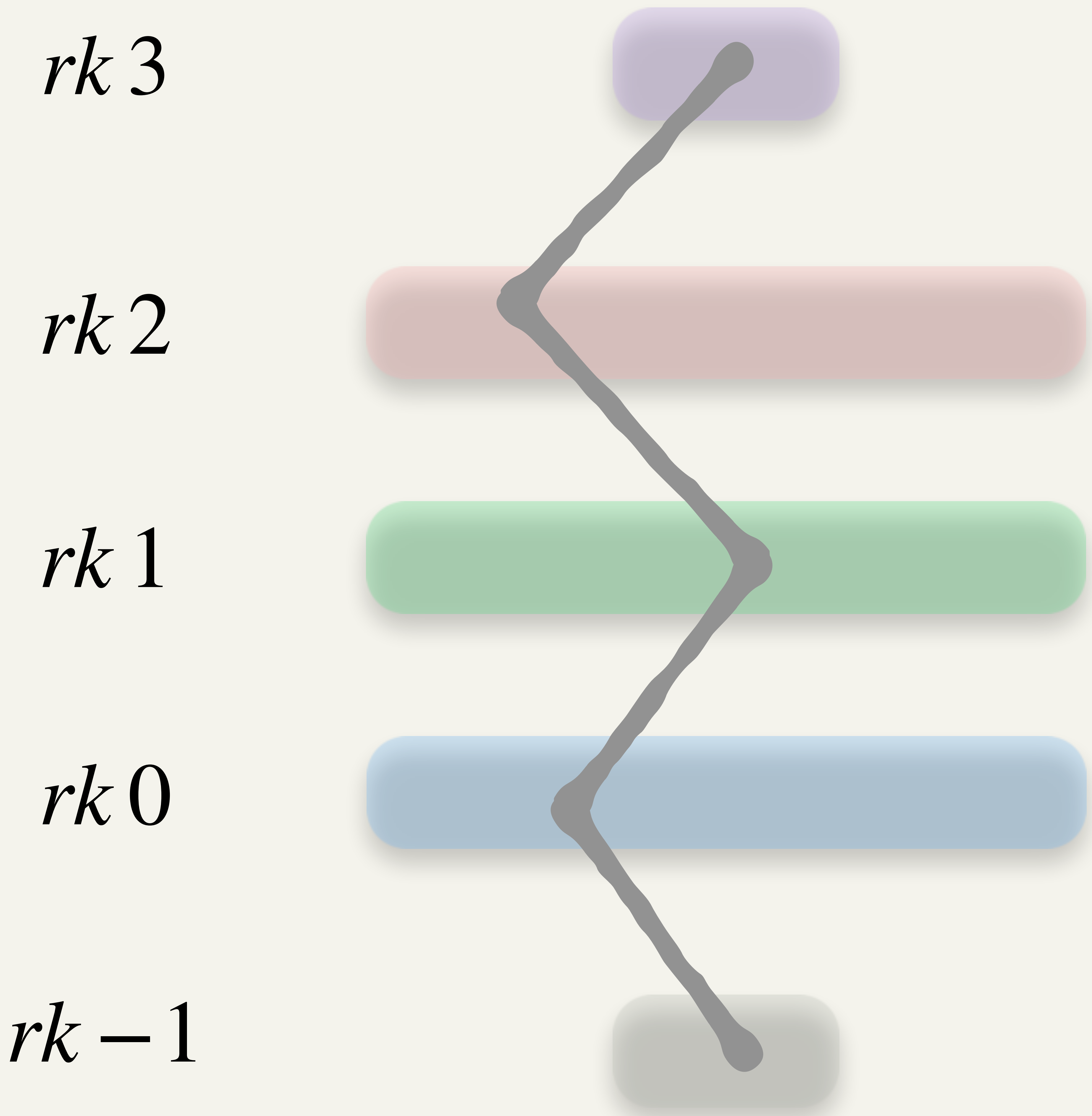

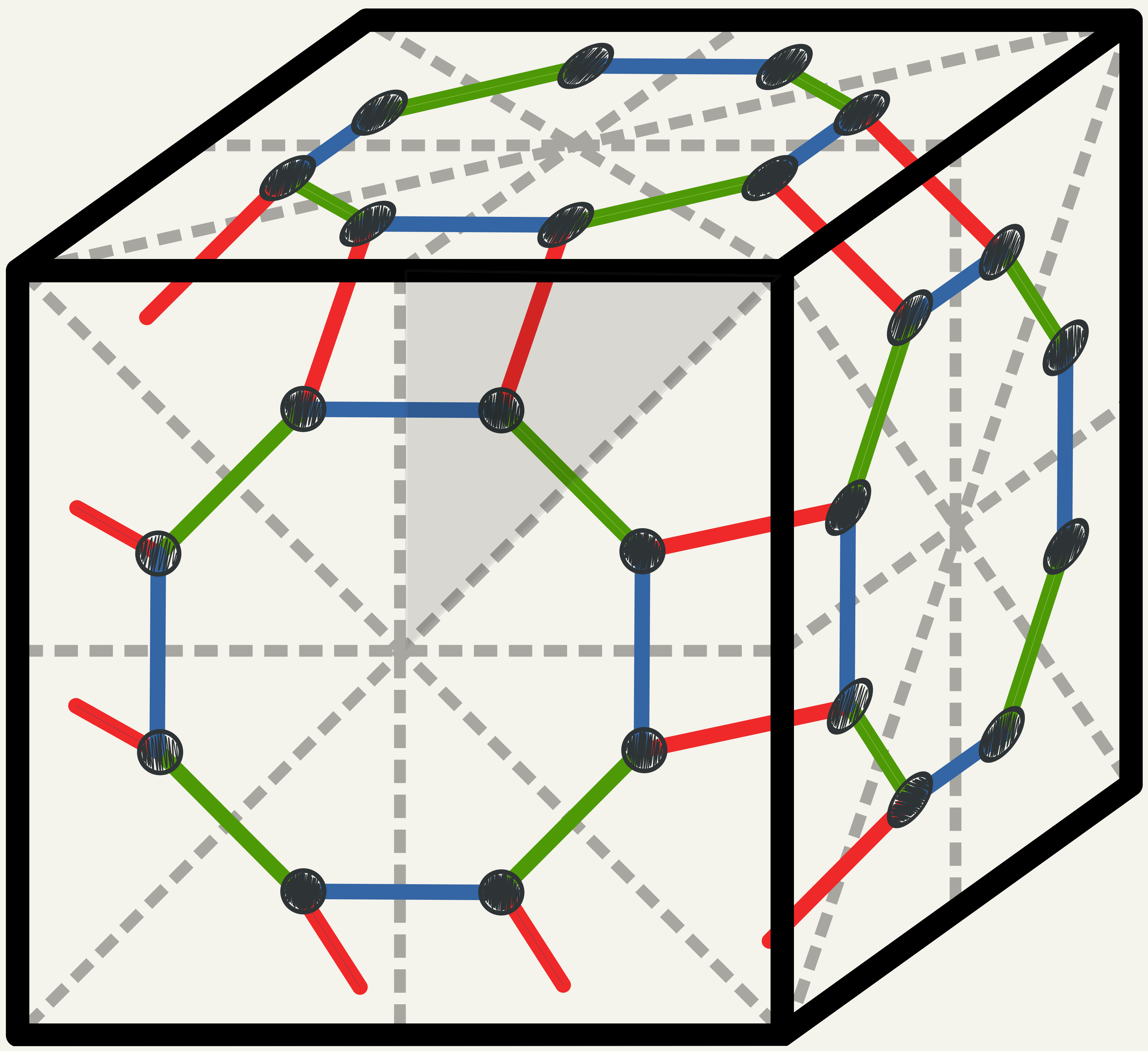

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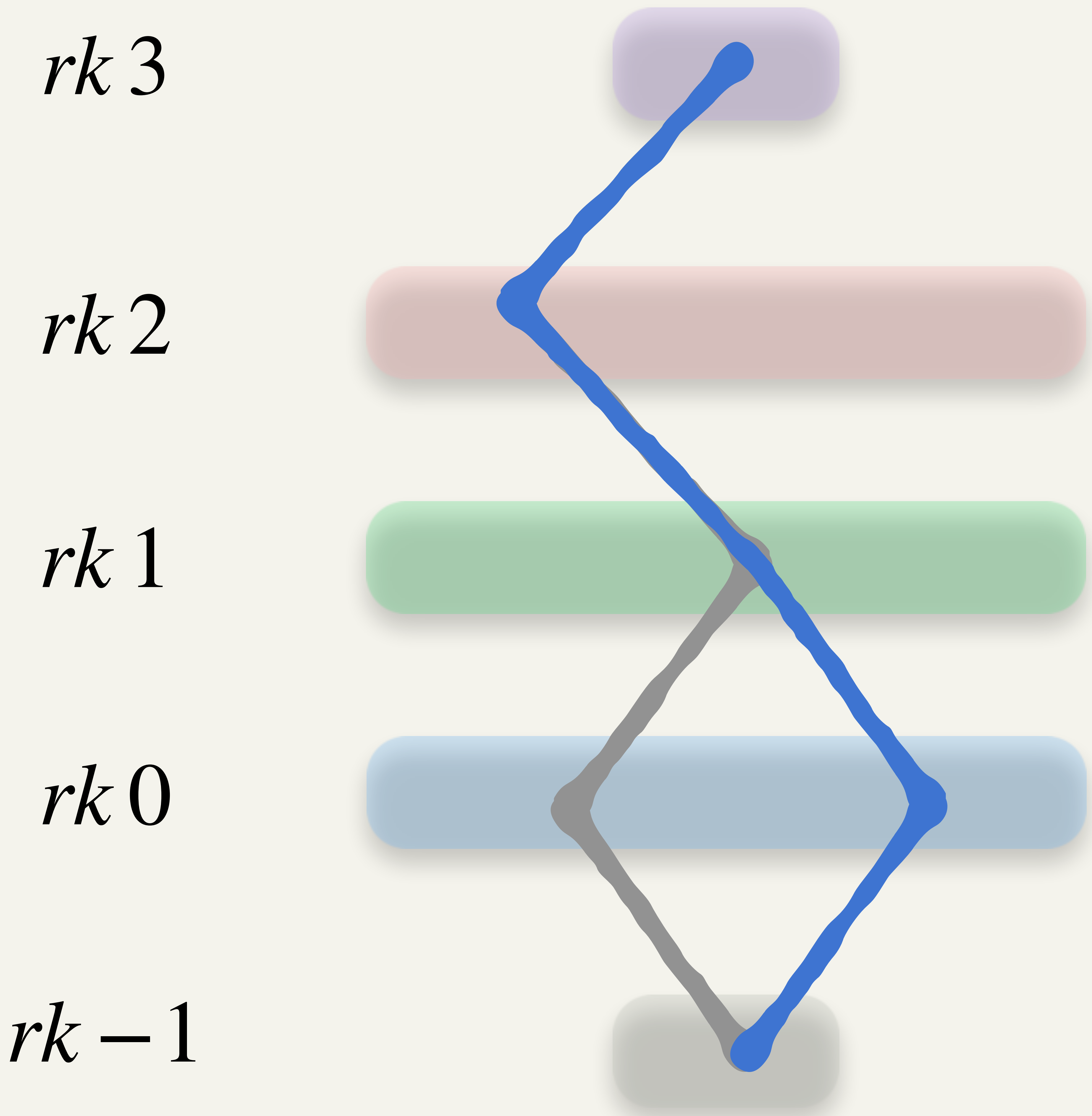

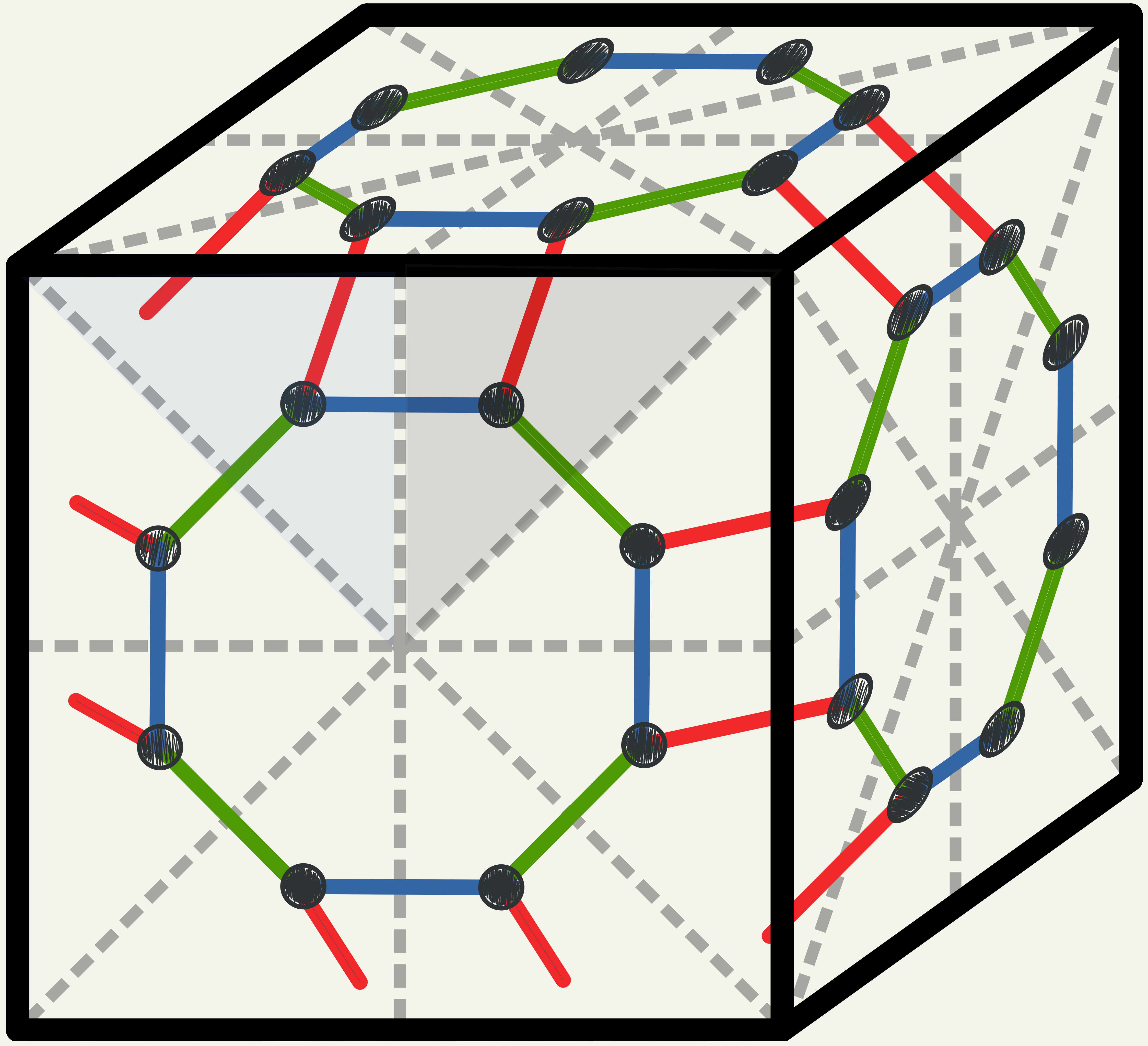

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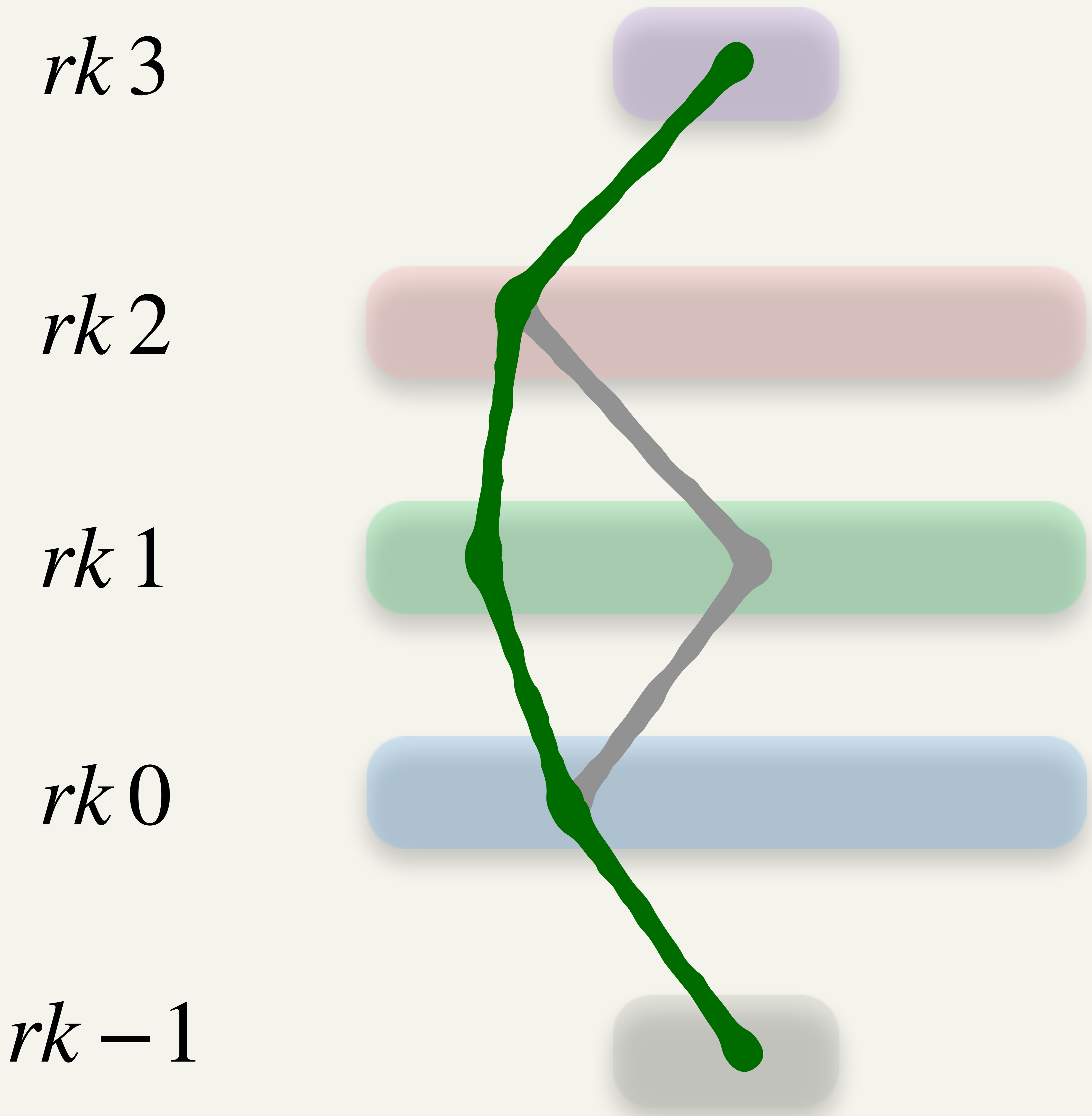

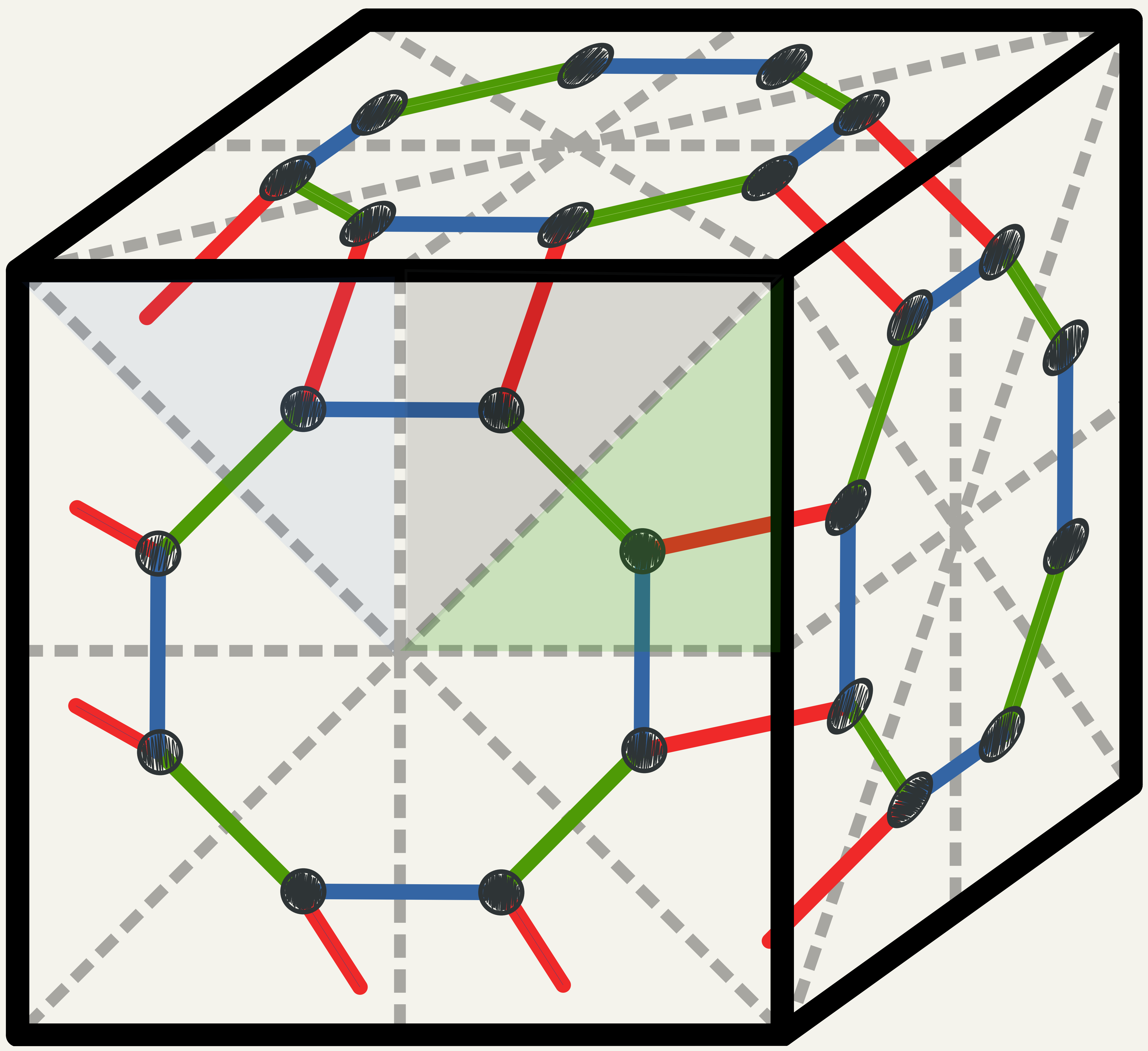

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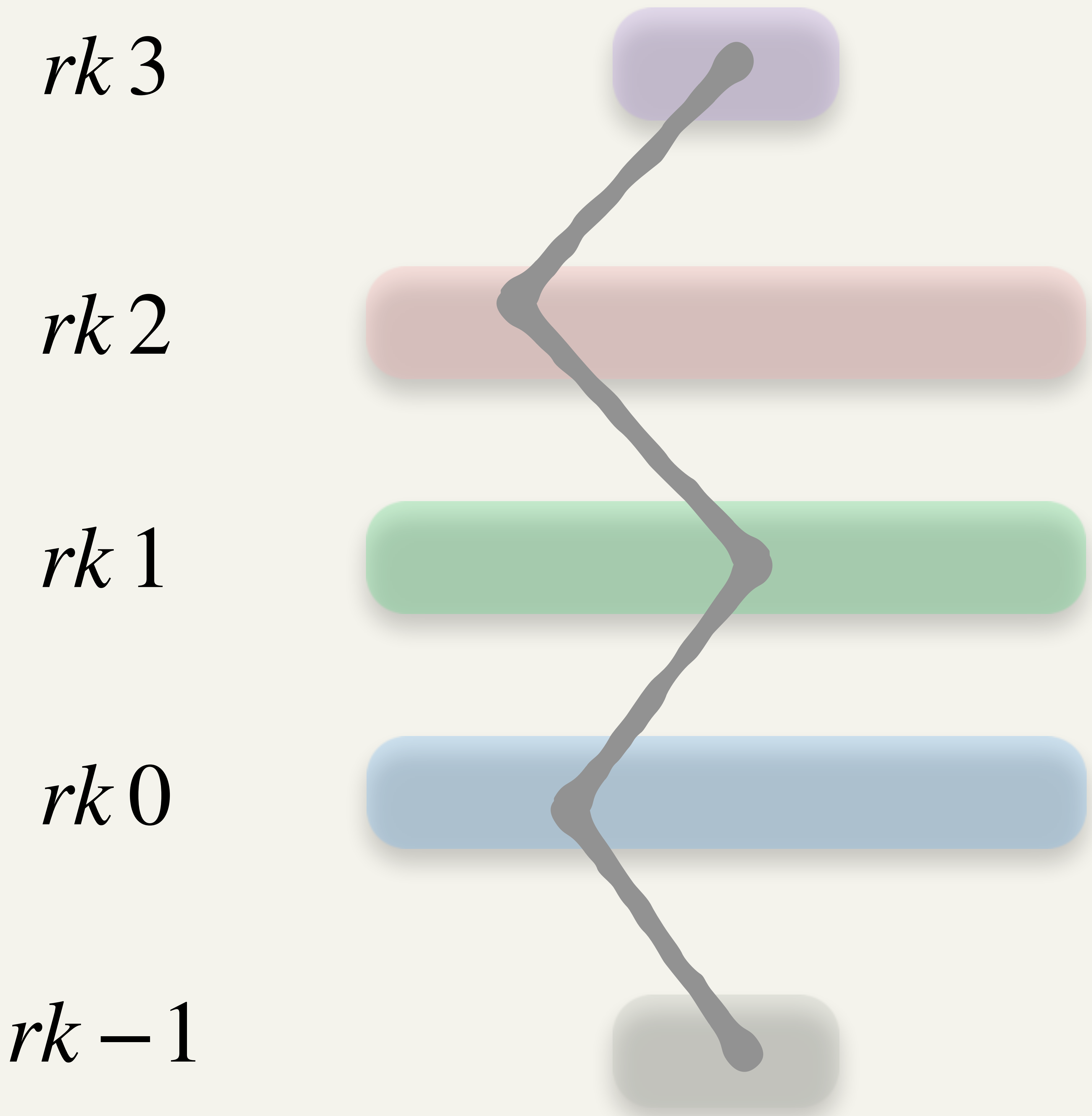

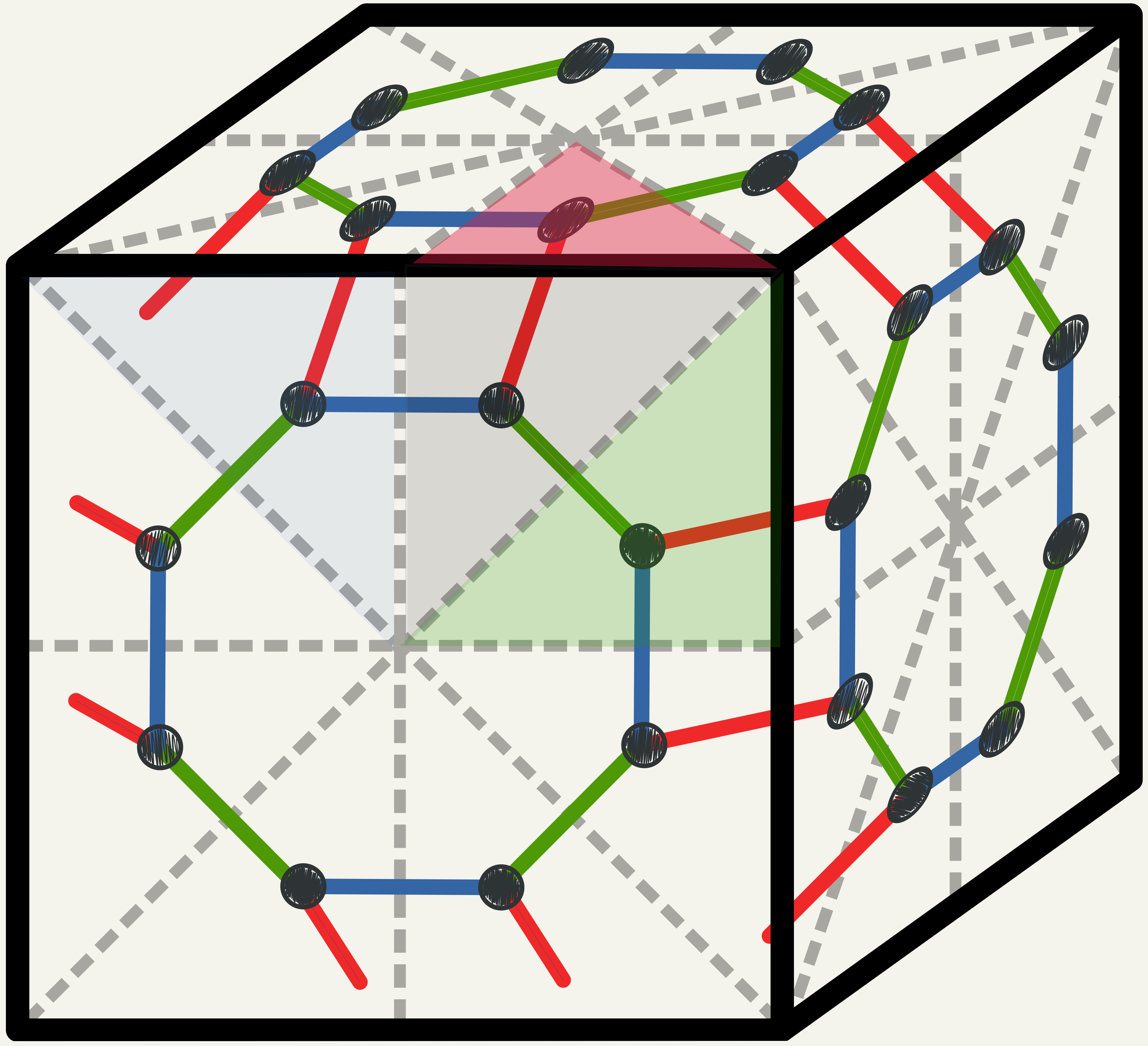

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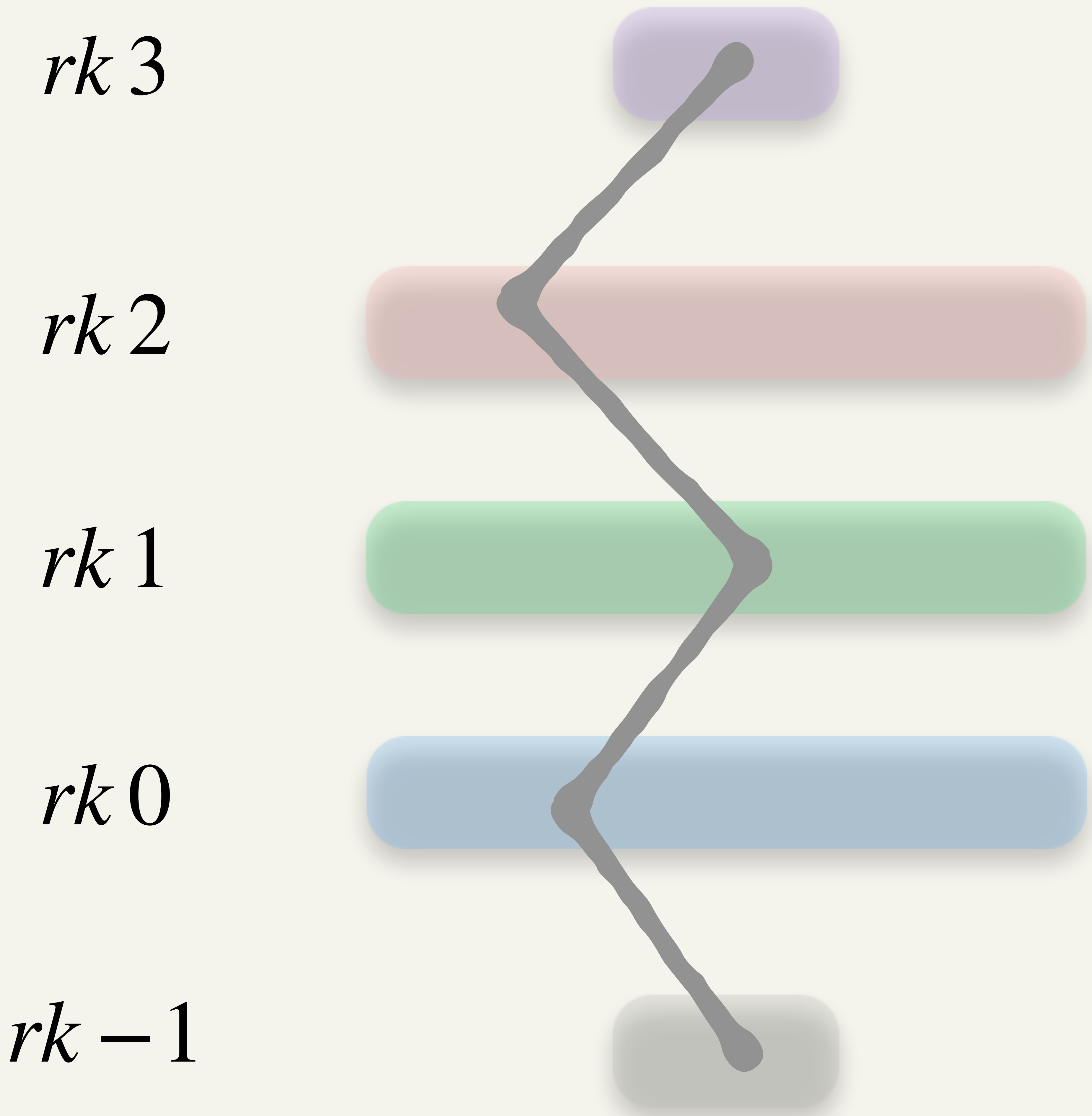

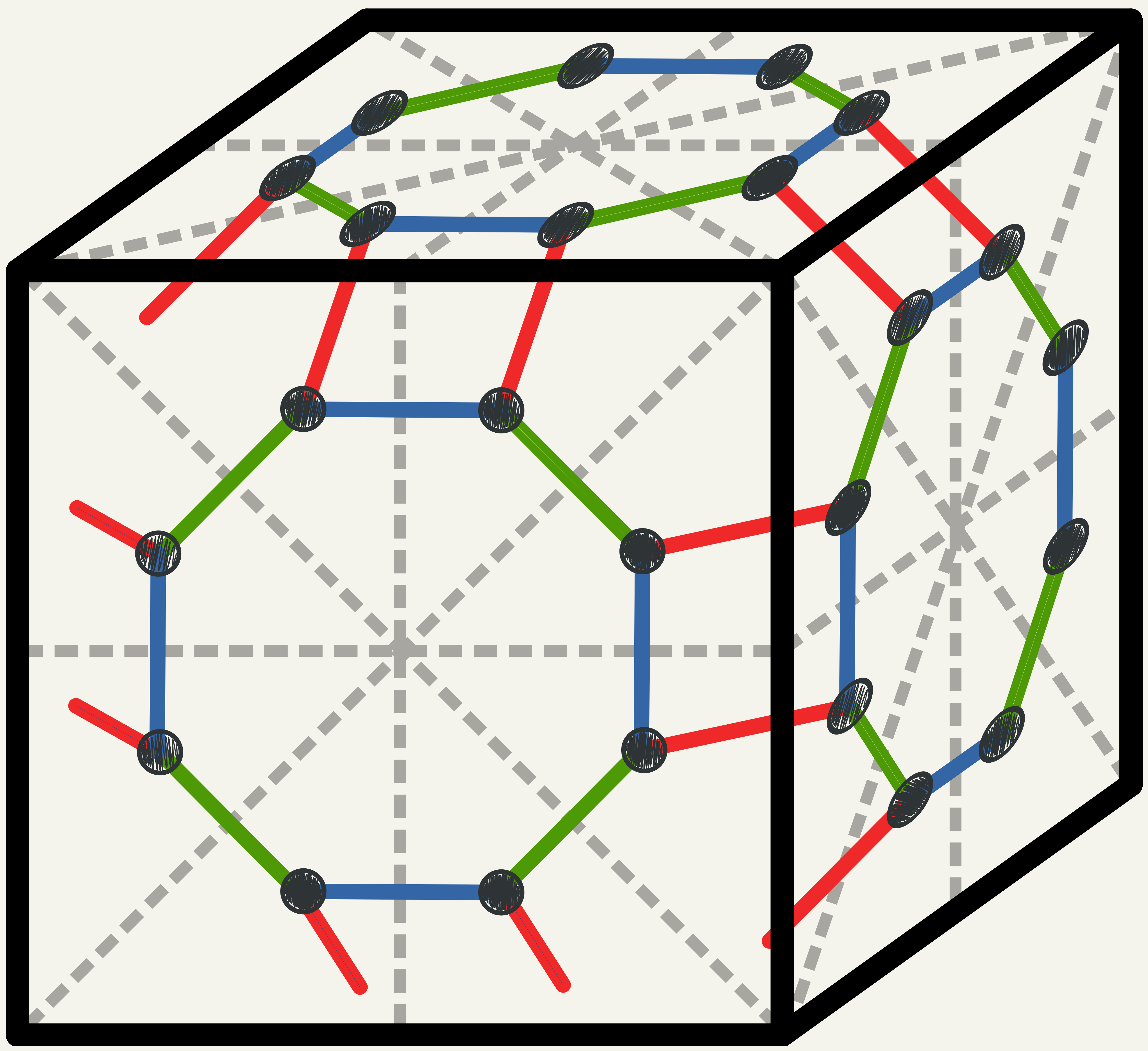

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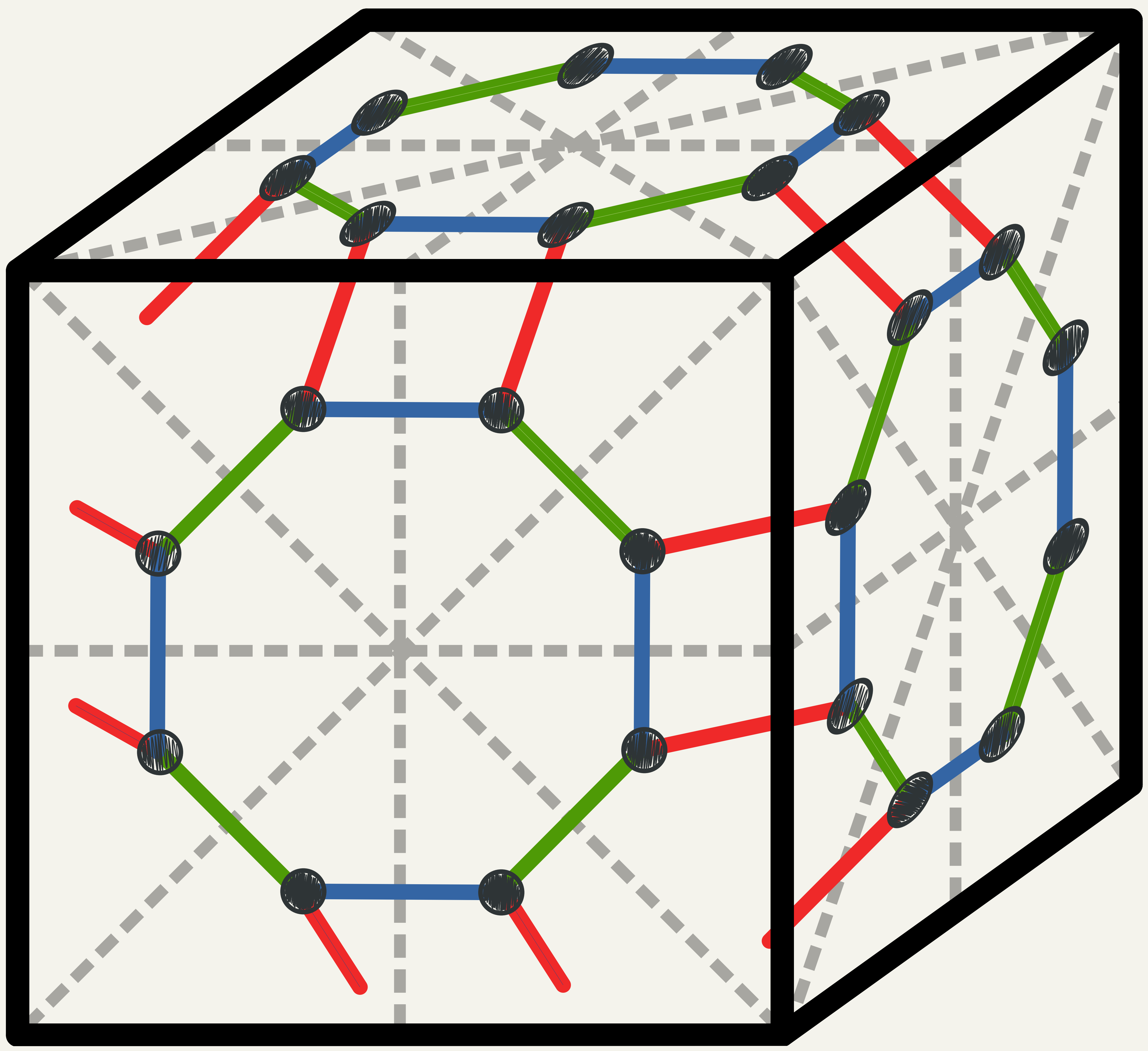

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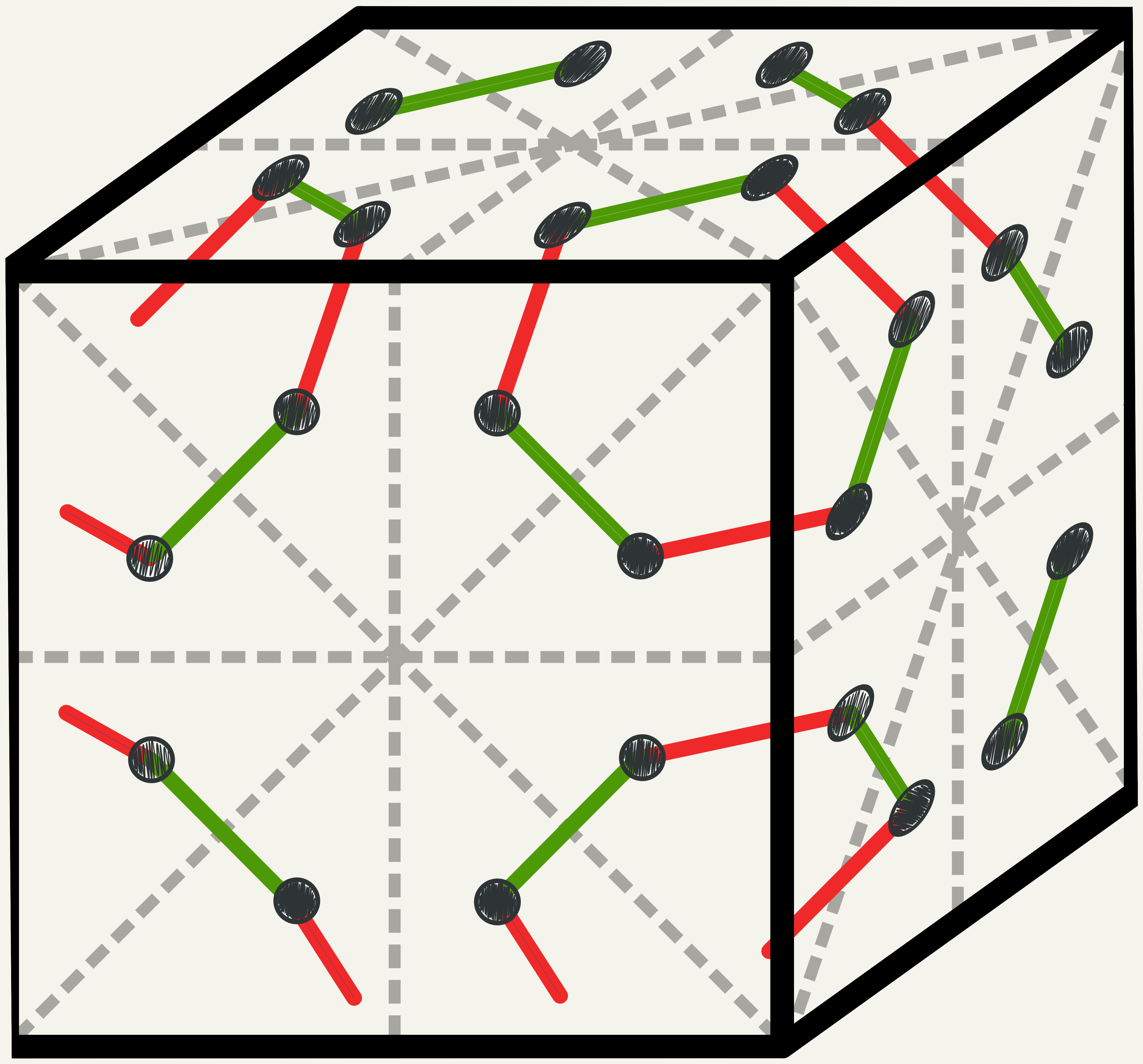

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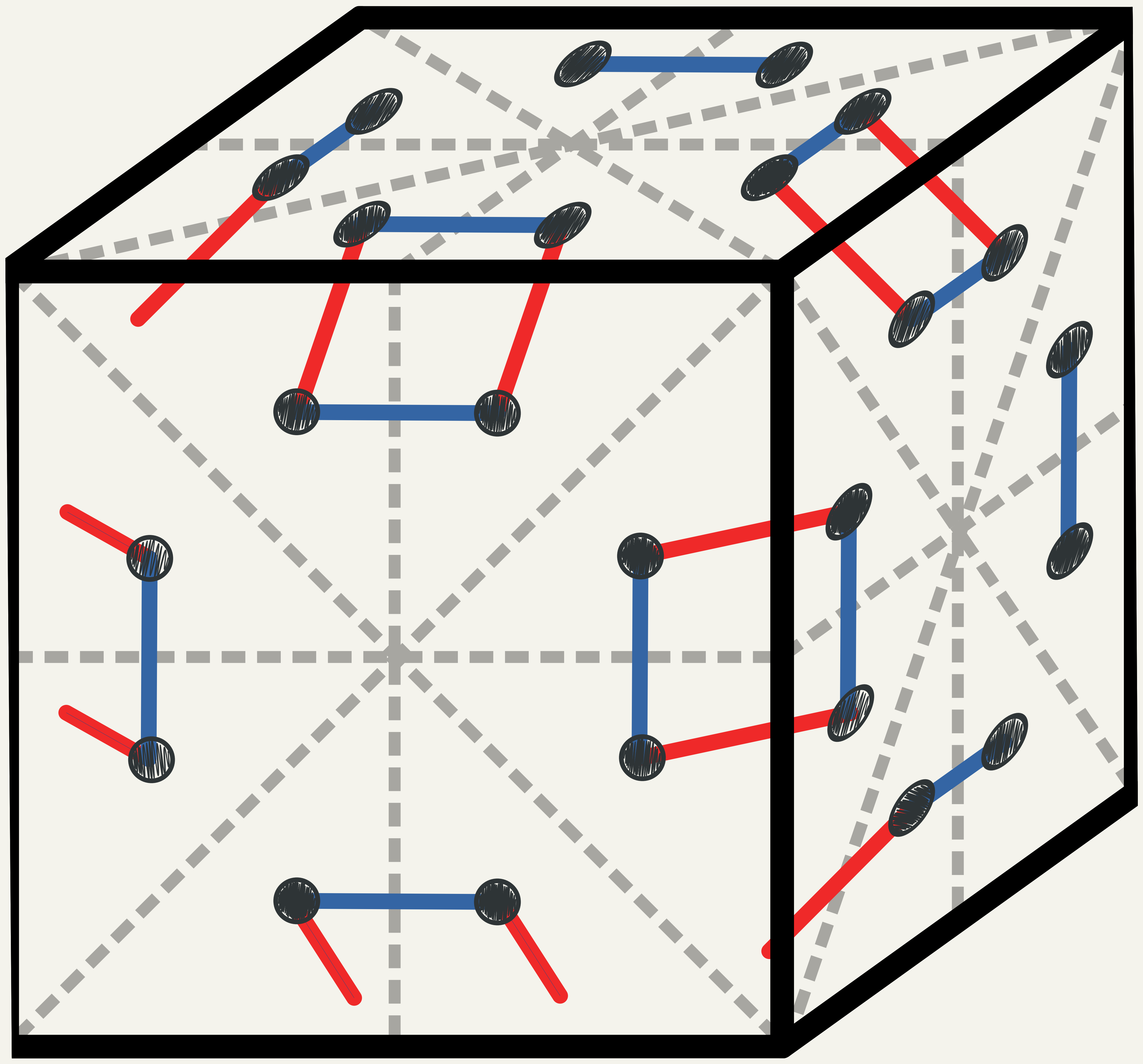

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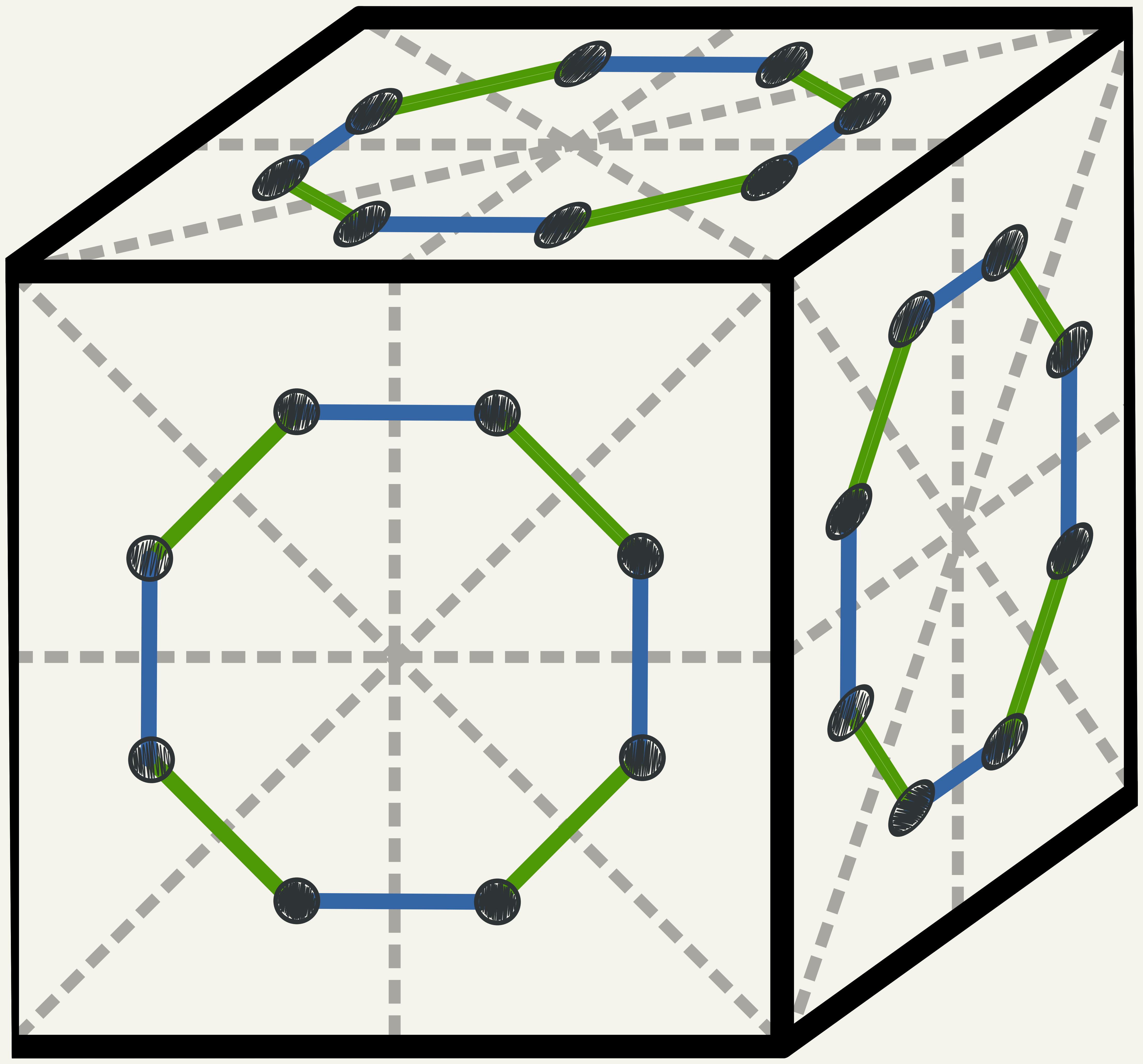

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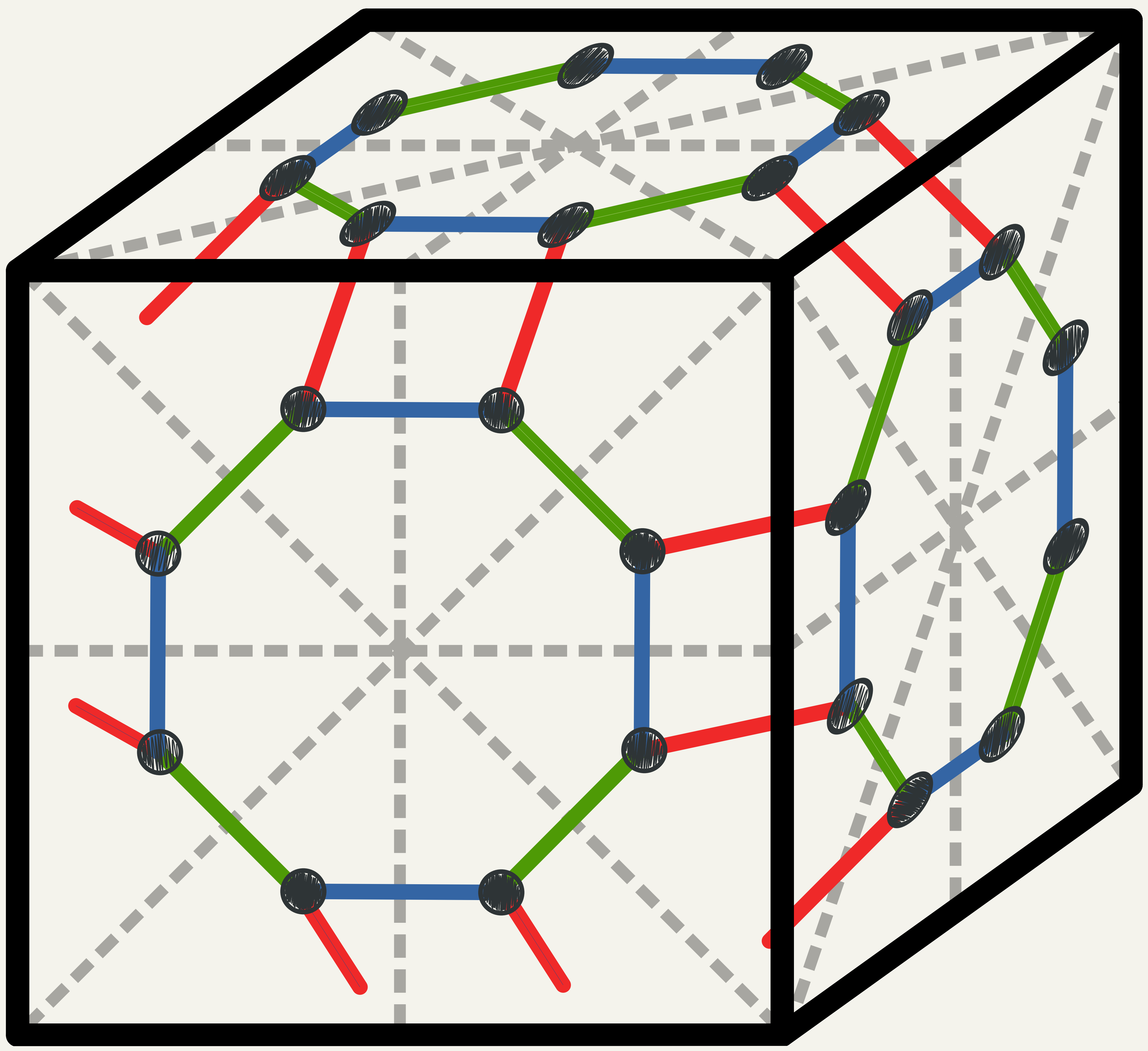

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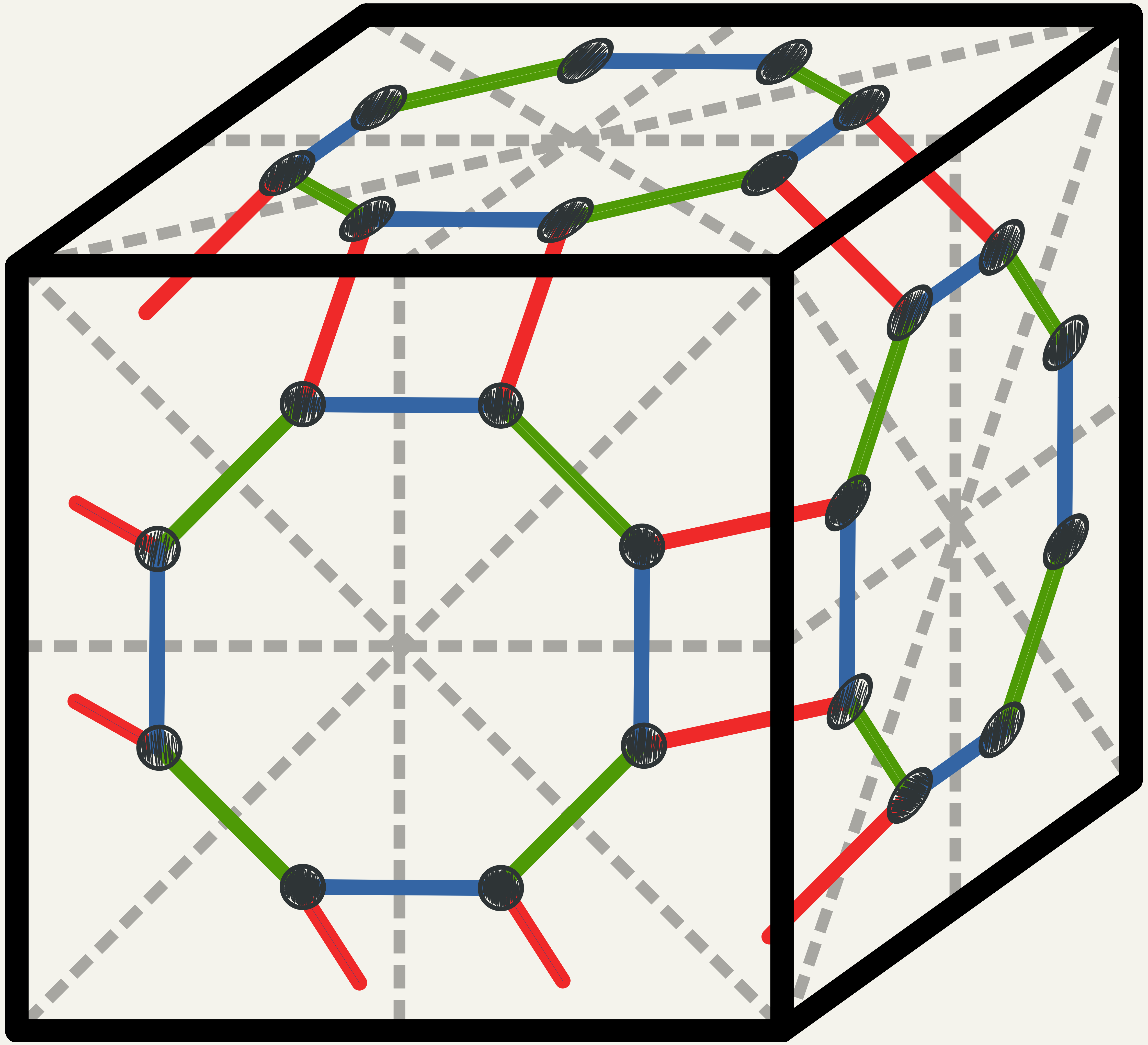

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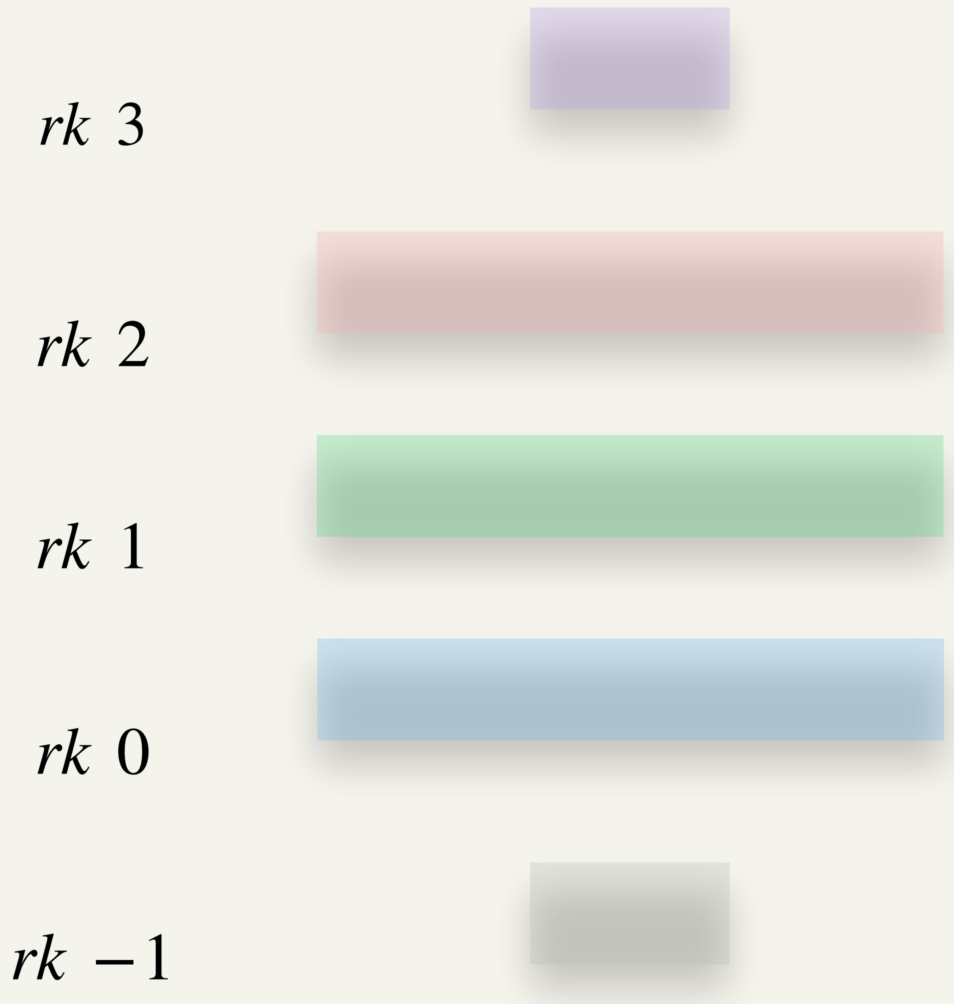
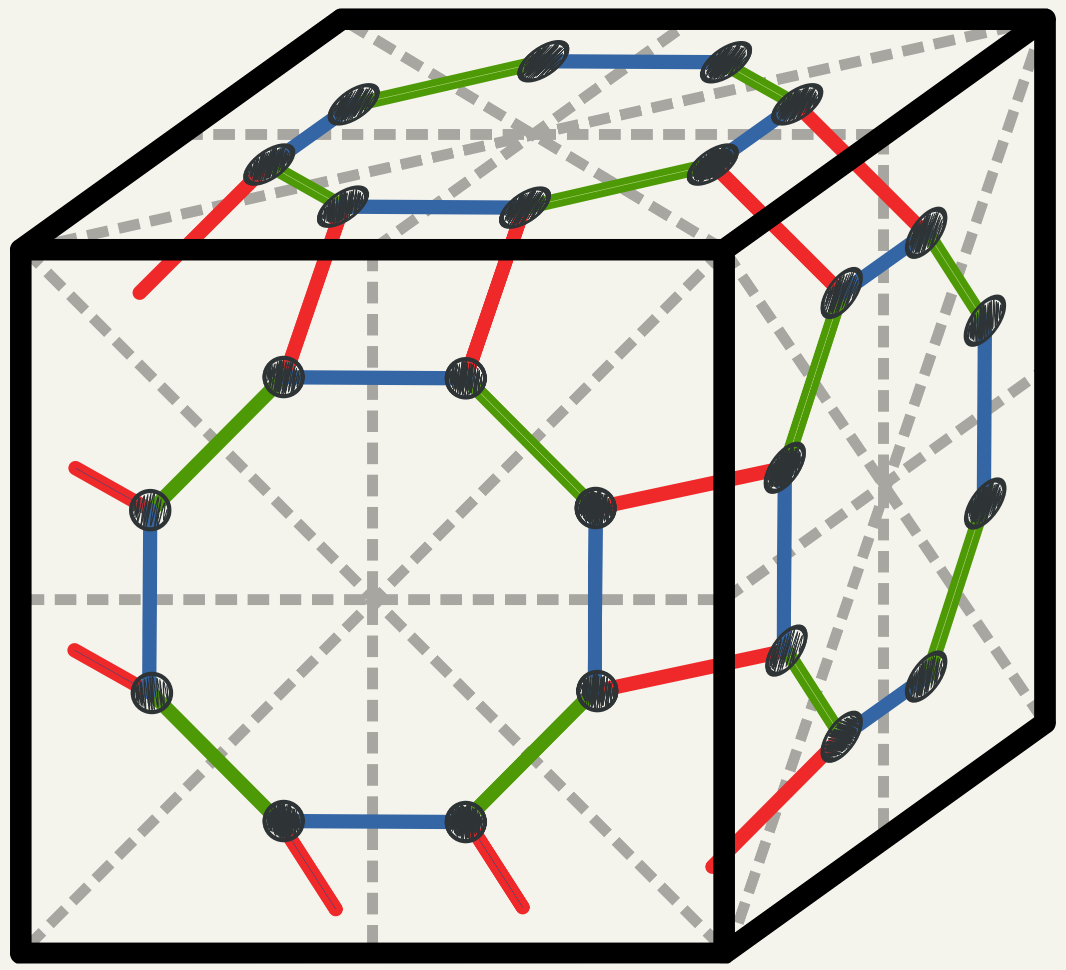
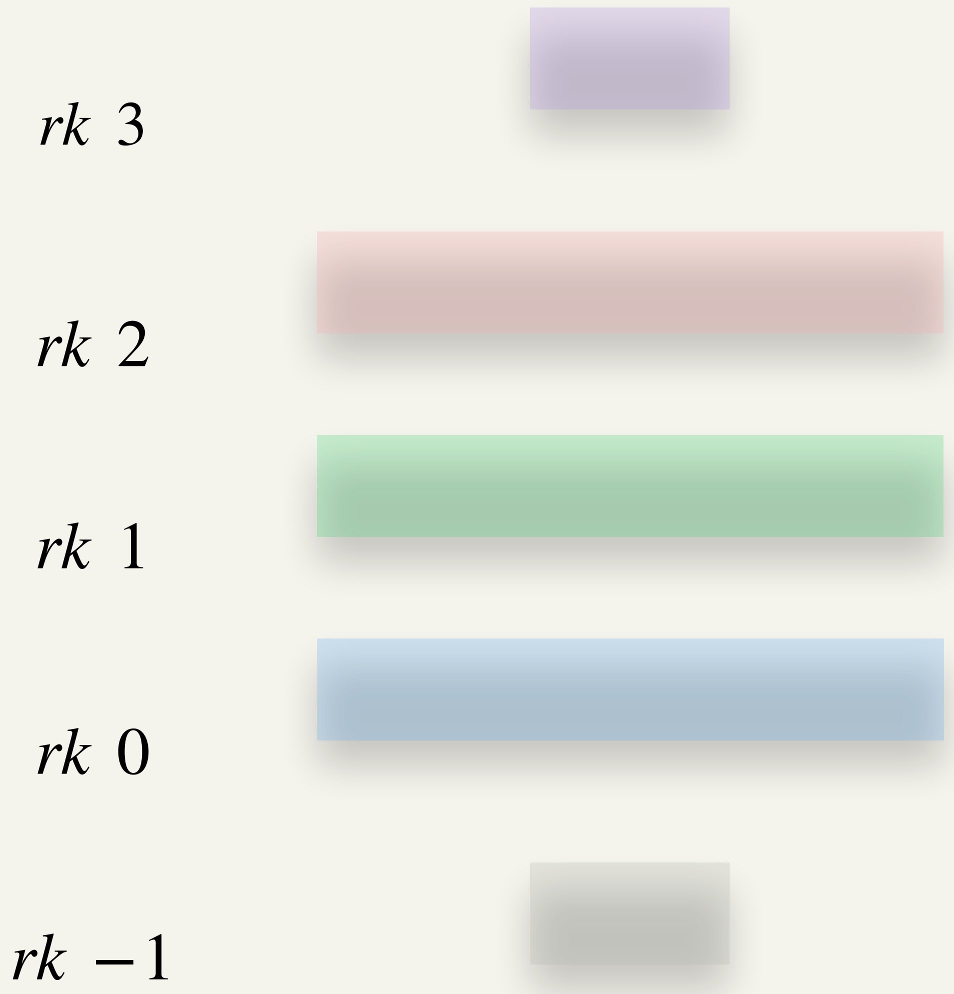
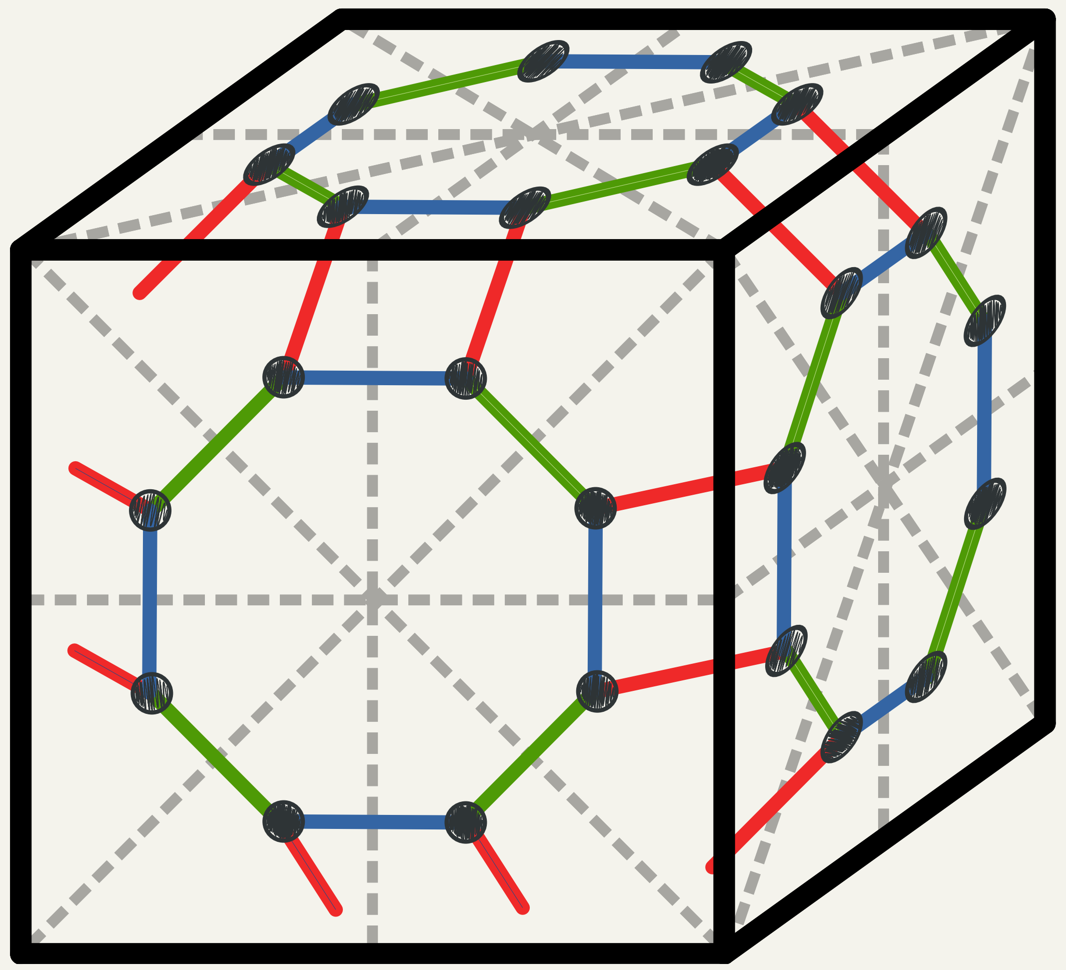




Polytopes vs maniplexes

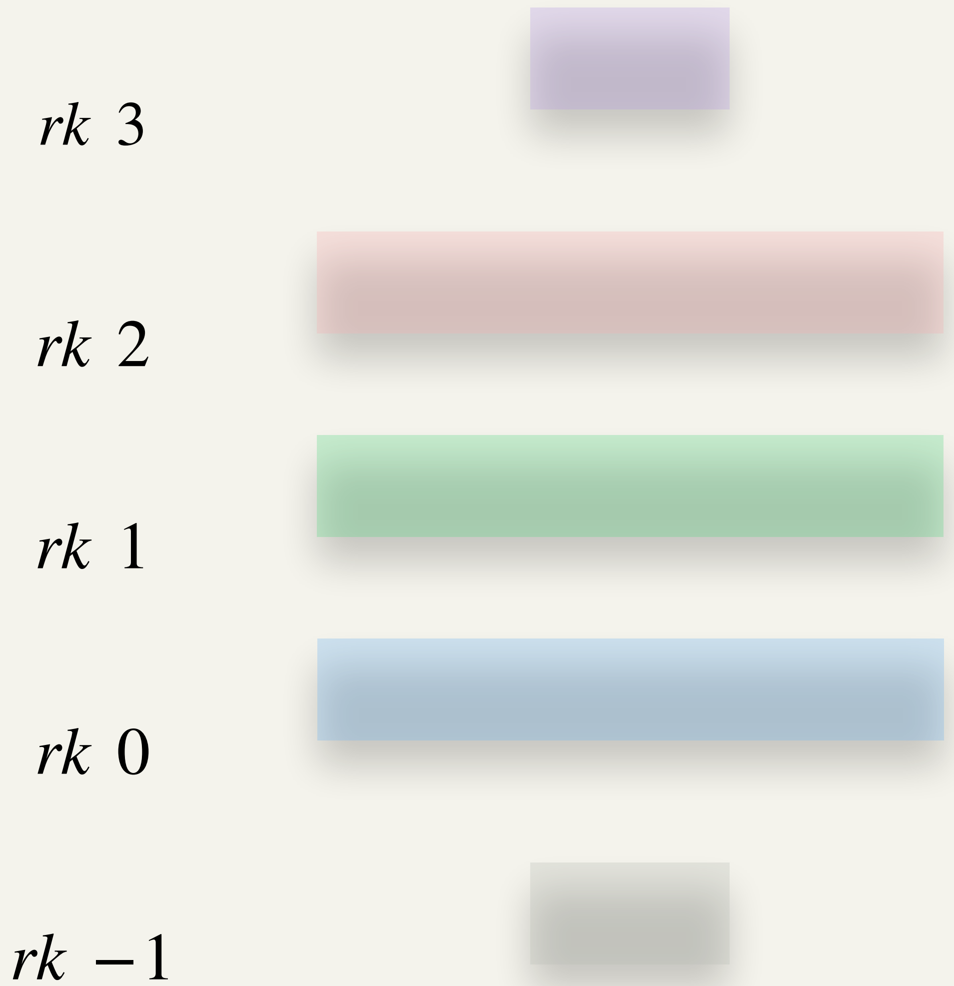
$\mathscr{P}$ $\longrightarrow$ $\mathrm{Fl}(\mathscr{P})$

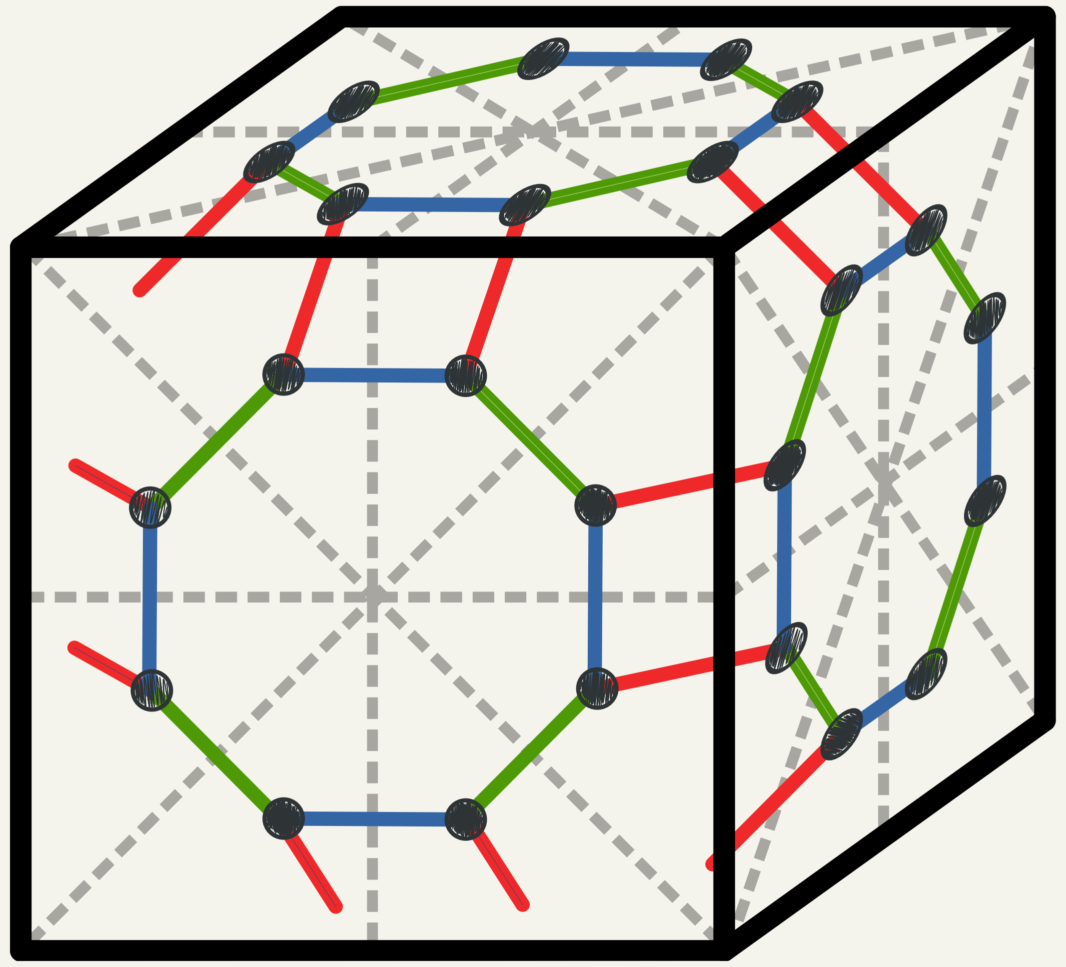




Polytopes vs maniplexes

$$\mathcal{P} \longrightarrow \mathrm{Fl}(\mathcal{P})$$



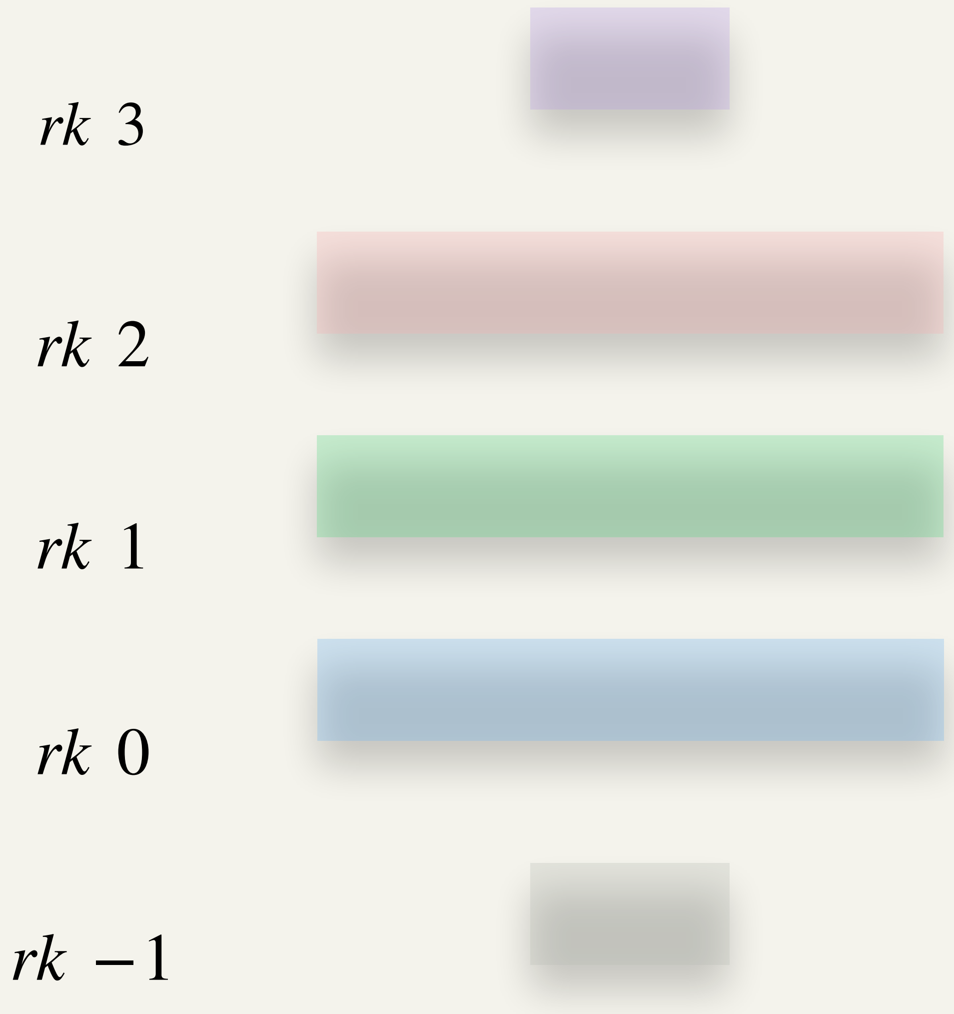


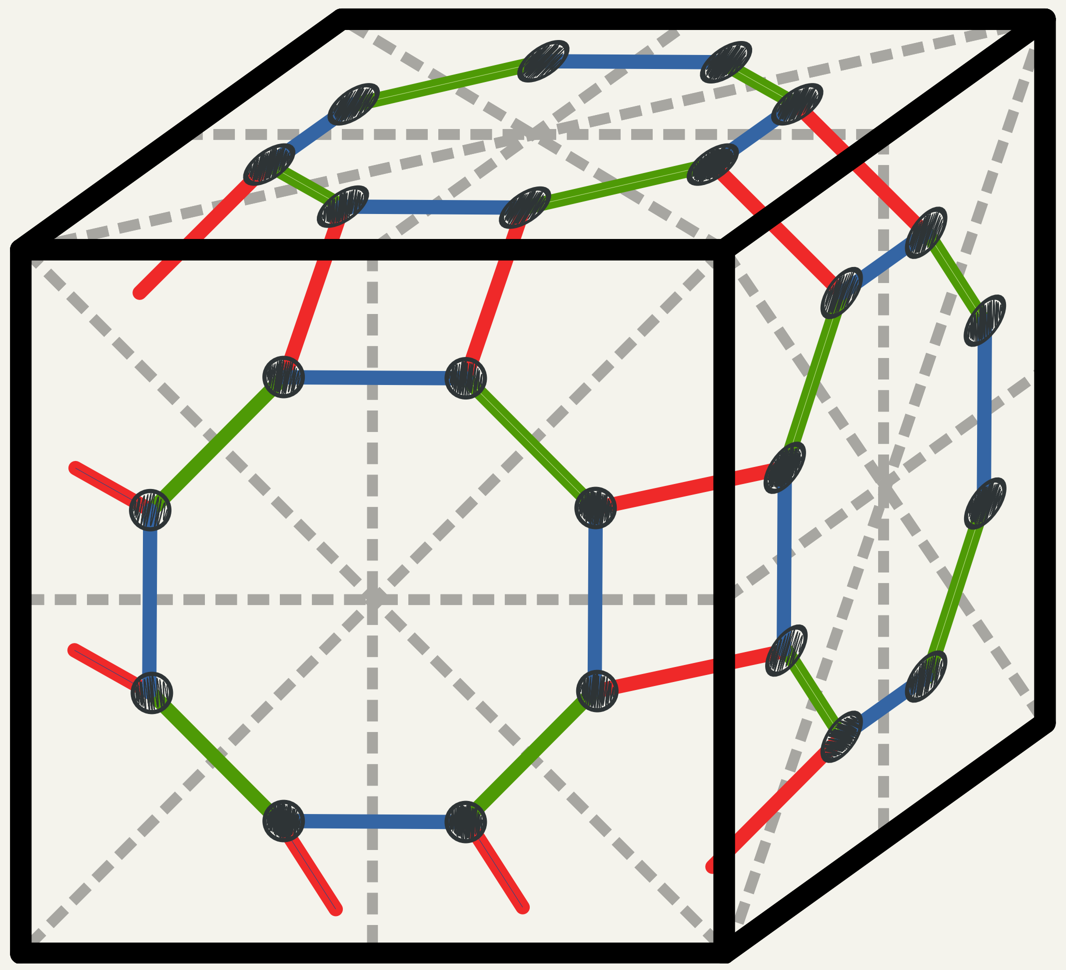
$$\mathrm{Pos}(\mathcal{M}) \longleftarrow \mathcal{M}$$

Polytopes vs maniplexes

Always a Maniplex

$$\mathscr{P} \longrightarrow \mathrm{Fl}(\mathscr{P})$$



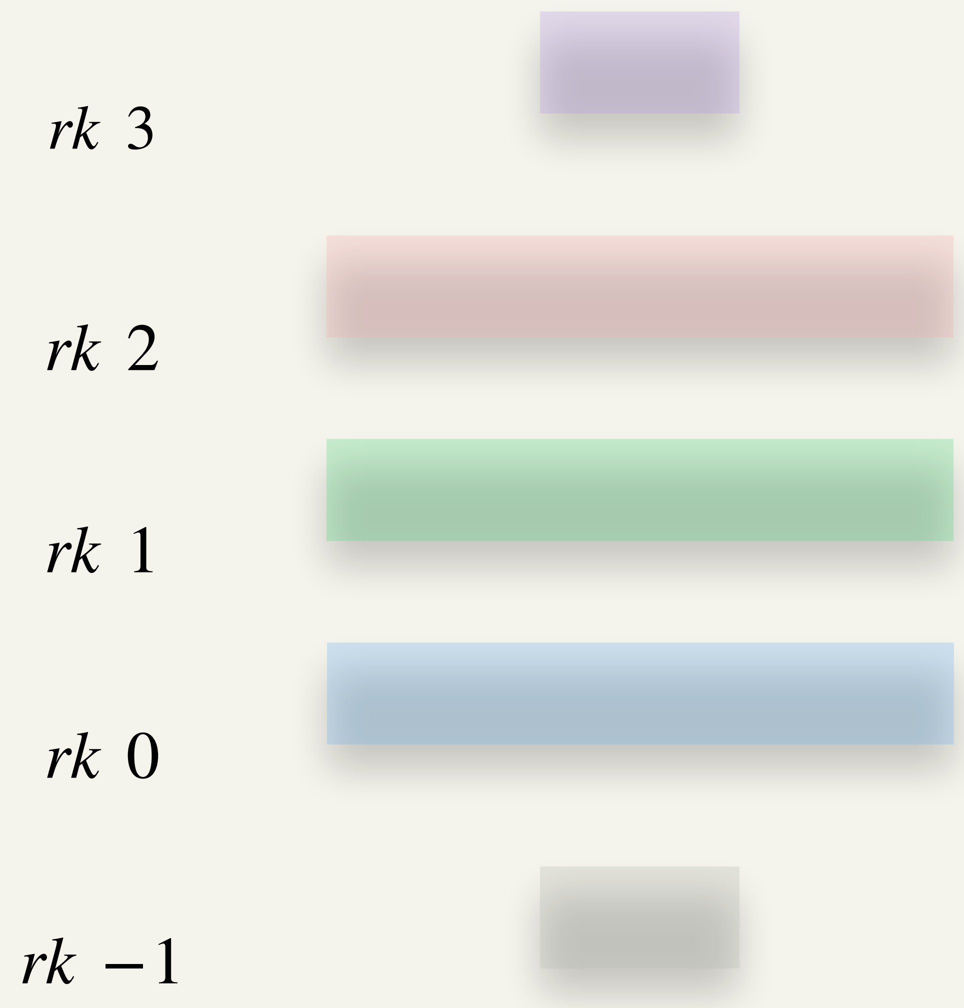


$$\mathrm{Pos}(\mathscr{M}) \longleftarrow \mathscr{M}$$

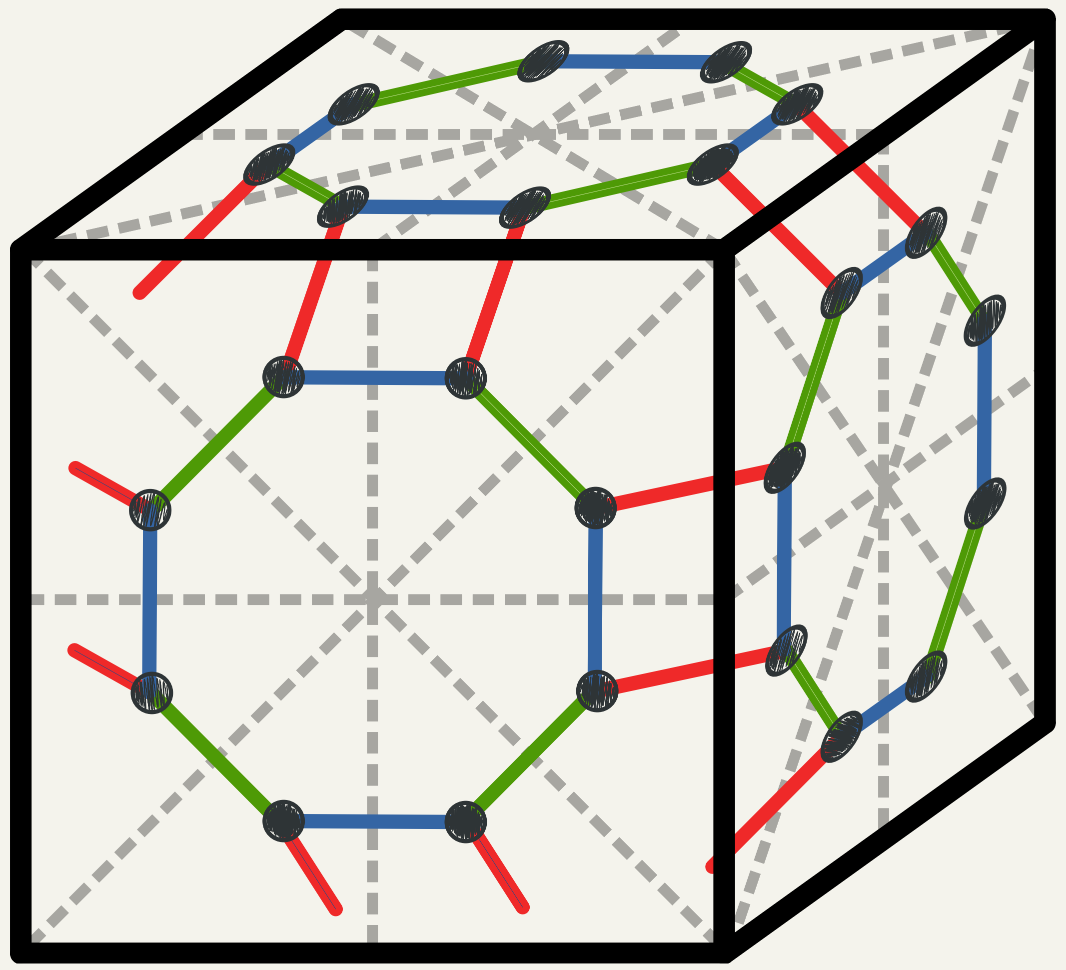
Polytopes vs maniplexes

Always a Maniplex

$\mathcal{P}$



$\mathrm{Fl}(\mathcal{P})$



$\mathrm{Pos}(\mathcal{M}) \longleftarrow \mathcal{M}$

Polytopes vs maniplexes

$\mathcal{M}$ $\qquad$ $\mathrm{Pos}(\mathcal{M})$

Polytopes vs maniplexes

$$\mathscr{M} \longrightarrow \mathrm{Pos}(\mathscr{M})$$

Polytopes vs maniplexes

$$\mathscr{M} \longrightarrow \mathrm{Pos}(\mathscr{M})$$

- Being a polytope is very nice, but very restrictive.

Polytopes vs maniplexes

$$\mathcal{M} \longrightarrow \mathrm{Pos}(\mathcal{M})$$

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- Classical operations on polytopes often do not yield polytopes.

Polytopes vs maniplexes

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  - $\delta(\mathscr{P})$ (dual) ✅

Polytopes vs maniplexes

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  - $\pi(\mathscr{P})$ (Petrie-dual)… not necessarily ⁉️

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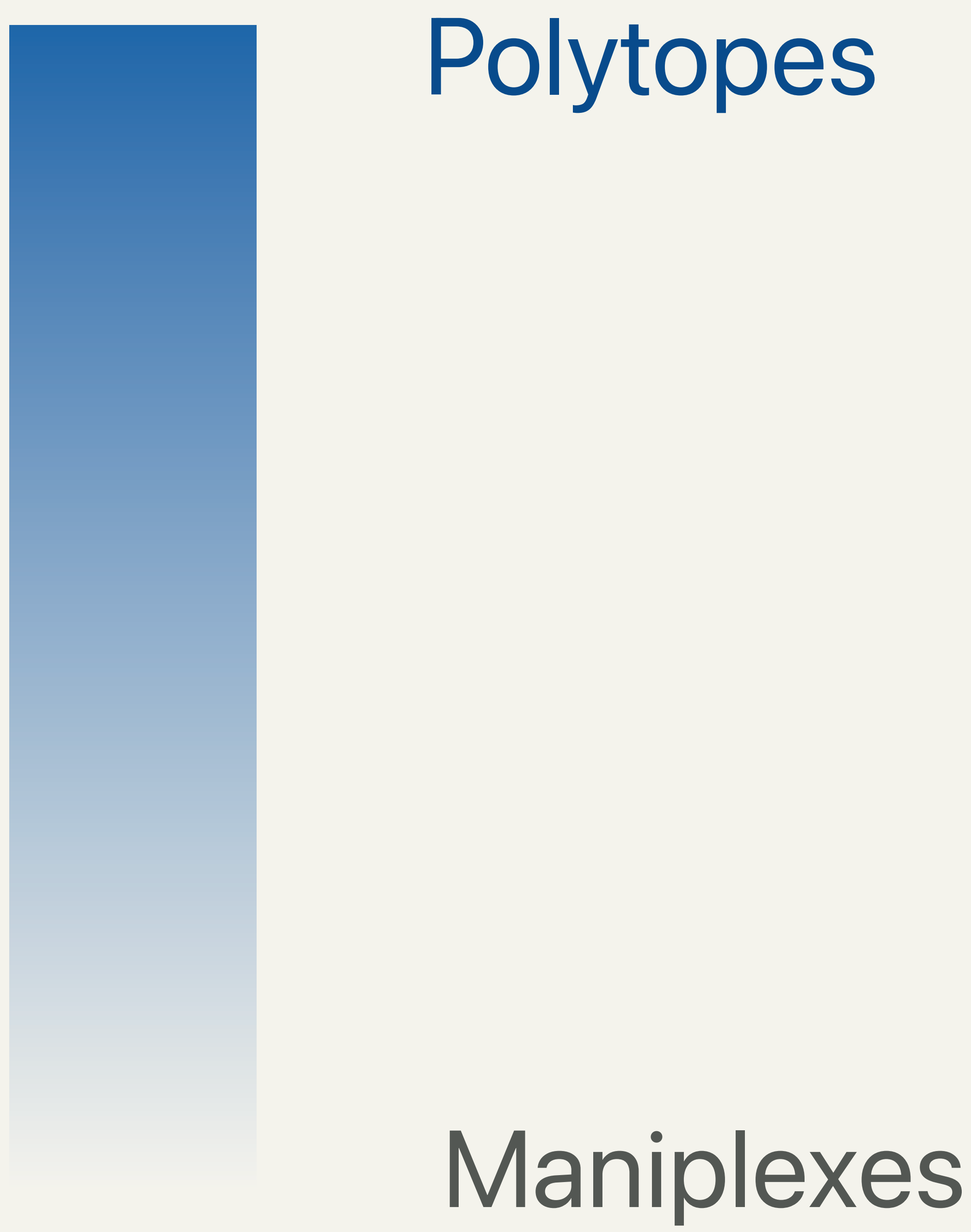

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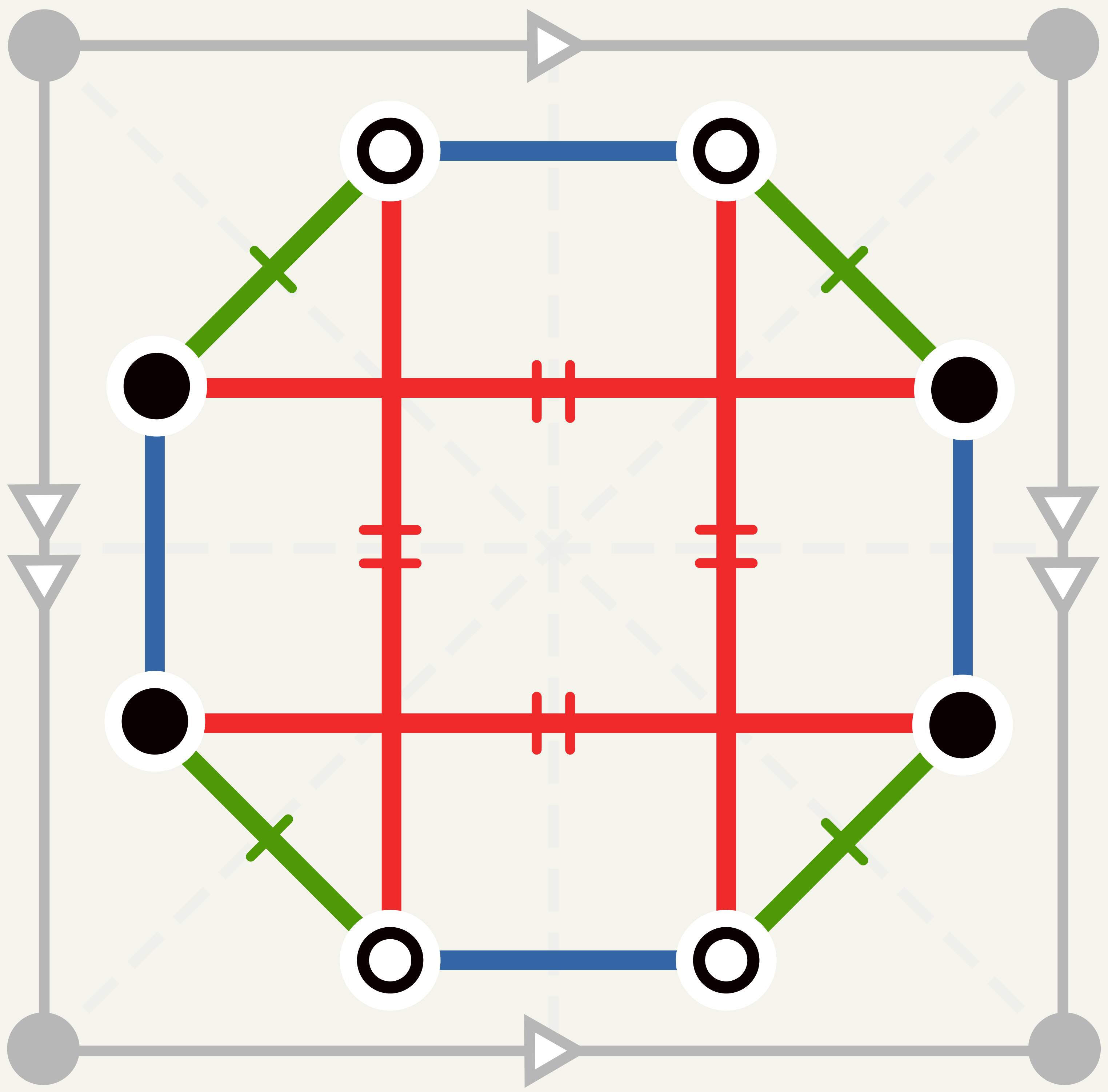

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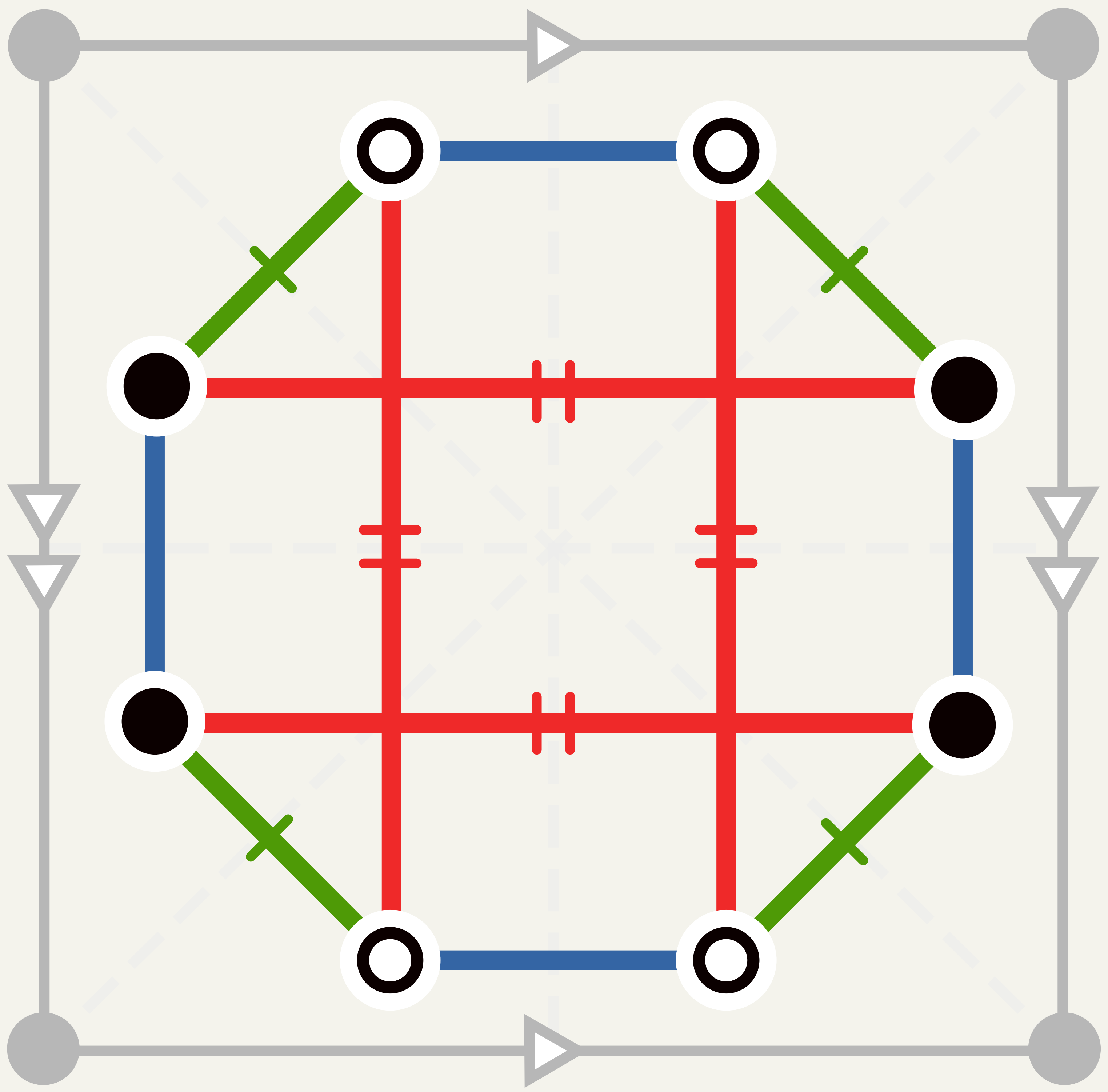

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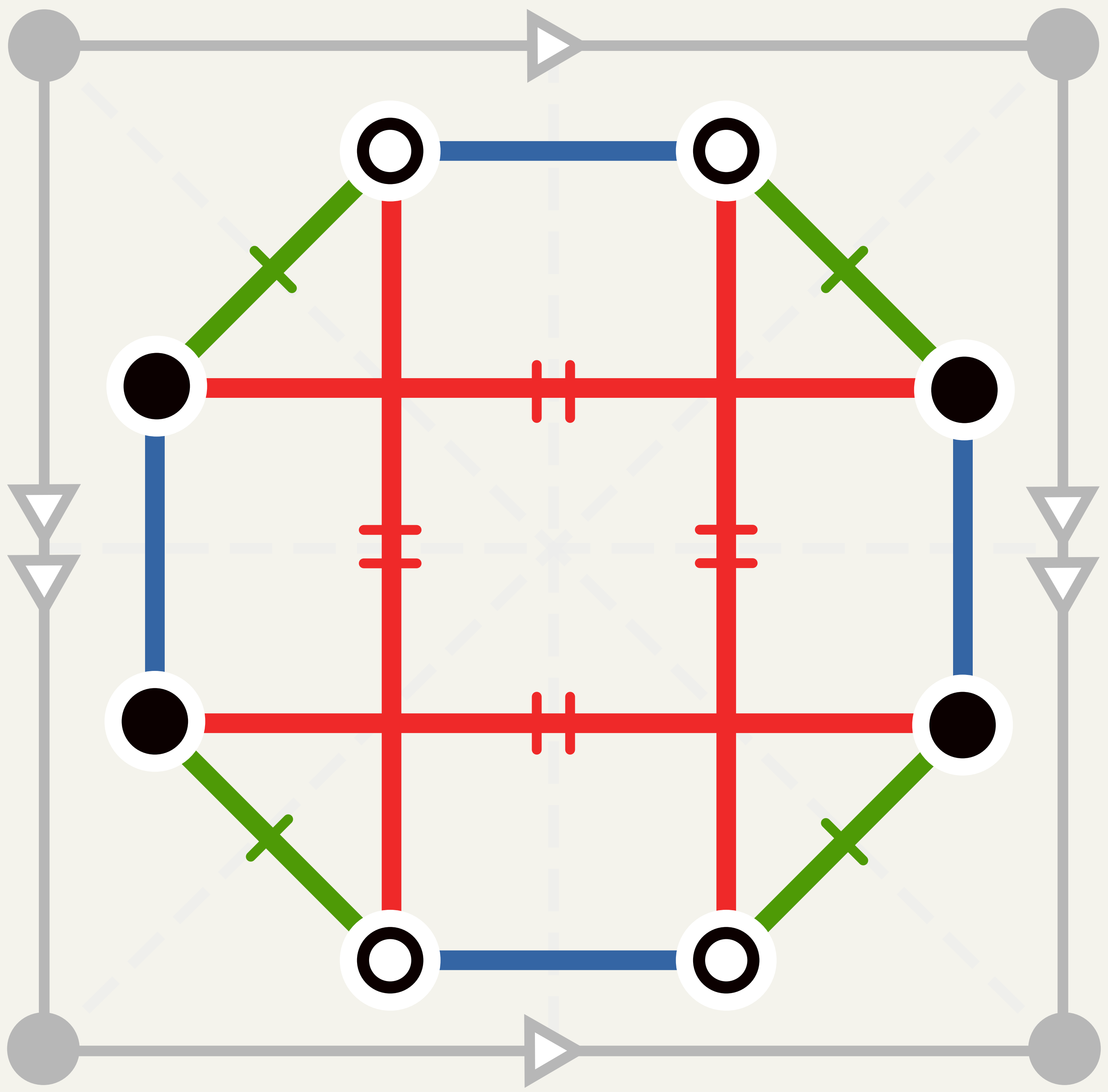

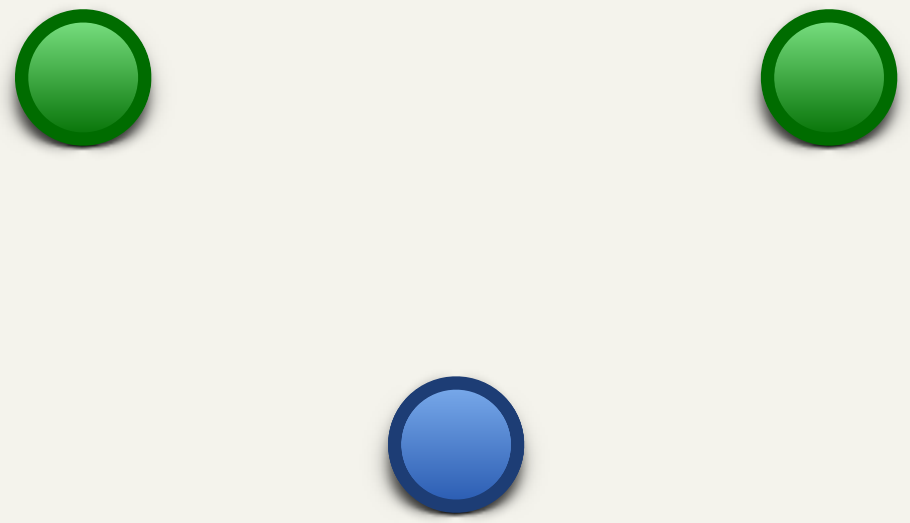

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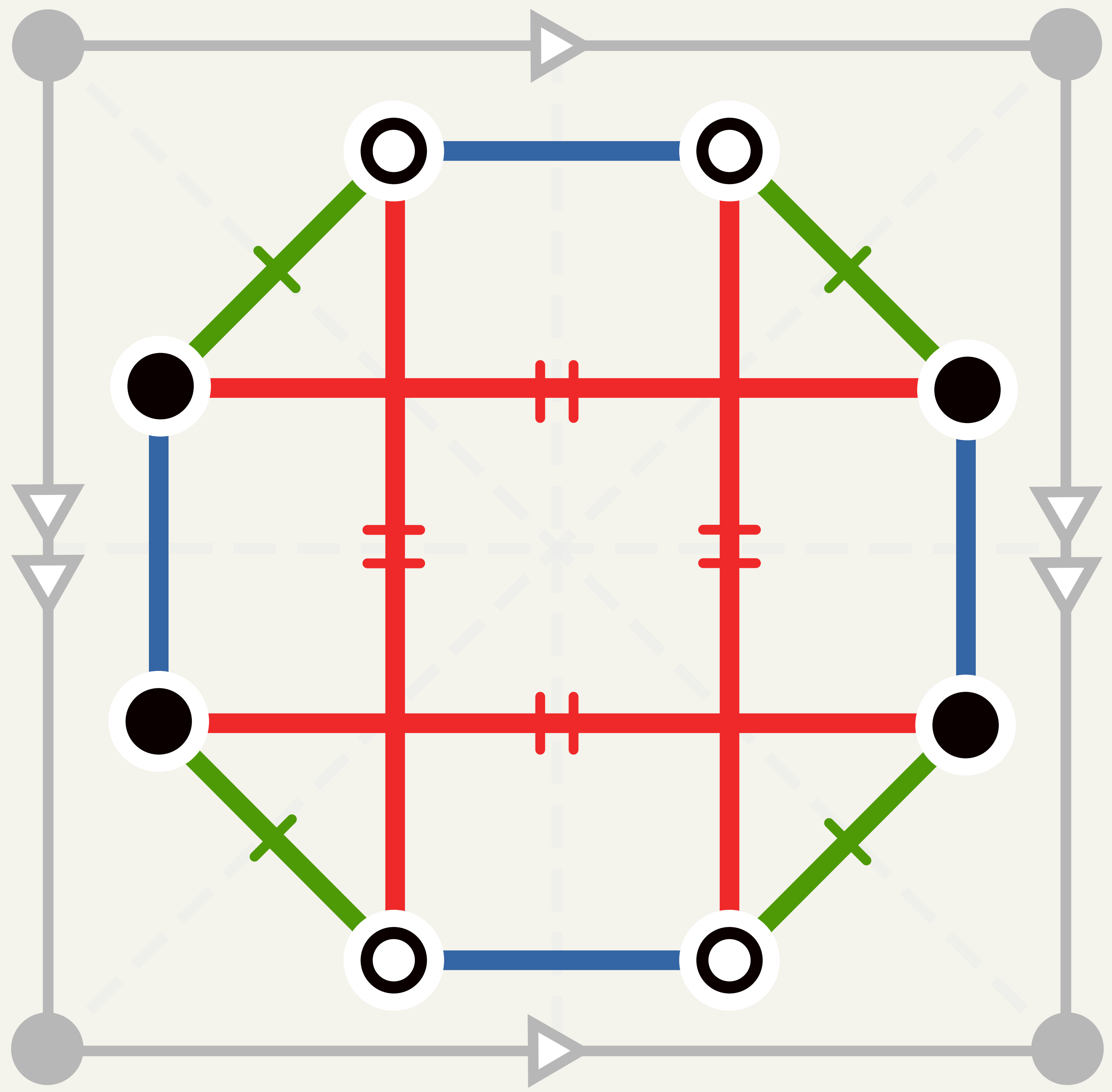
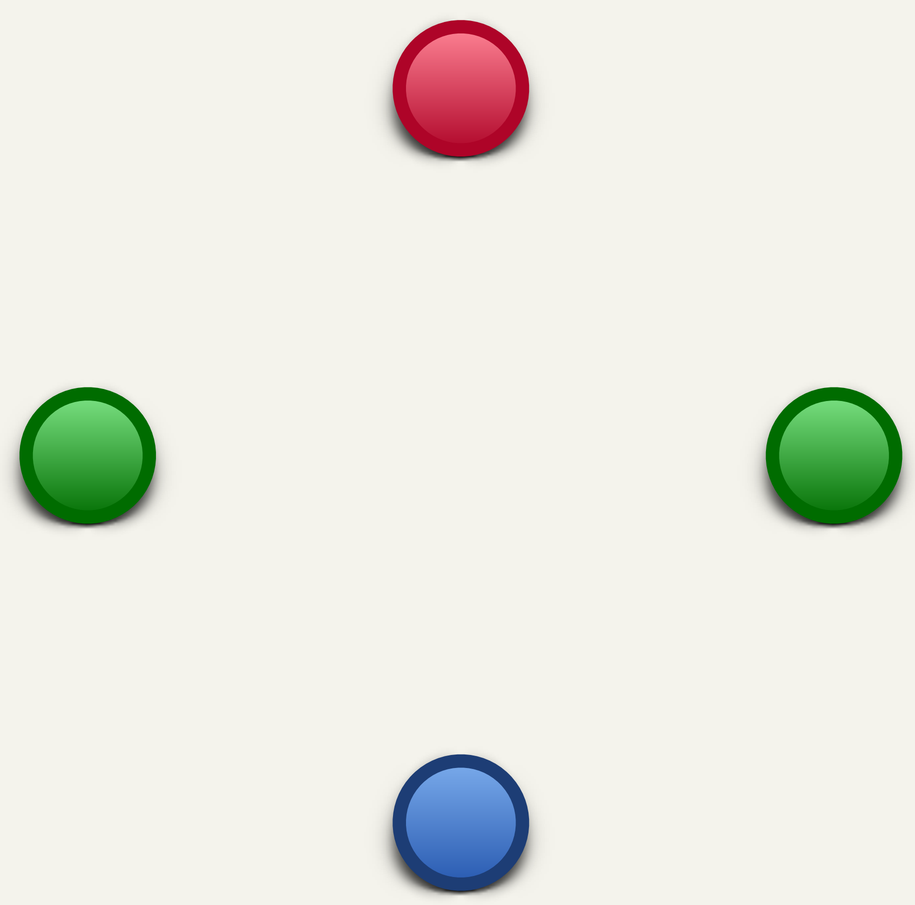
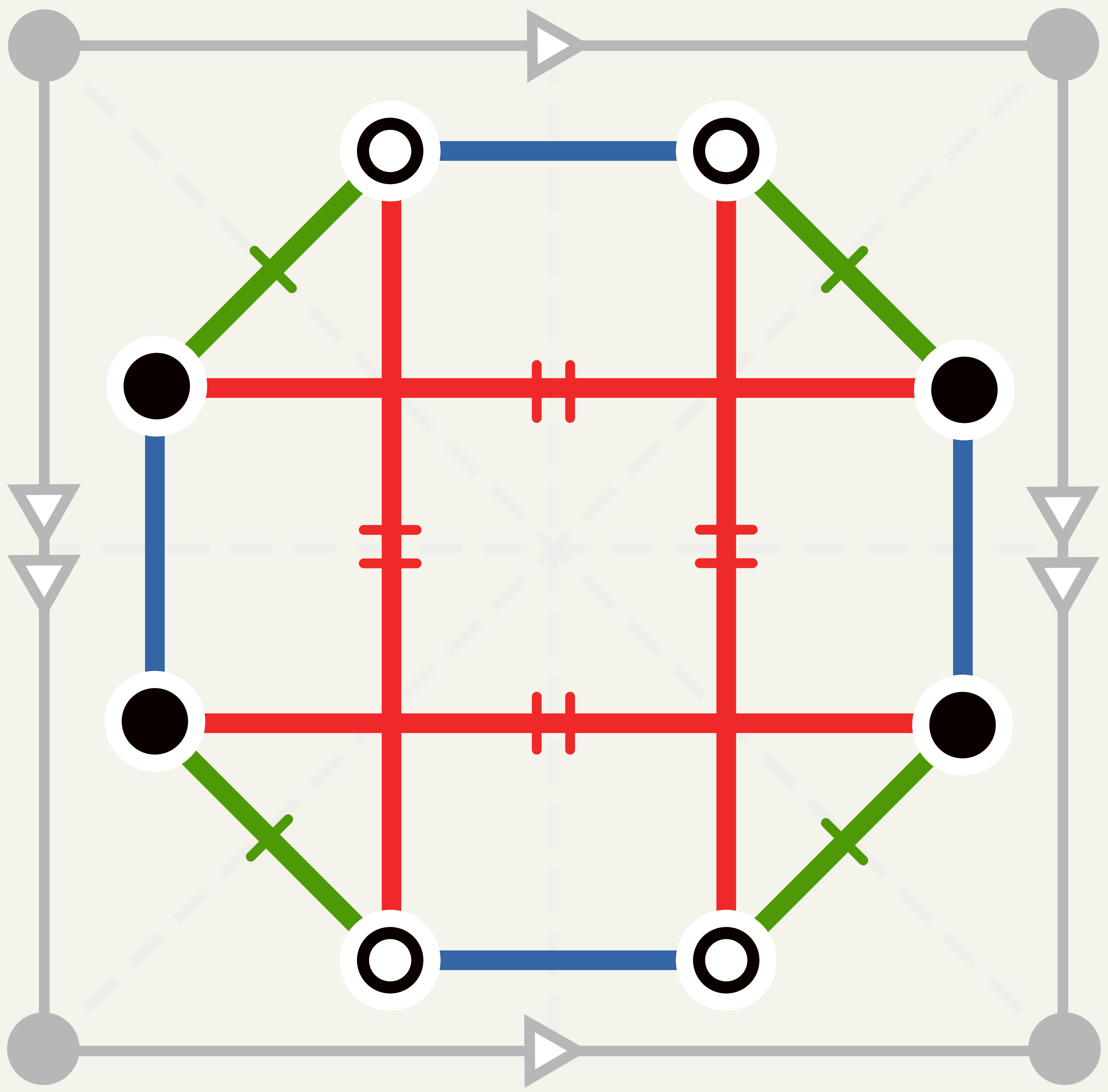


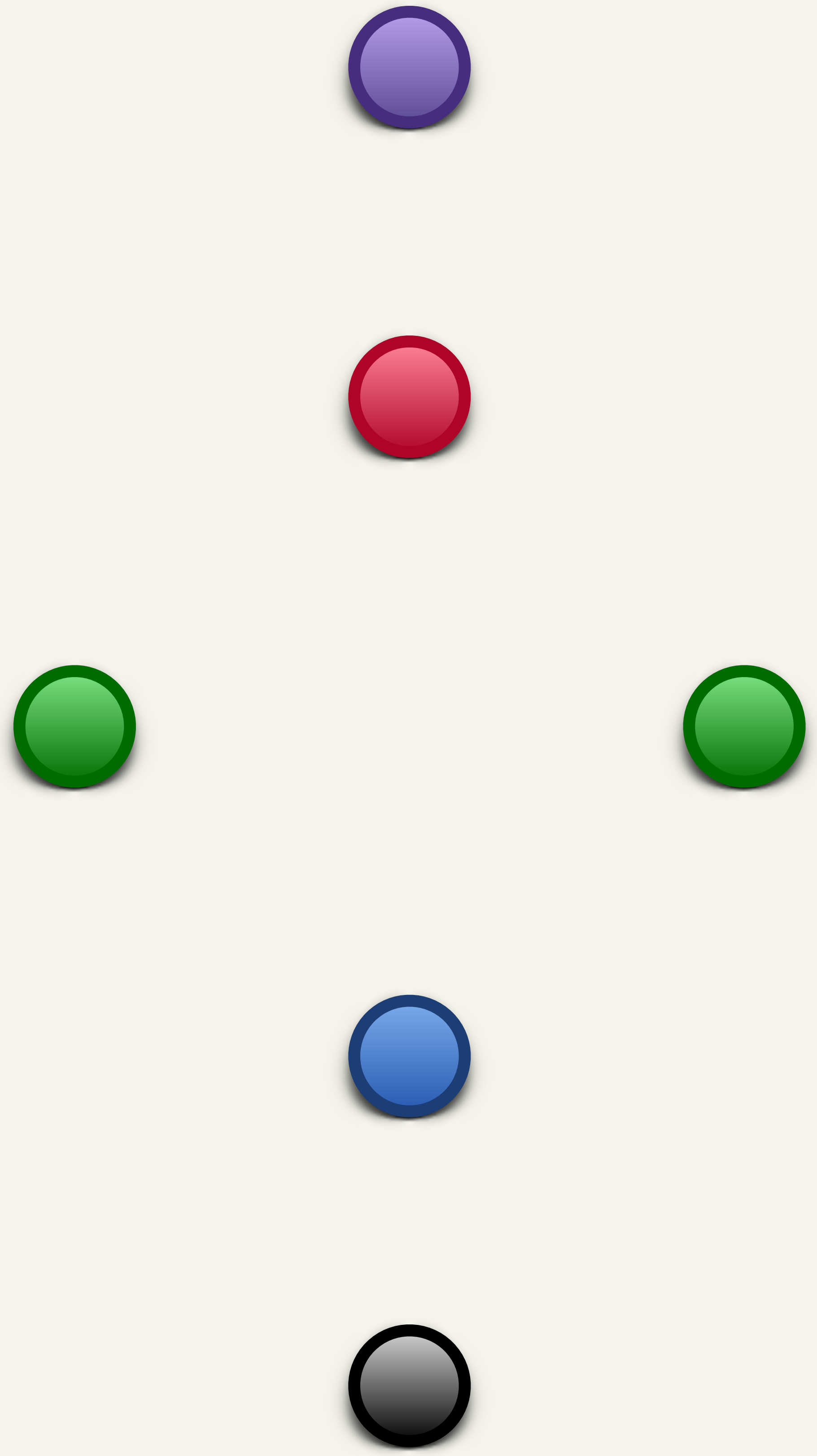


Polytopes vs maniplexes

$\mathcal{M}$



$\longrightarrow$ $\mathrm{Pos}(\mathcal{M})$



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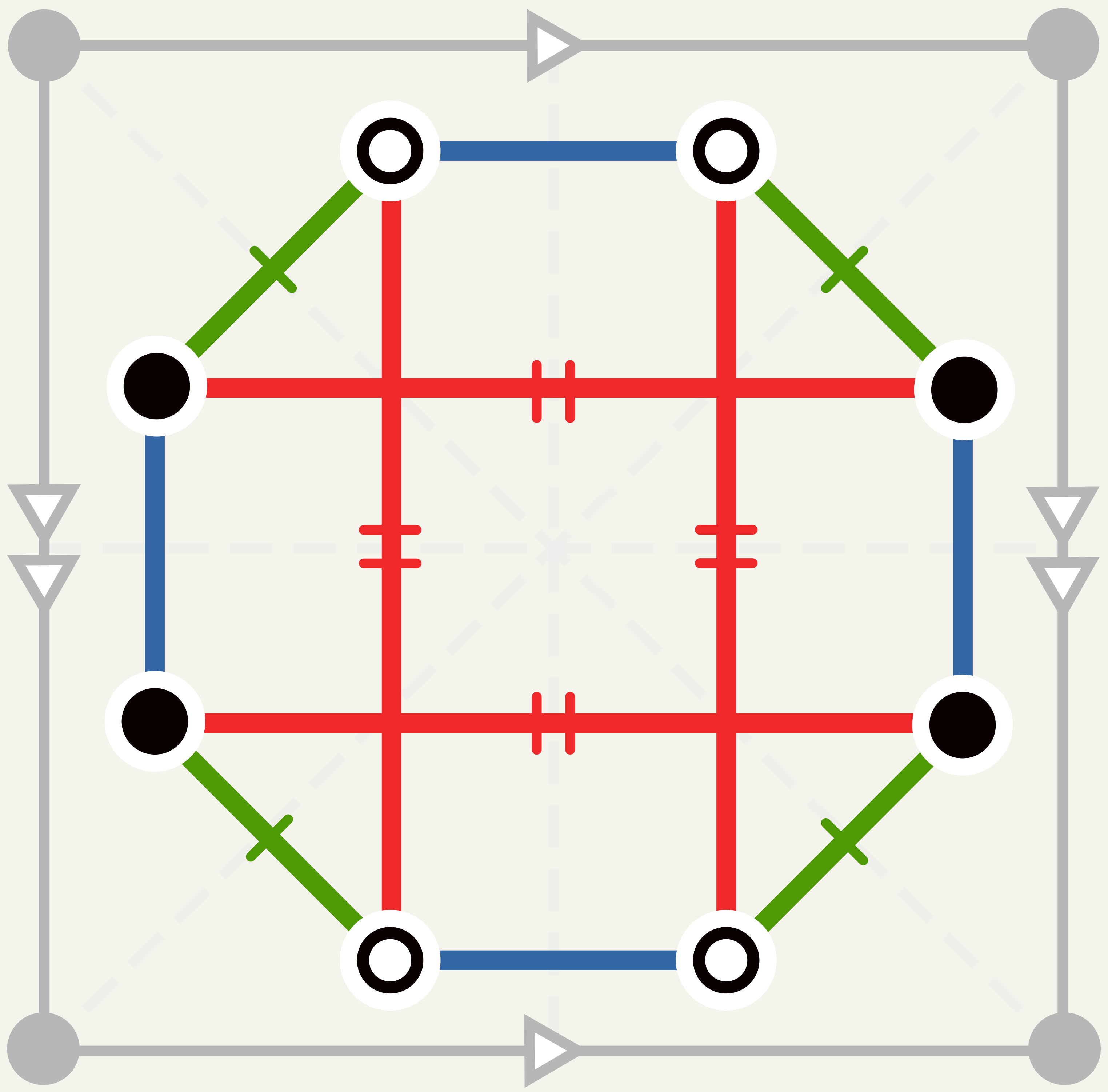

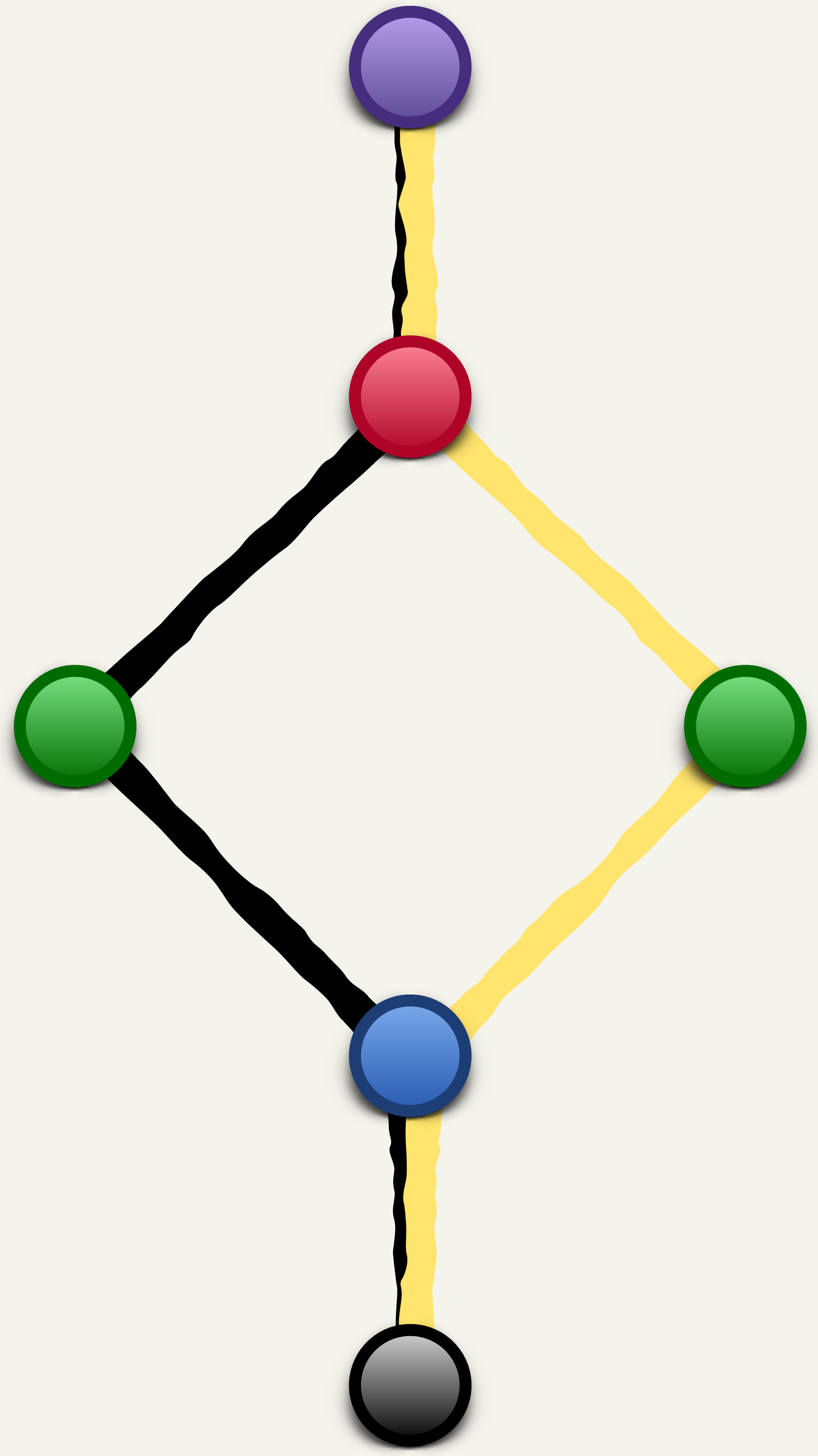

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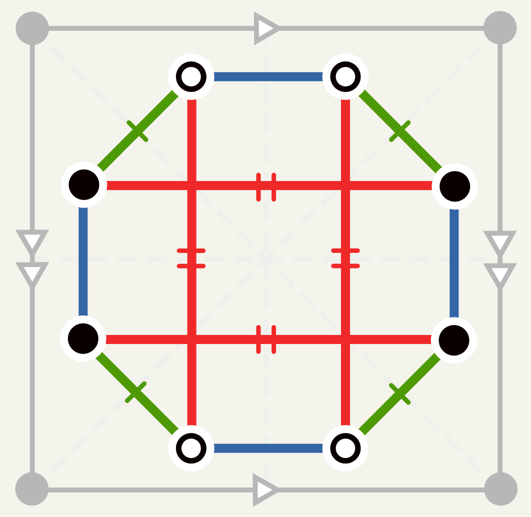

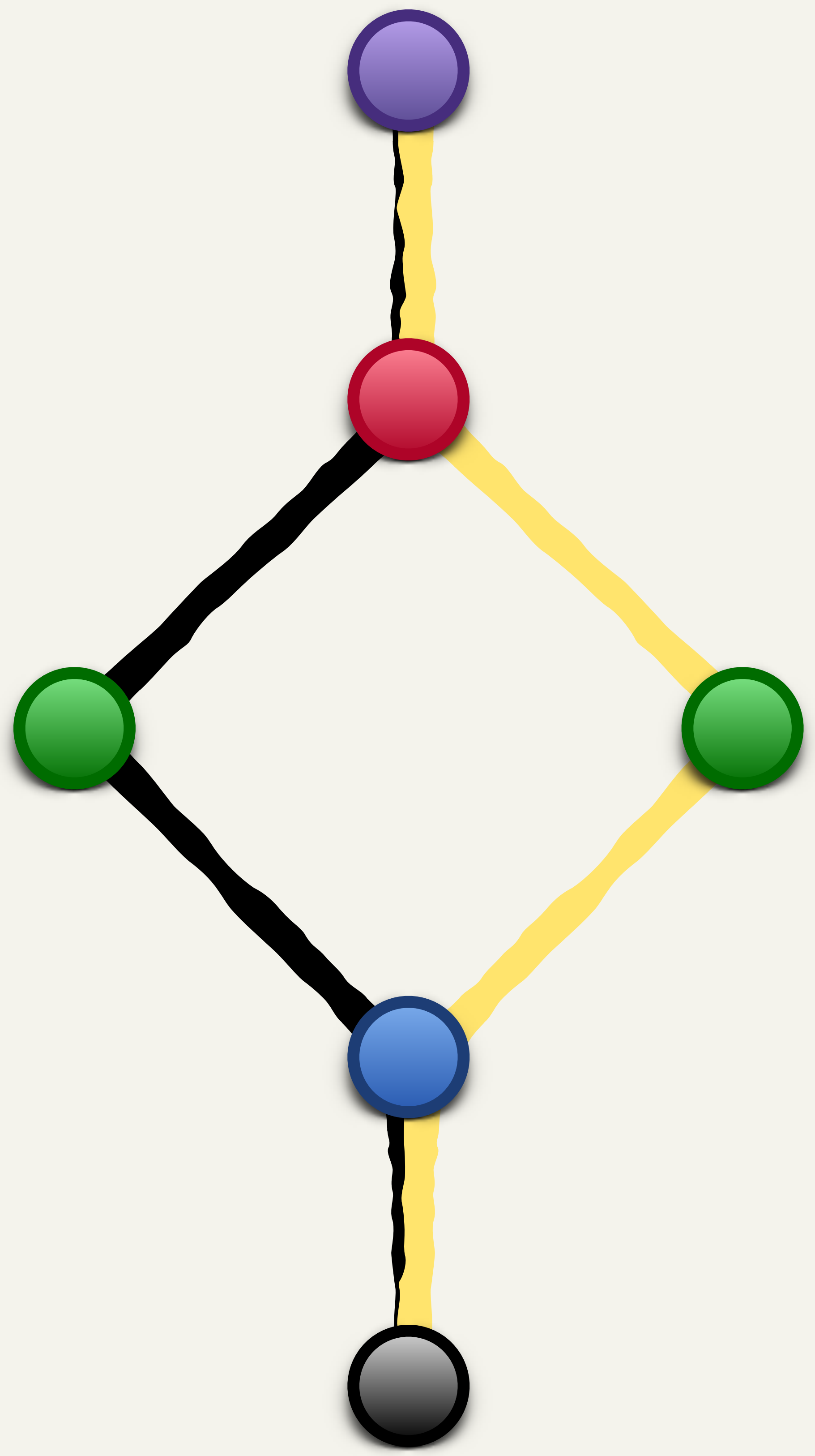

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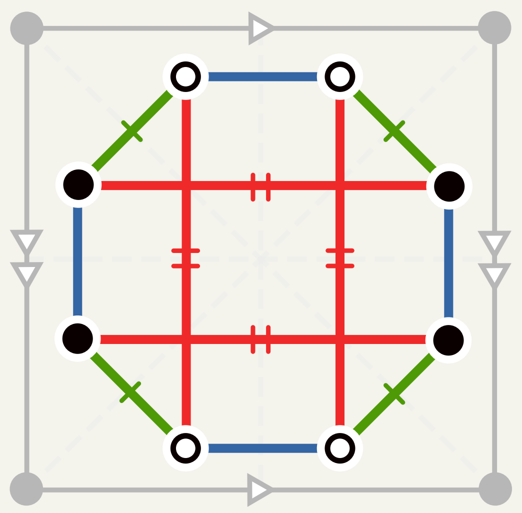

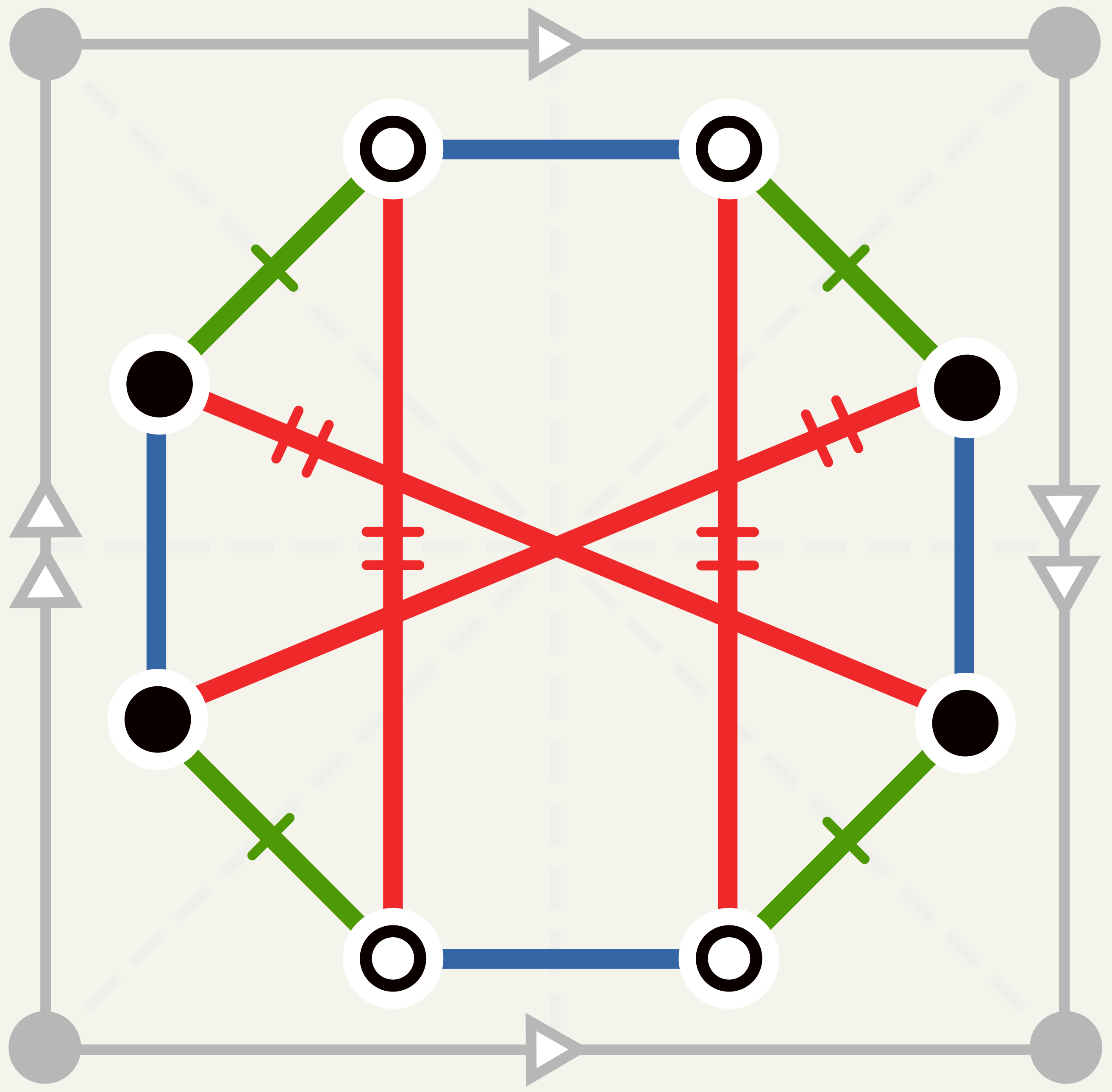

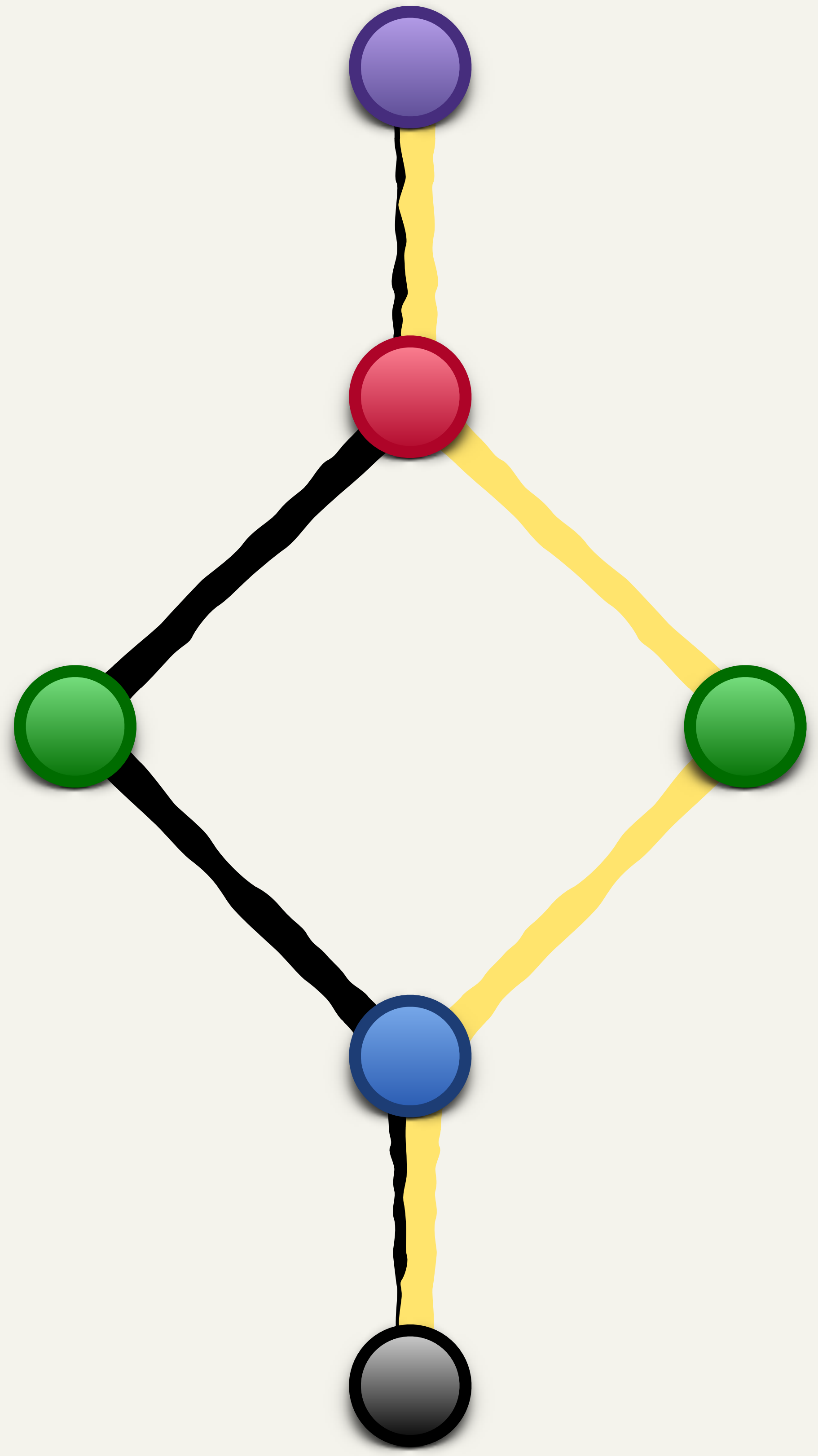

Faithfulness in maniplexes

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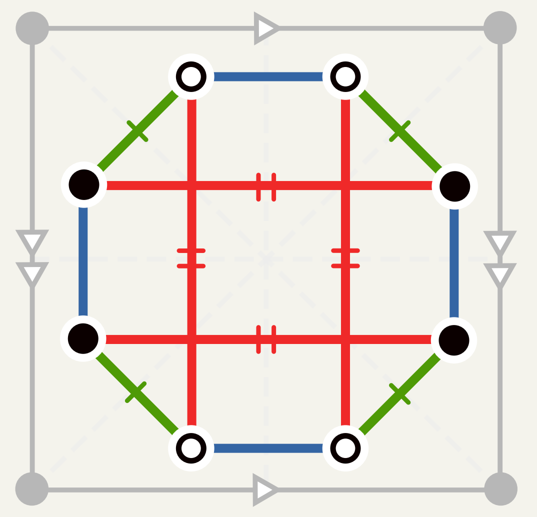

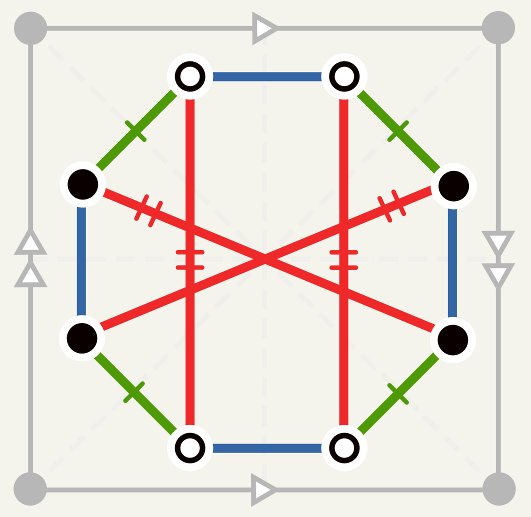

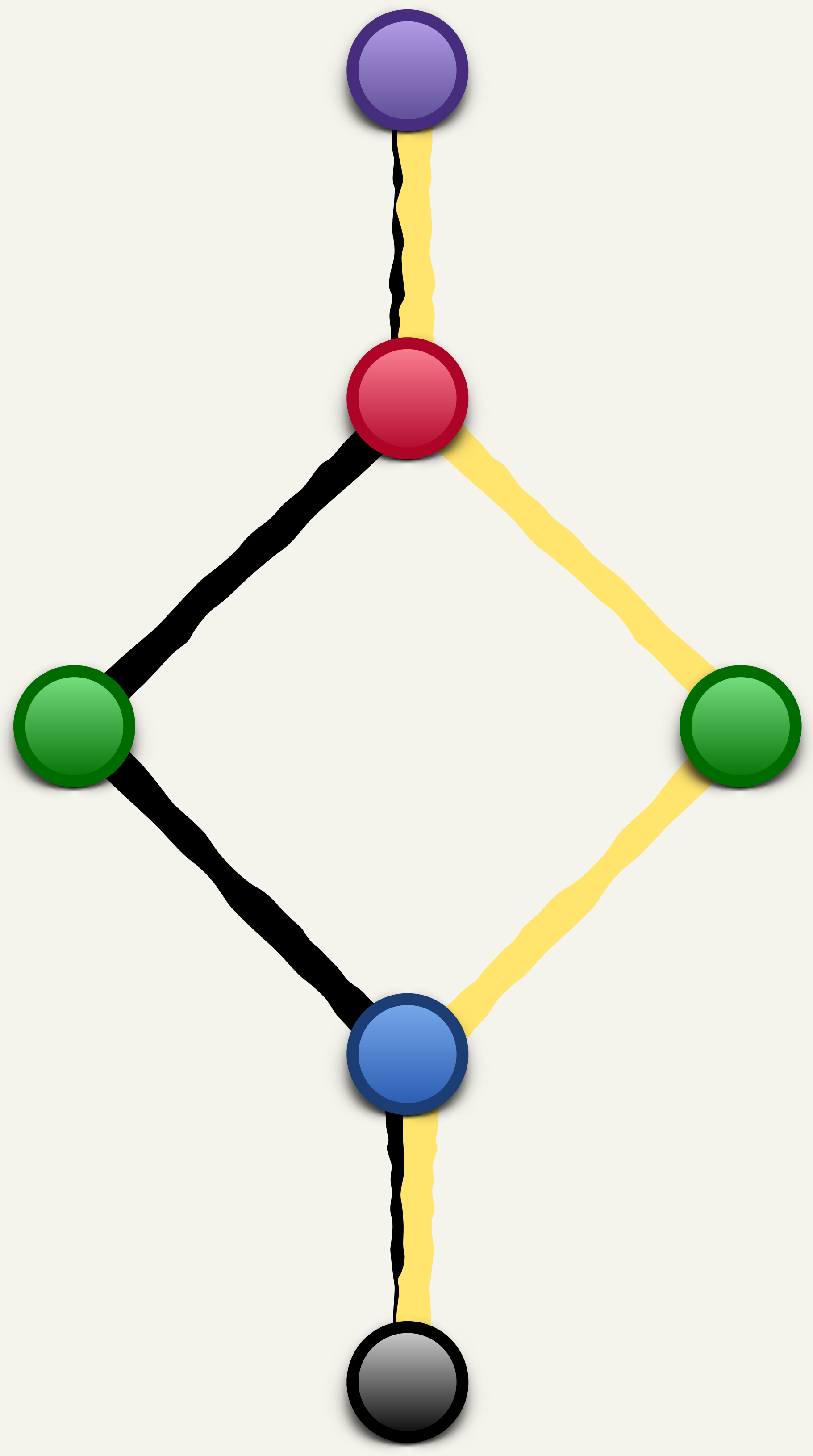

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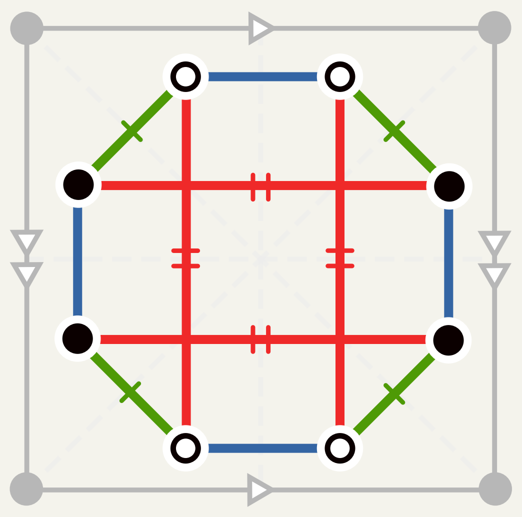

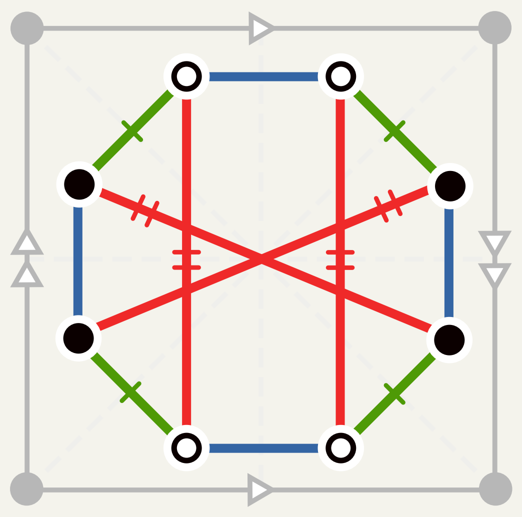

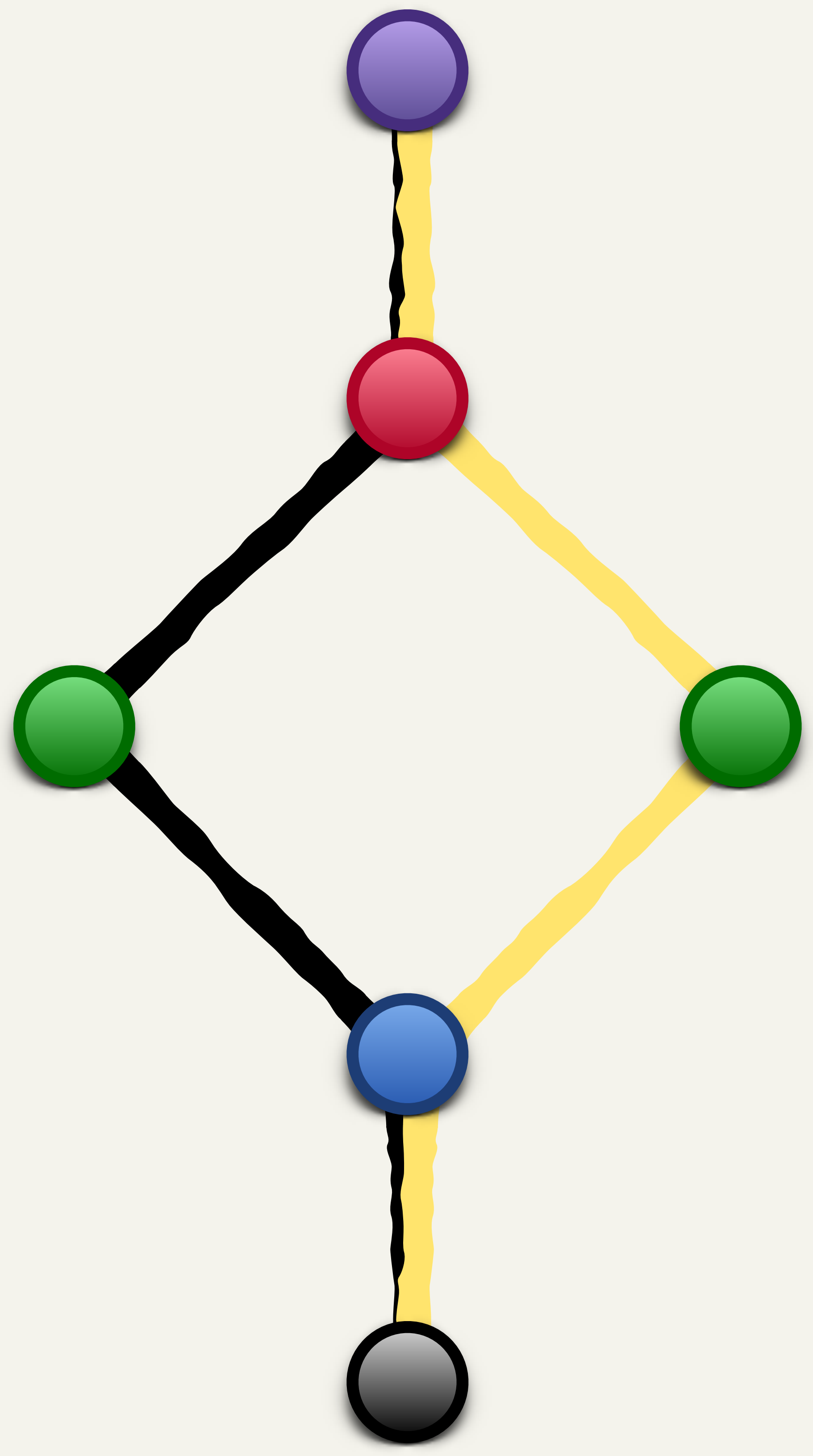

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Leemans, Toledo (2025):

- If $\mathscr{M}$ is faithful, has the DC and is of rank 3, then it has the PIP (is polytopal)
- There exist infinitely many maniplexes that are faithful, have the DC but are non-polytopal (of rank $n \geq 4$)

Faithfulness in maniplexes

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$$\mathcal{M} \longrightarrow \mathrm{Pos}(\mathcal{M})$$

✅

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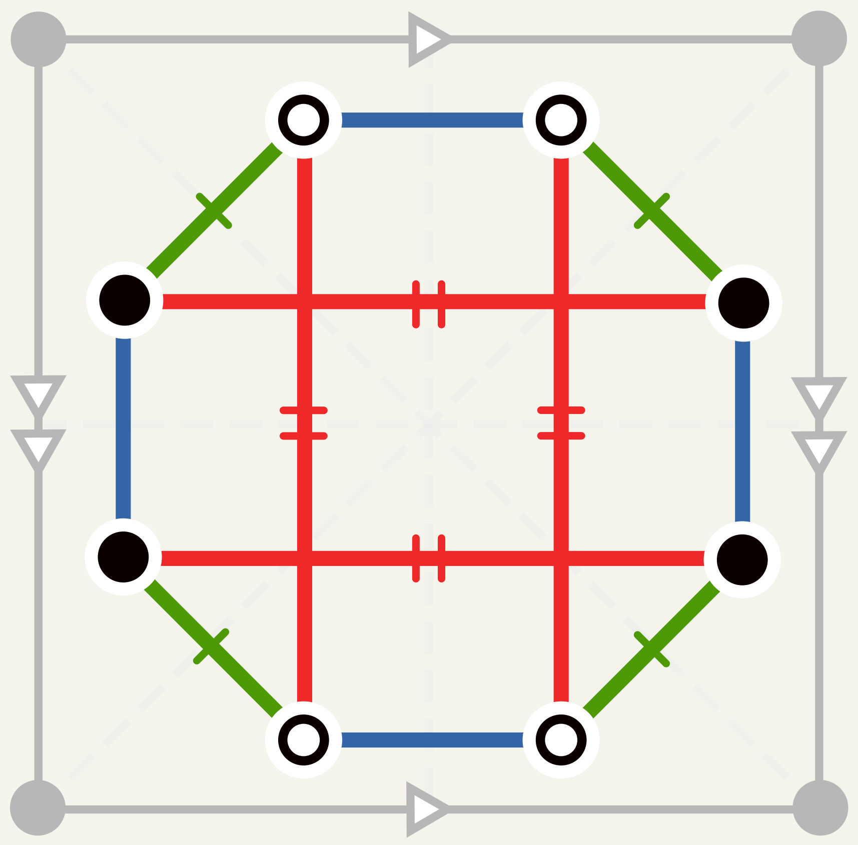

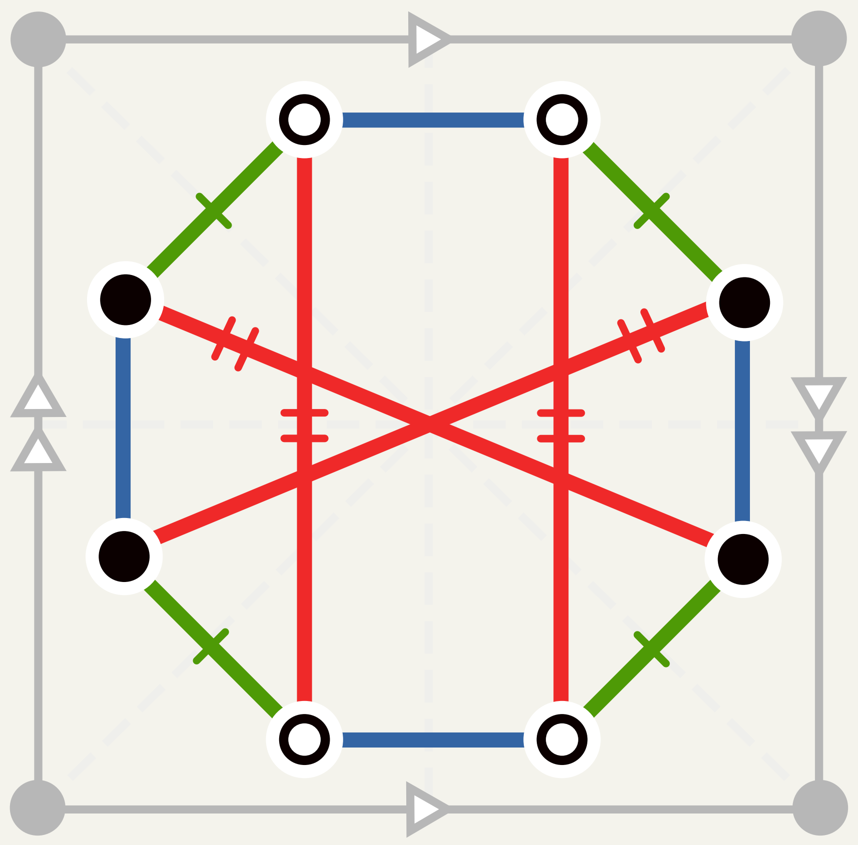

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Faithfulness and covers

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$\Gamma_{[\Phi]} = 1$ and $\sigma_1 \Gamma_0 \cap \Gamma_1 \cap \ldots \cap \Gamma_{n-1} = \varnothing$

* we also have similar results for other 2-orbit maniplexes and for arbitrary STG

Highly symmetric unfaithful maniplexes

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Classification of regular and 2-orbit unfaithful maps:

- Regular ones: embeddings of $2k$-gons on the projective plane or on an orientable surface of genus $\lfloor k/2 \rfloor$.
- There are no chiral unfaithful maps.
- There are no unfaithful maps in class 2_0 or 2_2.
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This was Elías's talk!

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Census of rotary 4-maniplexes

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graphsym.net

(Primož Potočnik)

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Census of Rot 4 Maniplexes

(Cun, Pot, Tol, Moc, M.)

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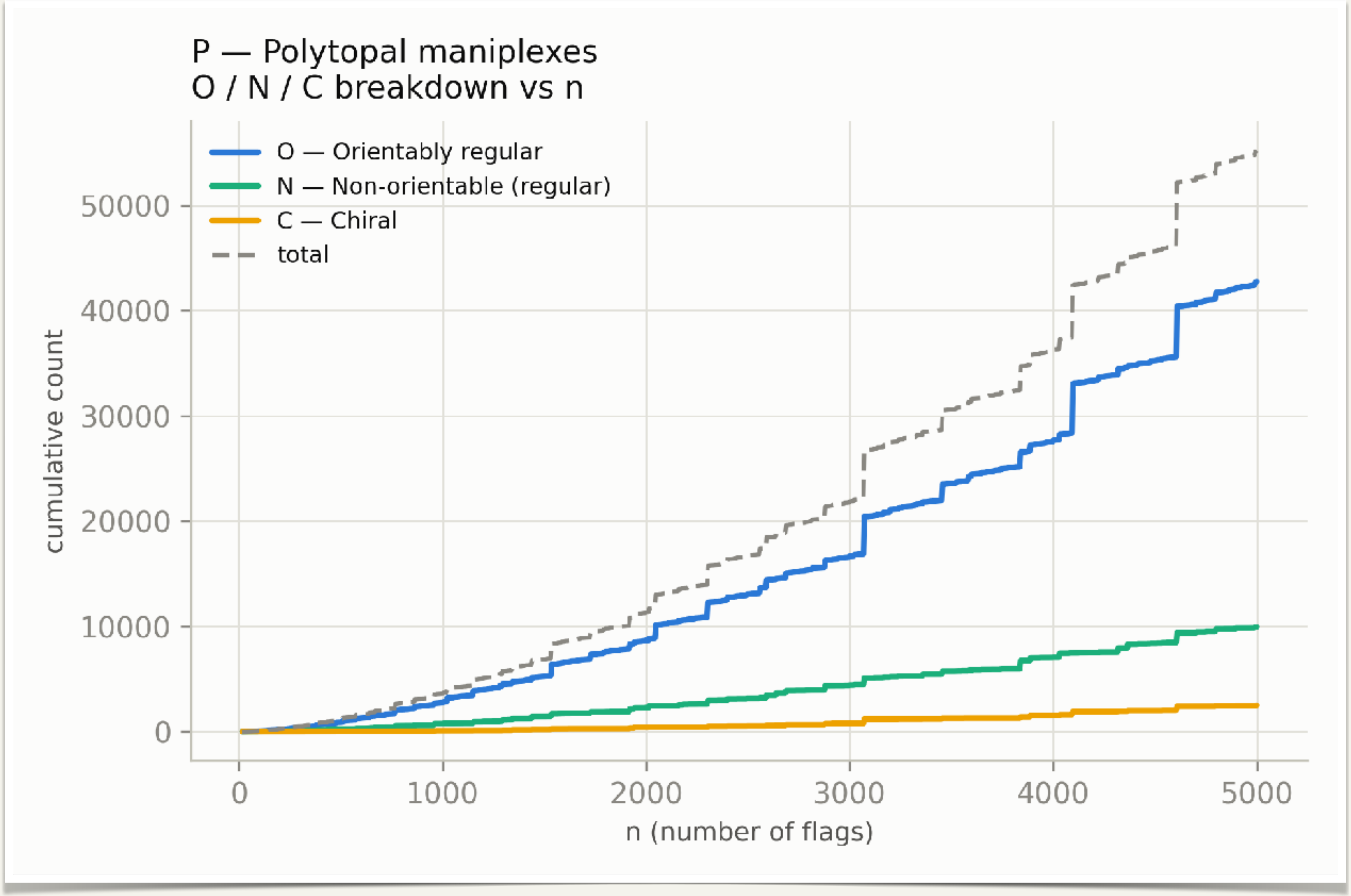
RAMP

Research Assistant for Maps and Polytopes

(G#P - Package)

Cunningham - Mixer - Williams

Census of rotary 4-maniplexes



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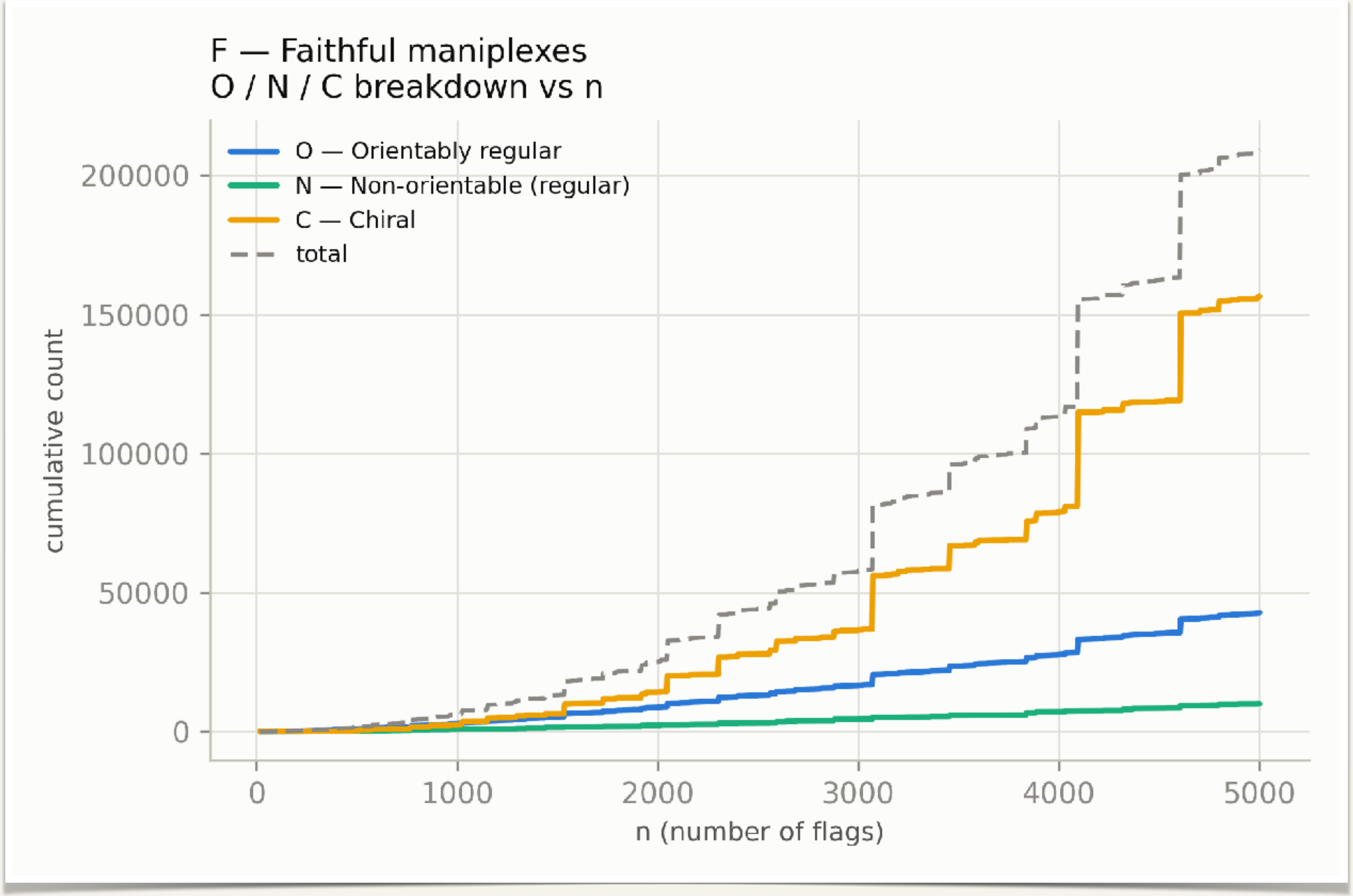

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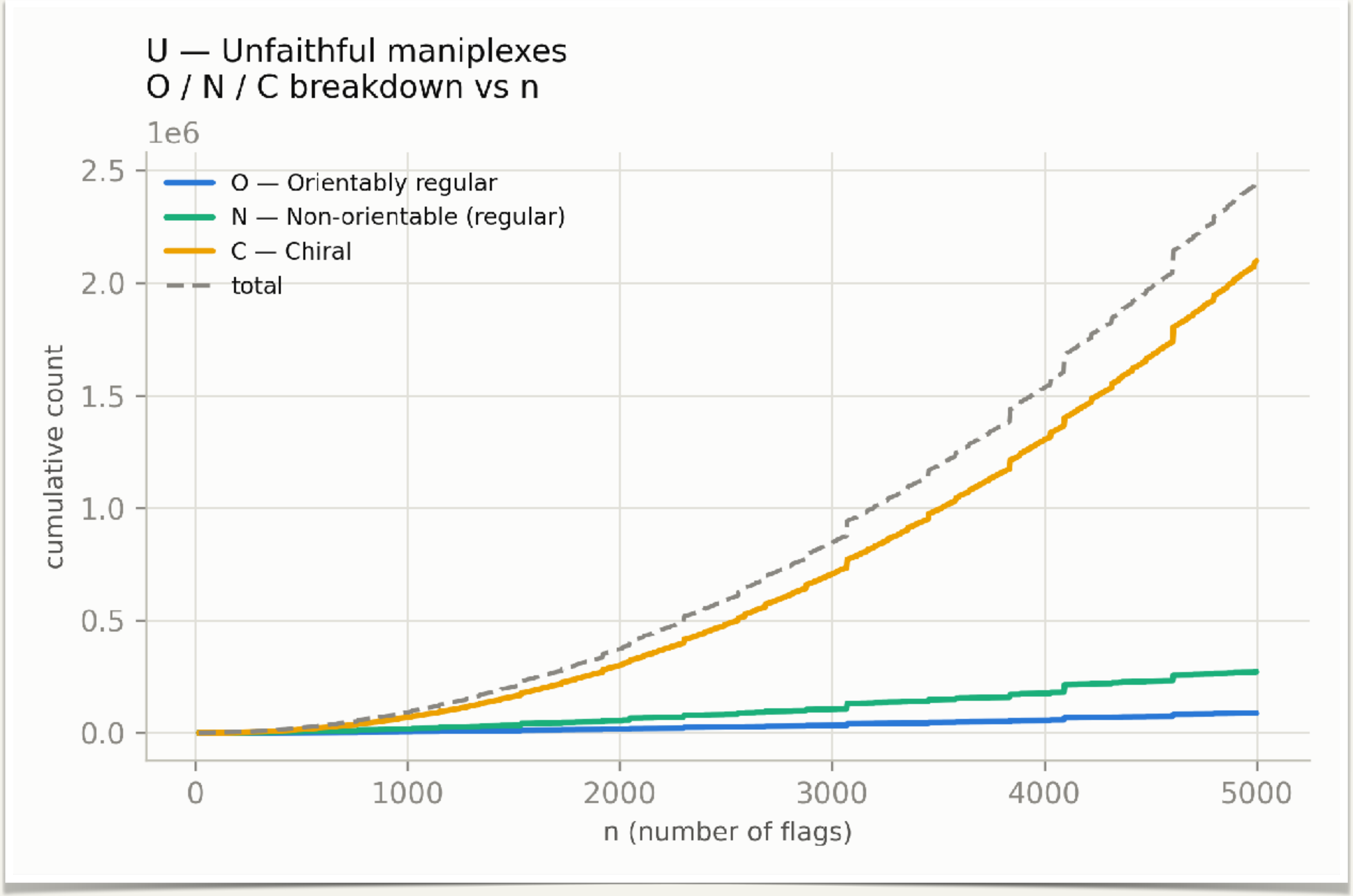
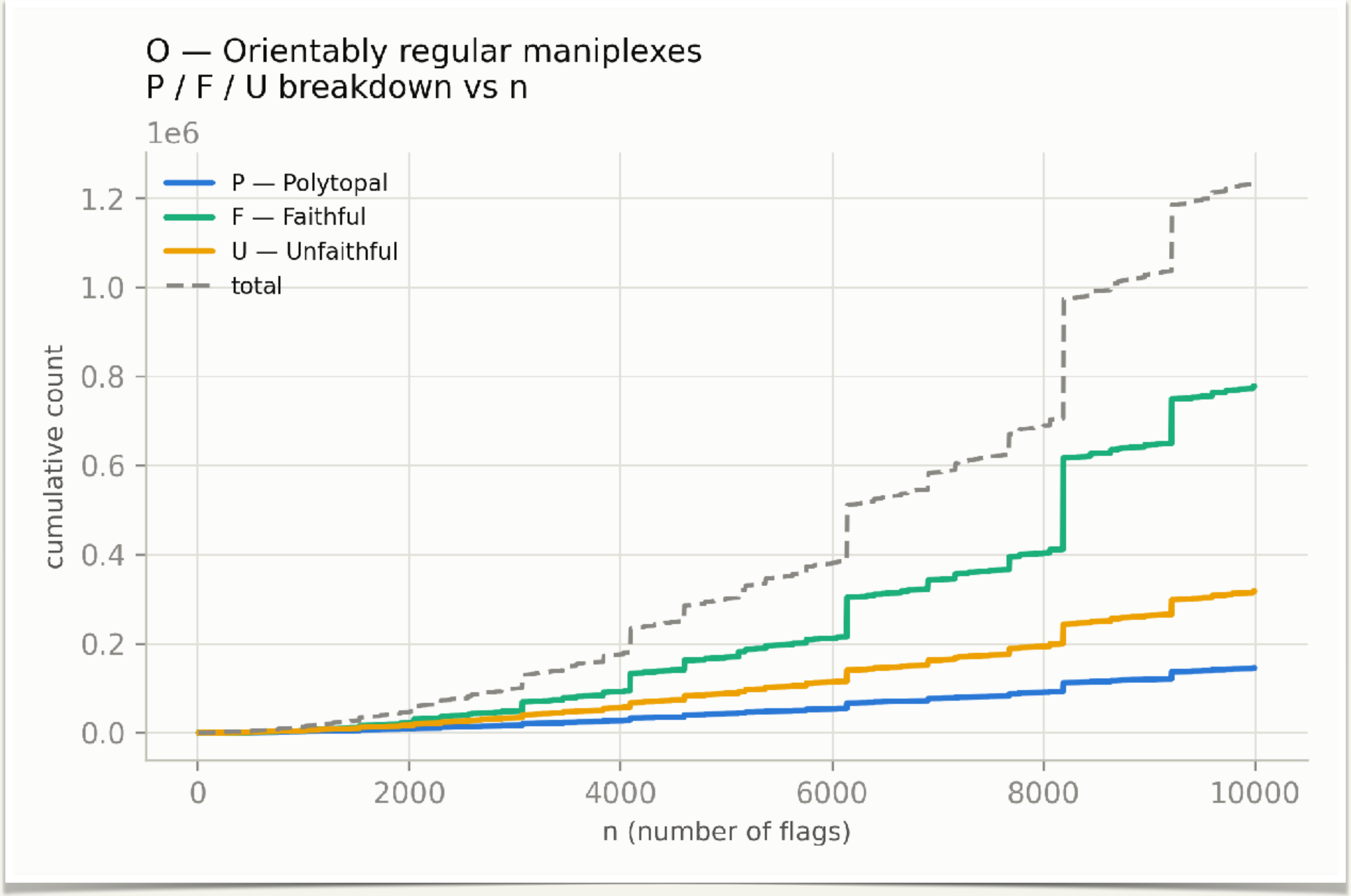
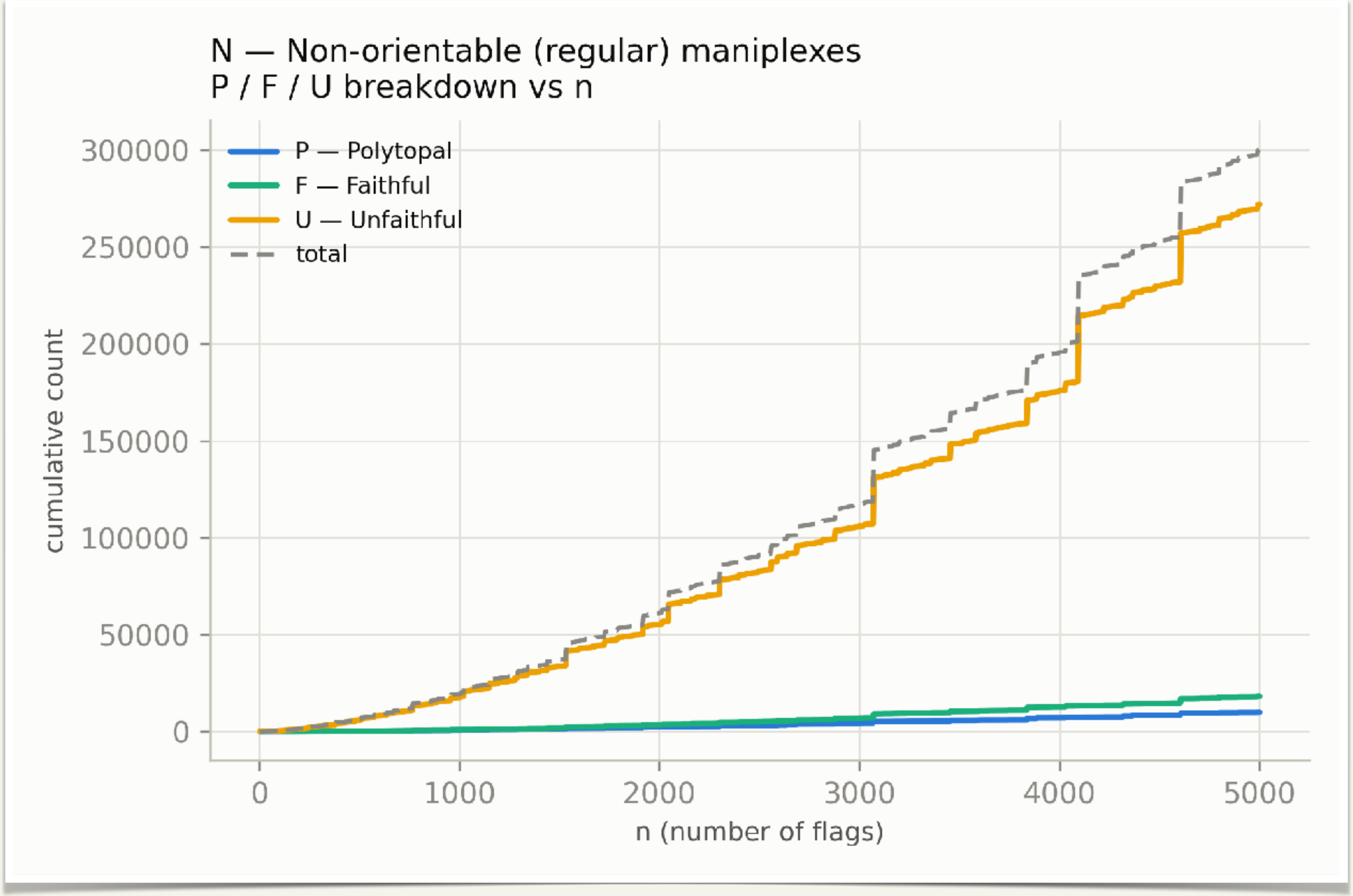


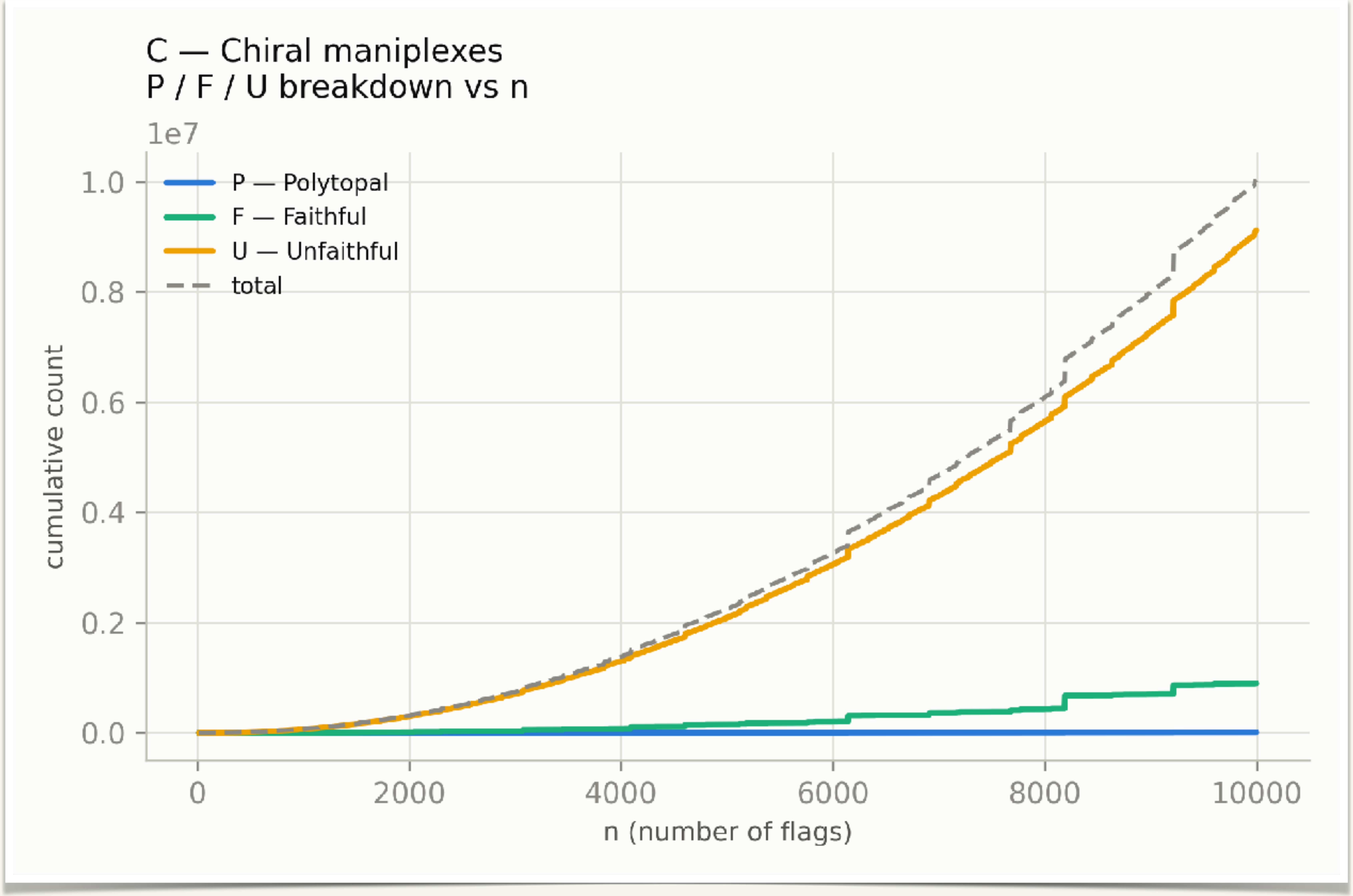
Census of rotary 4-maniplexes



Census of rotary 4-maniplexes



Census of rotary 4-maniplexes



Census of rotary 4-maniplexes

Charts

Thank you!

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RAMP GraphSym Slides Census Charts