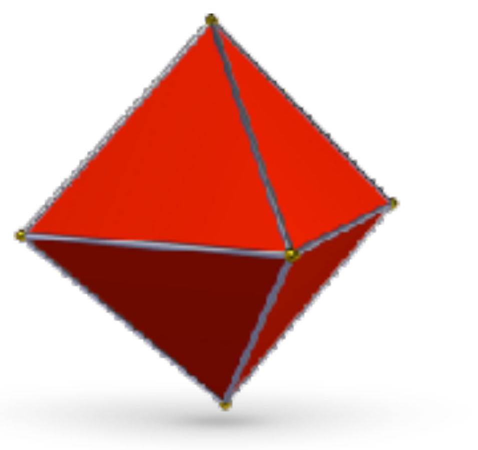
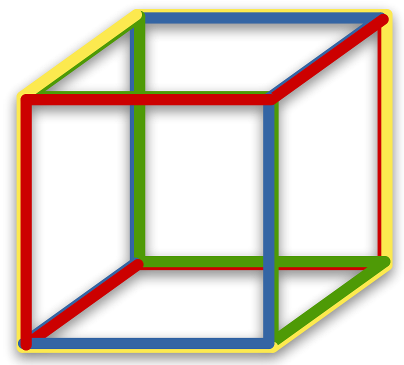
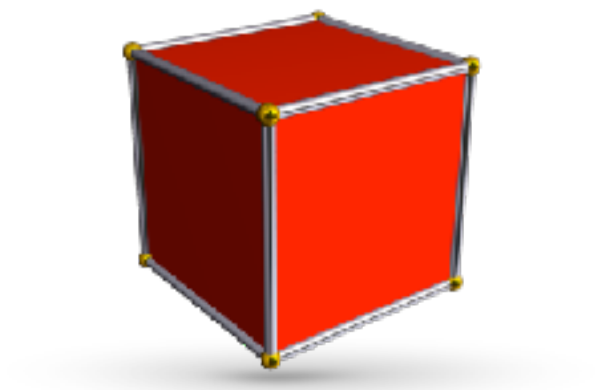
Symmetries of voltage operations on maniplexes and polytopes

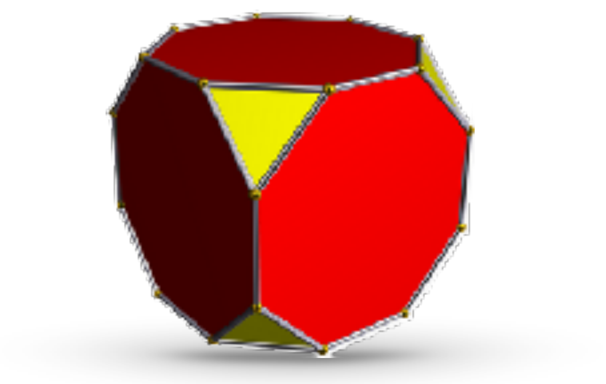
Antonio Montero
University of Ljubljana

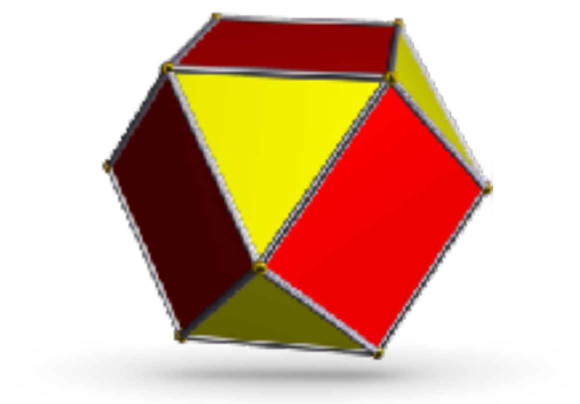




Based on joint work with Isabel Hubard and Elías Mochán







GEMS 2025: Graphs embeddings and Maps on Surfaces
June 2025, Trenčianske Teplice, Slovakia

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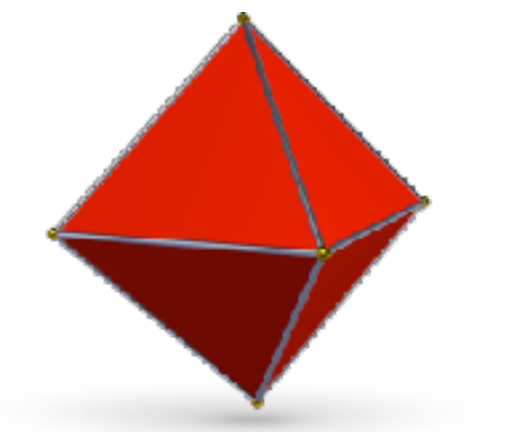
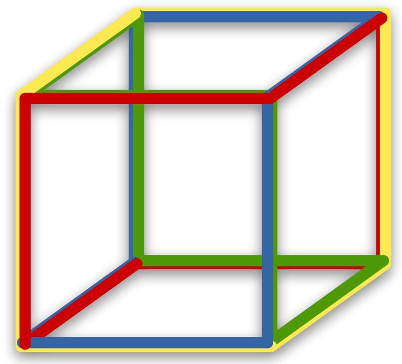
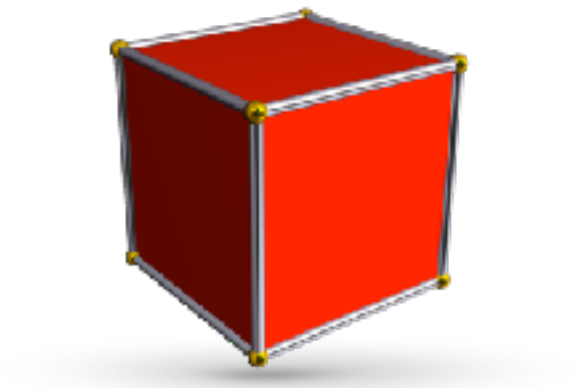
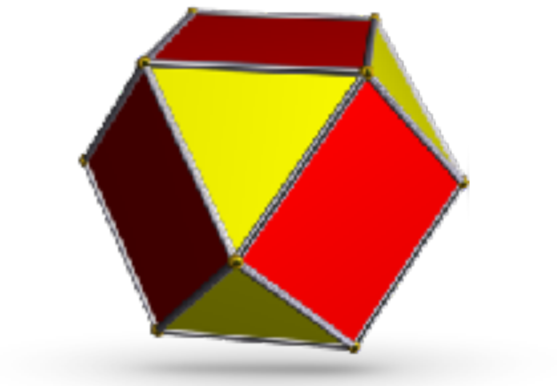
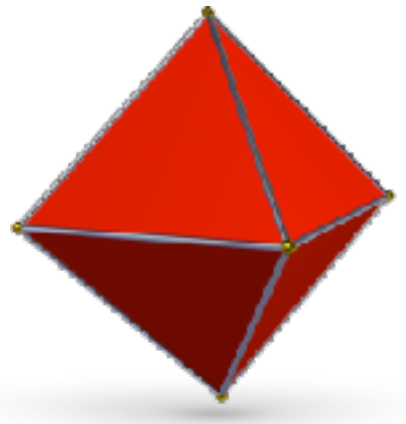


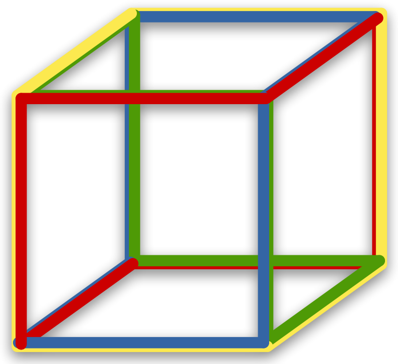


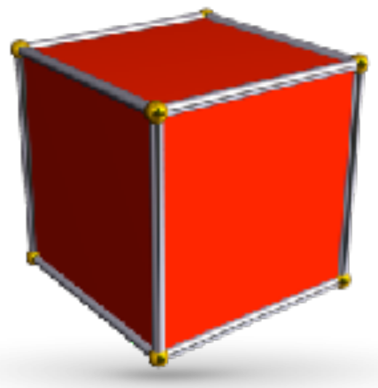


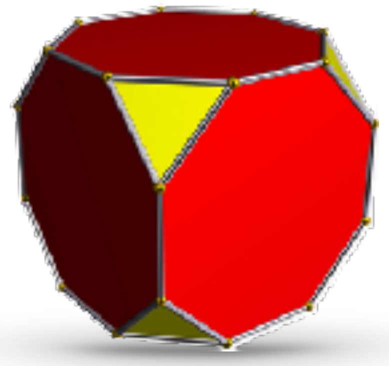


operations









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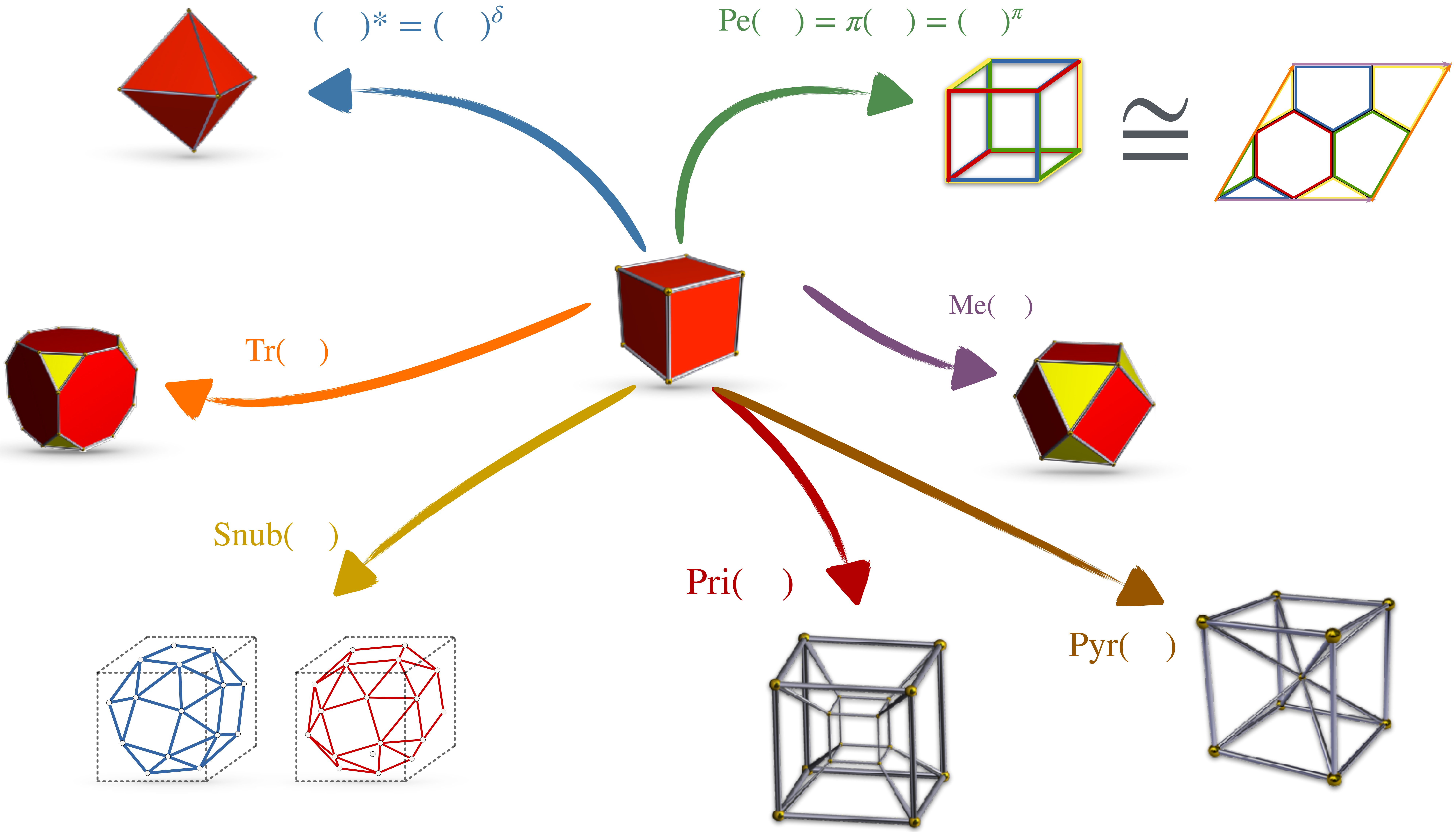

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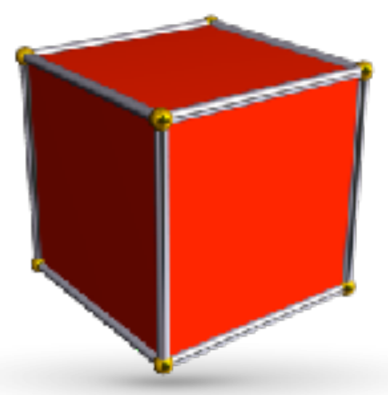
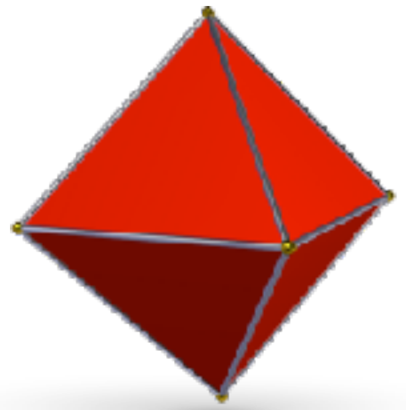
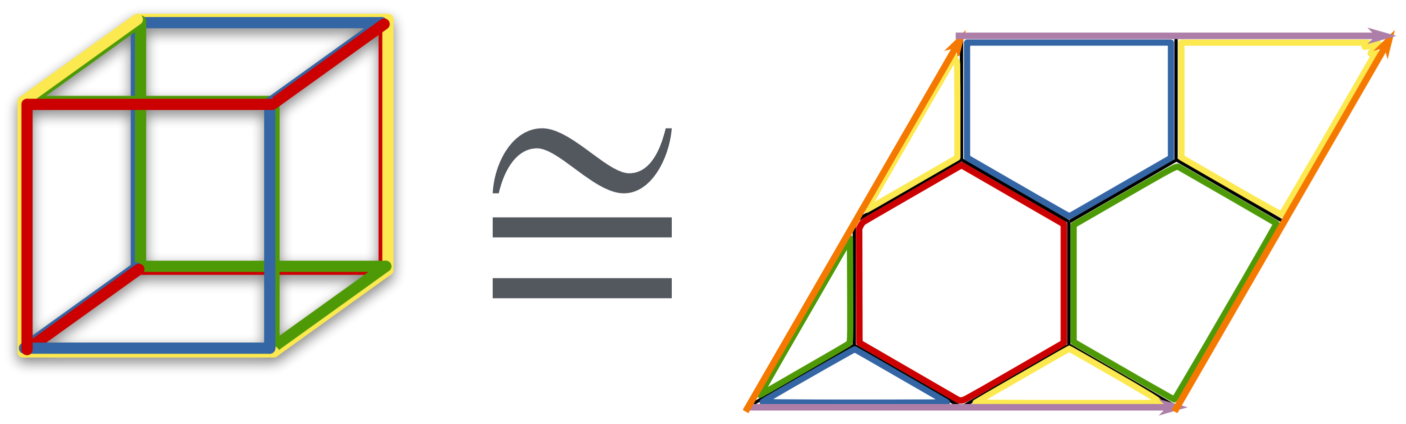
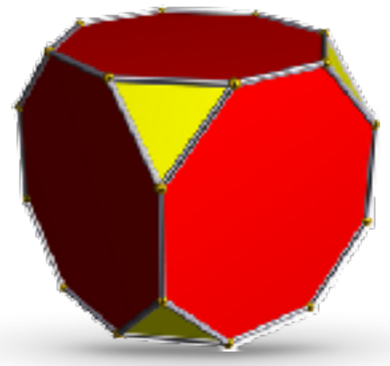




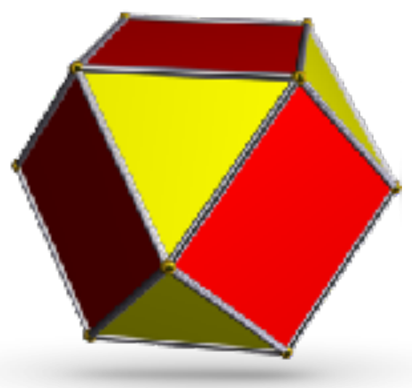
$(\quad)^* = (\quad)^\delta$



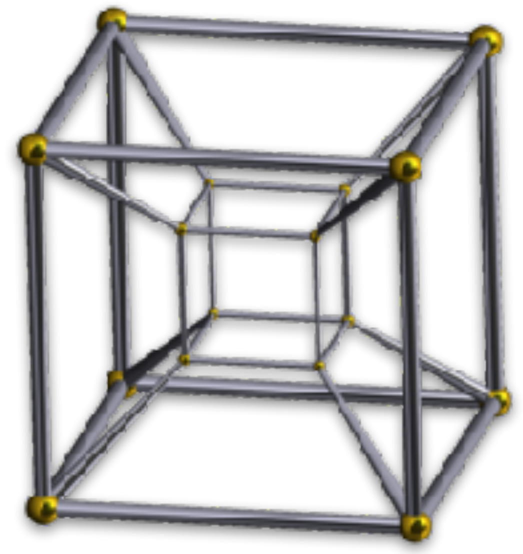
$\mathrm{Pe}(\quad) = \pi(\quad) = (\quad)^\pi$



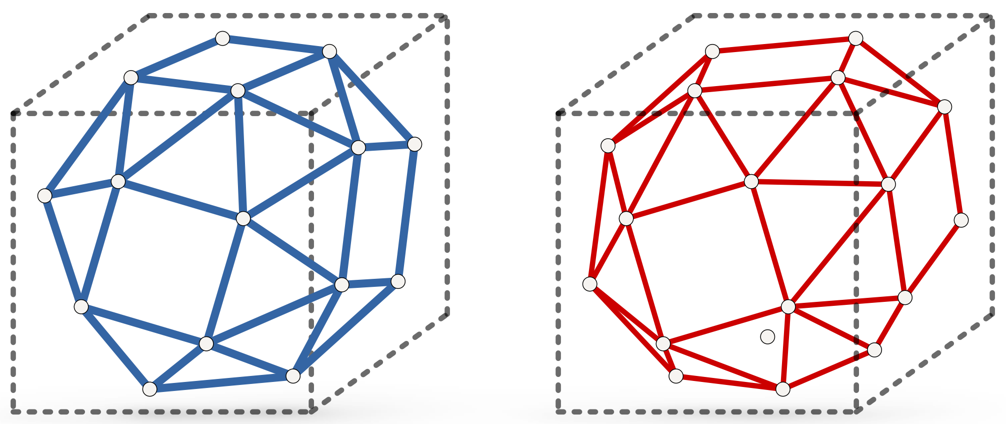
Tr(   )



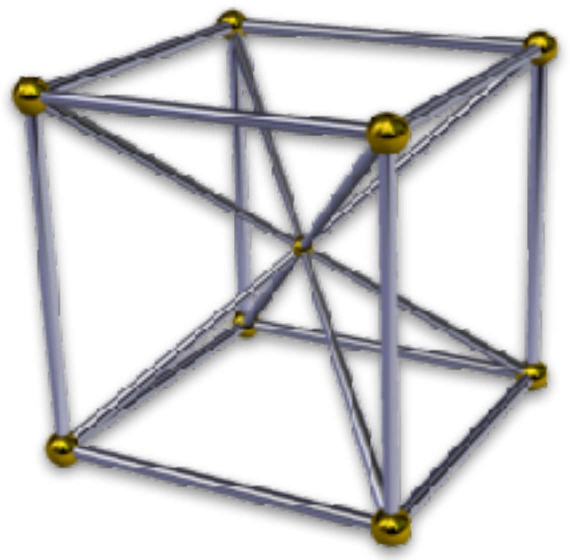
Me(   )



Pri(   )

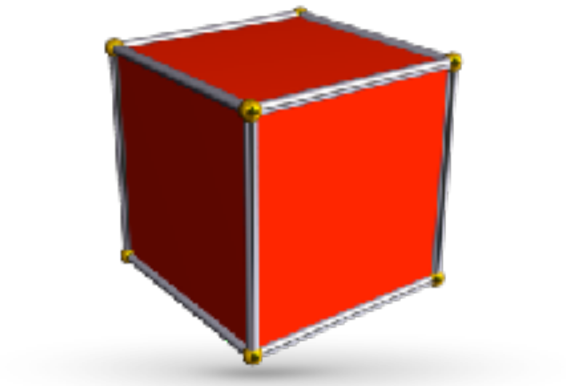


Snub(   )

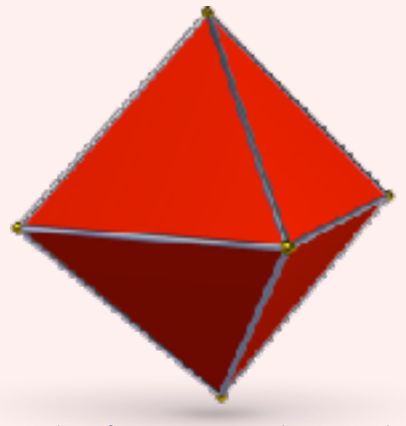


Pyr(   )

operations



- Automorphism group



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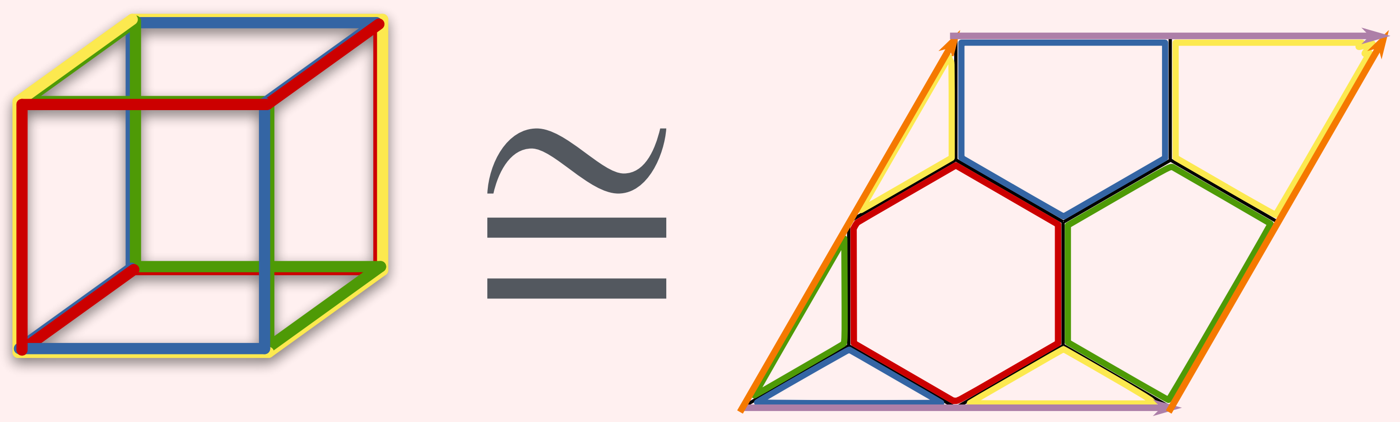

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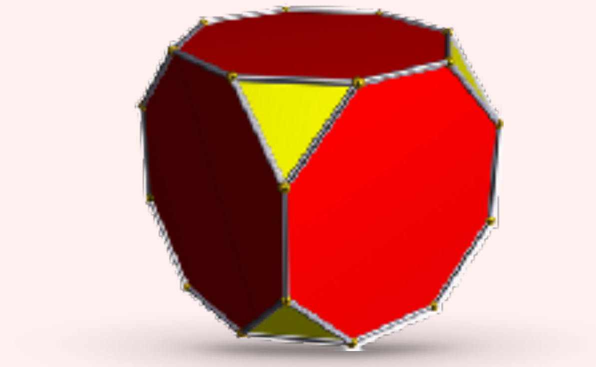
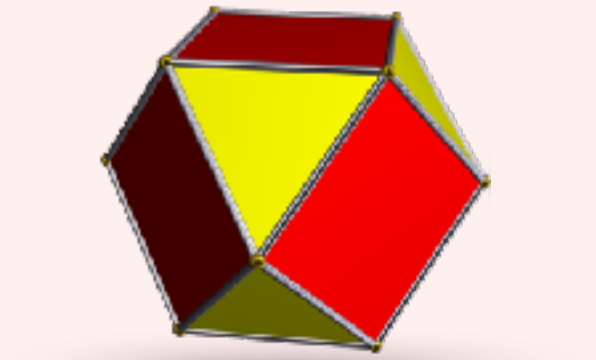
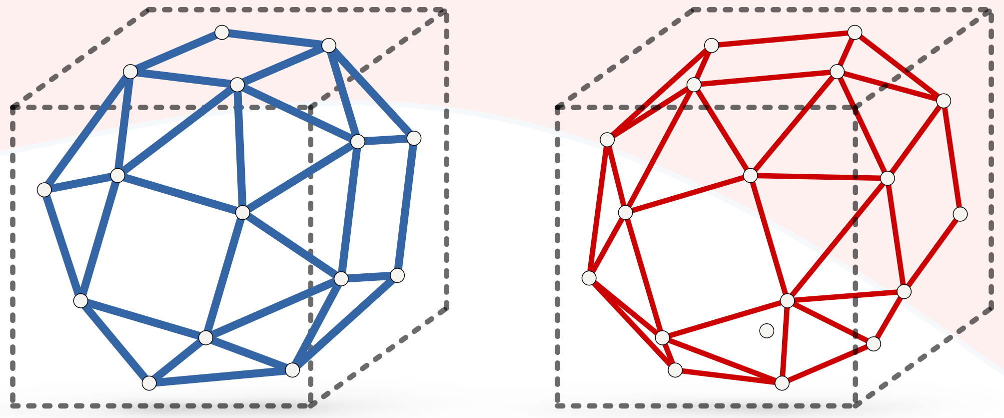


Tr( )



Me( )



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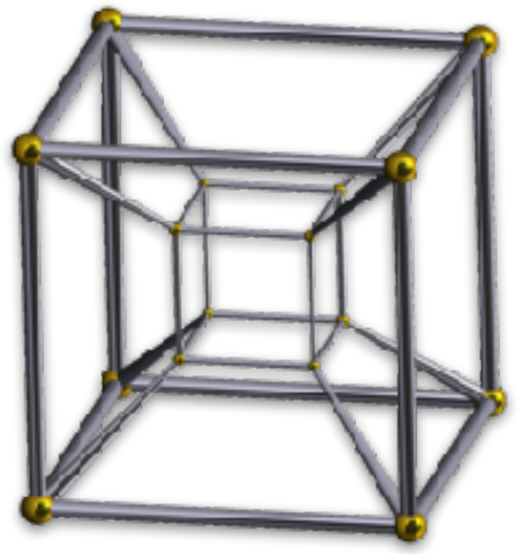

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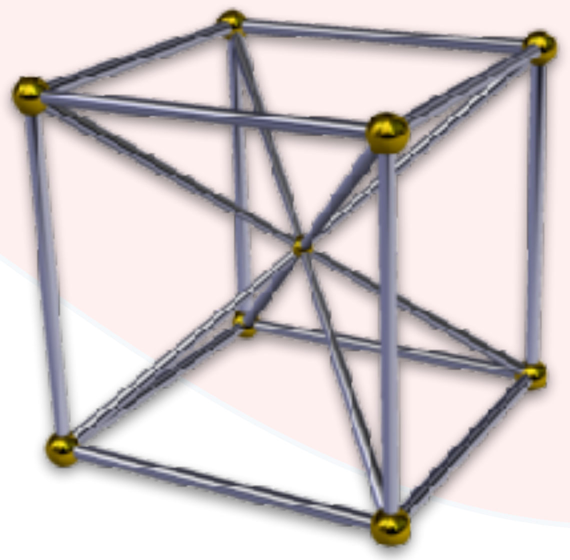

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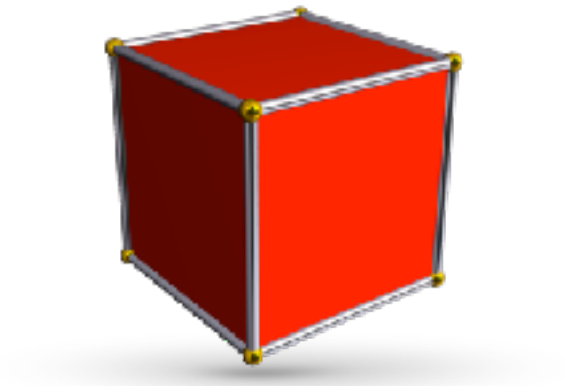
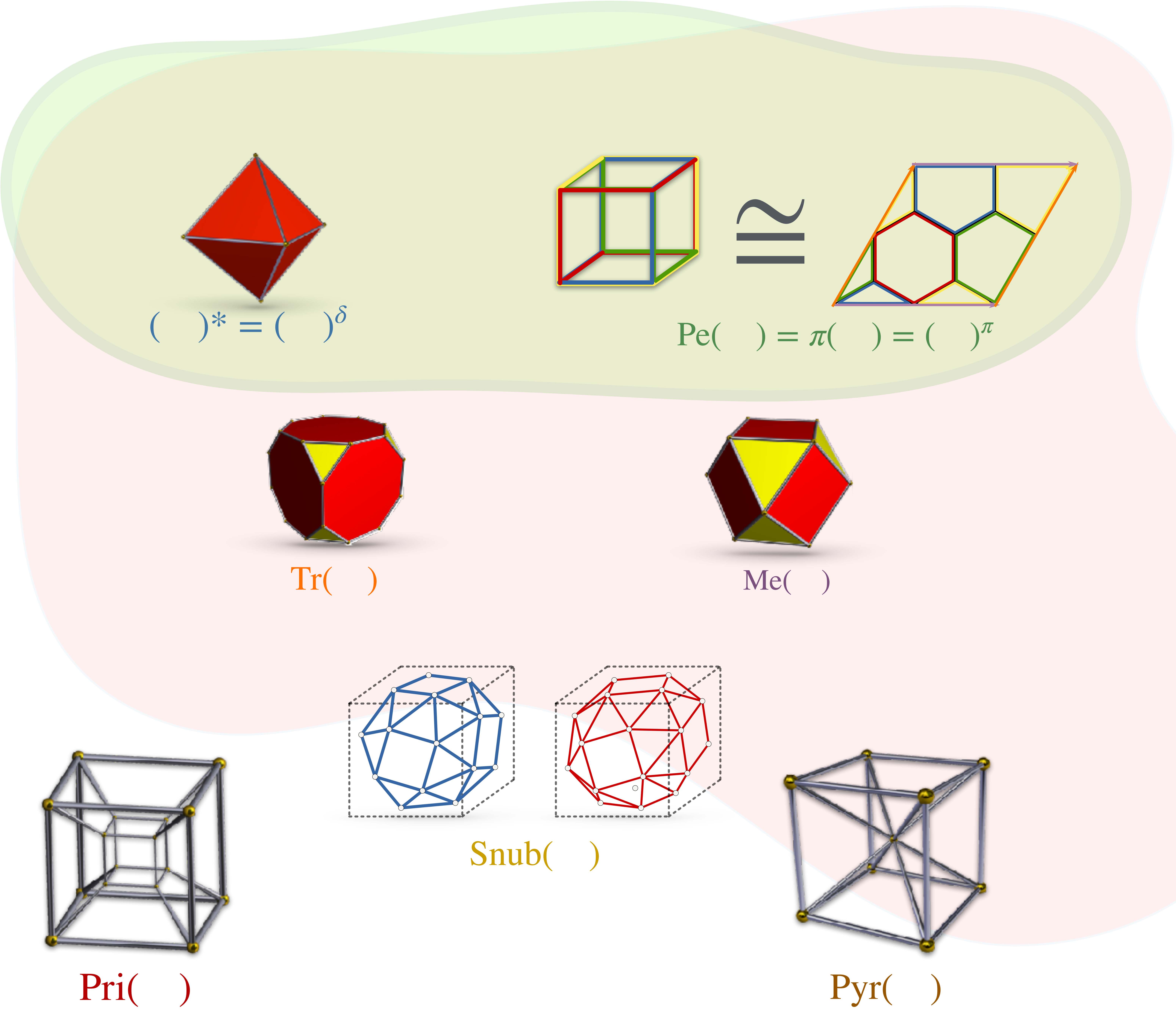
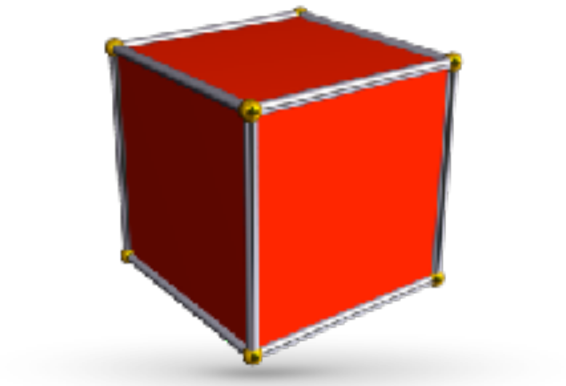
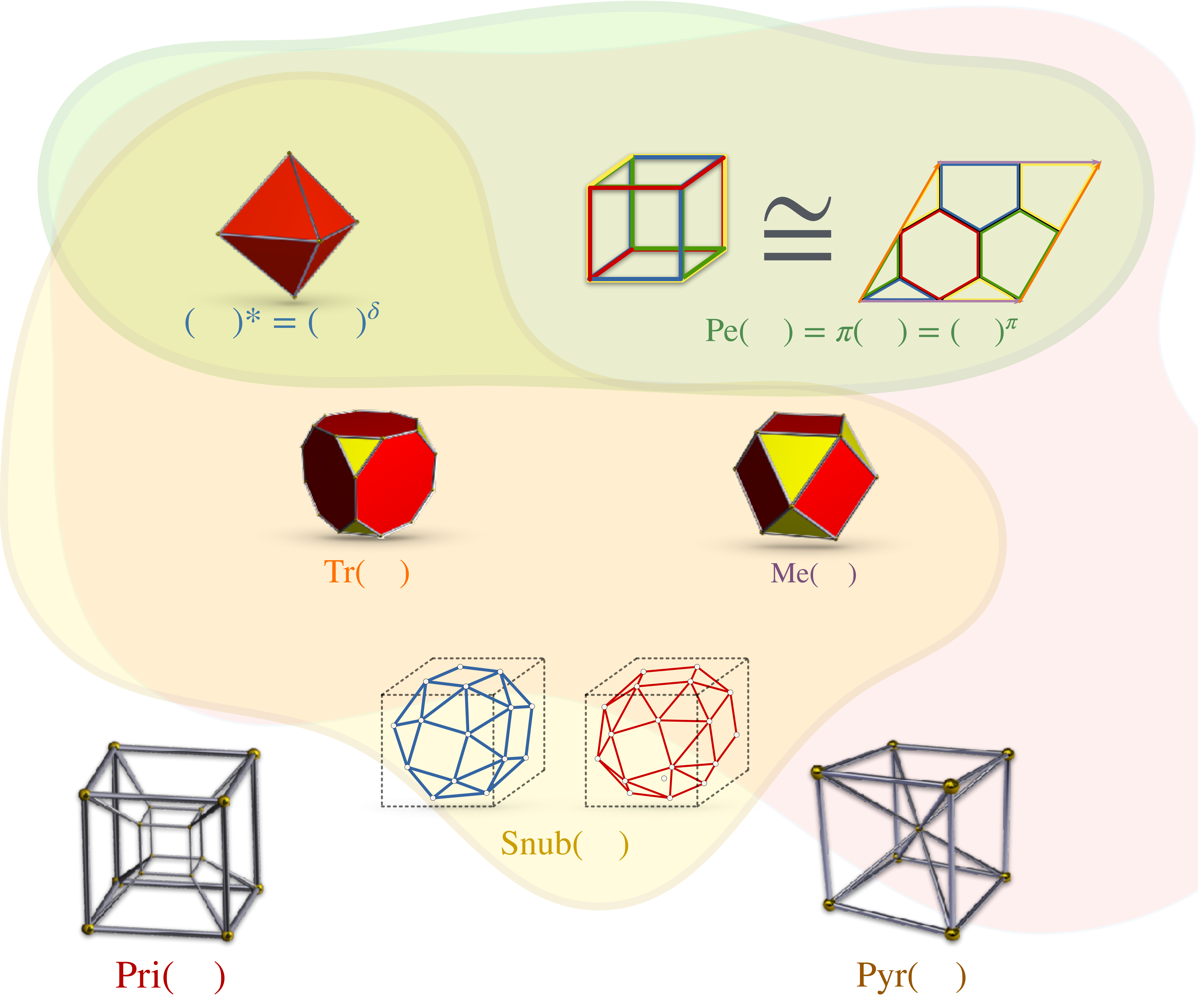


- Automorphism group
- Size (number of flags)



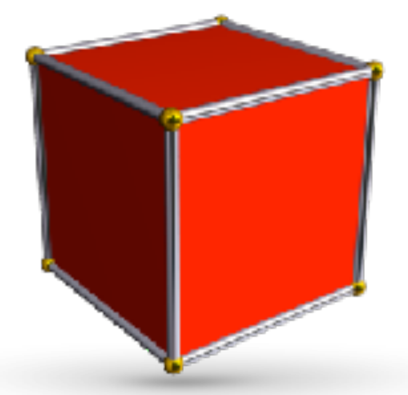
operations



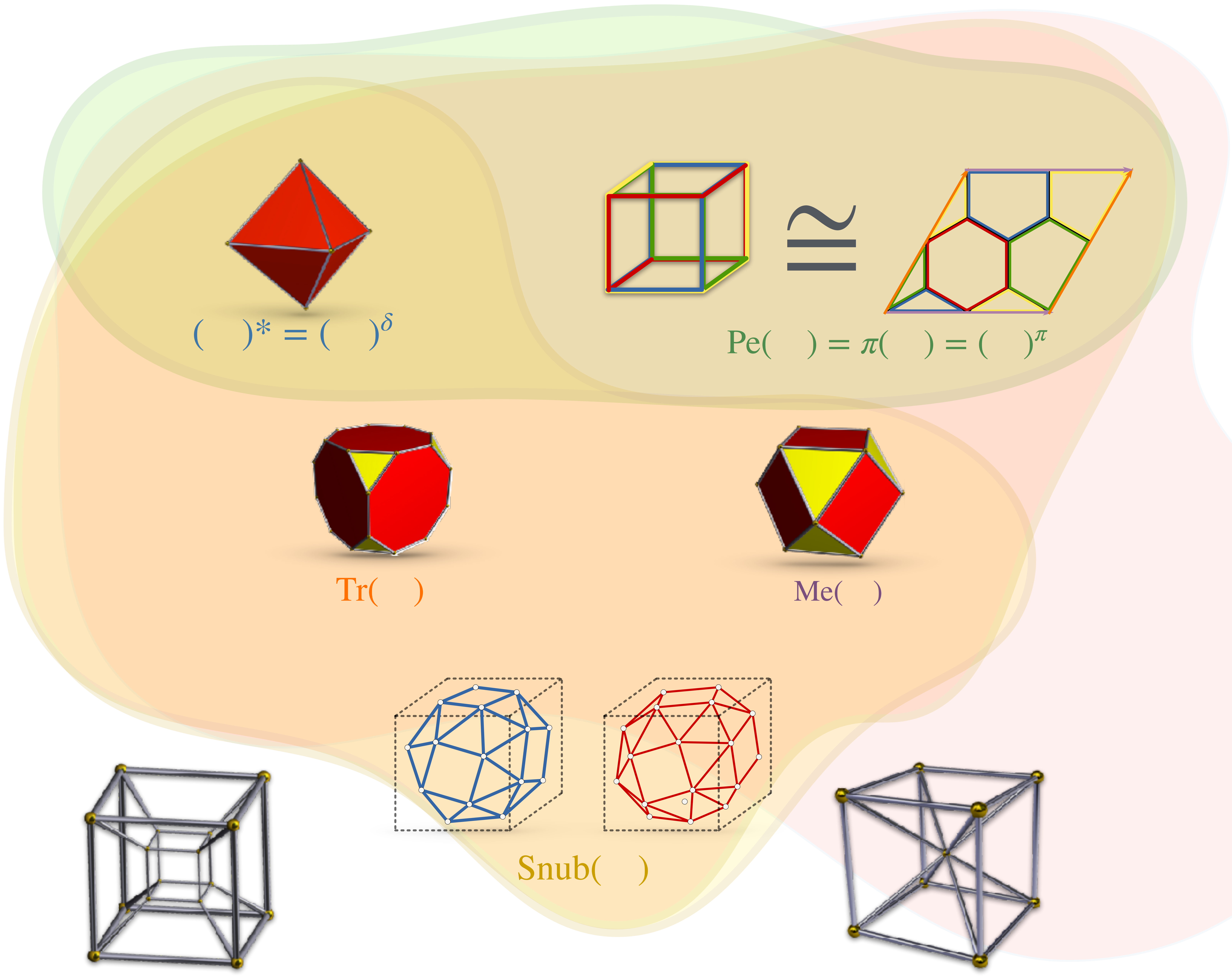


- Automorphism group
- Size (number of flags)
- Surface

operations



- Automorphism group
- Size (number of flags)
- Surface
- Rank (dimension)



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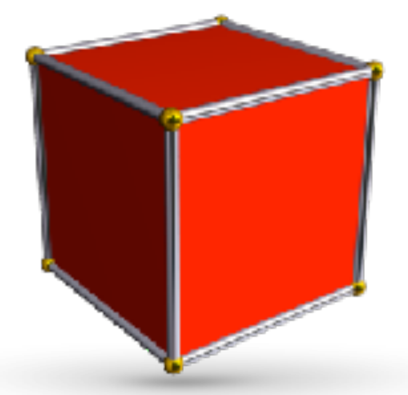
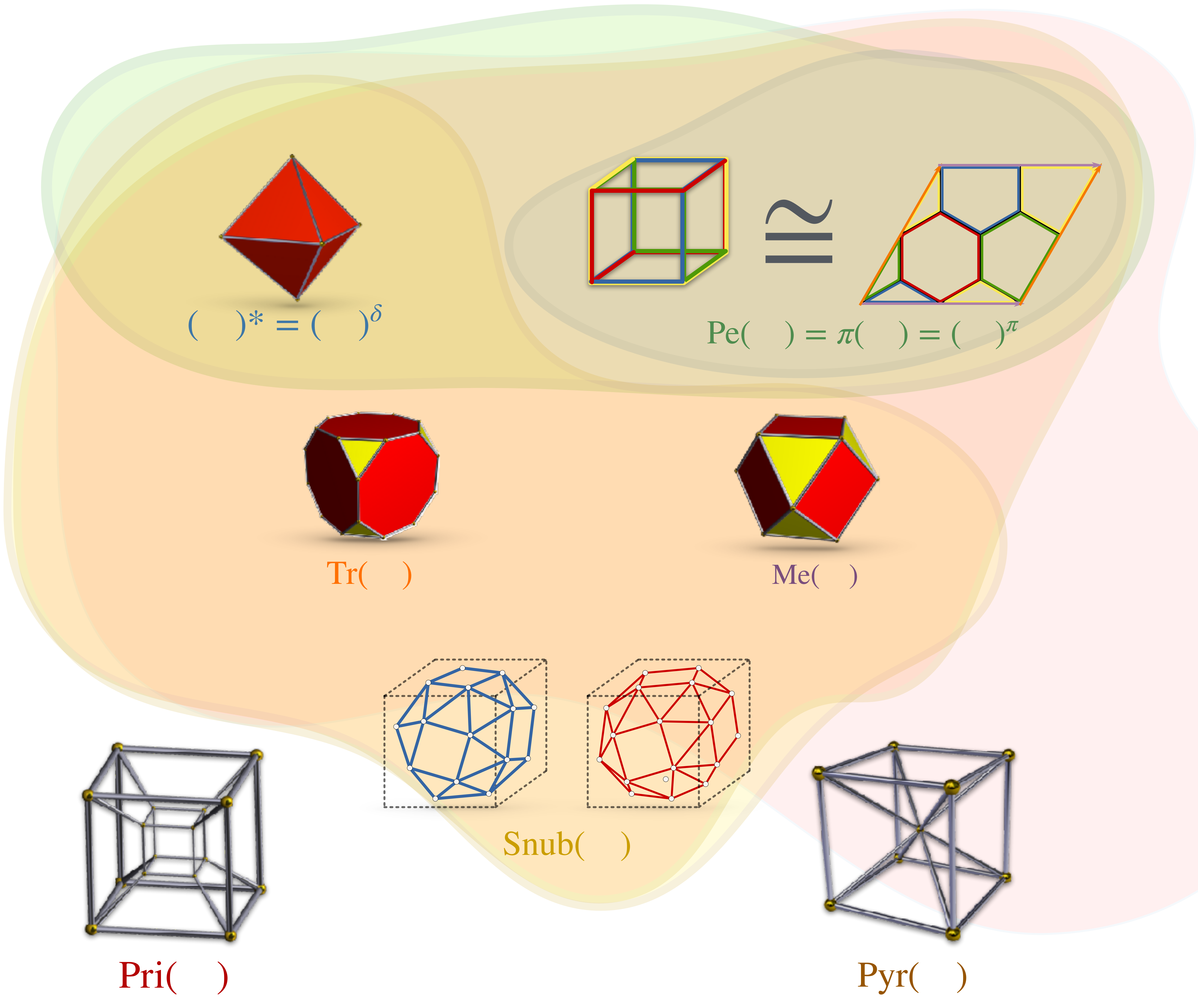


- Automorphism group
- Size (number of flags)
- Surface
- Rank (dimension)
- 1-skeleton



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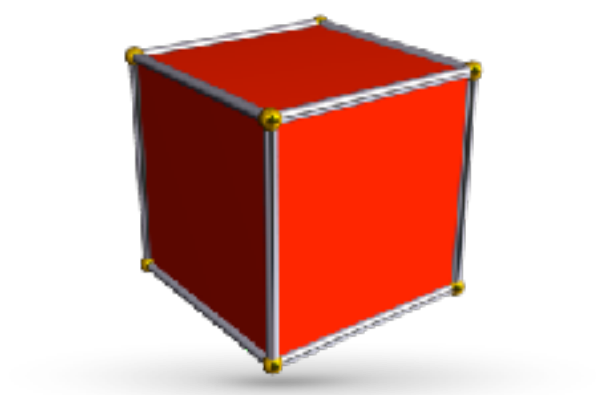

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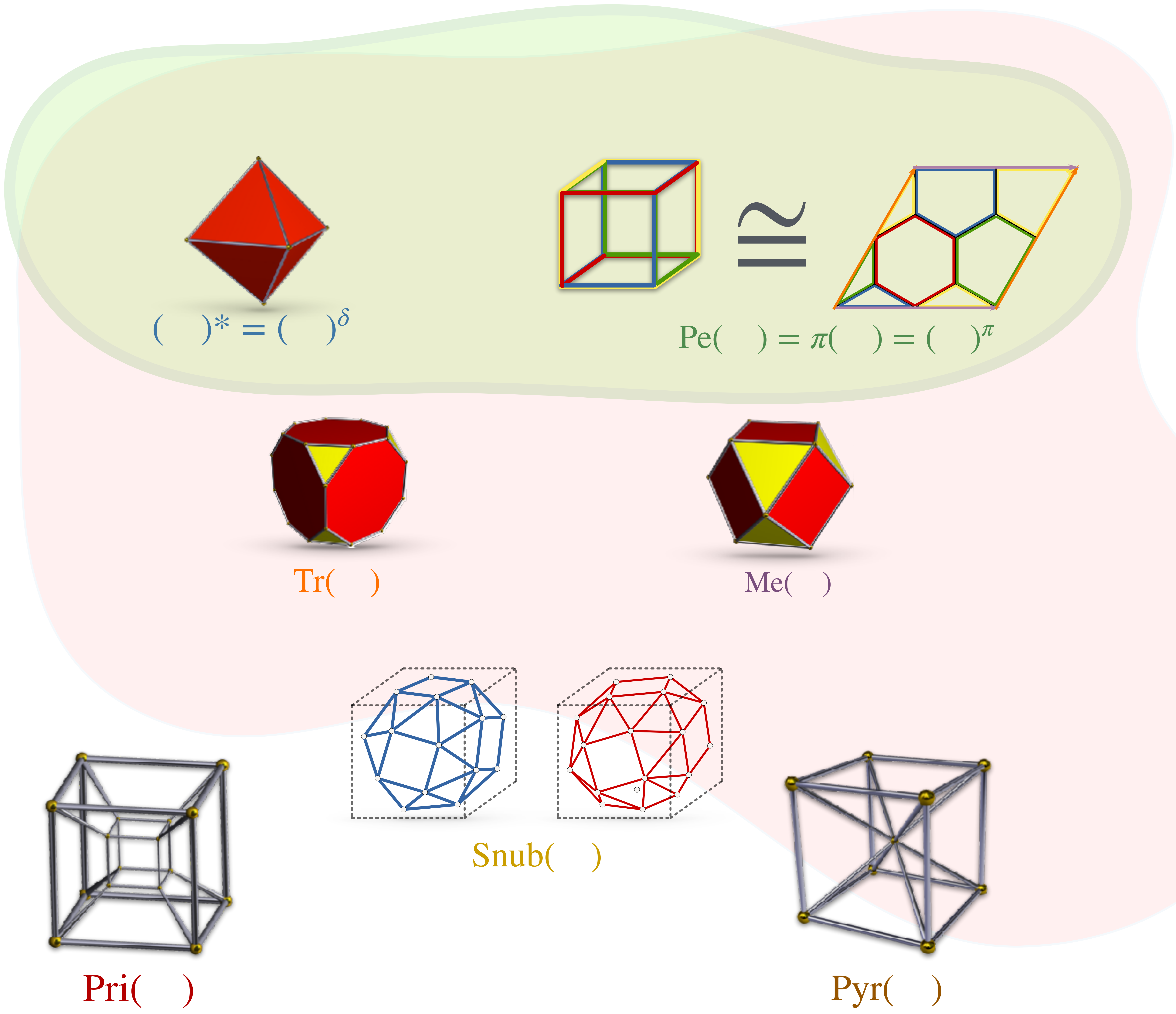

Symmetries of voltage operations on maniplexes and polytopes

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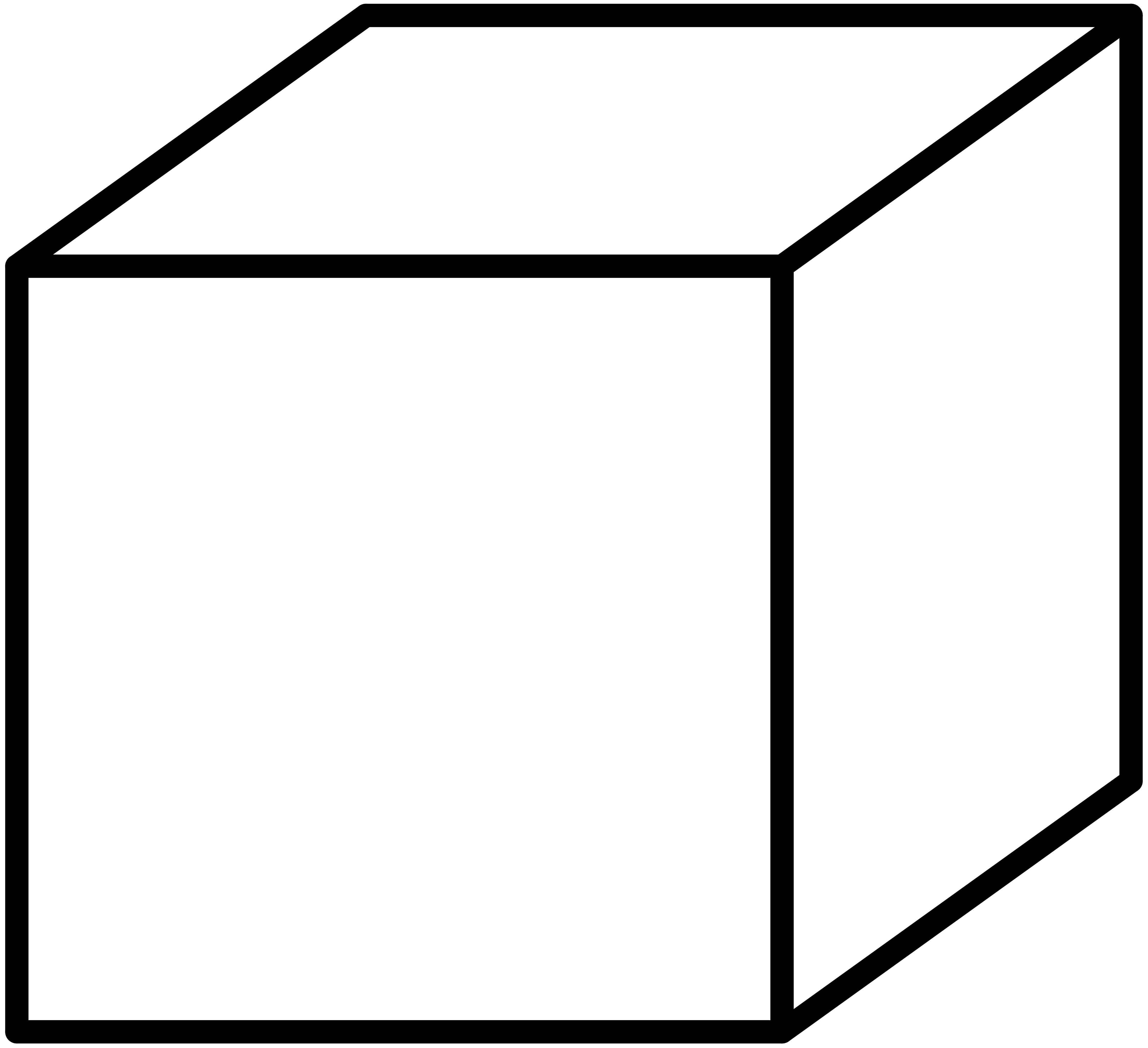
maniplexes and polytopes



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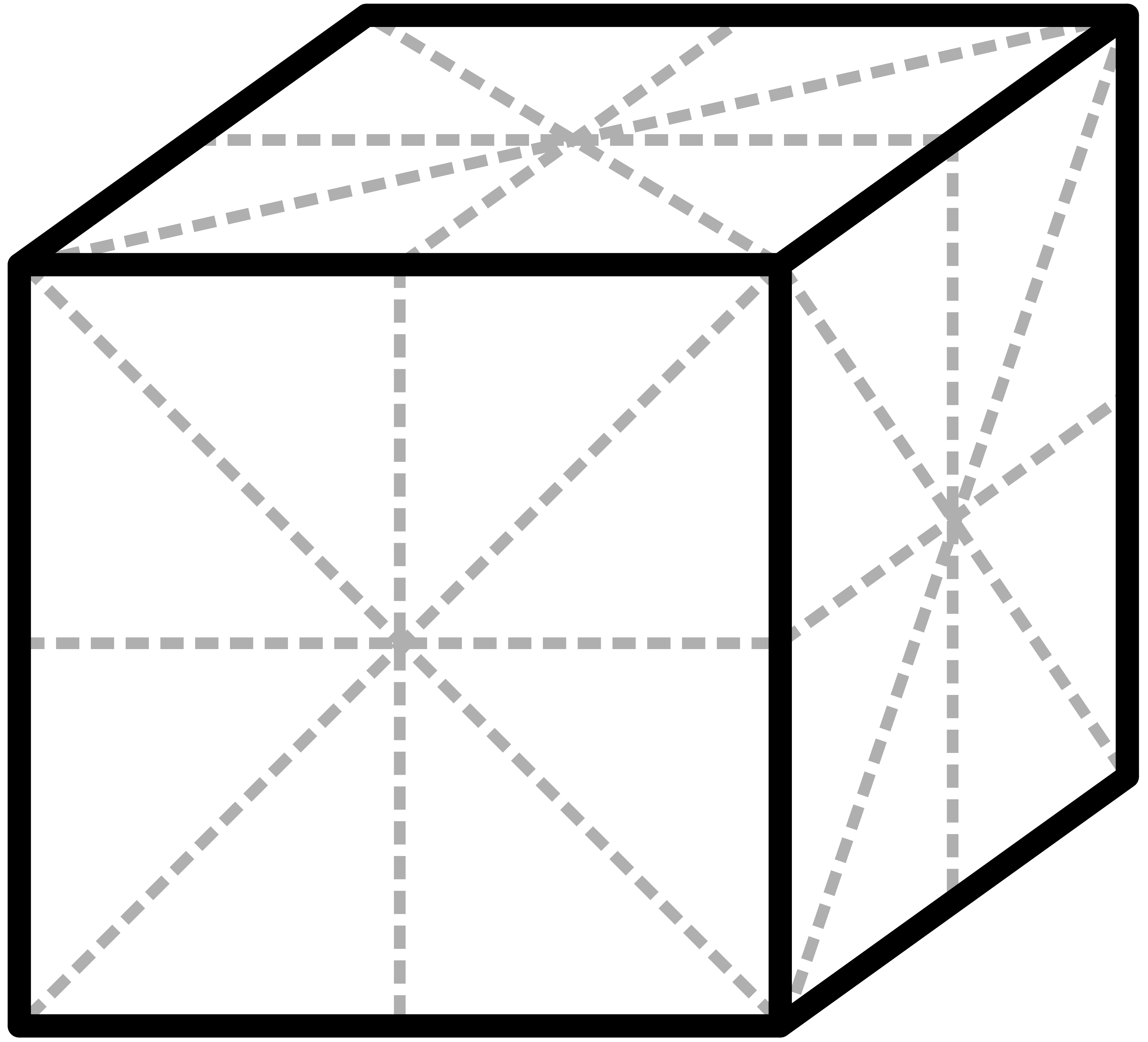

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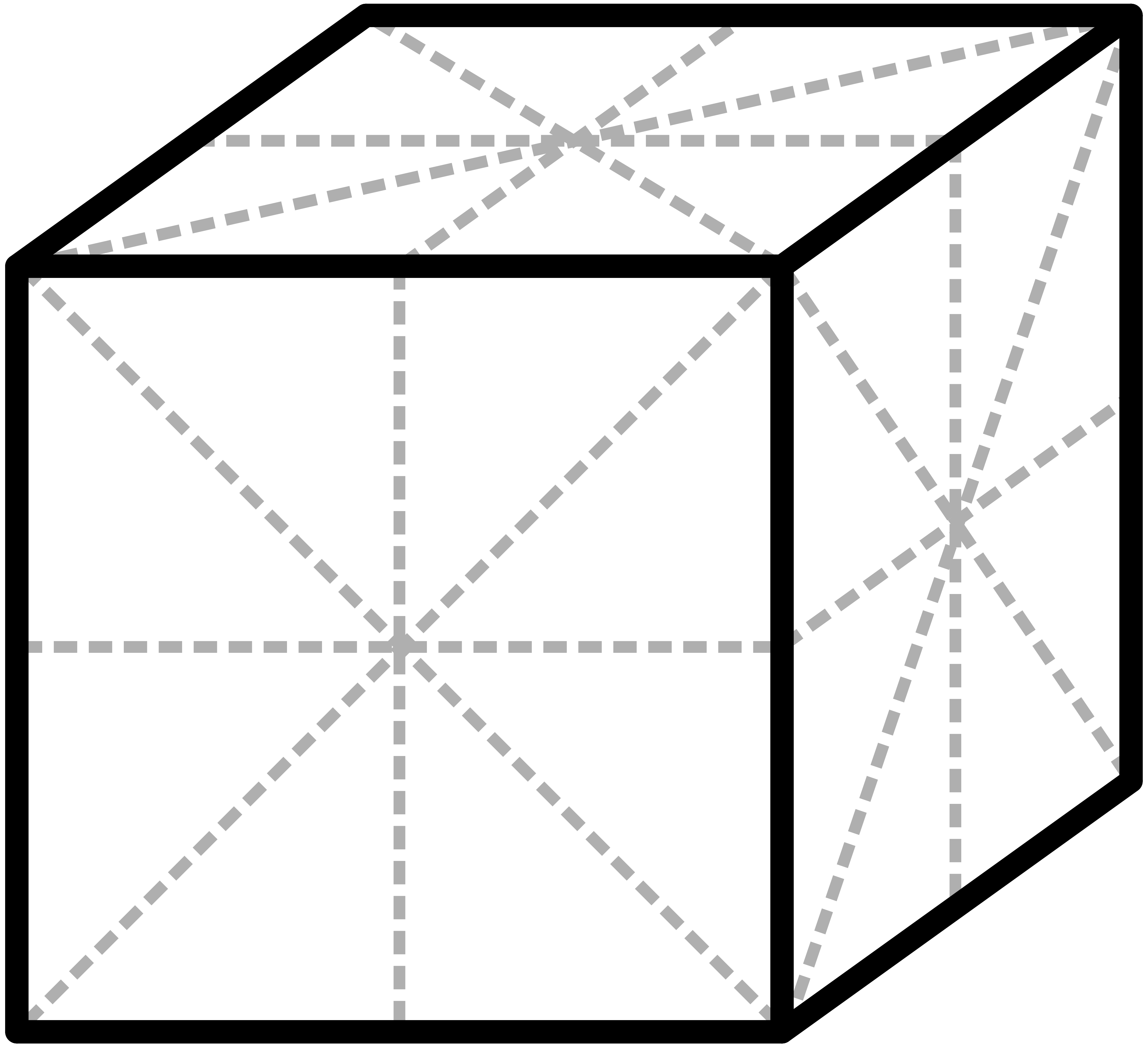

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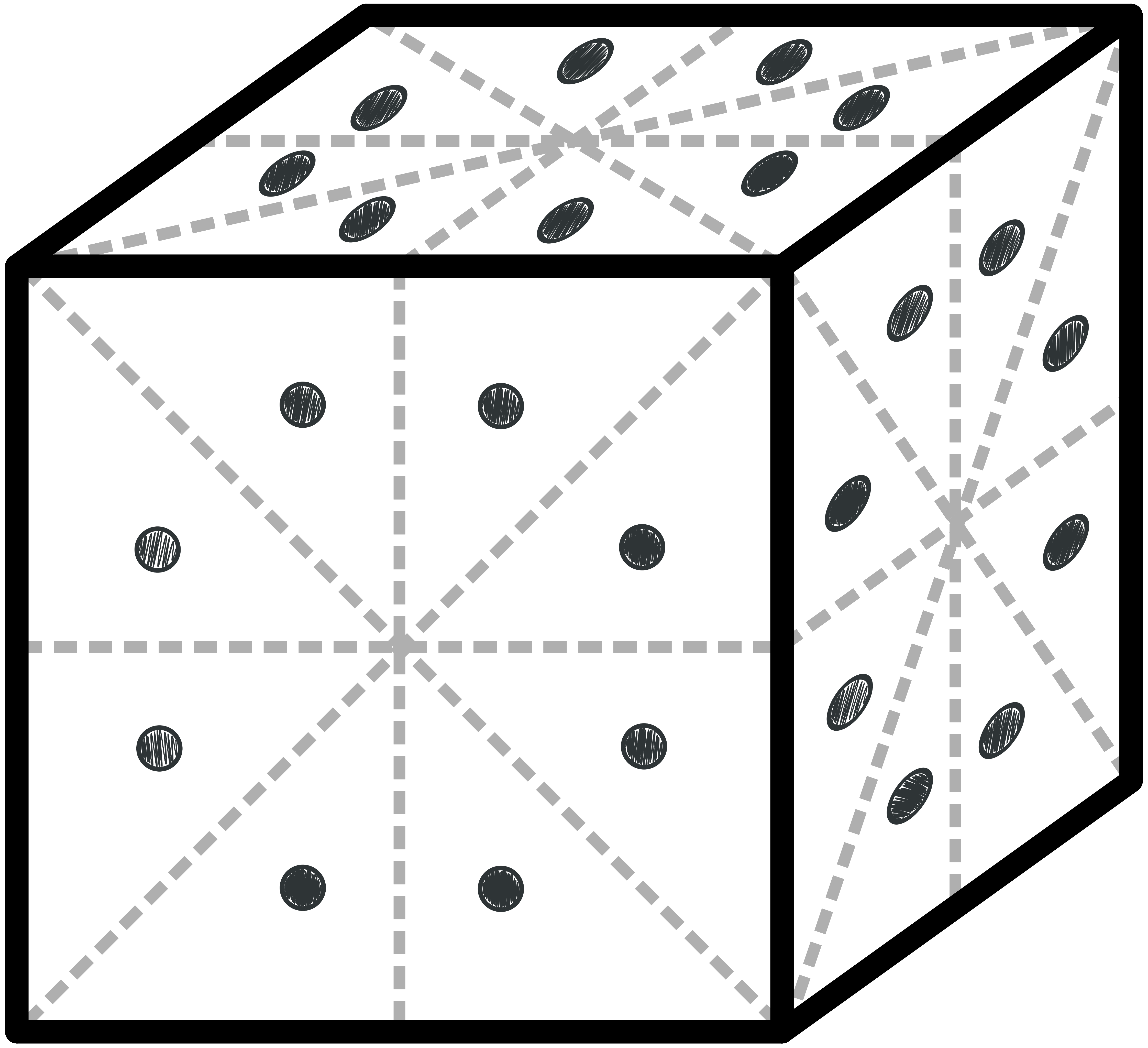

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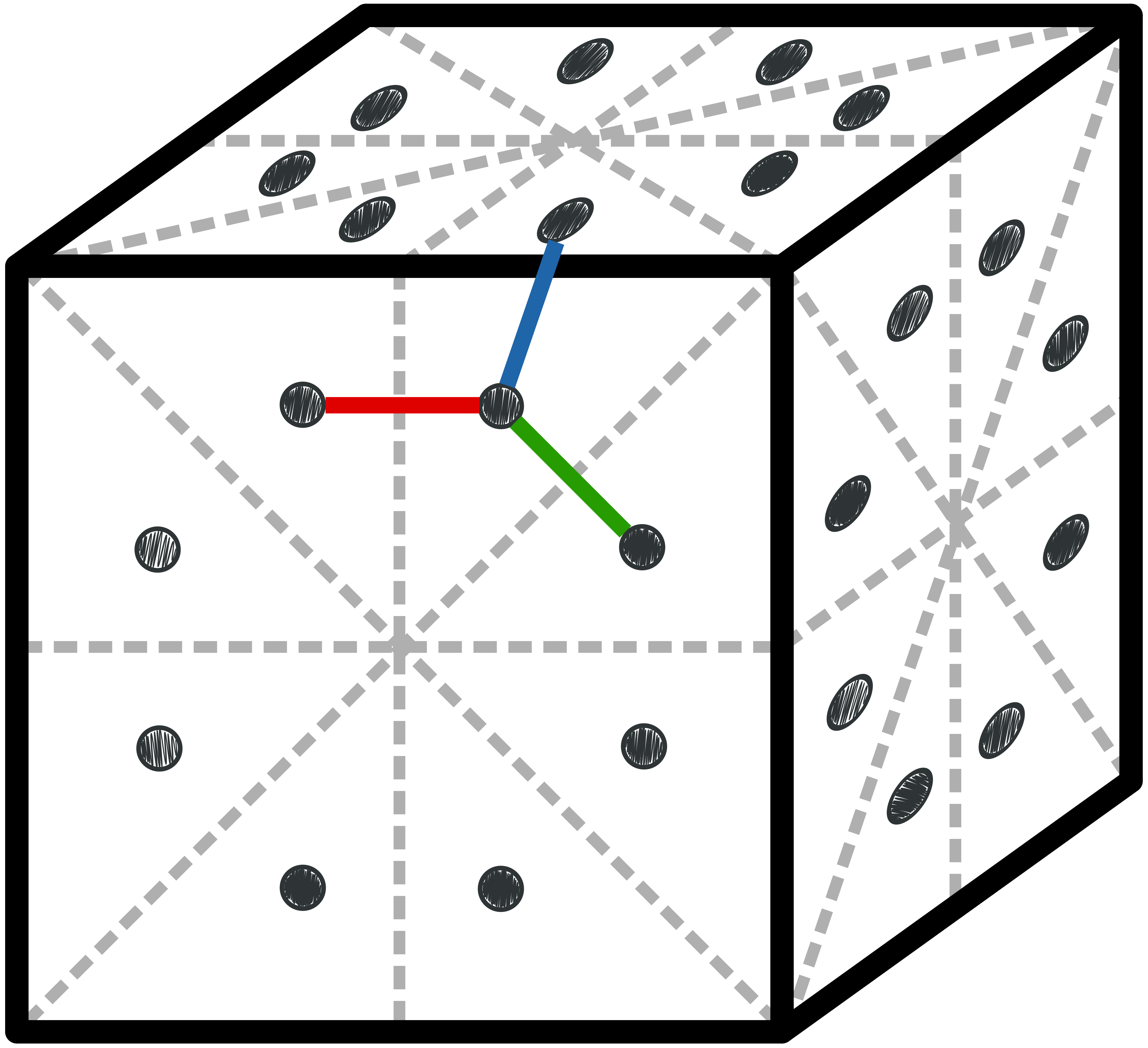

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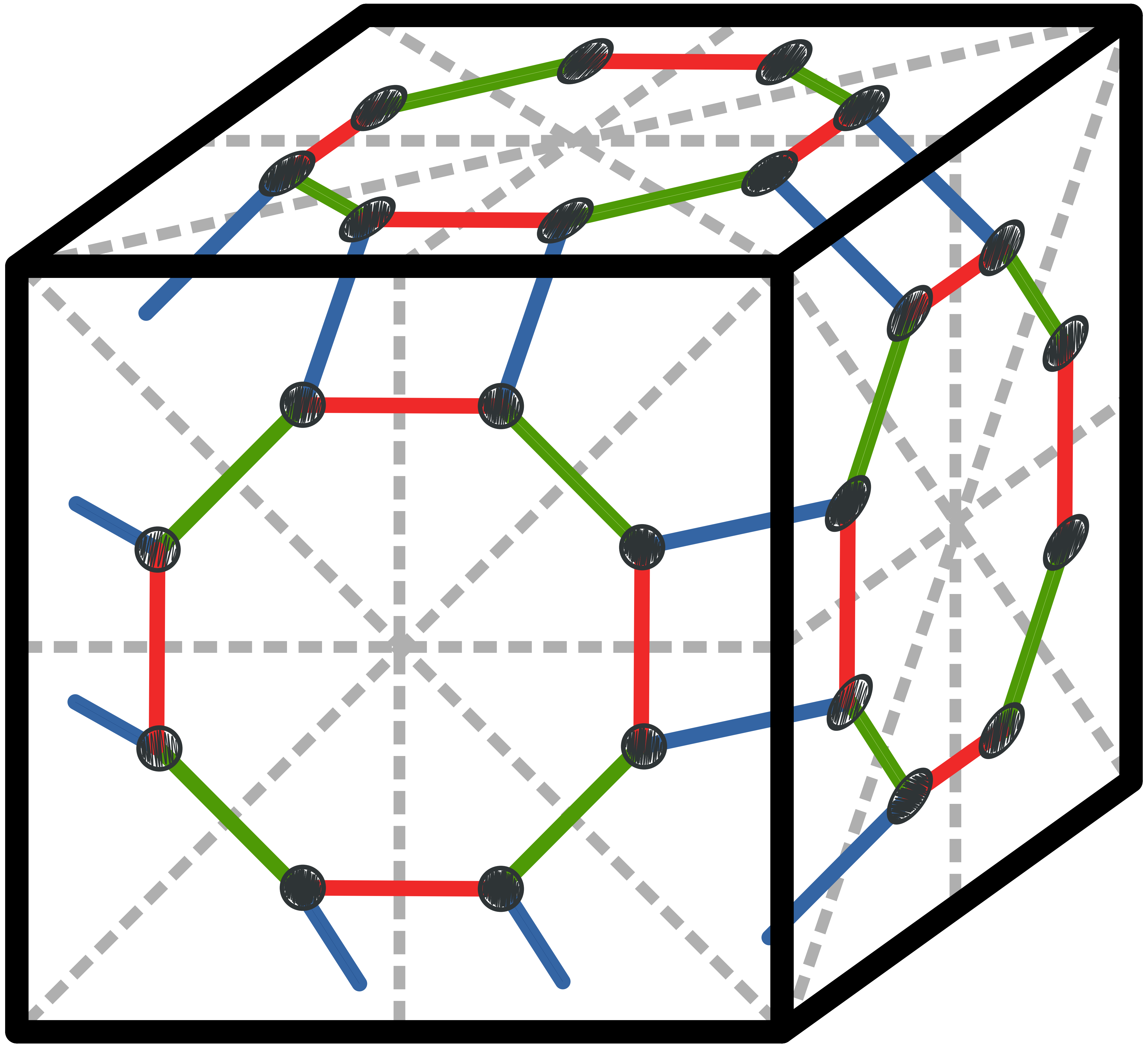
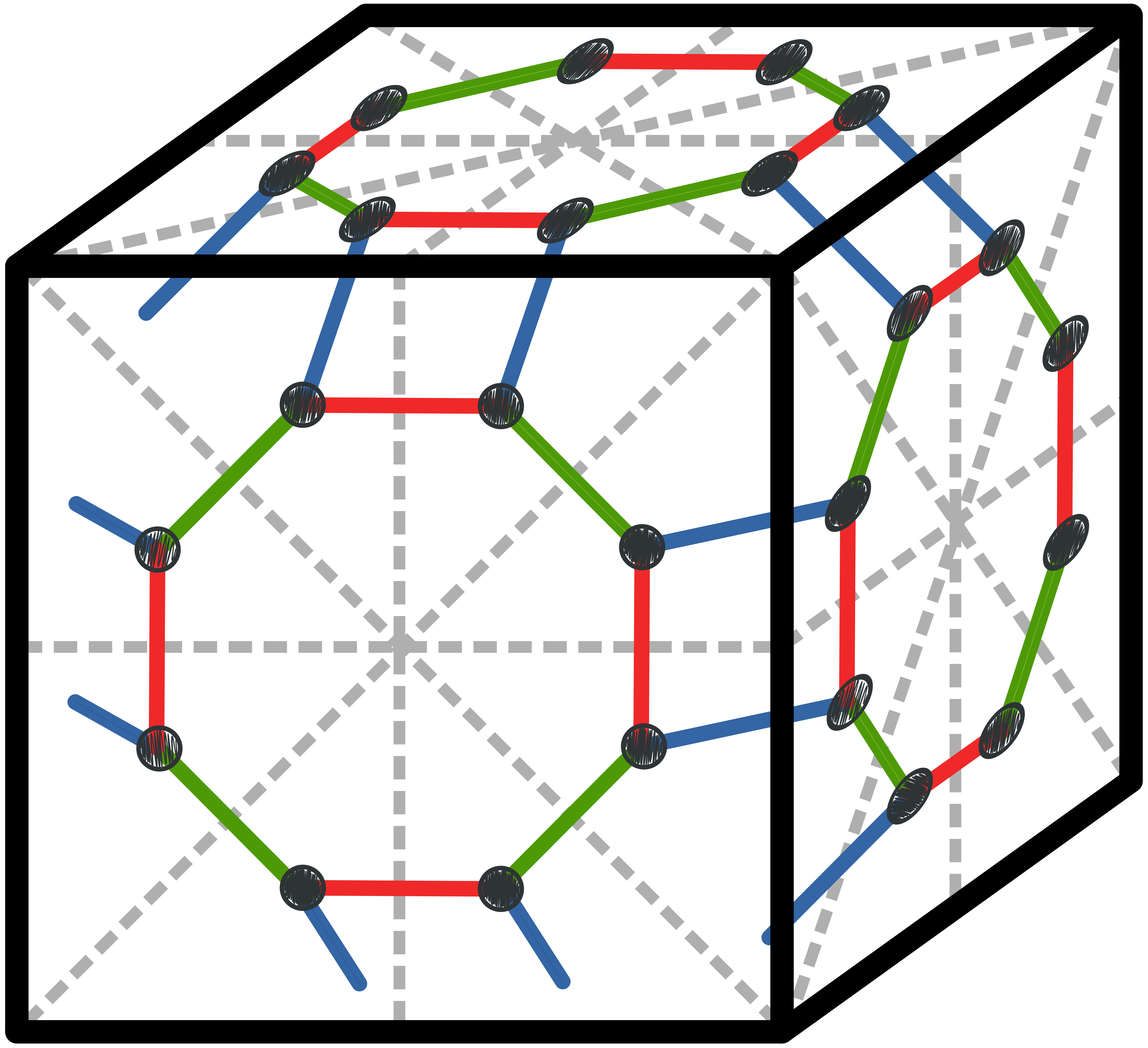


maniplexes and polytopes

The flag graph $\mathscr{F}(\mathscr{M})$ of a map $\mathscr{M}$:

- Connected and simple,
- Valency 3,
- Properly-edge 3-coloured,
- The (0,2,0,2)-paths are alternating squares.

$\mathscr{F}(\mathscr{M})$ contains all the combinatorial information of $\mathscr{M}$



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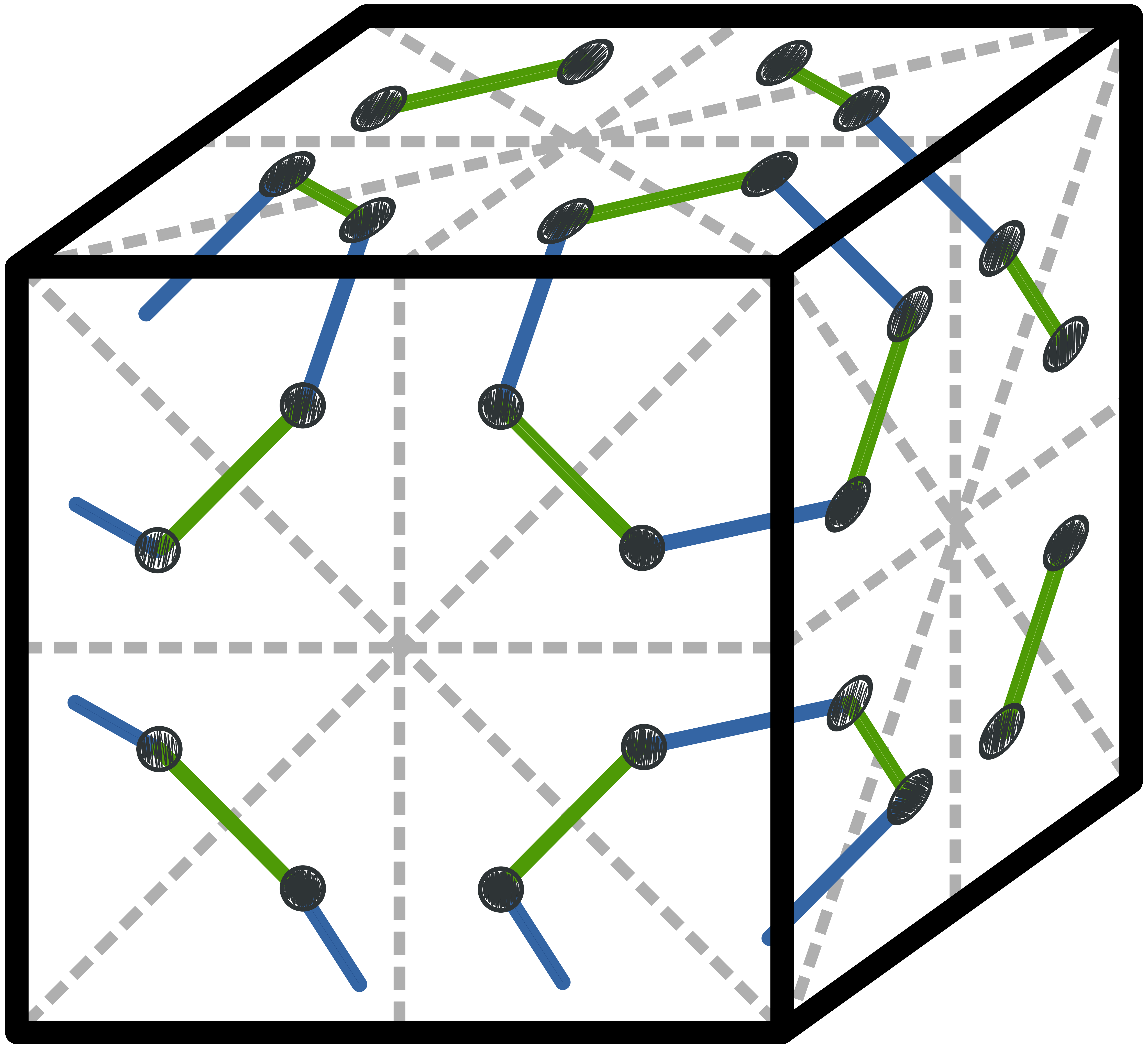

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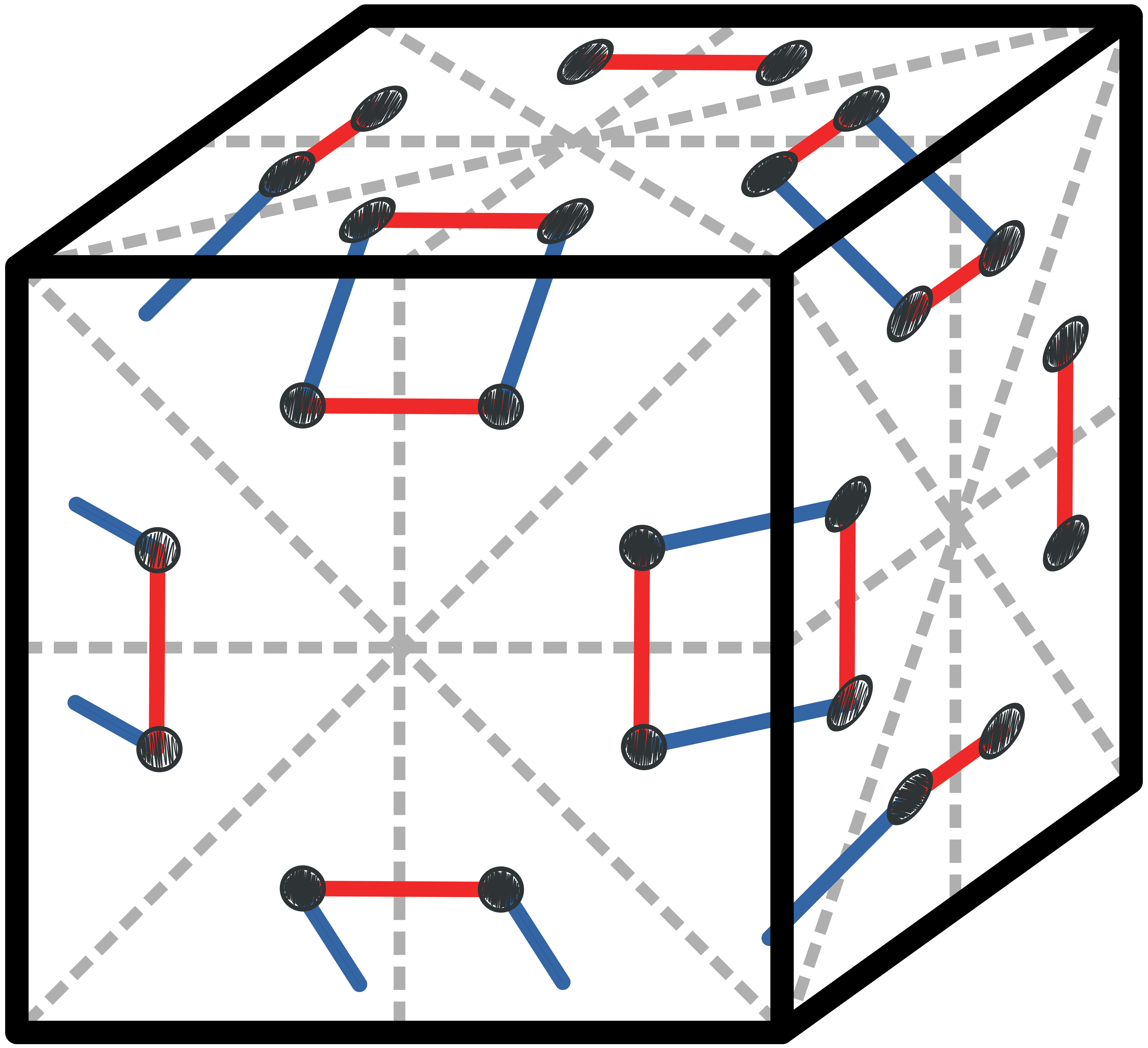

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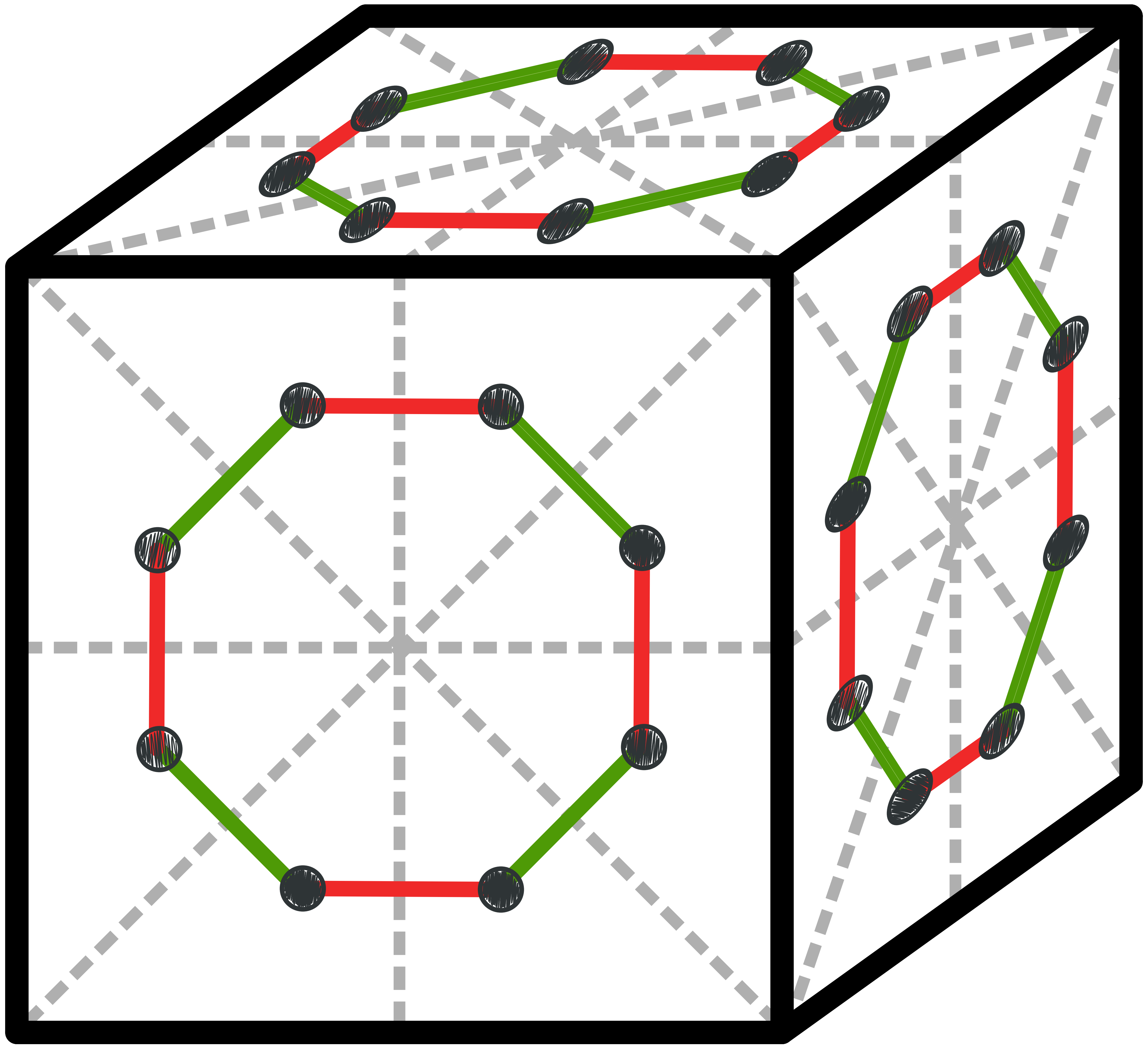

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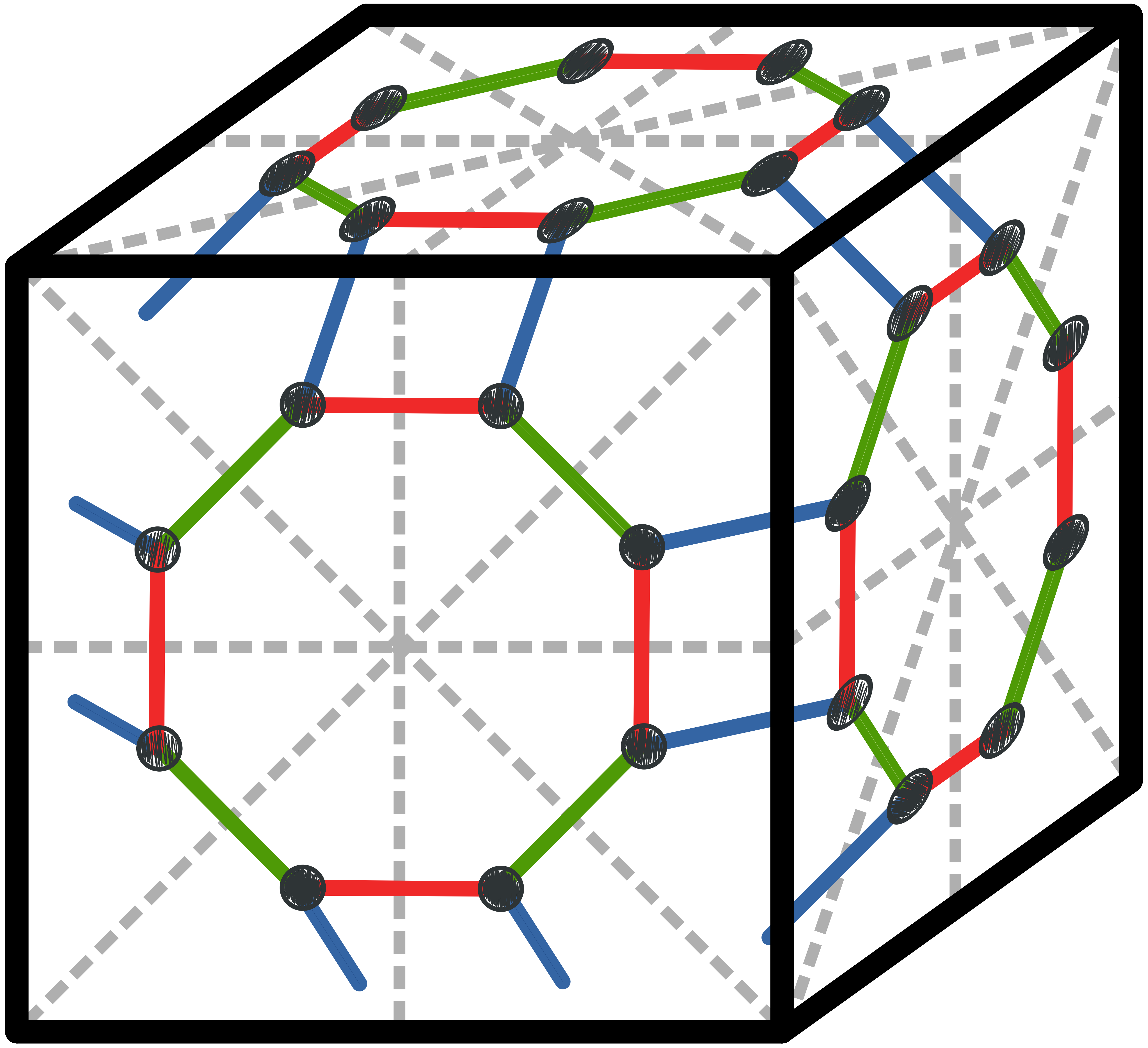

maniplexes and polytopes

A 3-maniplex is a graph $\mathscr{M}$:

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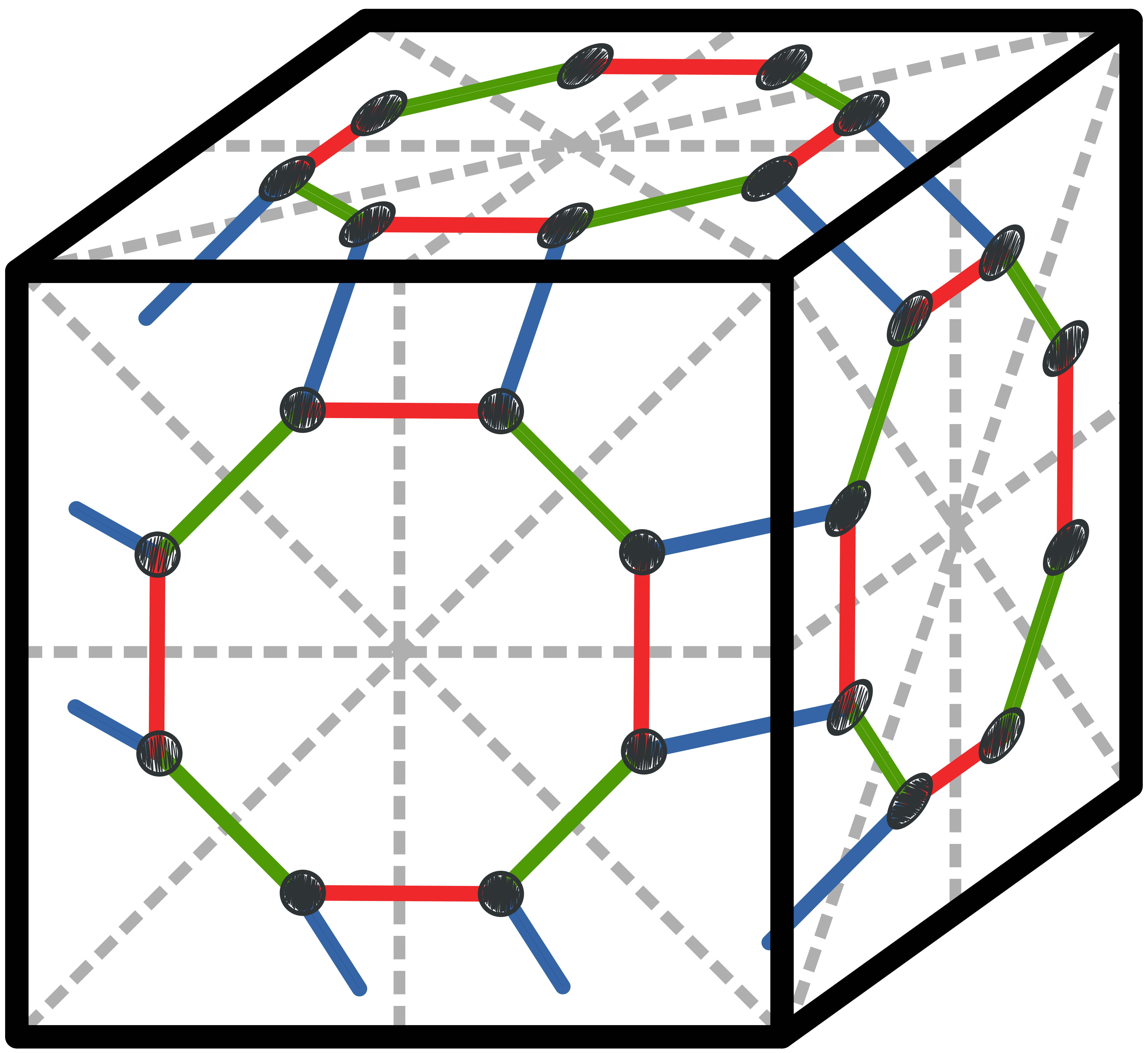
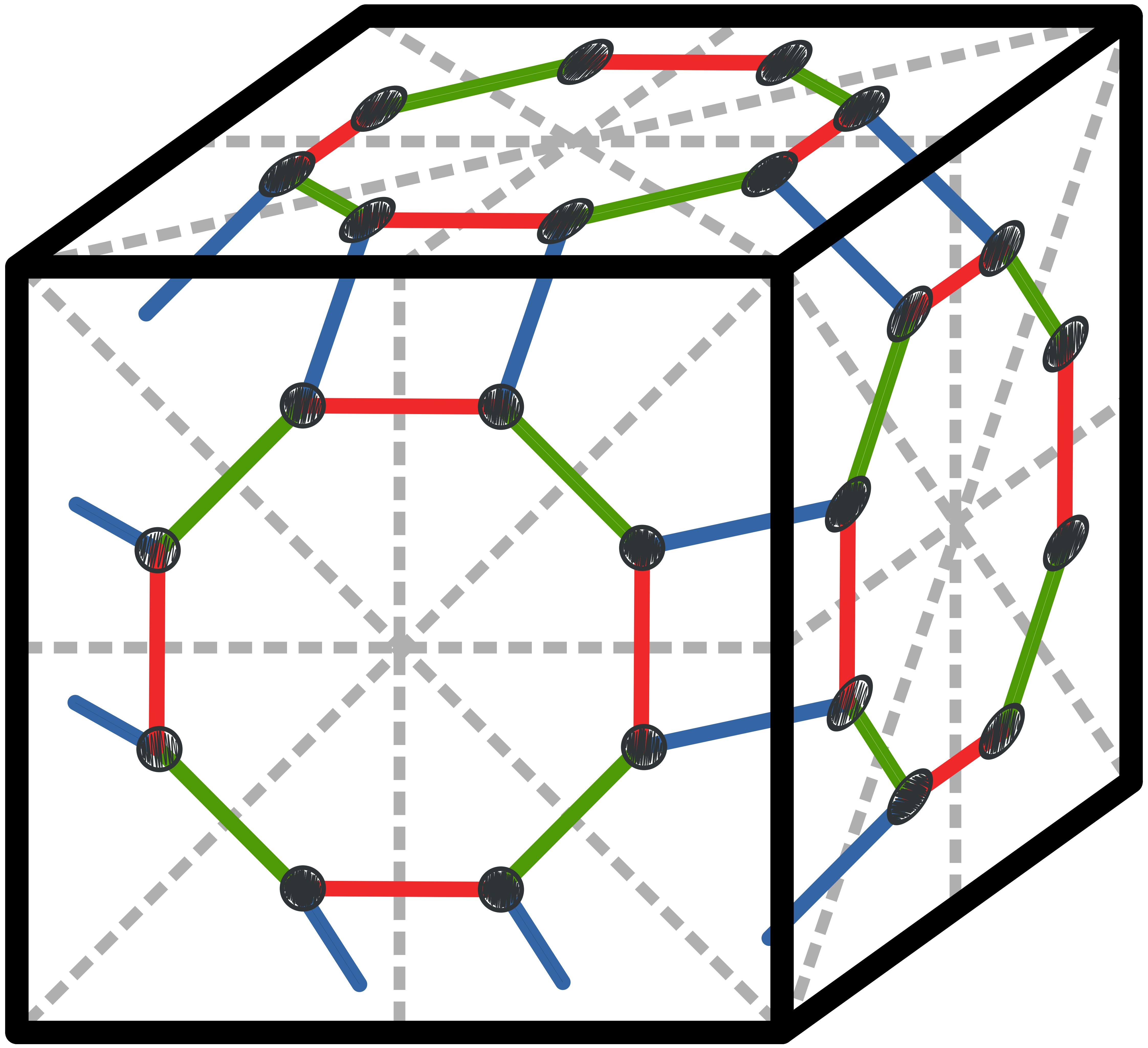


maniplexes and polytopes

A n-maniplex is a graph $\mathscr{M}$:

- Connected and simple,
- Valency n,
- Properly-edge n-coloured,
- The (i, j, i, j)-paths are alternating squares, whenever $|i - j| > 1$



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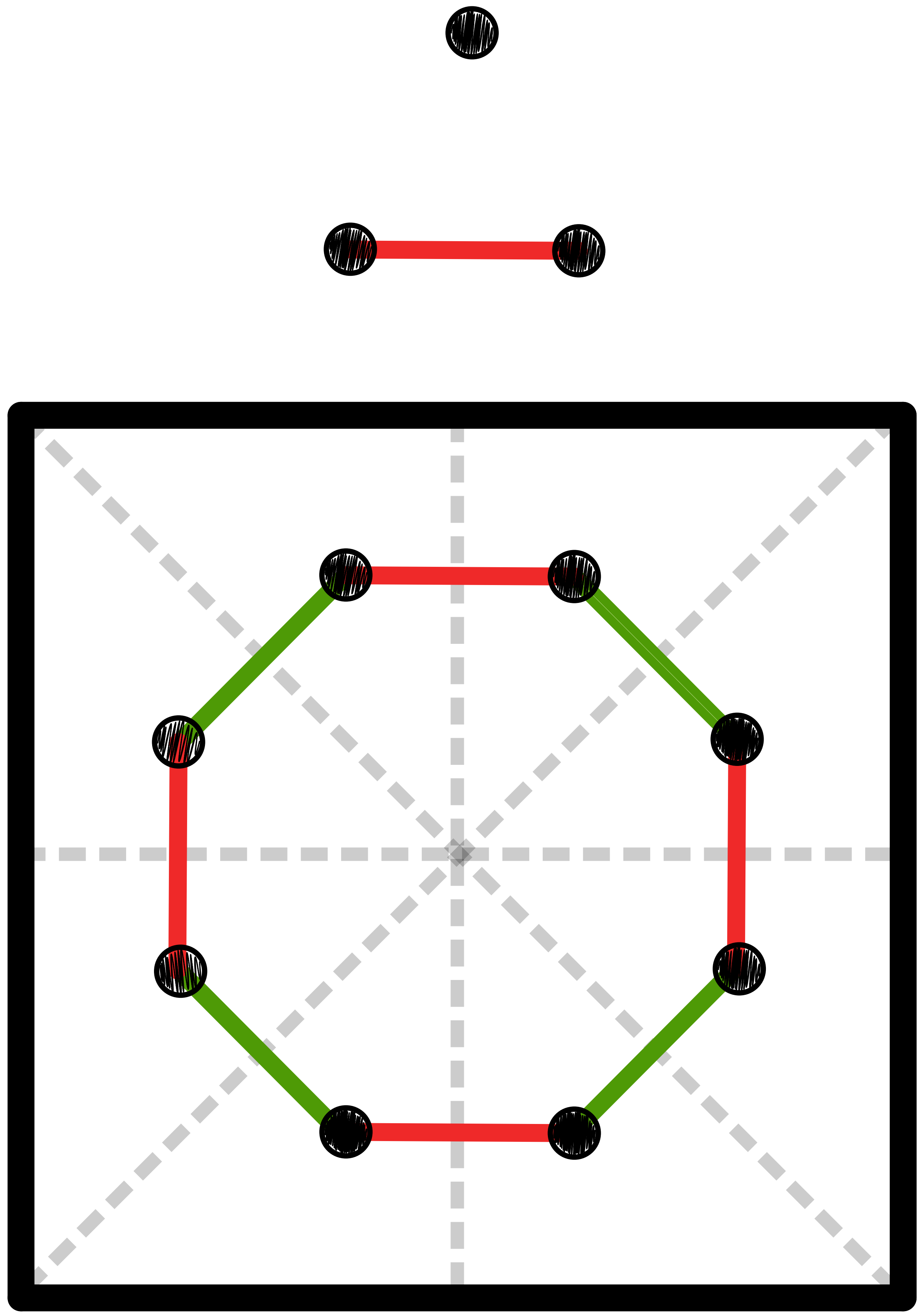

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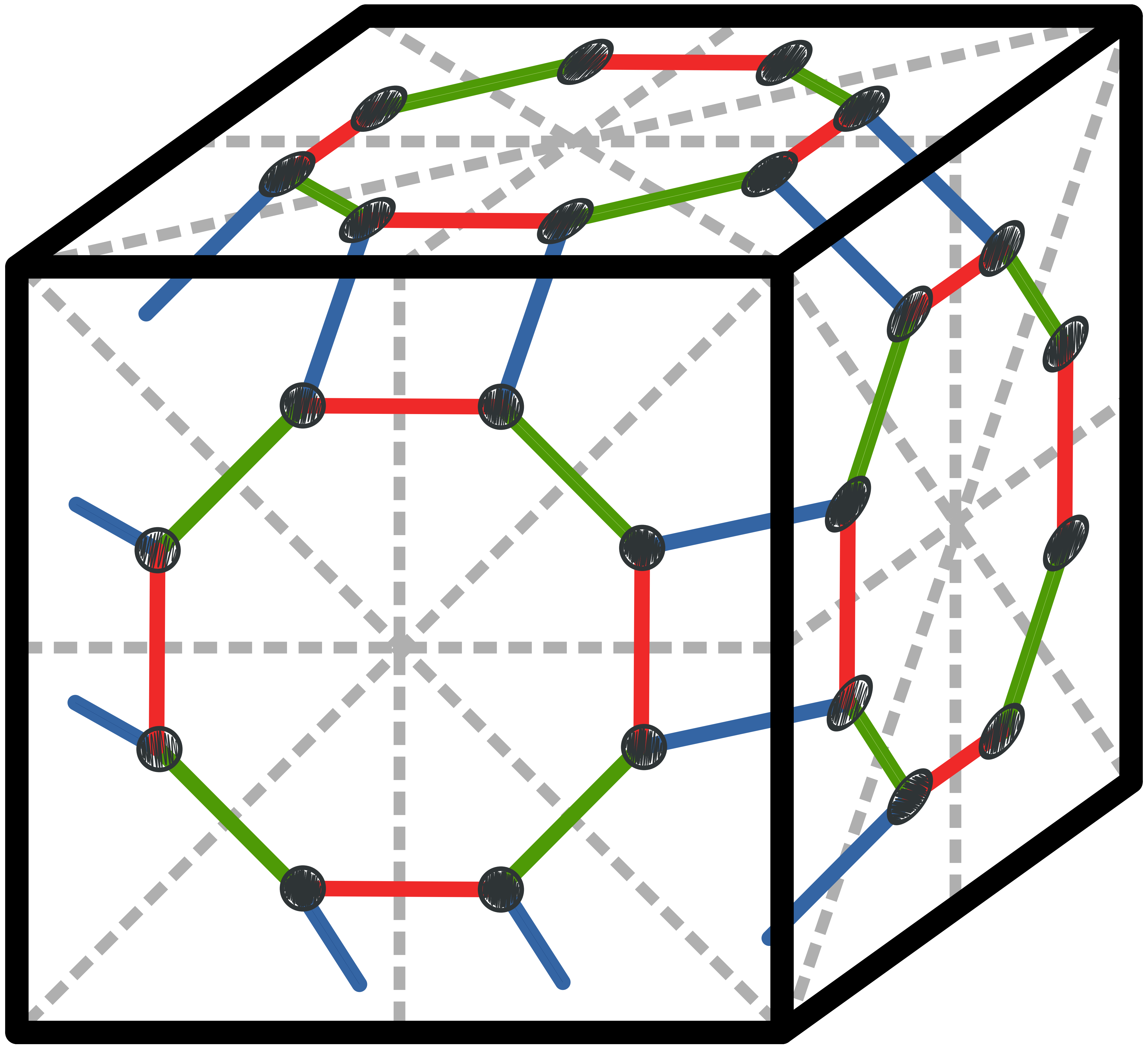

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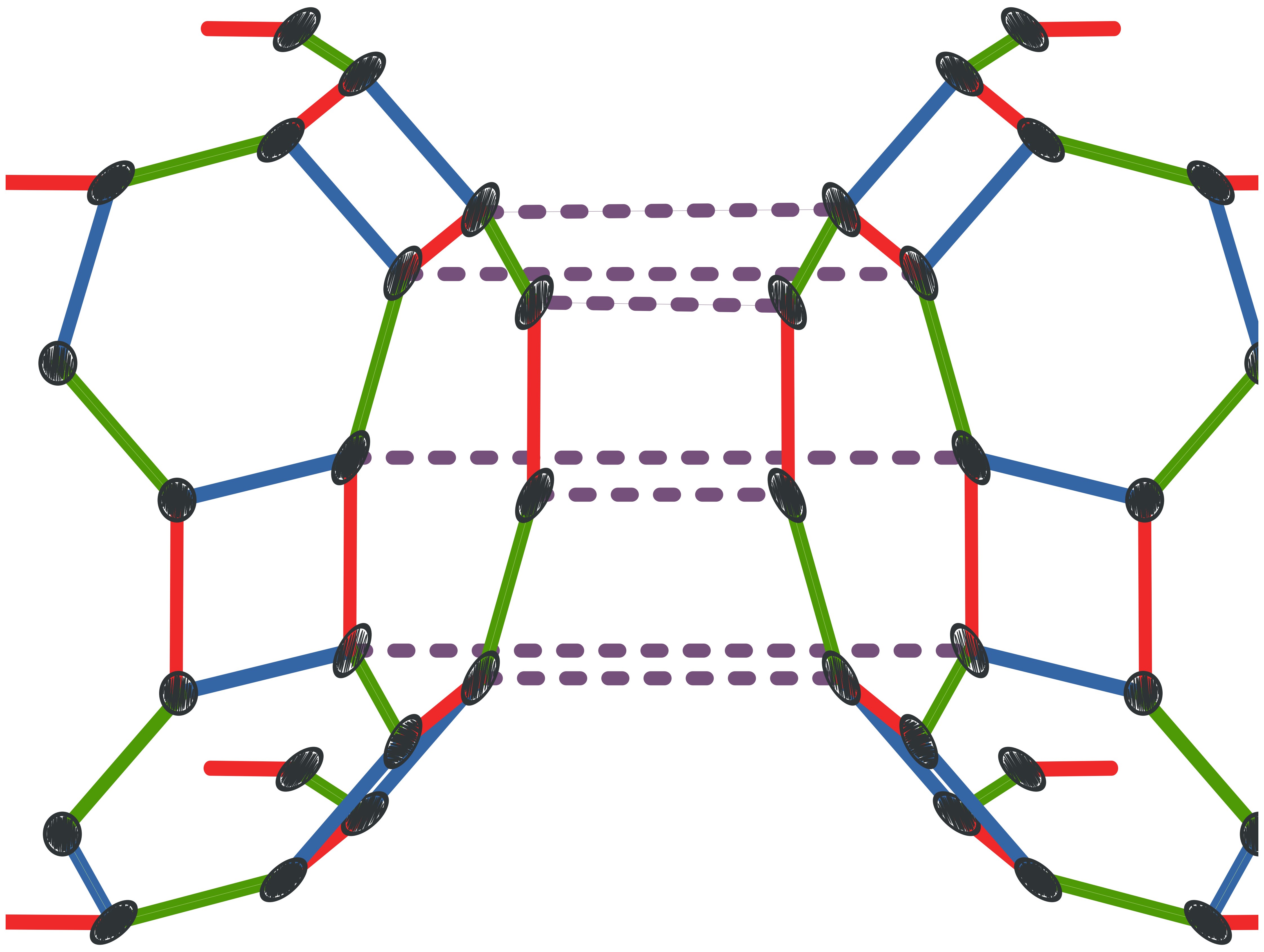

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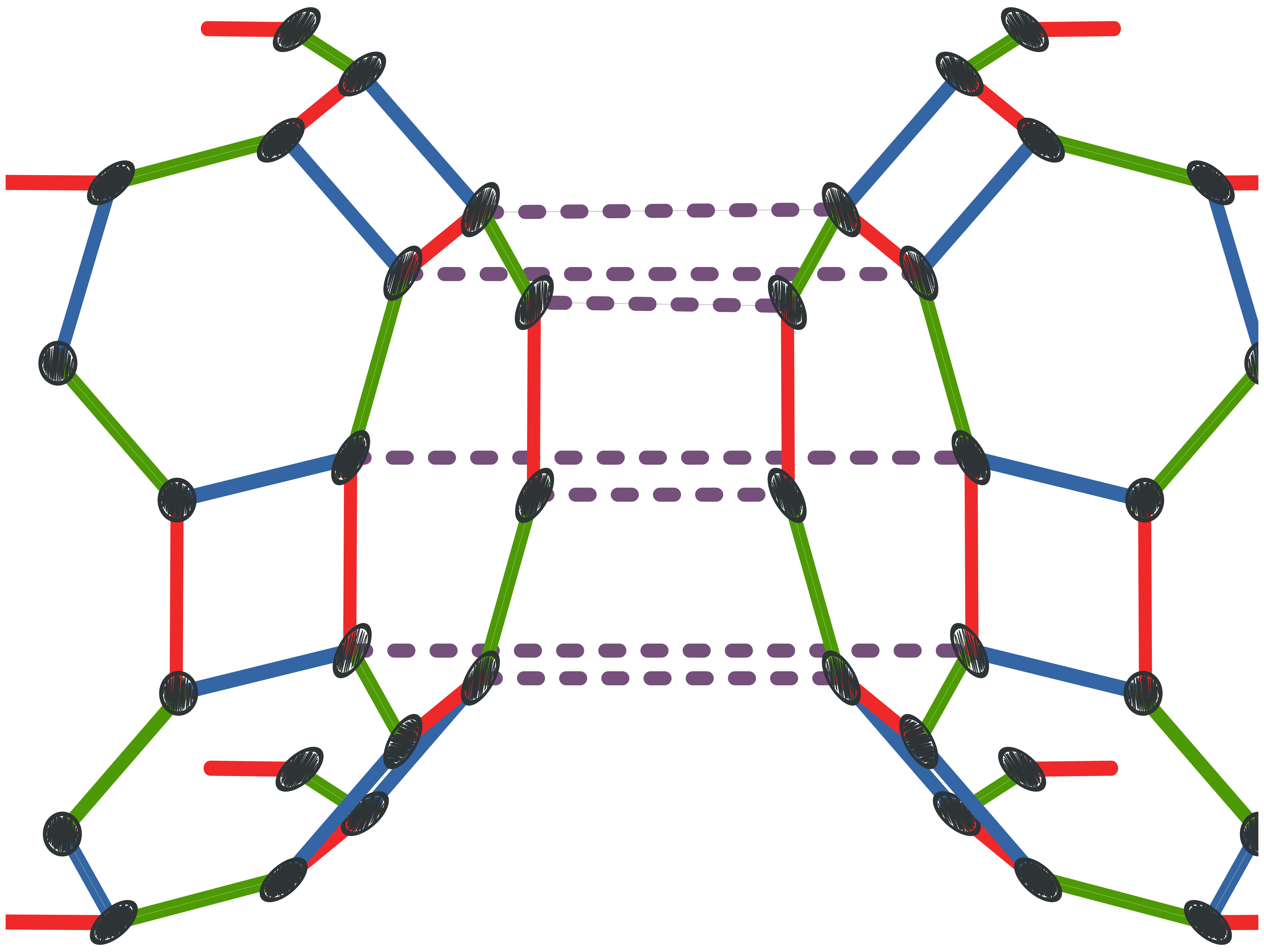
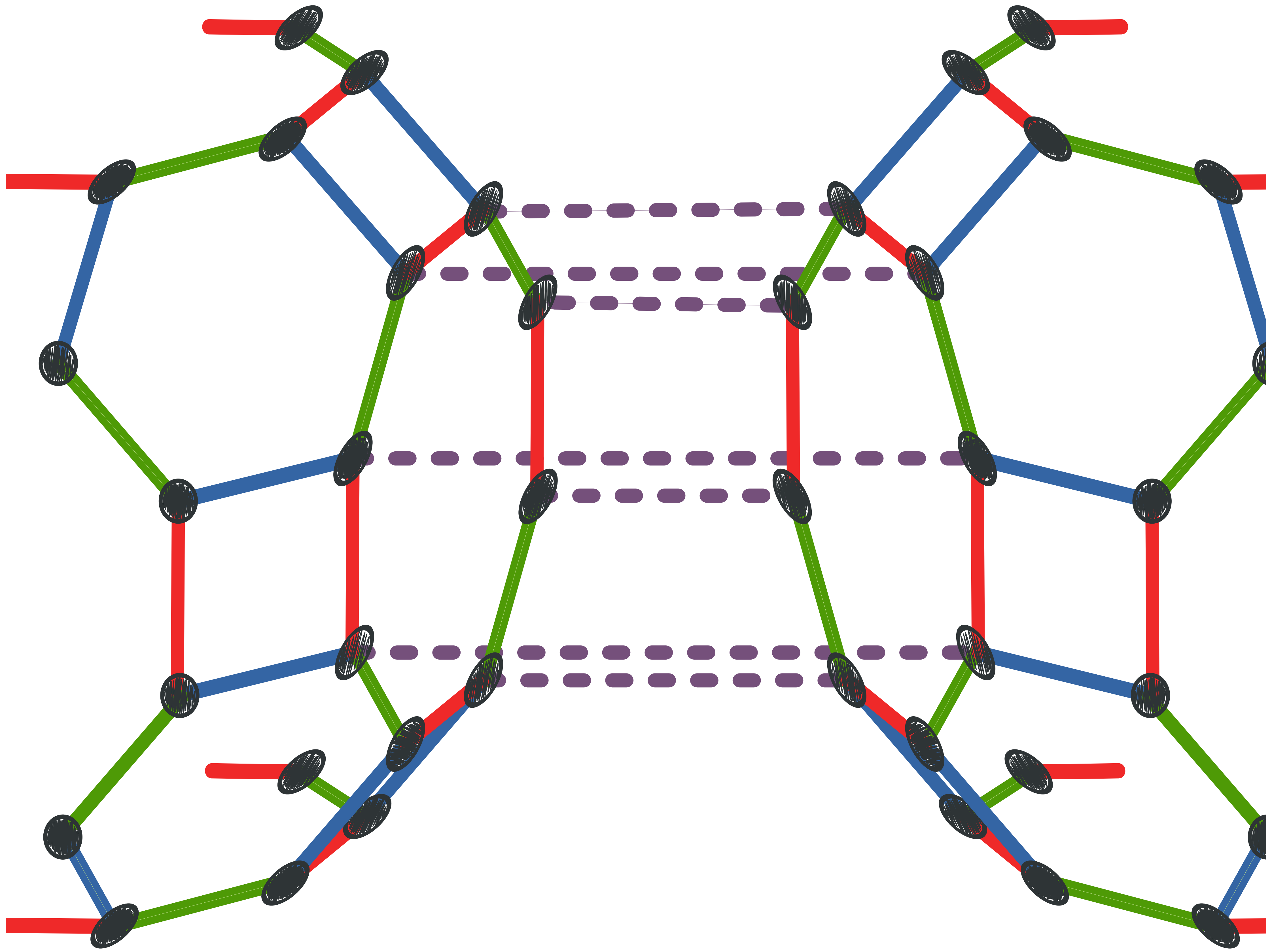


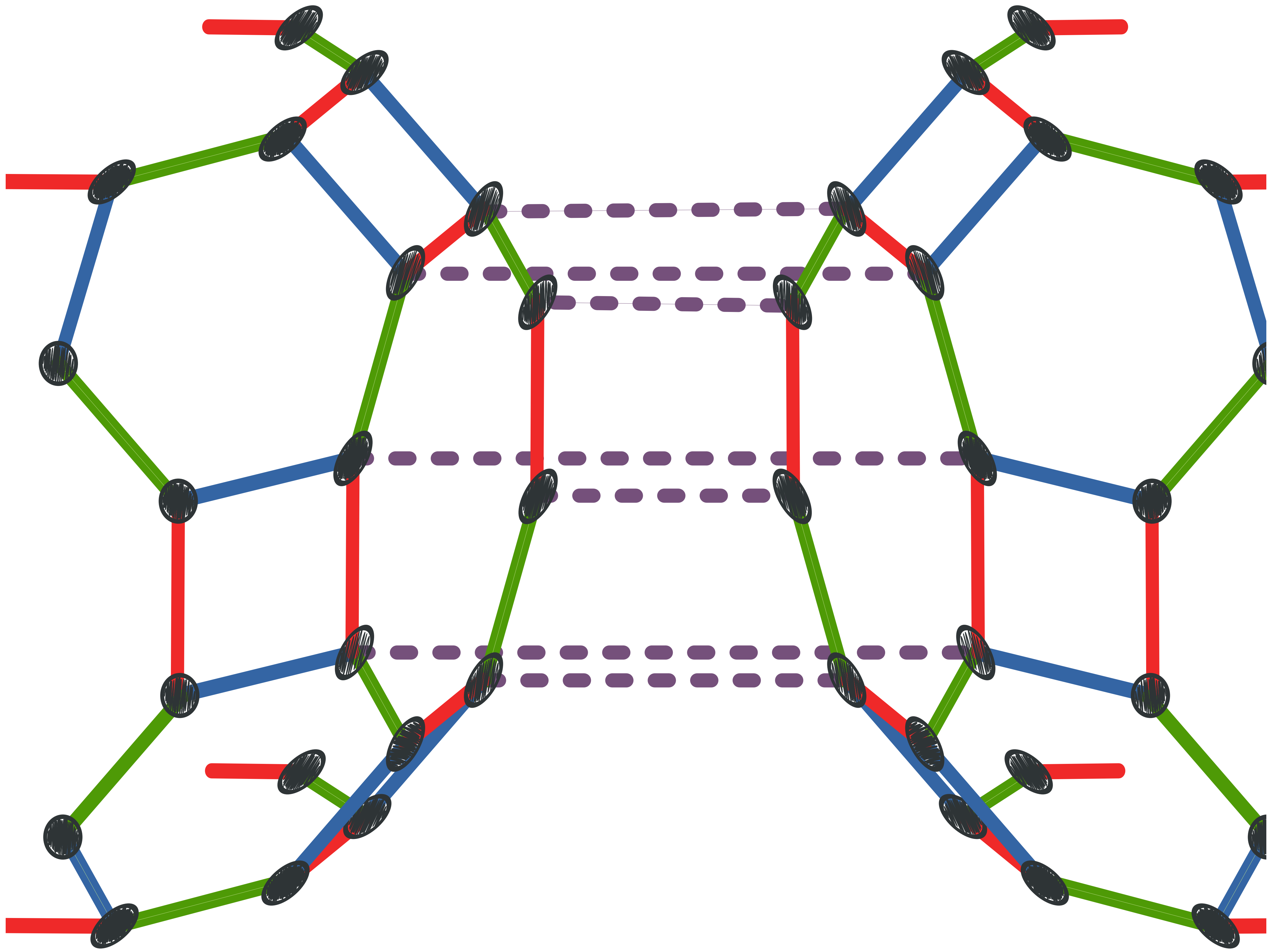
Hubard, Garza-Vargas (2017):
Abstract polytopes are faithful maniplexes that satisfy the PIC

Gabe might or might not have talked about this.

maniplexes and polytopes

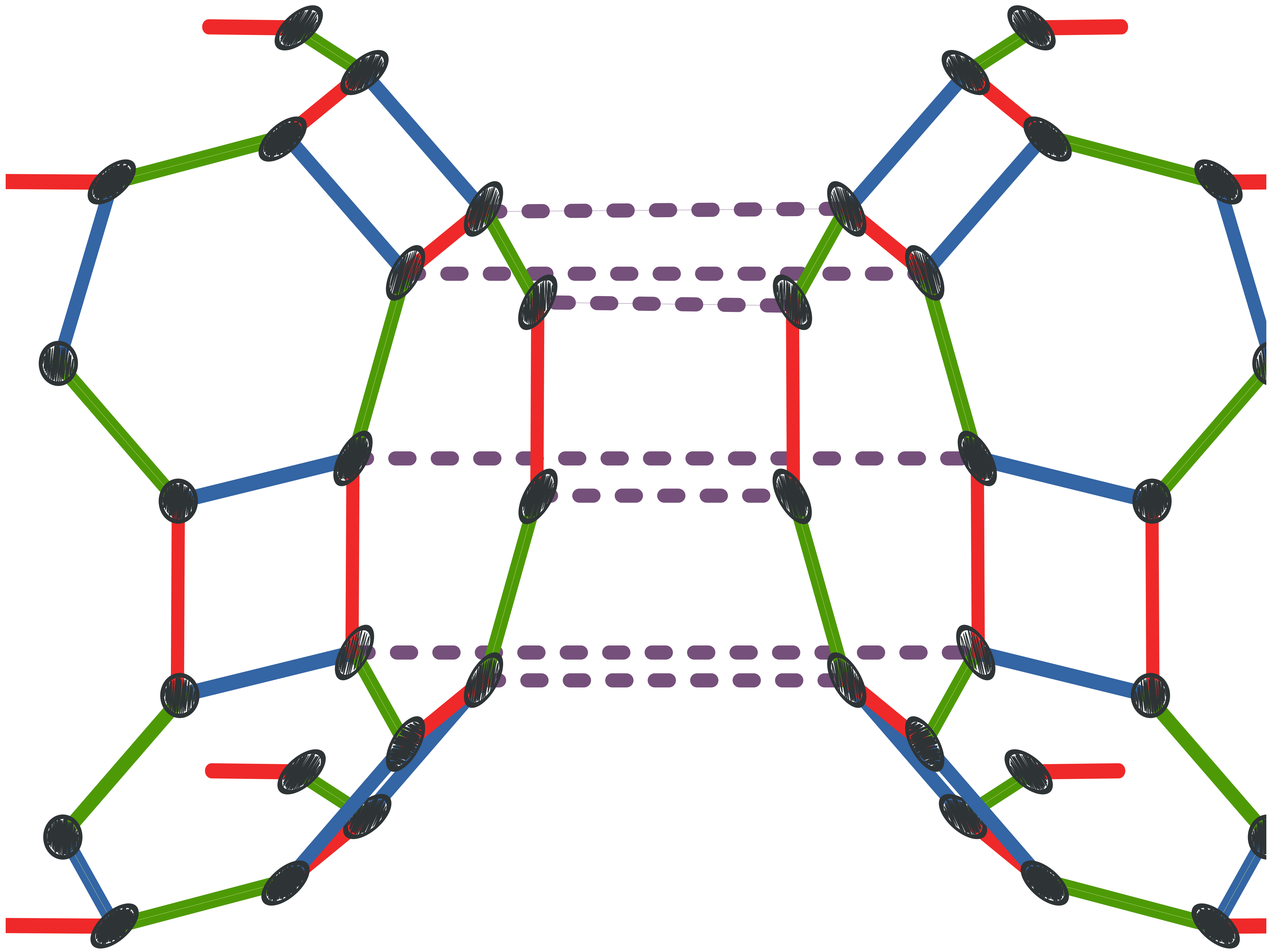


maniplexes and polytopes



- An automorphism of $\mathscr{M}$ is a colour-preserving automorphism.
- The group Aut($\mathscr{M}$) acts freely on the flags.

maniplexes and polytopes



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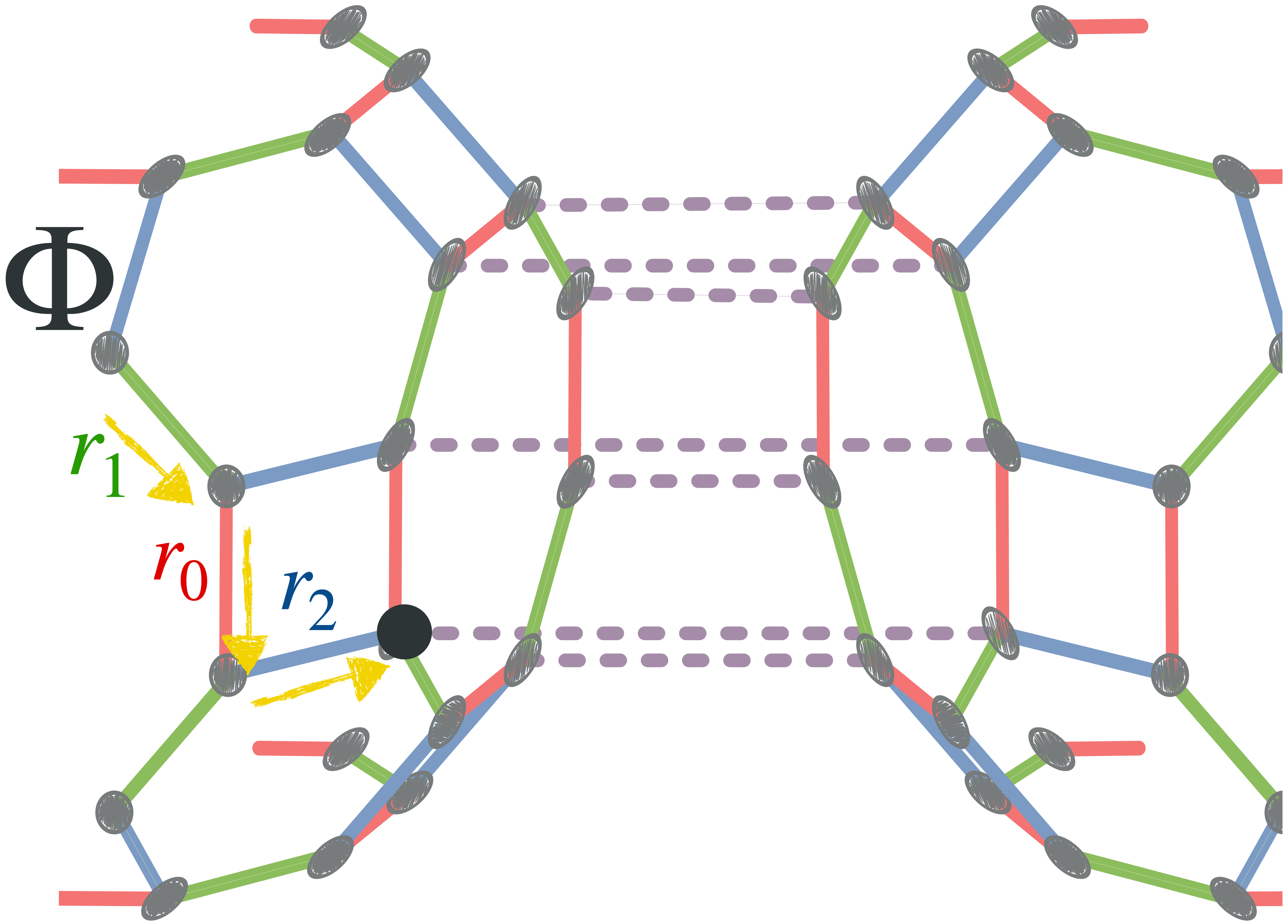
- The group

$$W_n = \langle r_0, \ldots, r_{n-1} \mid (r_i)^2 = (r_i r_j)^2 = 1 \rangle$$

acts on $\mathscr{M}$ by connections.

maniplexes and polytopes



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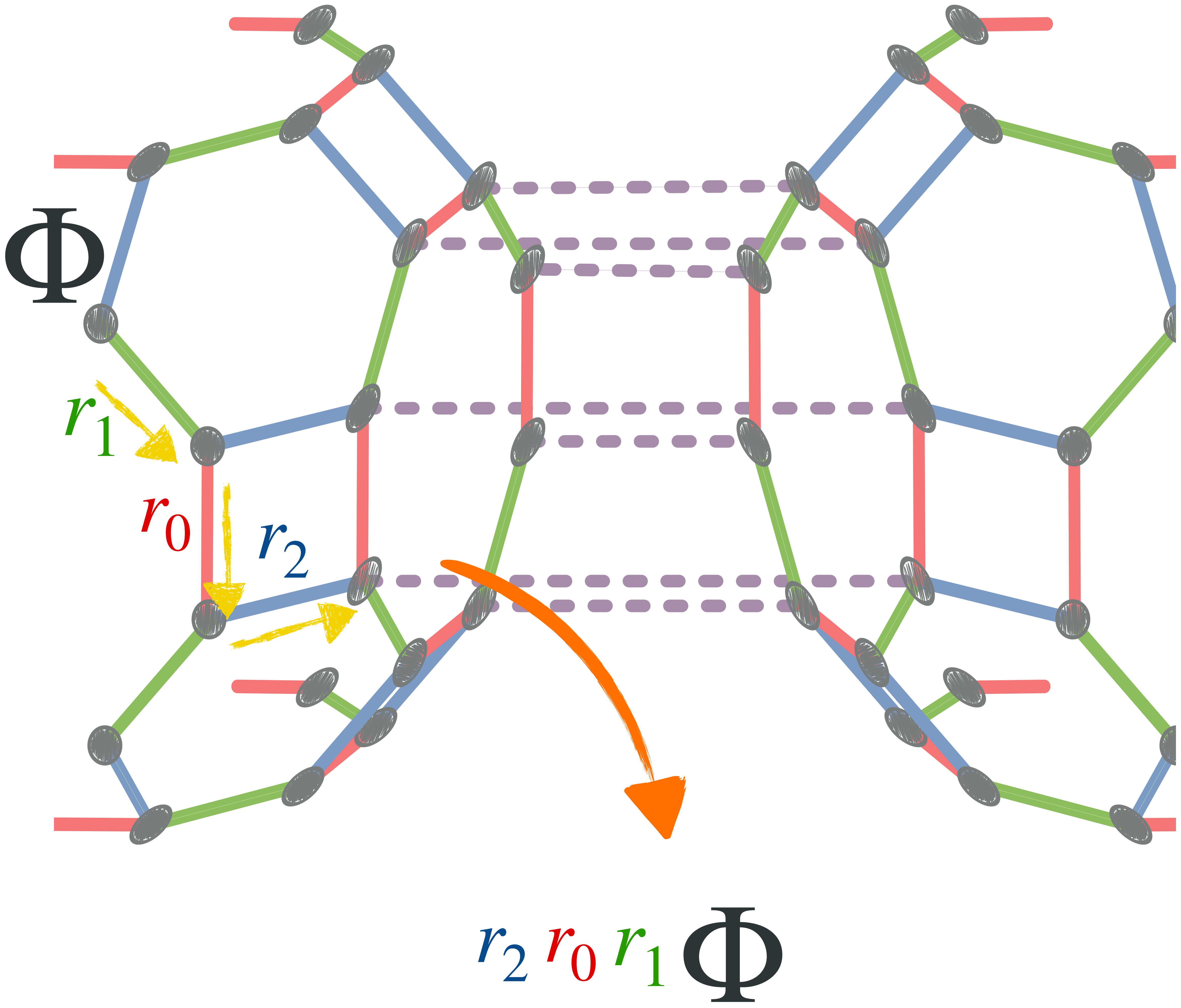

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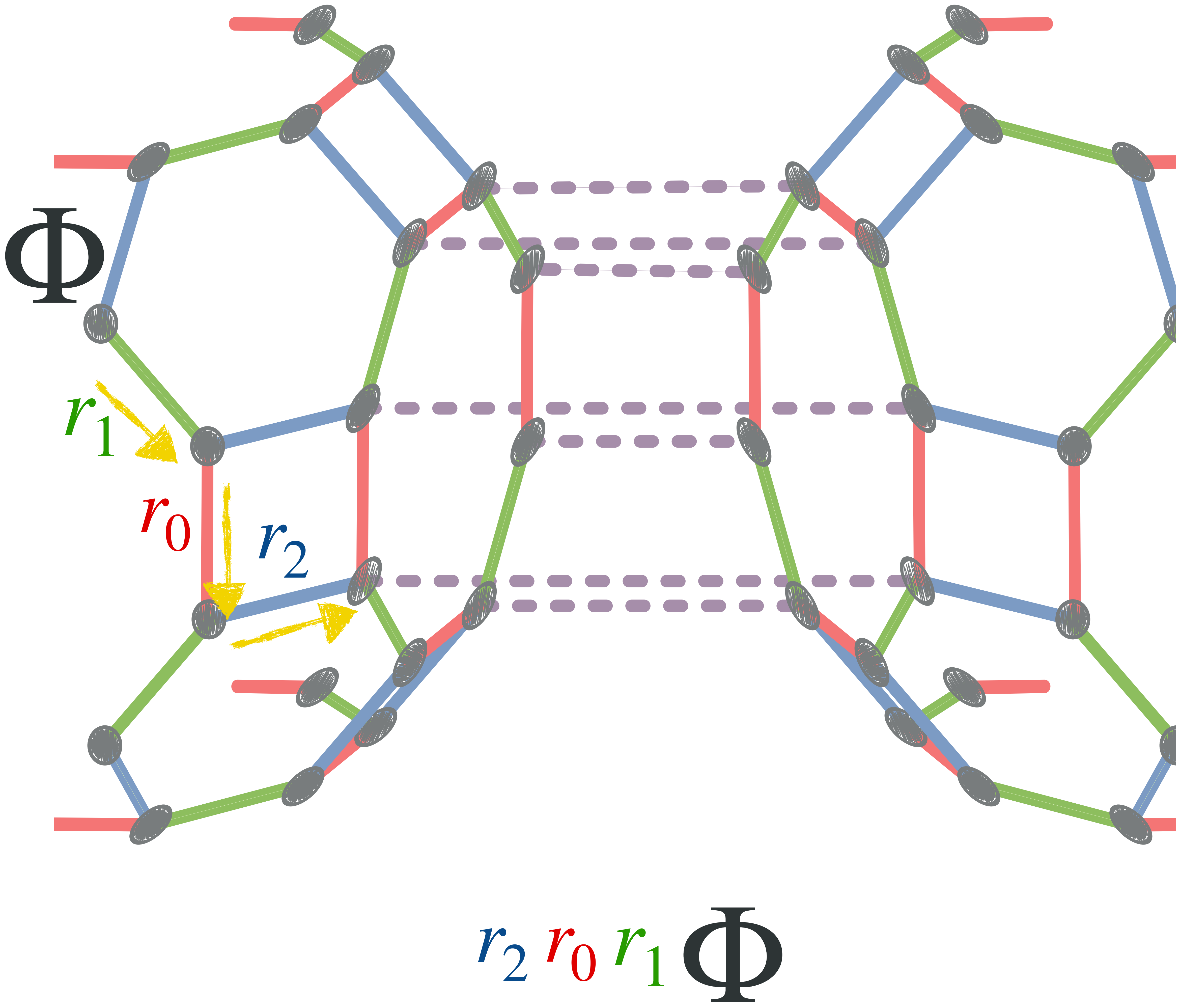
- The group $\mathrm{Aut}(\mathscr{M})$ acts freely on the flags.
- The group

$$W_n = \langle r_0, \ldots, r_{n-1} \mid (r_i)^2 = (r_i r_j)^2 = 1 \rangle$$

acts on $\mathscr{M}$ by connections.

maniplexes and polytopes



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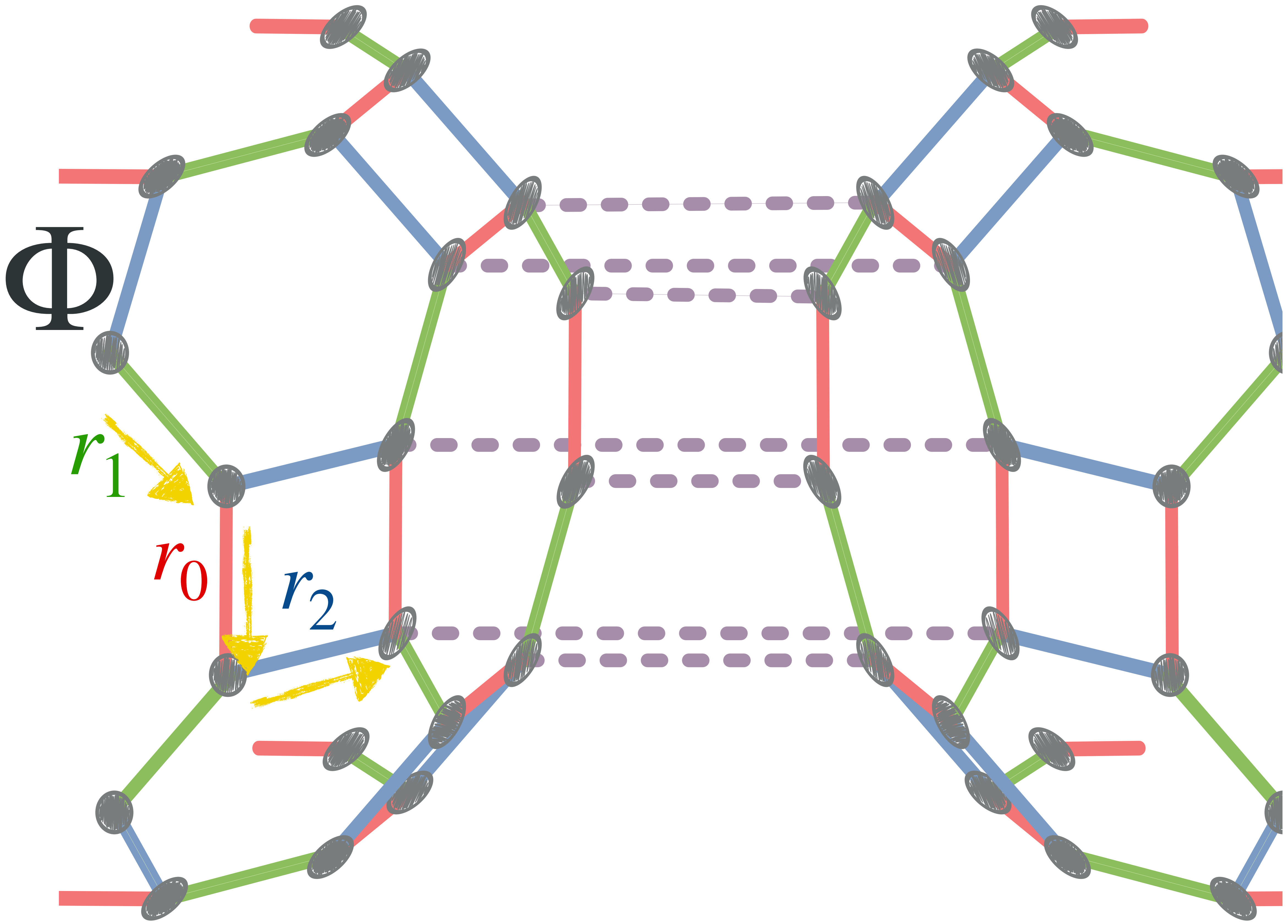

$r_2 \, r_0 \, r_1 \, \Phi$

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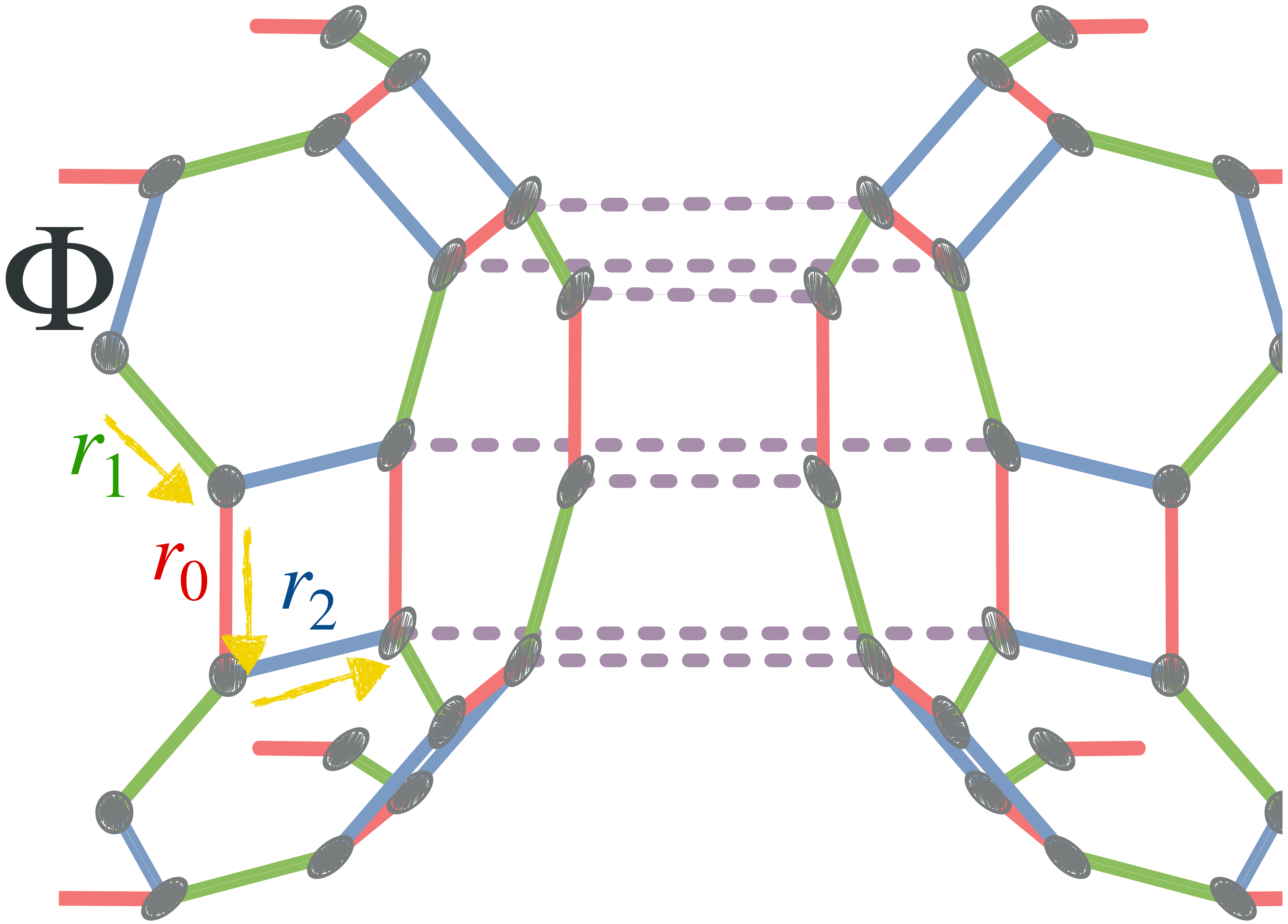
- The universal n-maniplex is

$$\mathscr{U}^n = \mathrm{Cay}(W_n)$$

$$\mathrm{Aut}(\mathscr{U}^n) = W_n$$

maniplexes and polytopes



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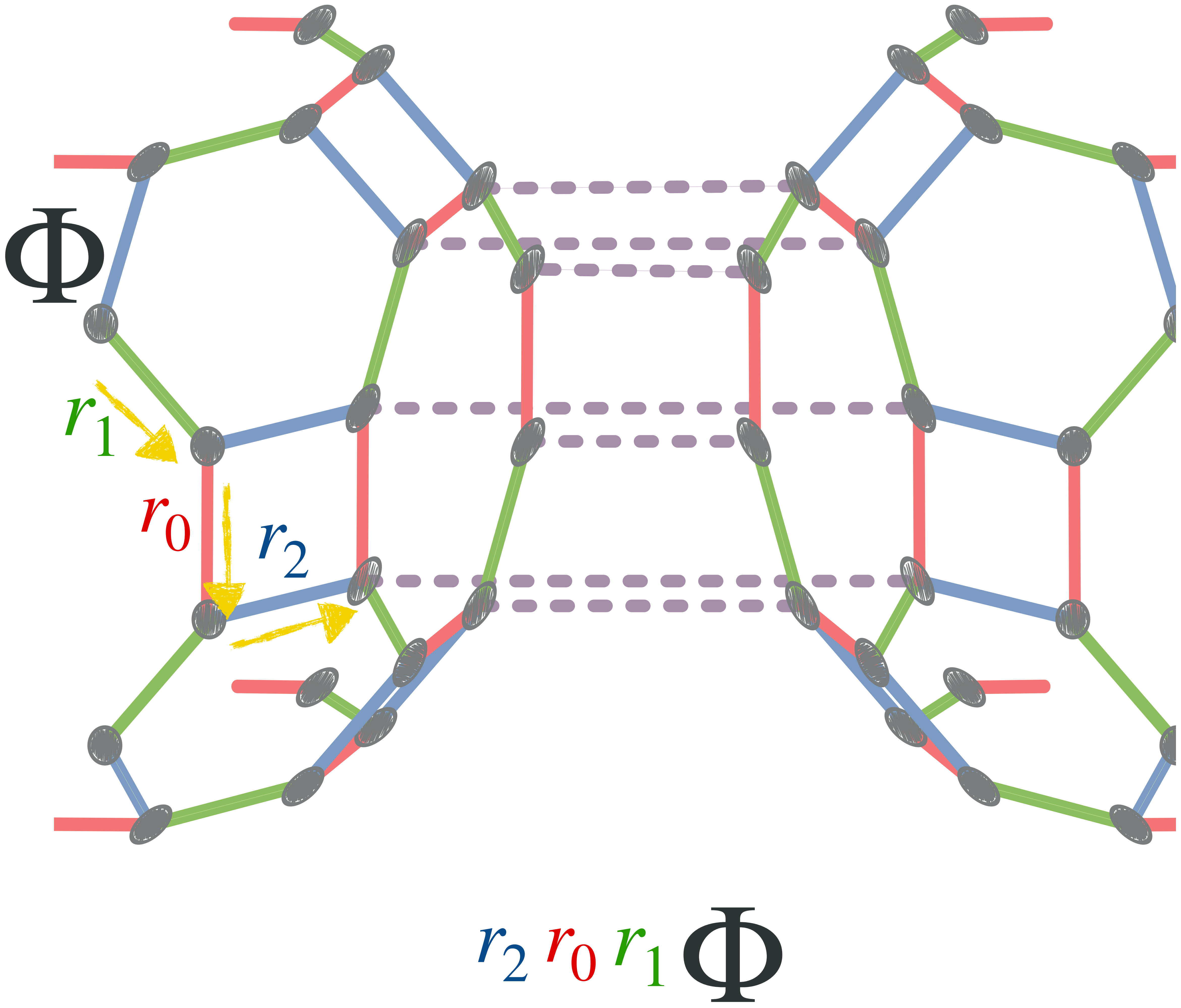

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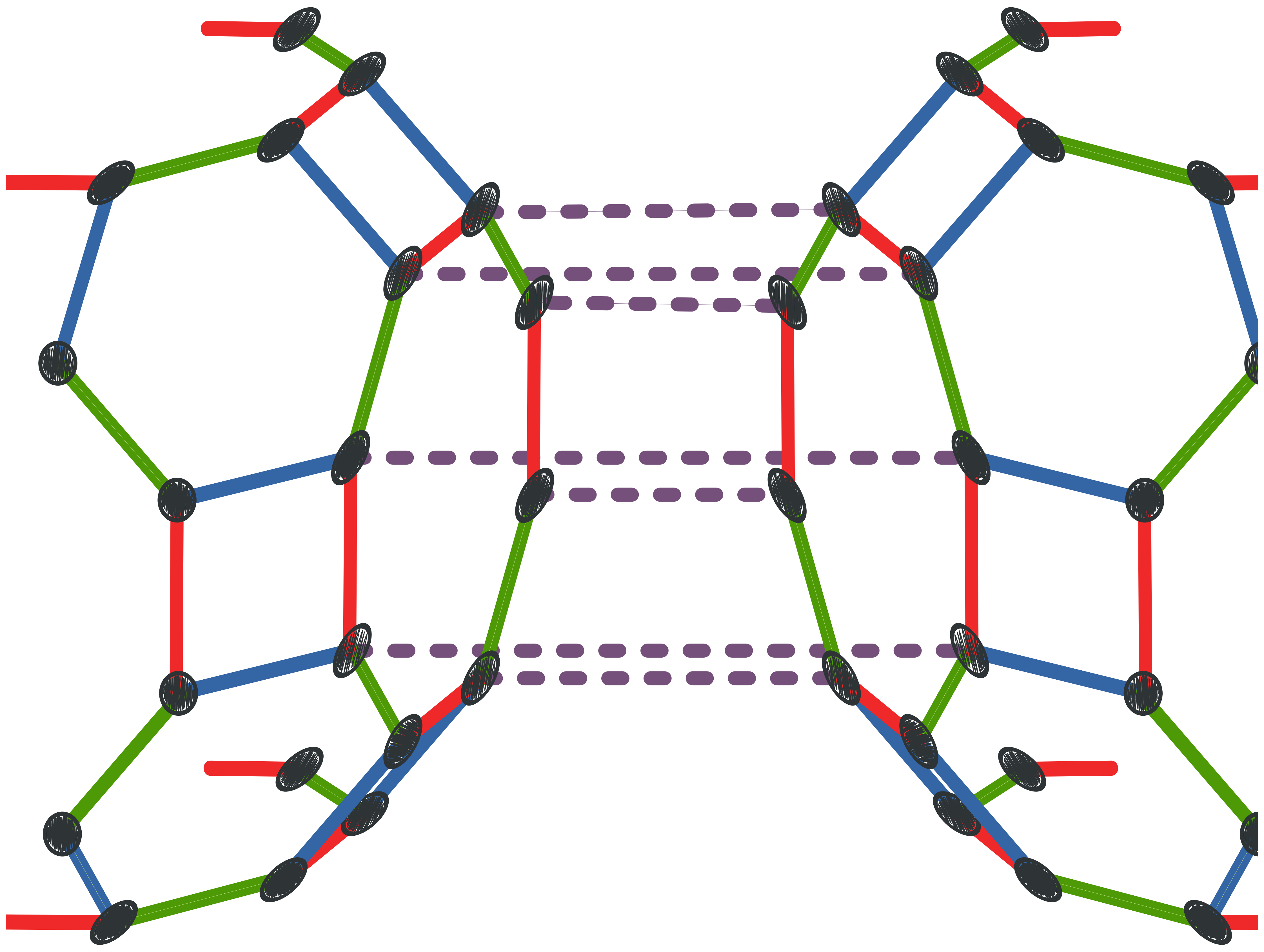
Hartley,... (1999, ...):
Every n-maniplex is a quotient of $\mathcal{U}^n$.

$$\mathcal{U}^n/M \searrow \mathcal{U}^n/N \iff M \leq N$$

maniplexes and polytopes

A n-maniplex is a graph $\mathscr{M}$:

- Connected and simple,
- Valency n,
- Properly-edge n-coloured,
- The (i, j, i, j)-paths are alternating squares, whenever $|i - j| > 1$



premaniplexes

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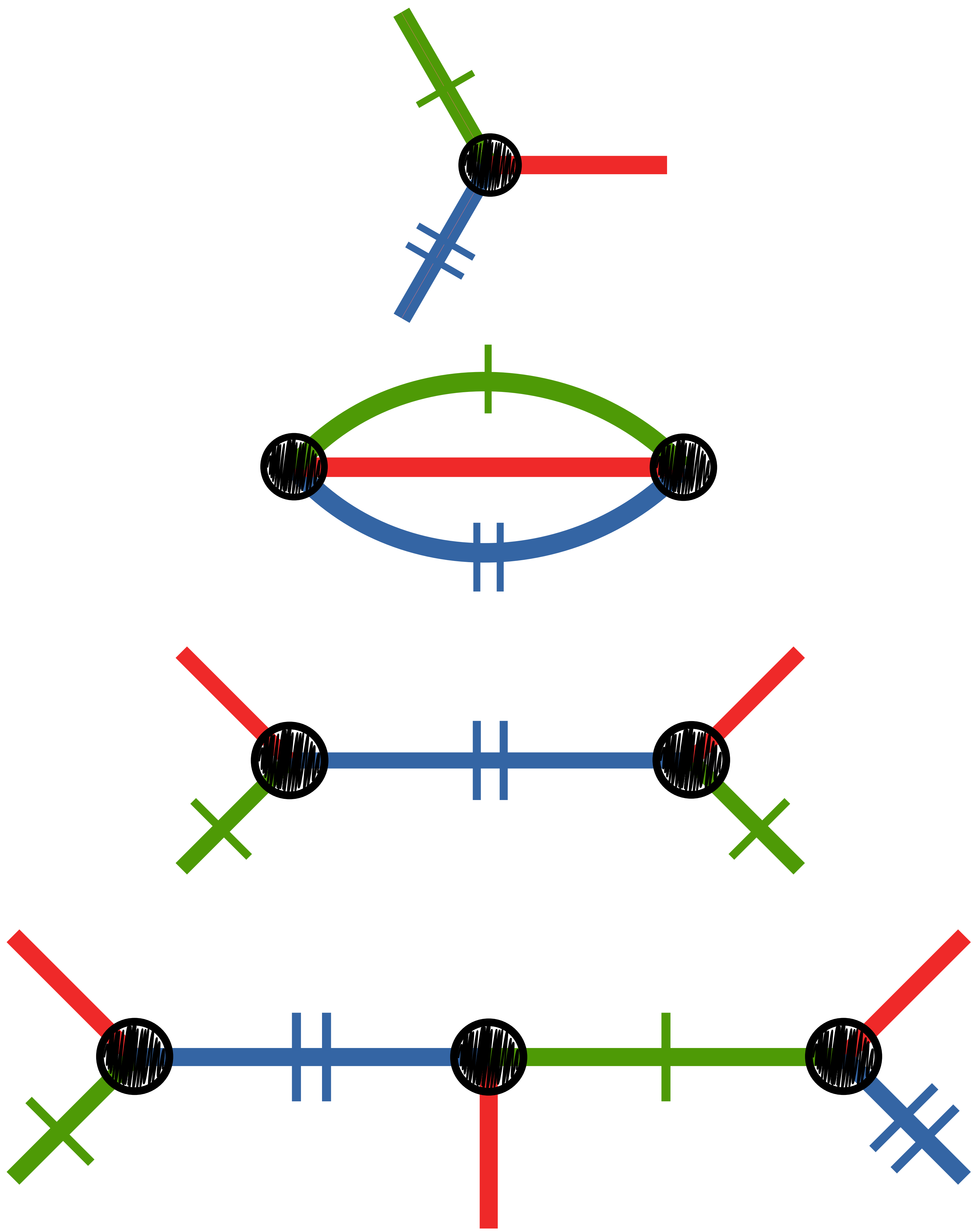

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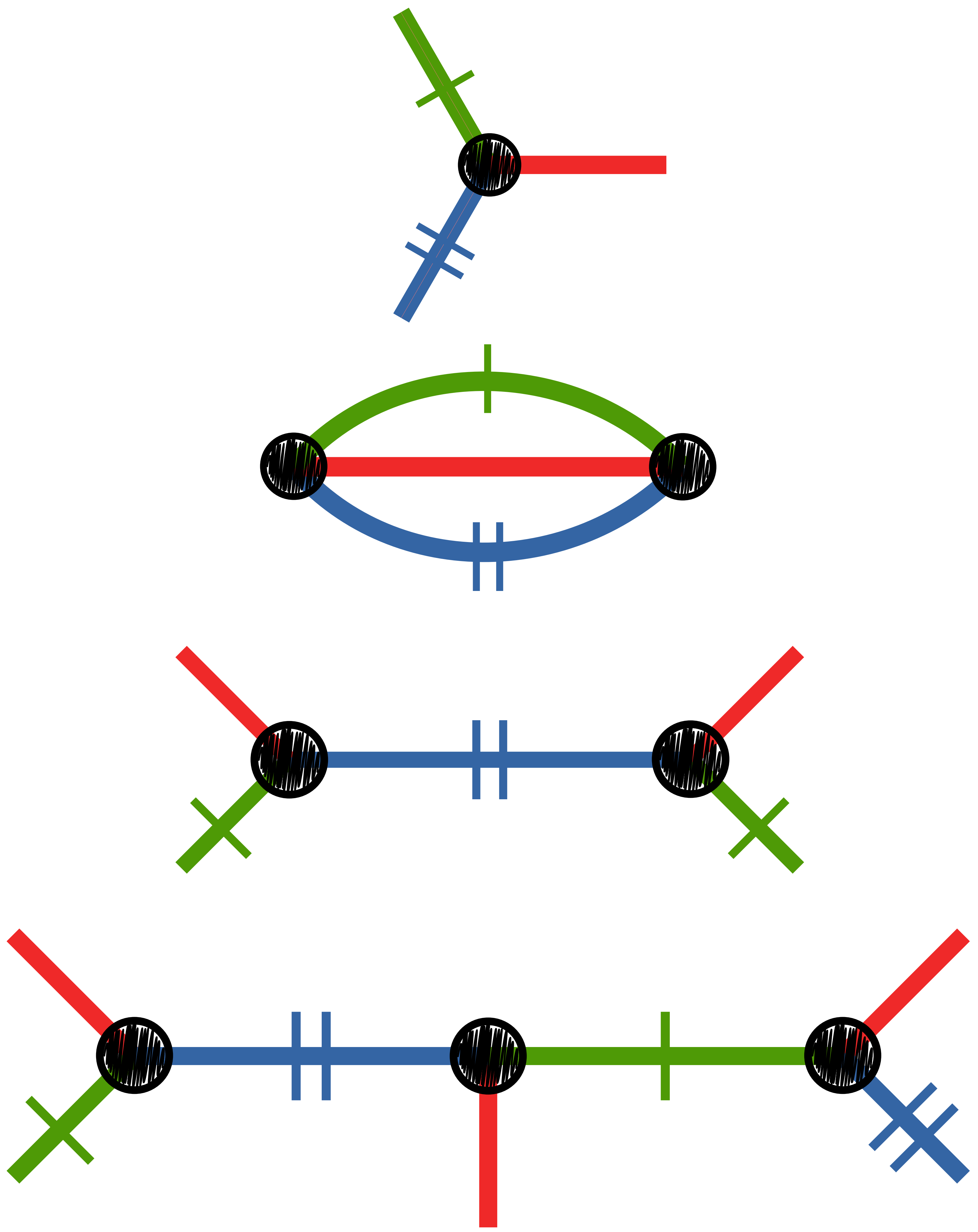
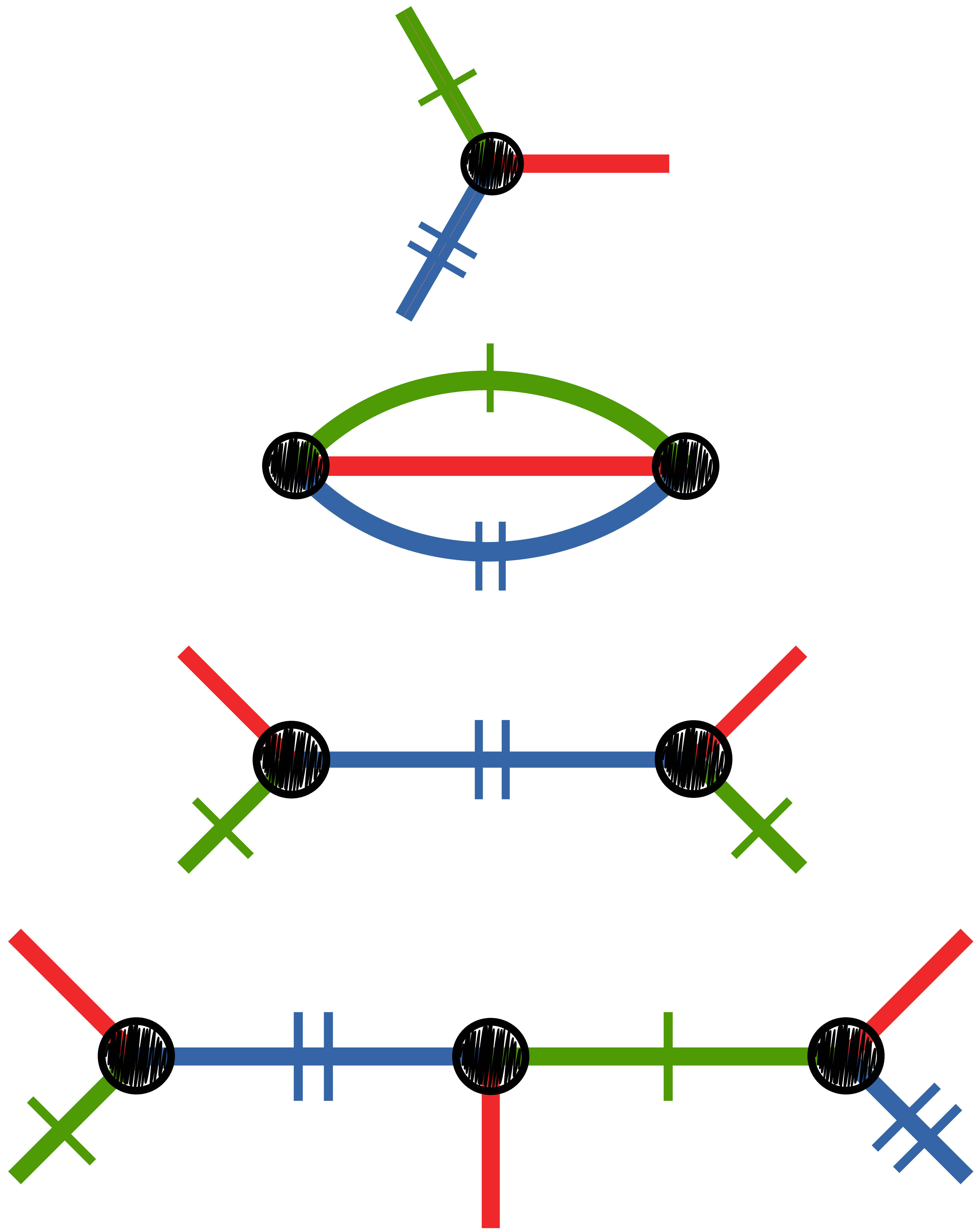
The notions of cover and automorphism extend naturally from maniplexes to premaniplexes



premaniplexes



- The Symmetry-type Graph (STG) of a maniplex $\mathscr{M}$ is the quotient $\mathscr{M}/\mathrm{Aut}(\mathscr{M})$

premaniplexes

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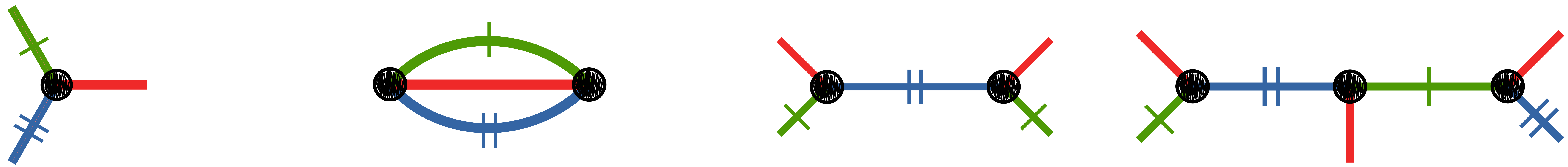

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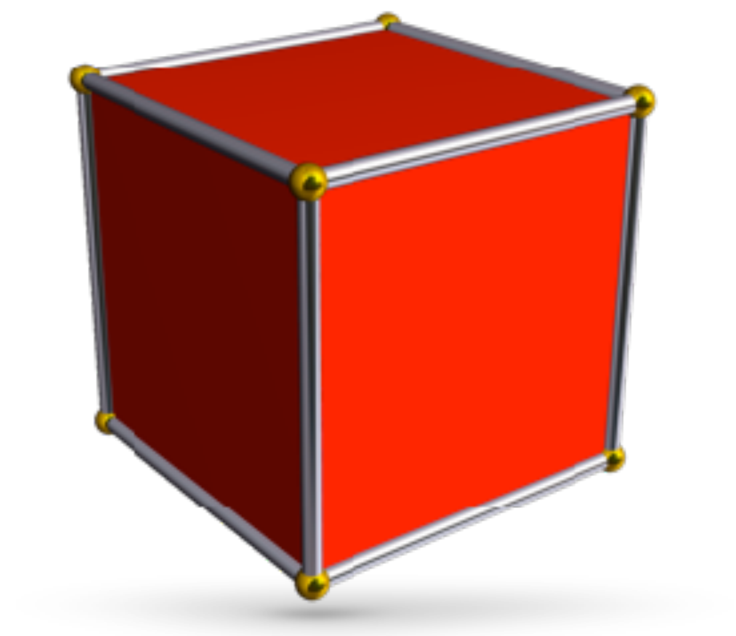

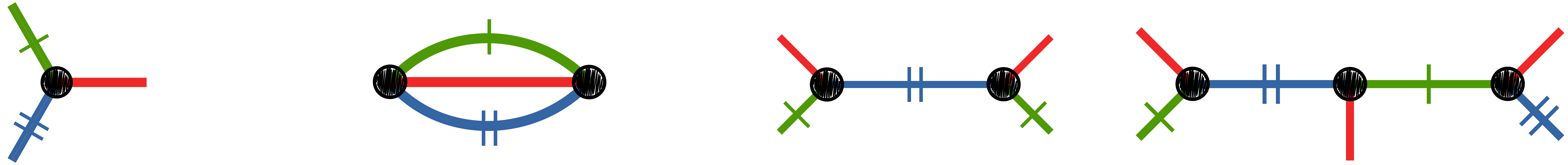

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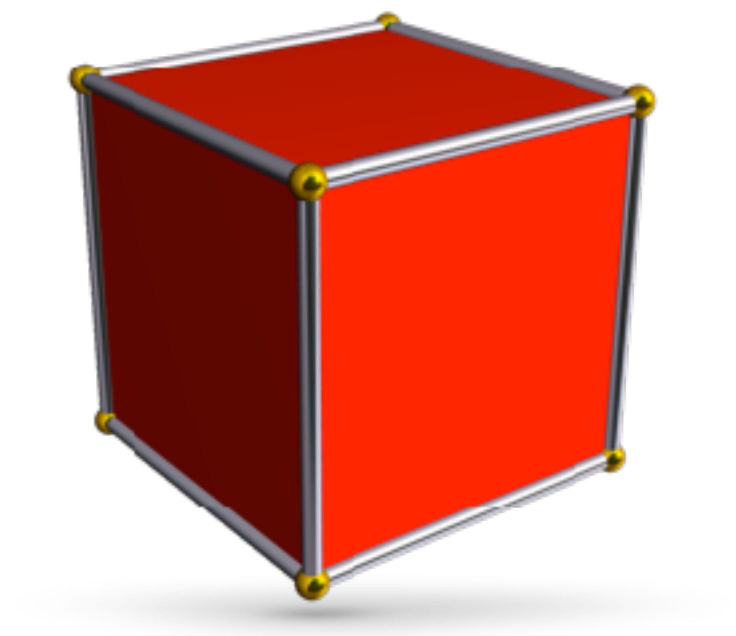

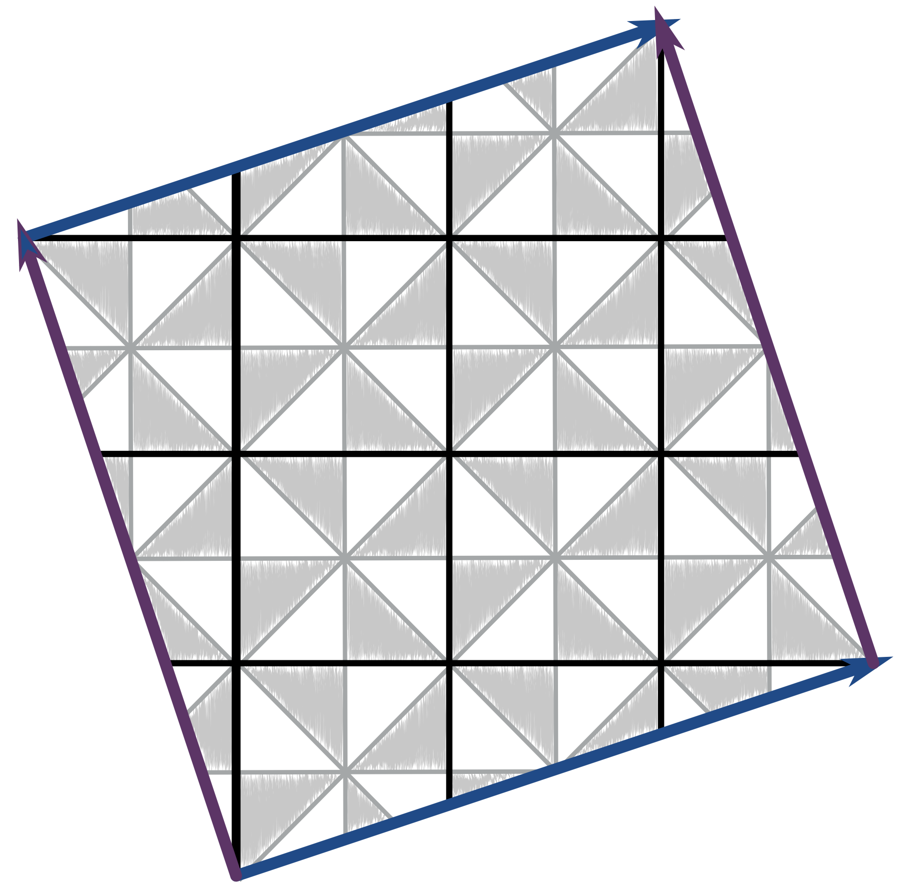

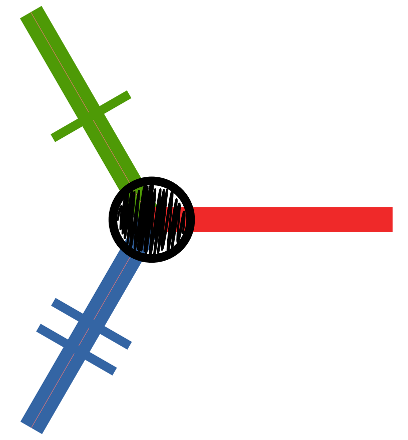

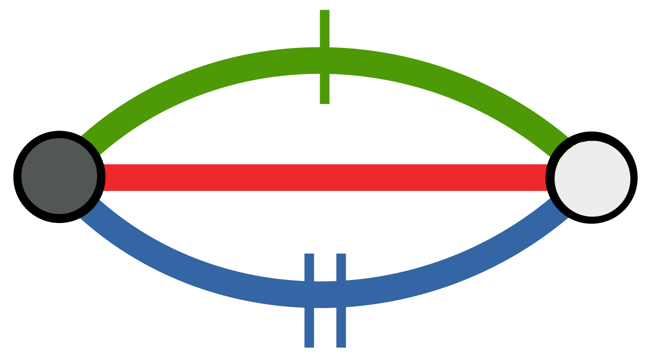

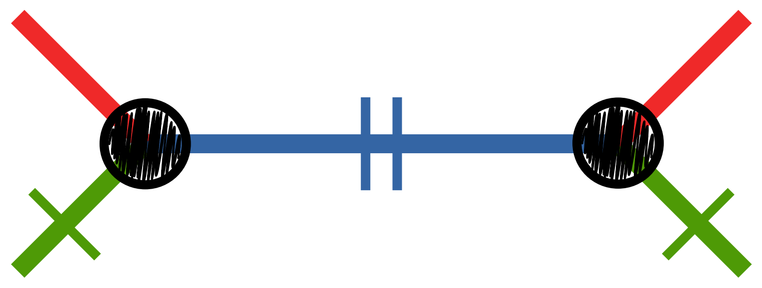

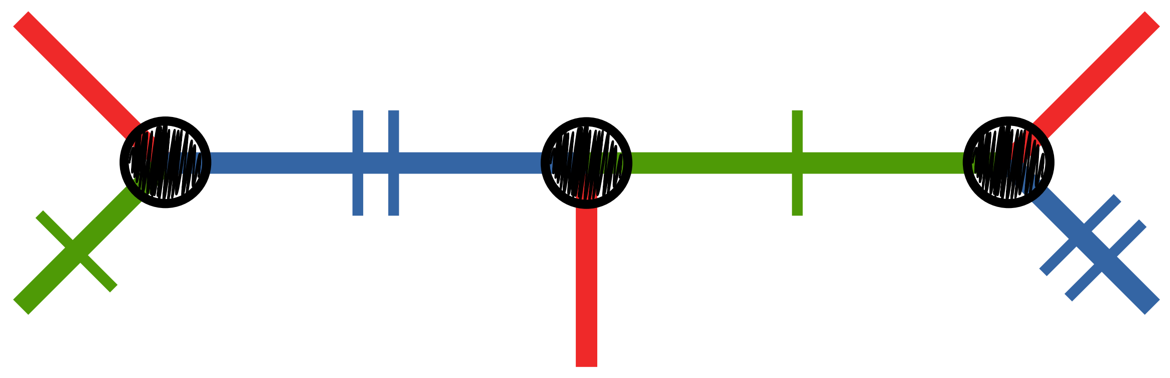

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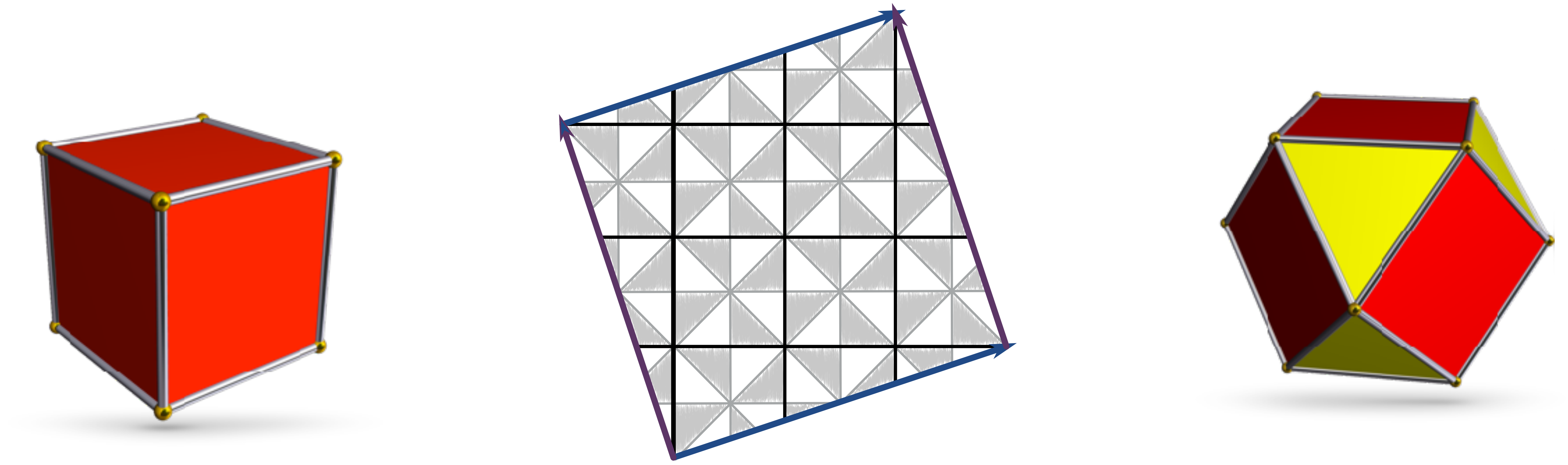

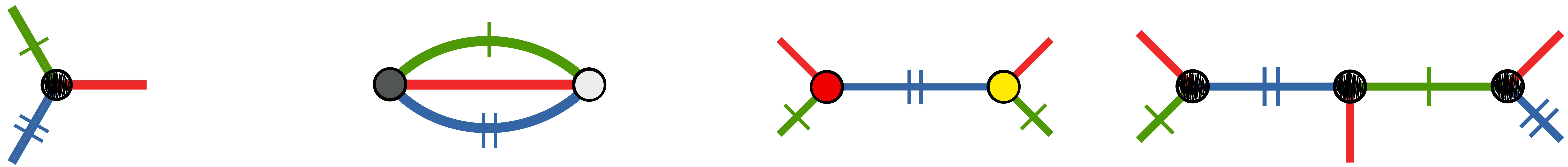

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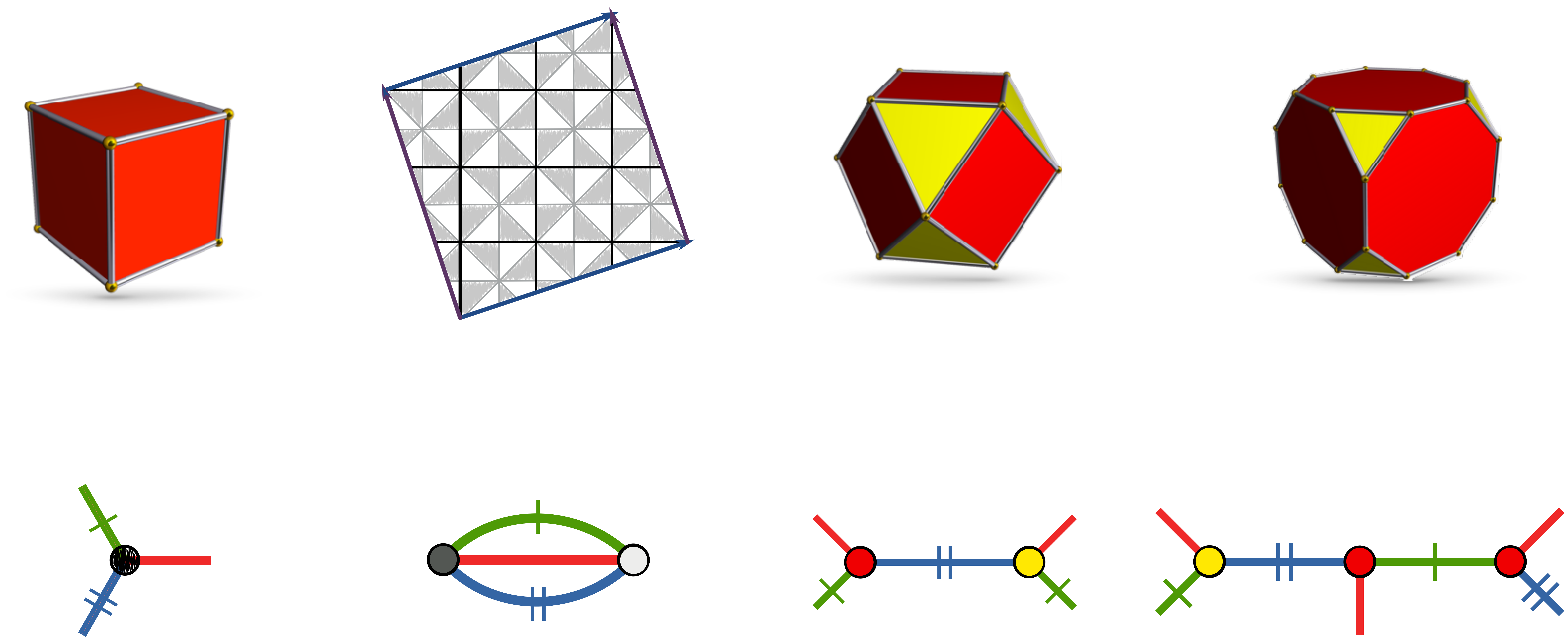
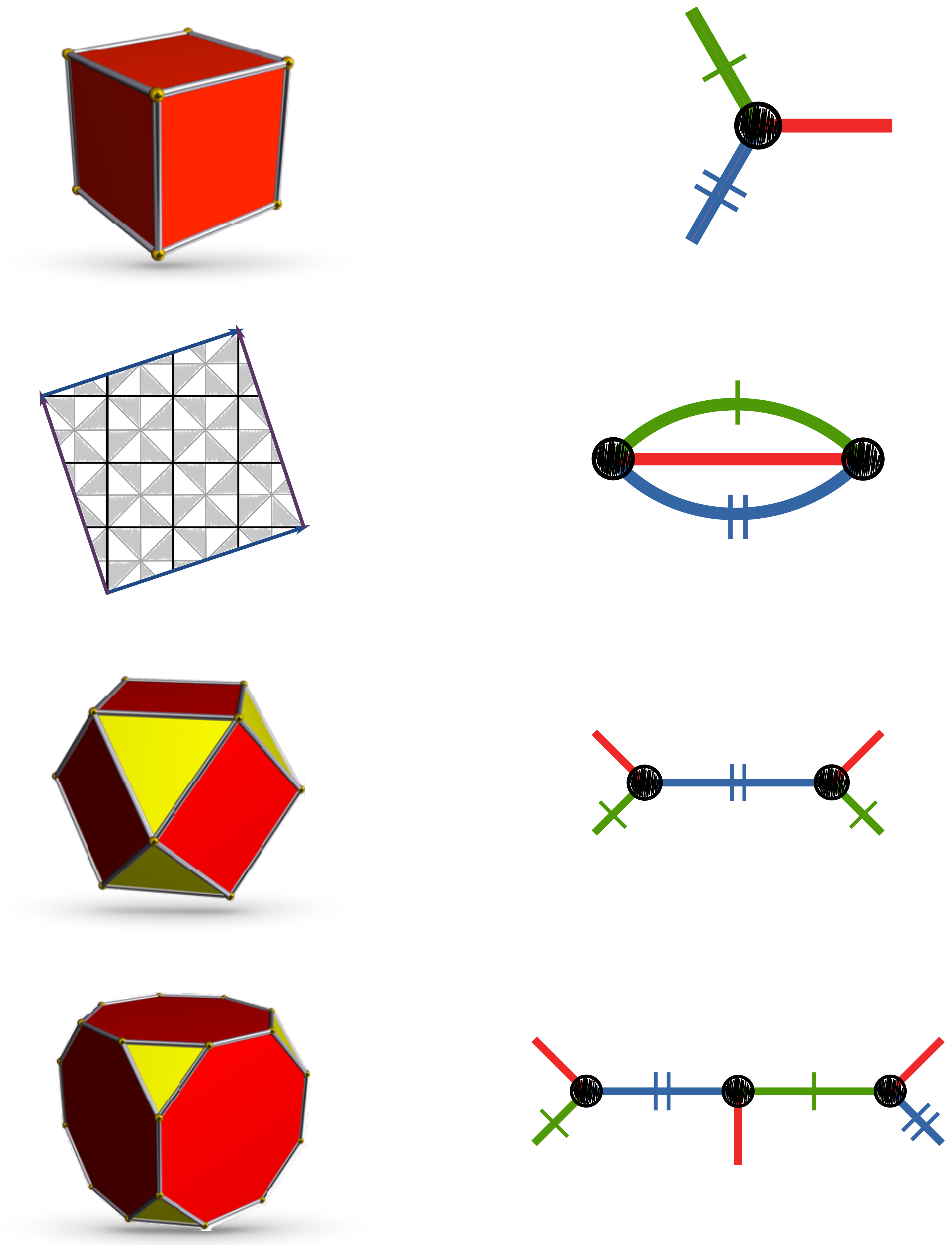


premaniplexes

Symmetry-type conjecture:

Given a connected n-premaniplex $\mathscr{T}$, there exists a n-maniplex (polytope) $\mathscr{M}$ such that

$$\mathrm{STG}(\mathscr{M}) = \mathscr{T}$$



Symmetries of voltage operations on maniplexes and polytopes

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voltage operations

- An (m, n)- voltage operator is a pair $(\mathscr{Y}, \eta)$

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$\mathscr{Y}$

n-premaniplex

voltage operations

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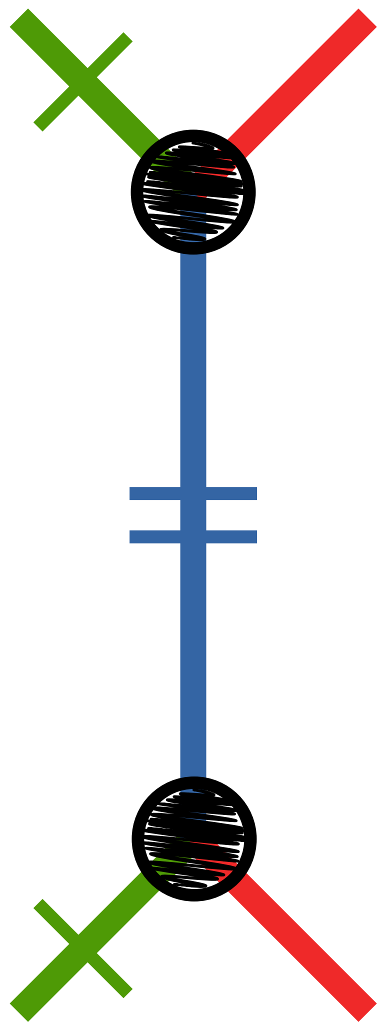

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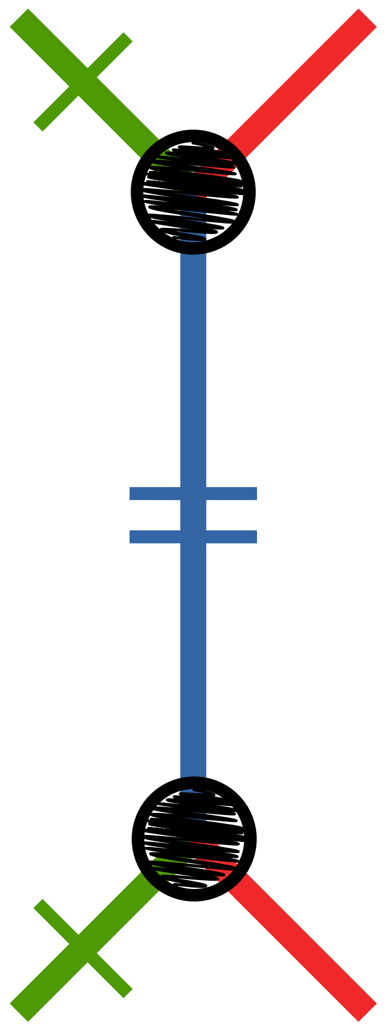

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

voltage assignment

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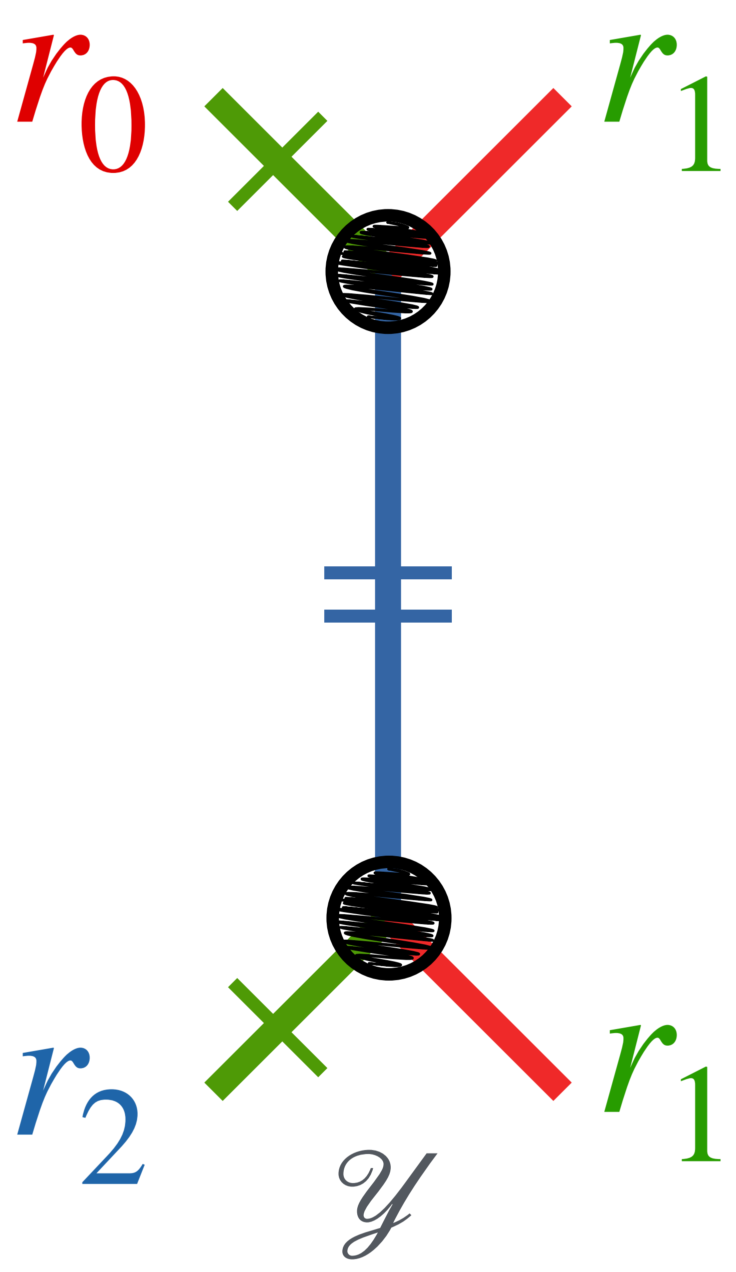

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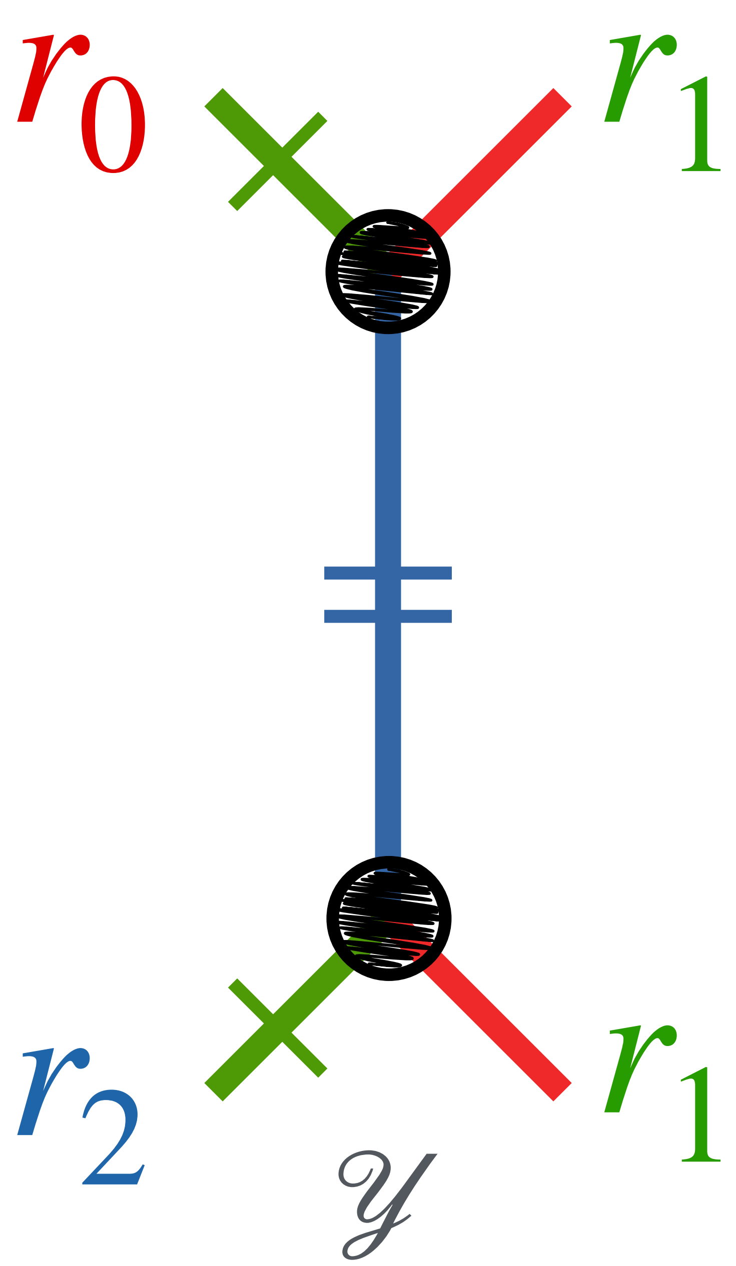

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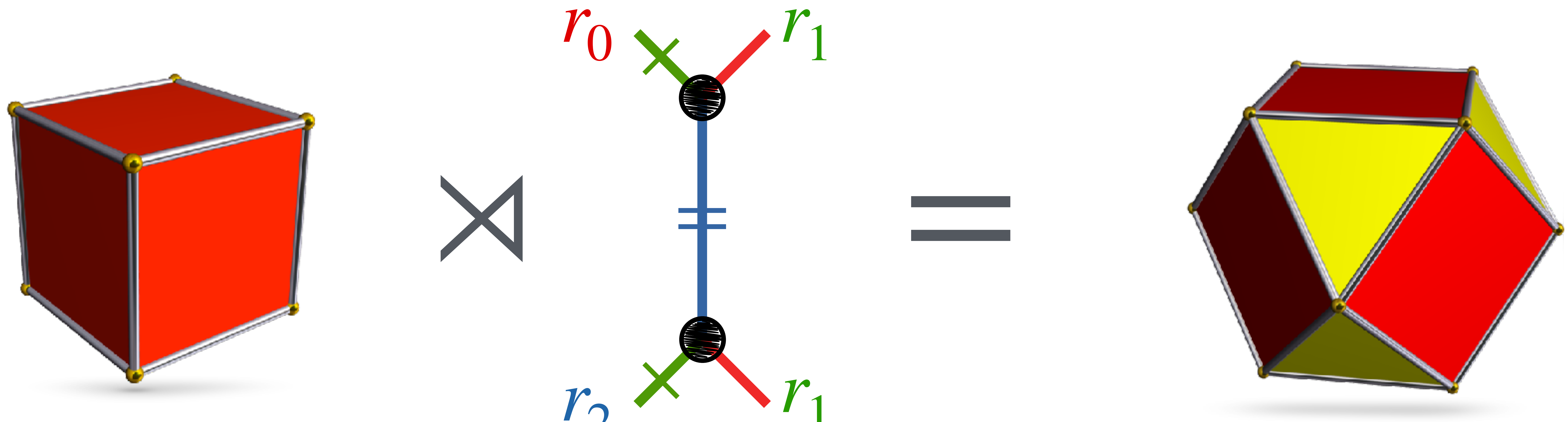

$\mathscr{X}$ | $\mathscr{Y}$ | $\mathscr{X} \rtimes_\eta \mathscr{Y}$

m-premaniplex

n-premaniplex

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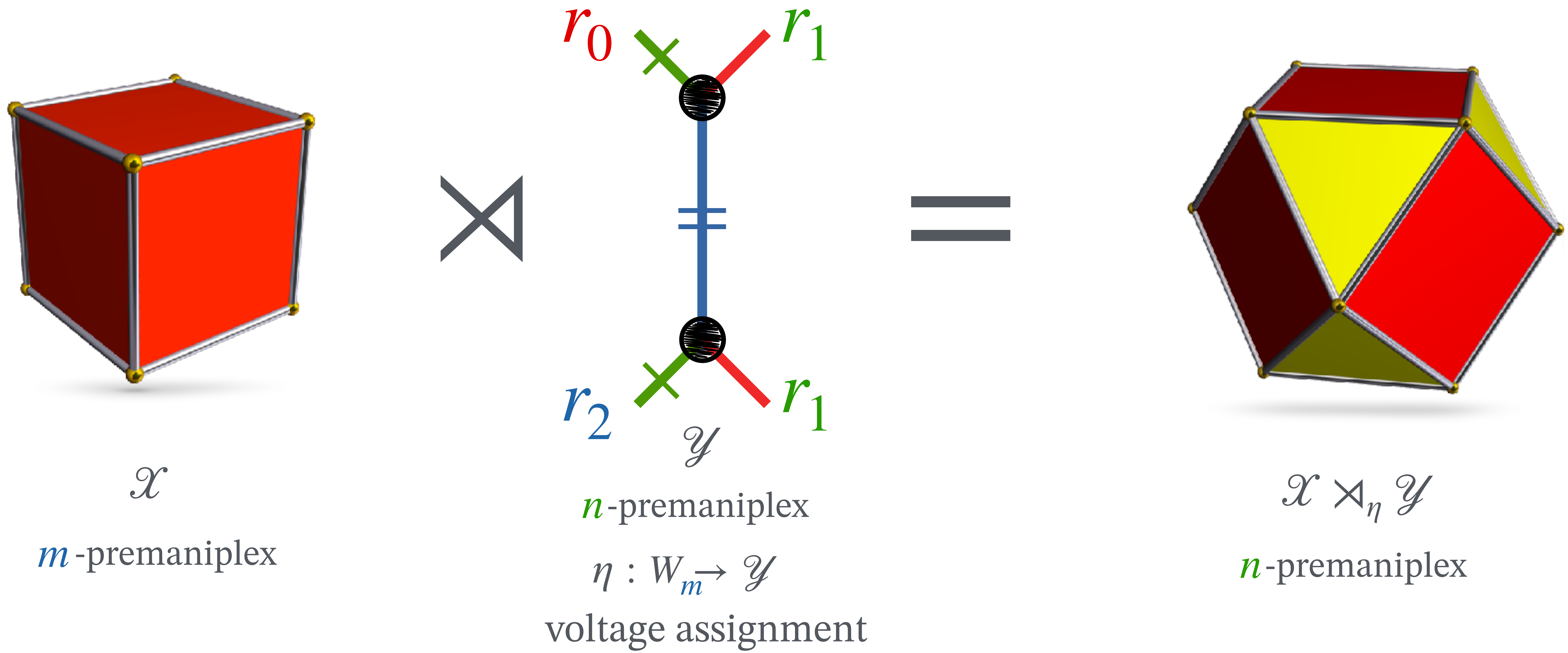

voltage operations

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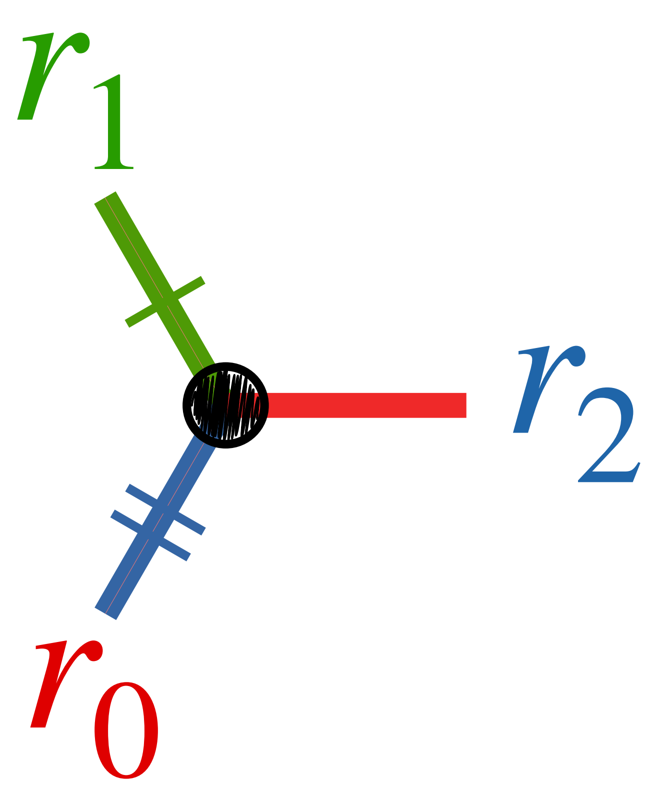
$\ltimes$



$=$

$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

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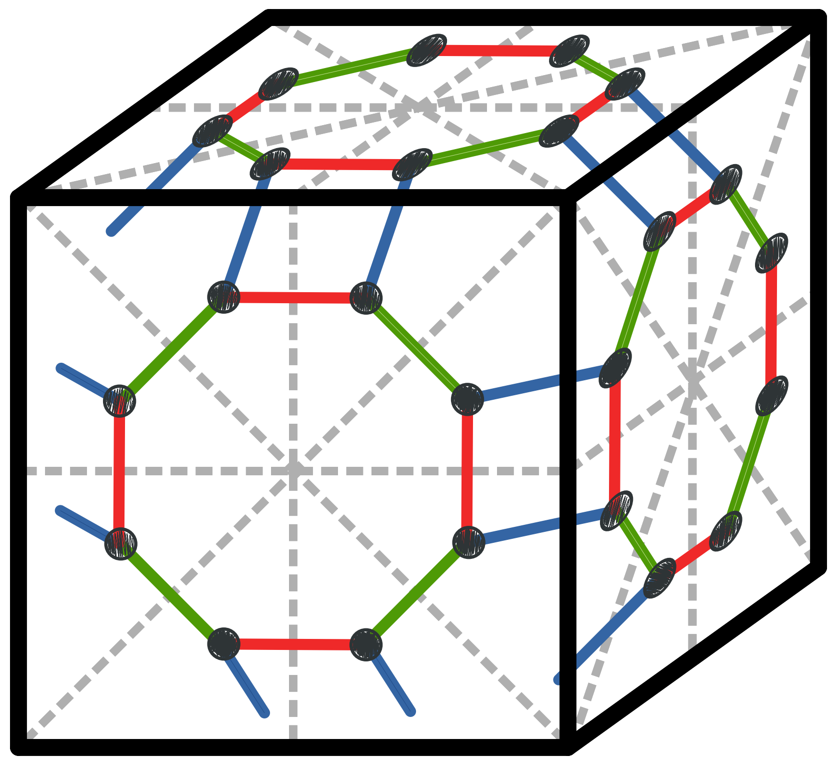

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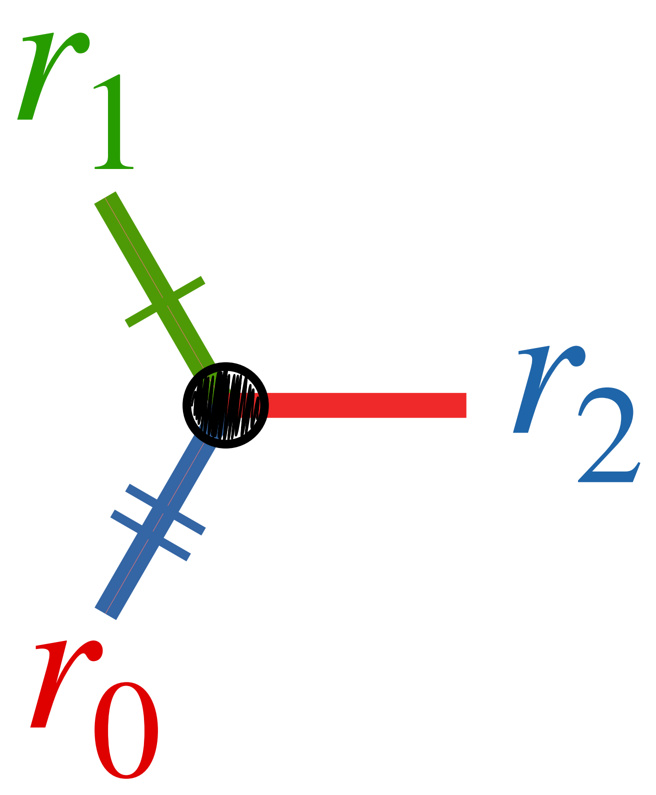

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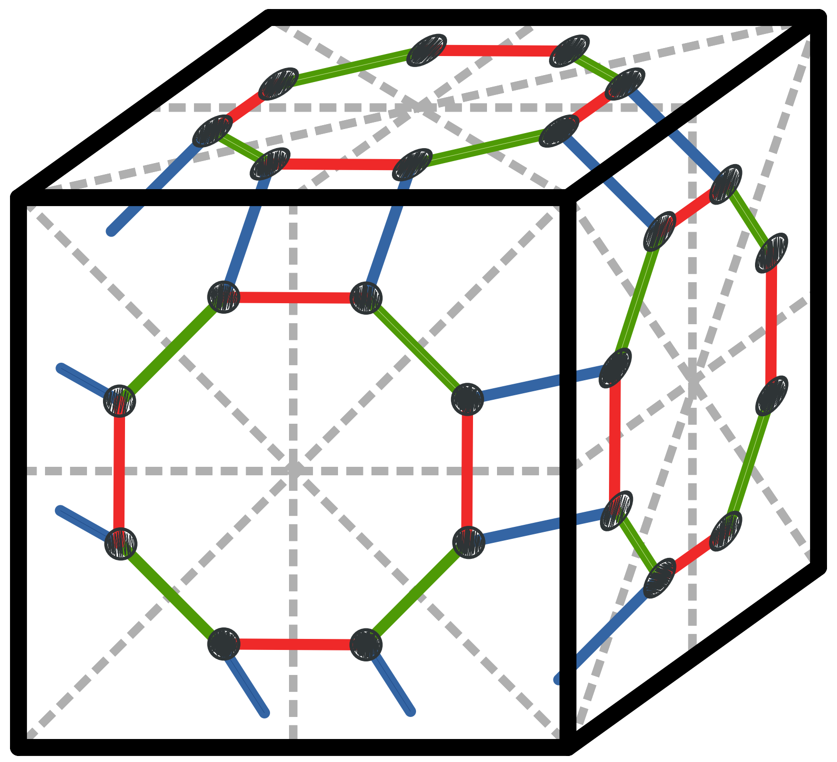
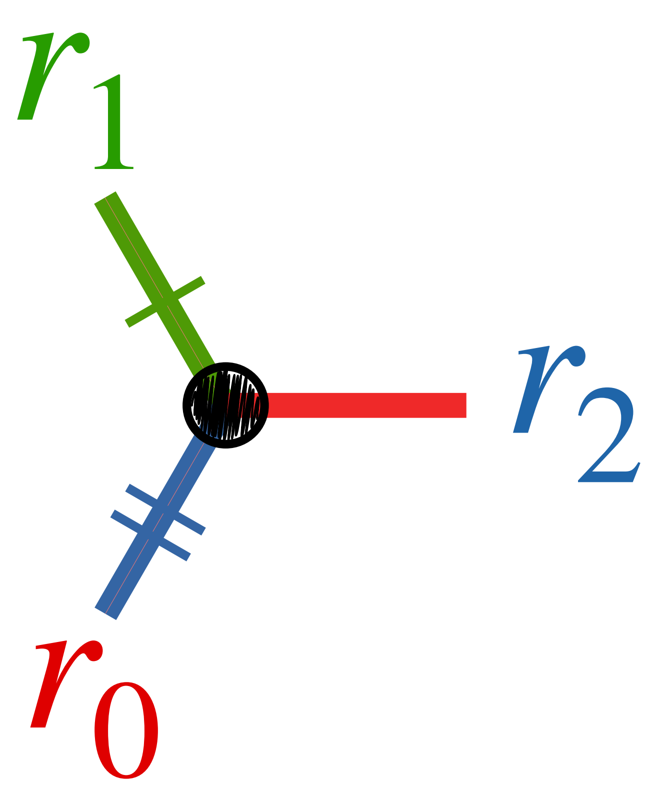


$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

n-premaniplex

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voltage assignment

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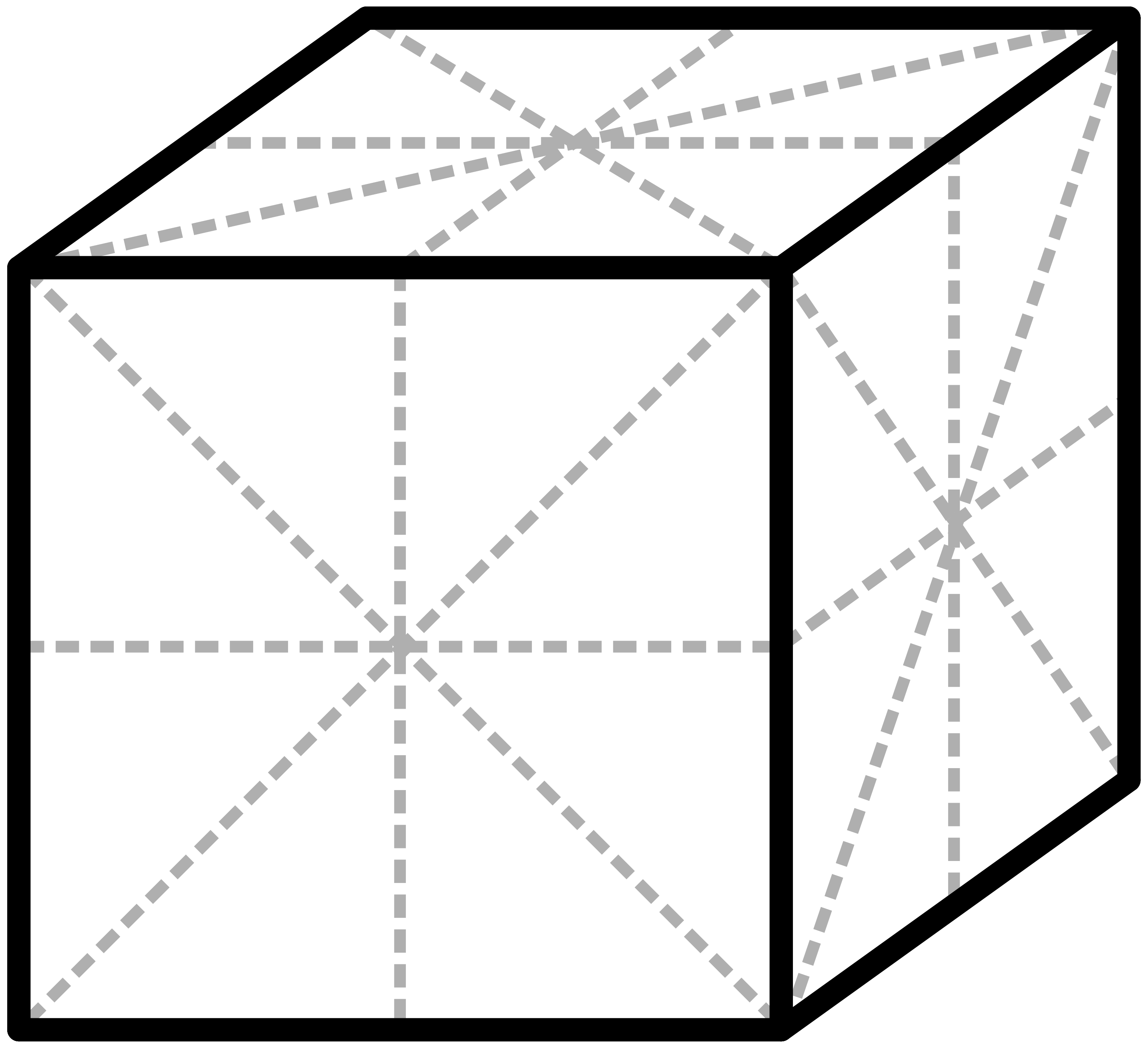

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

voltage operations

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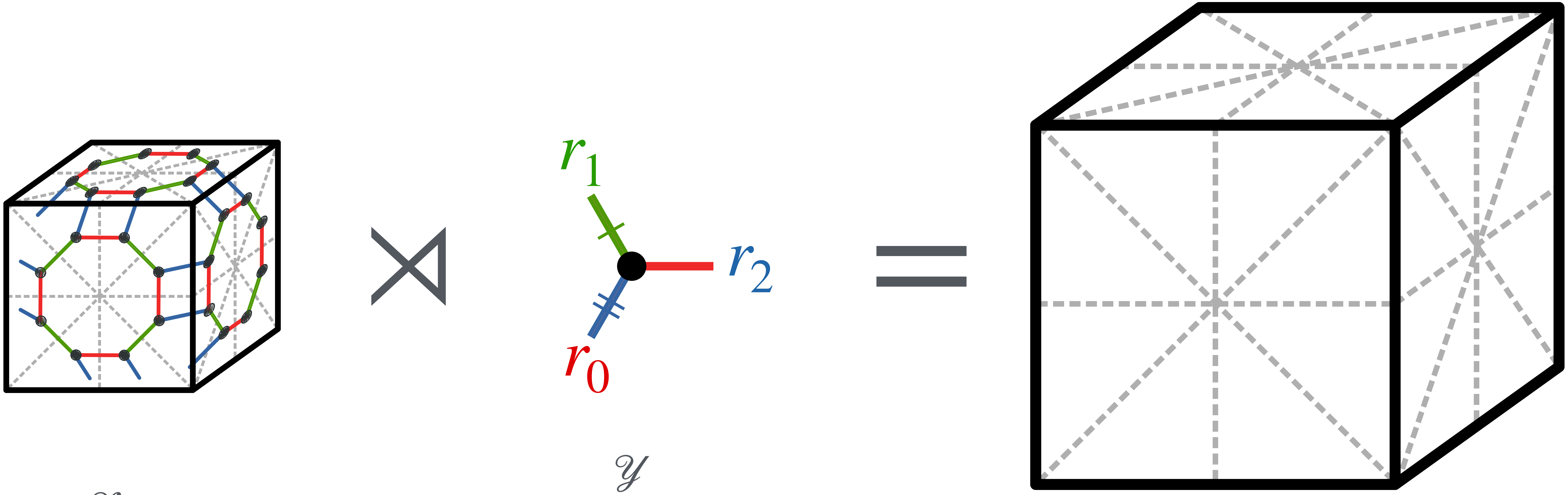

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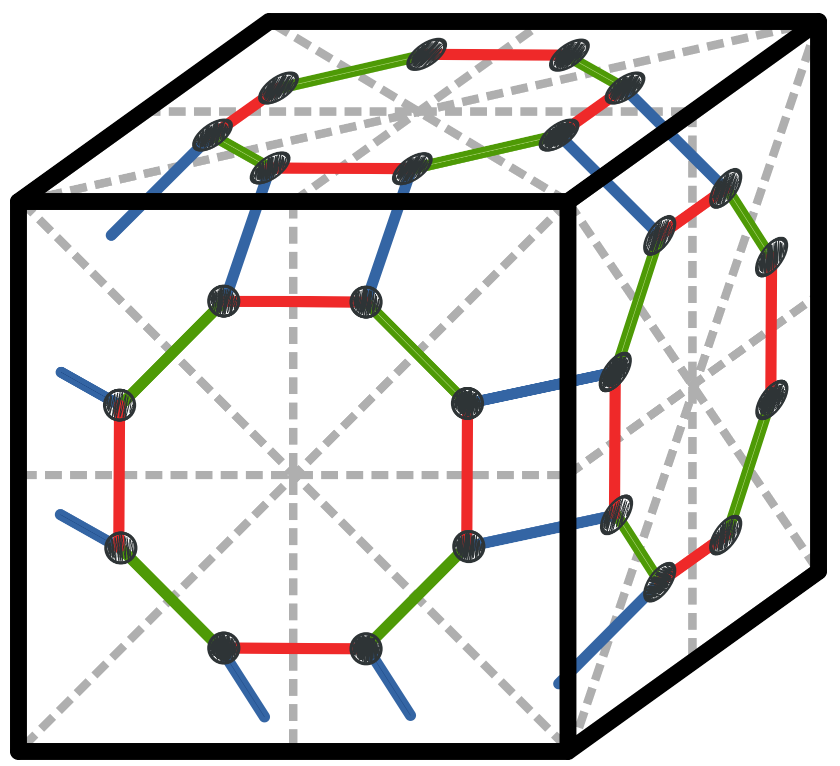

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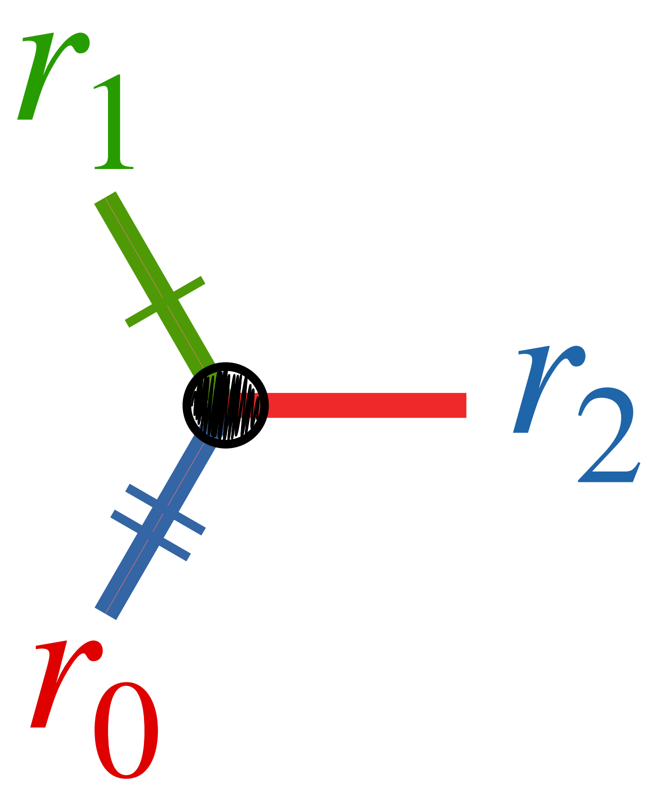

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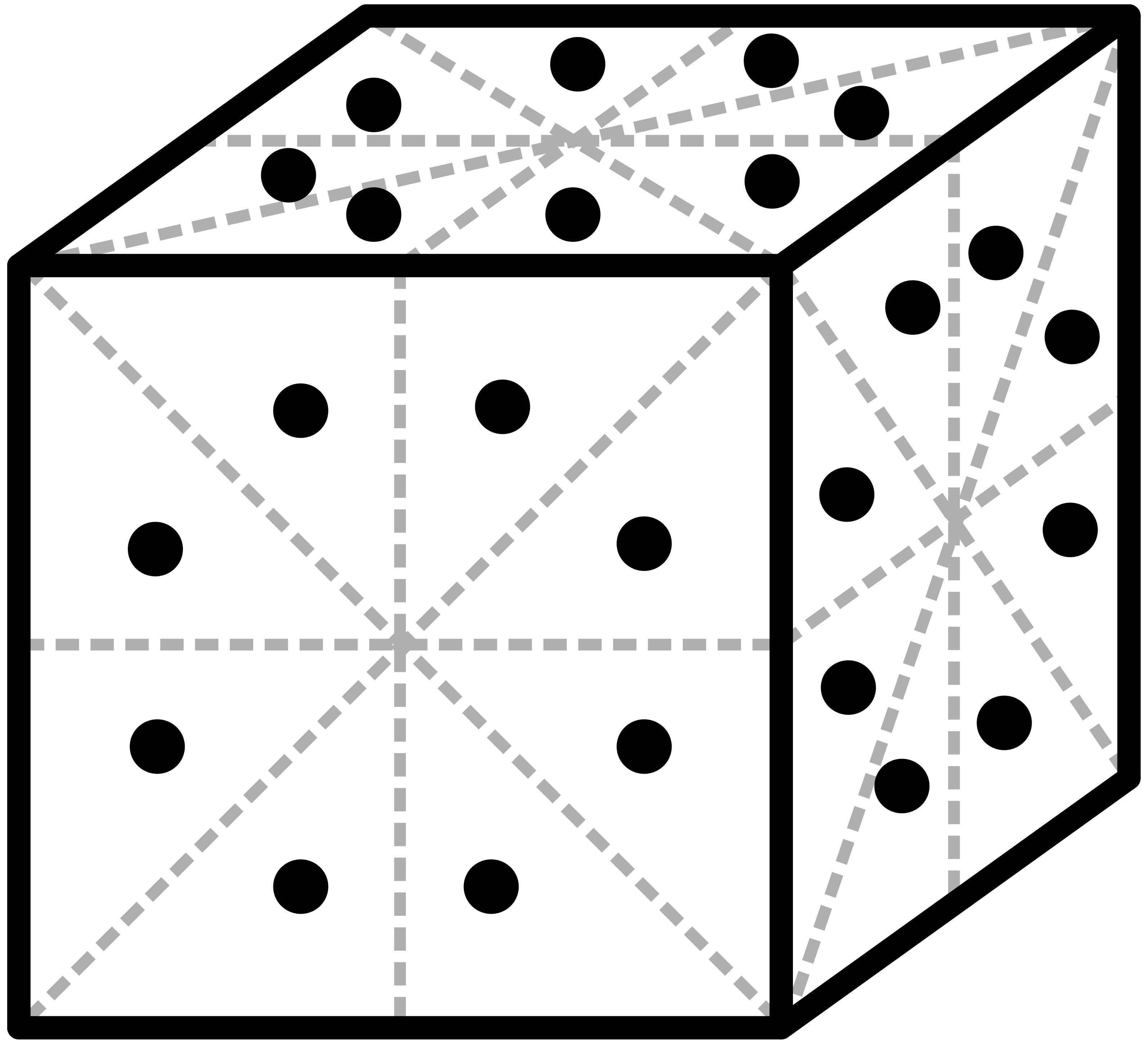

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n-premaniplex

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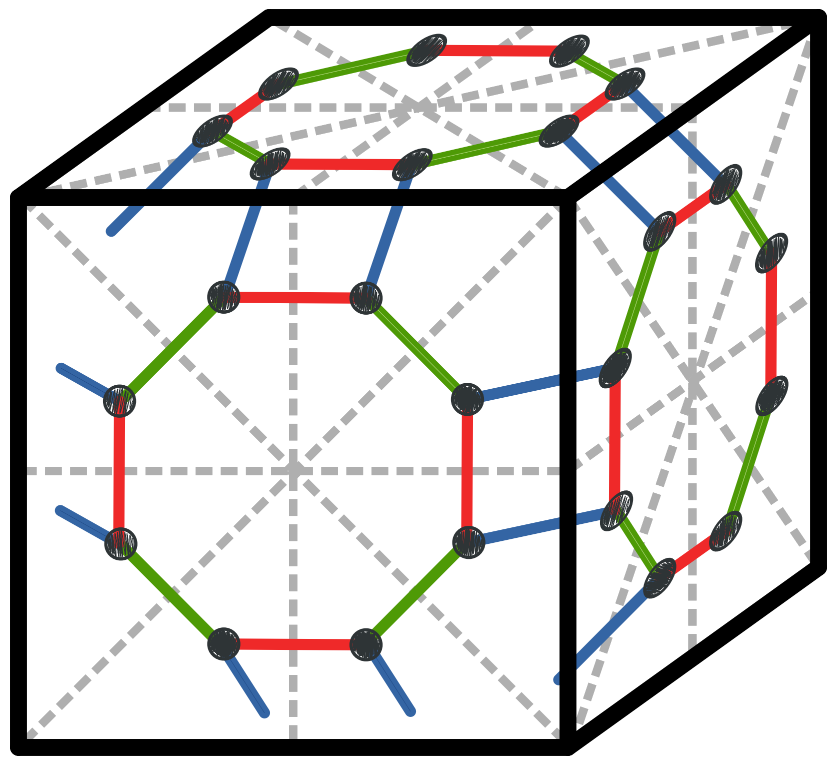
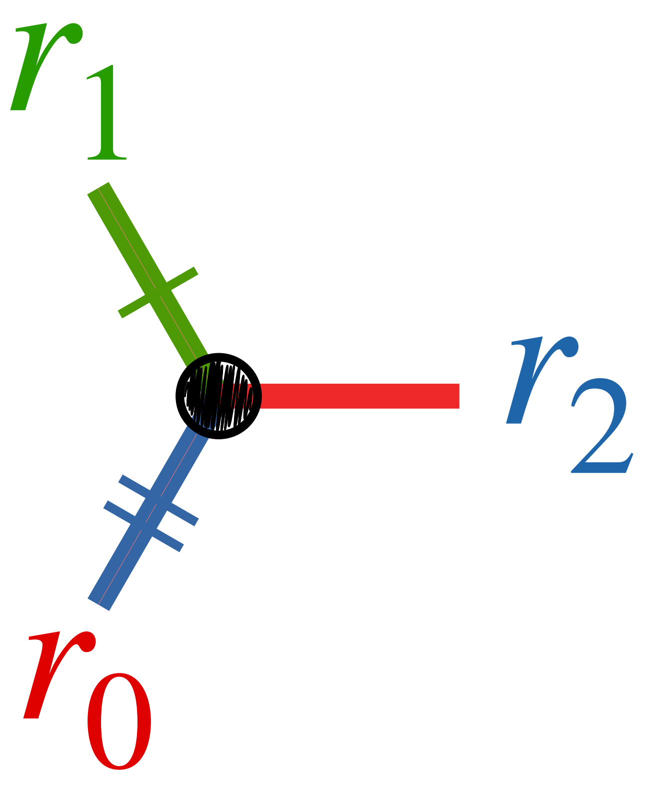


$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

n-premaniplex

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voltage assignment

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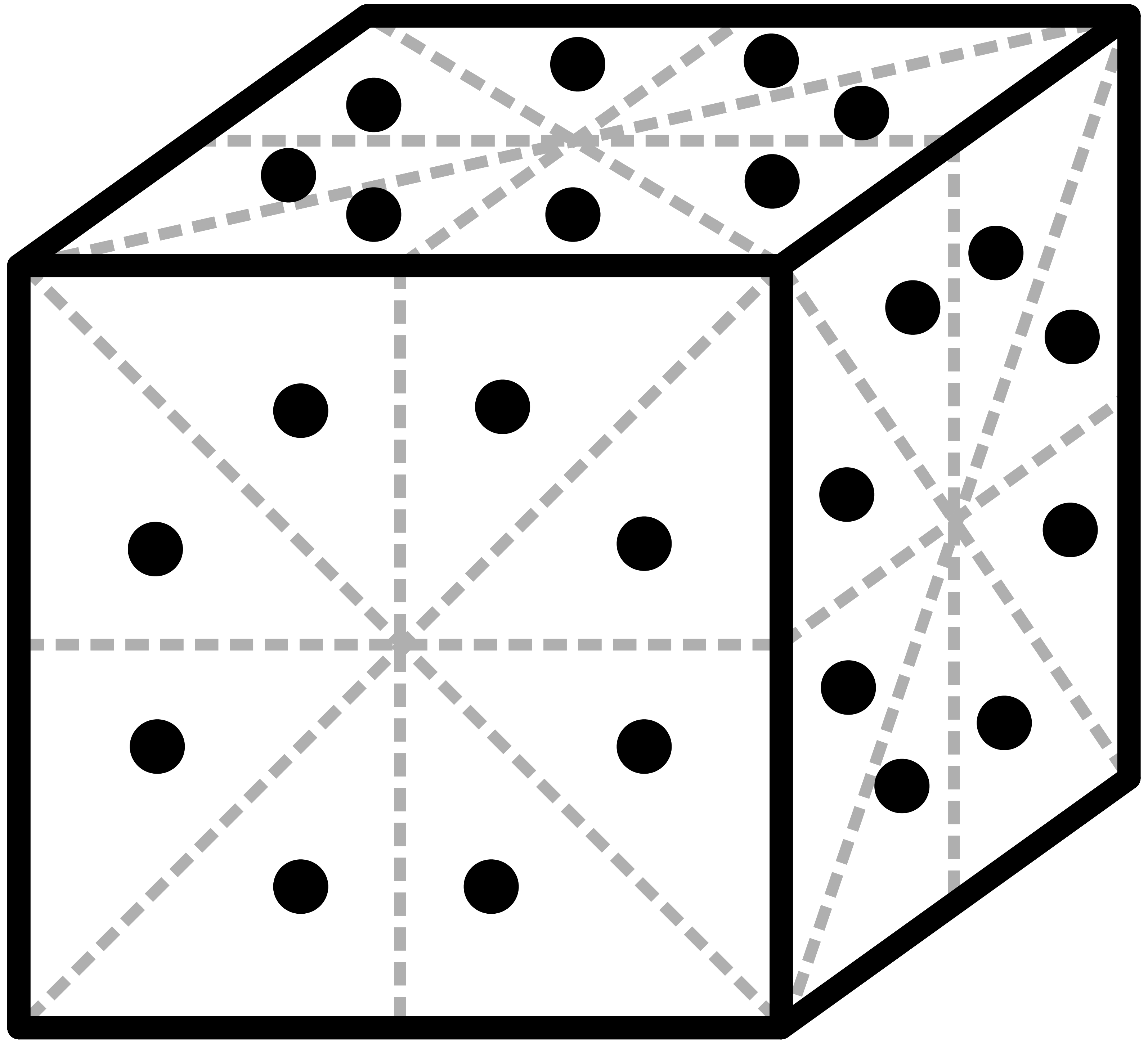

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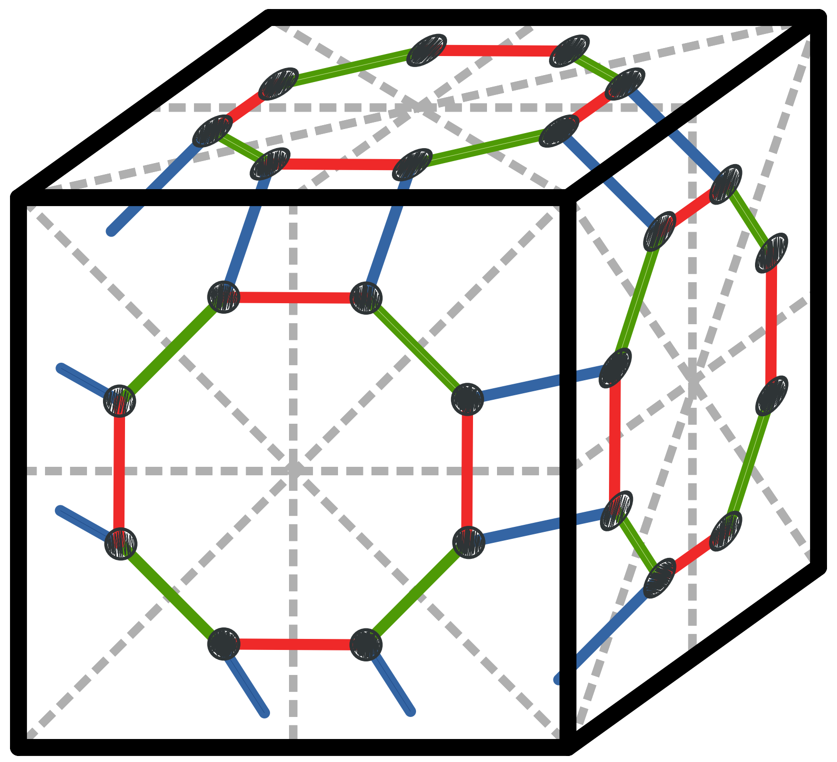

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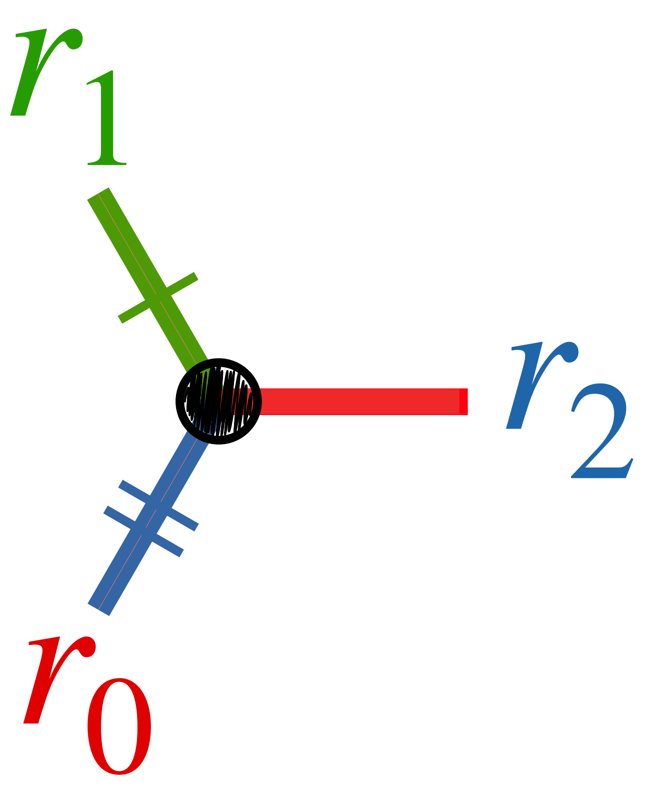

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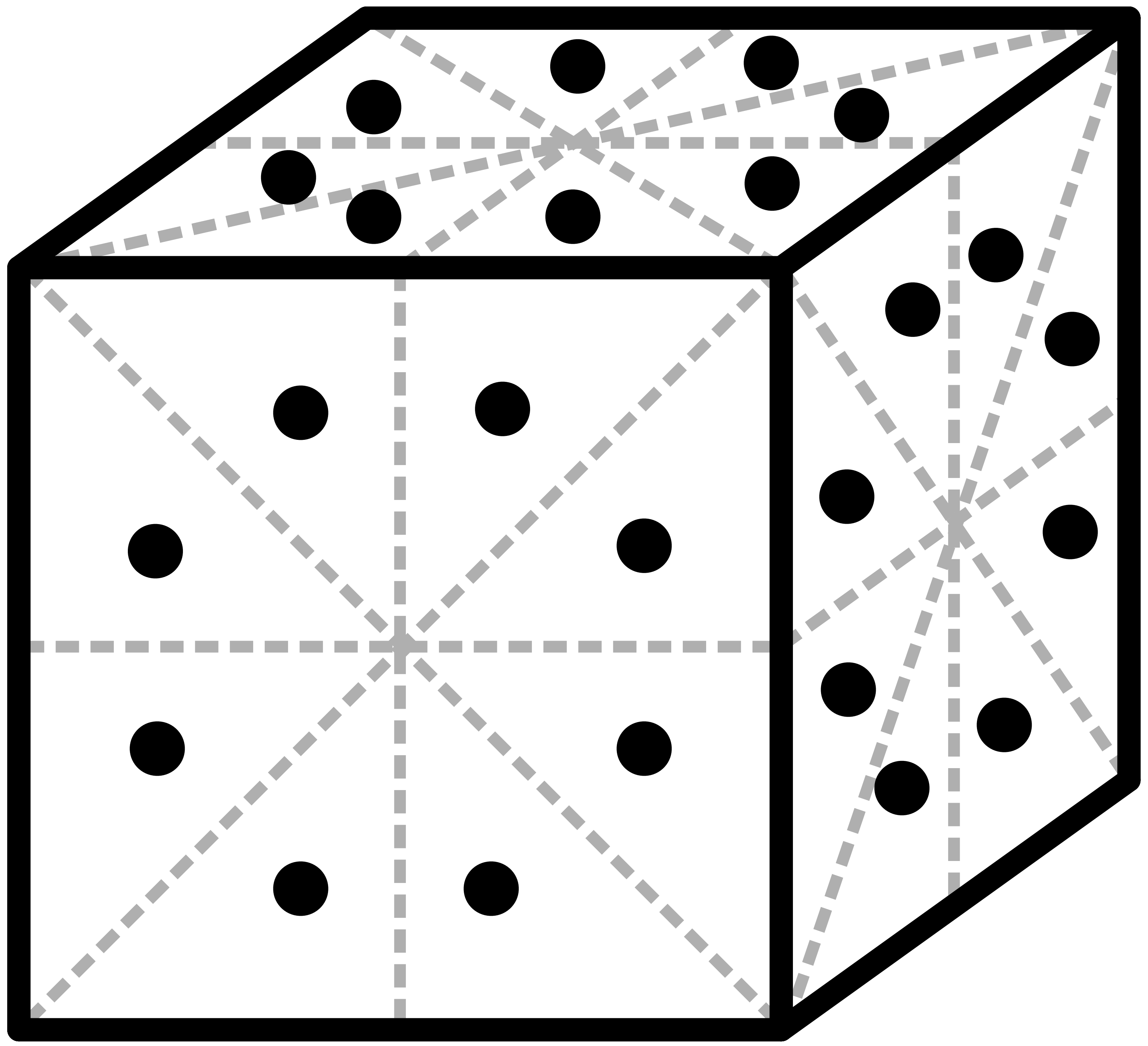

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m-premaniplex

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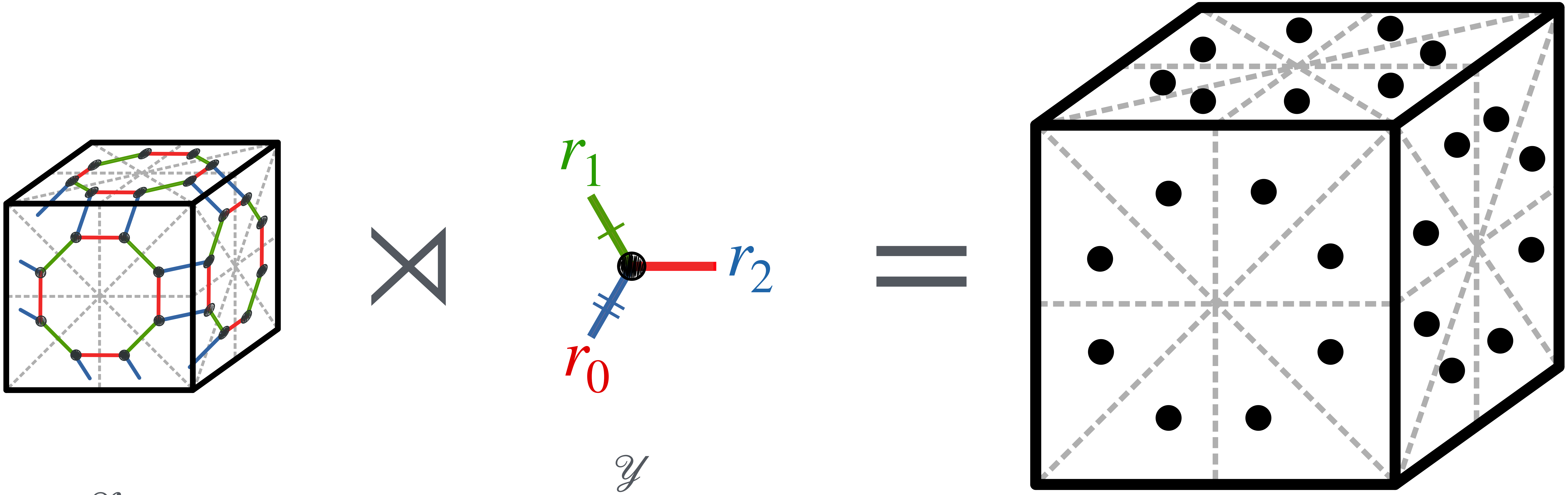

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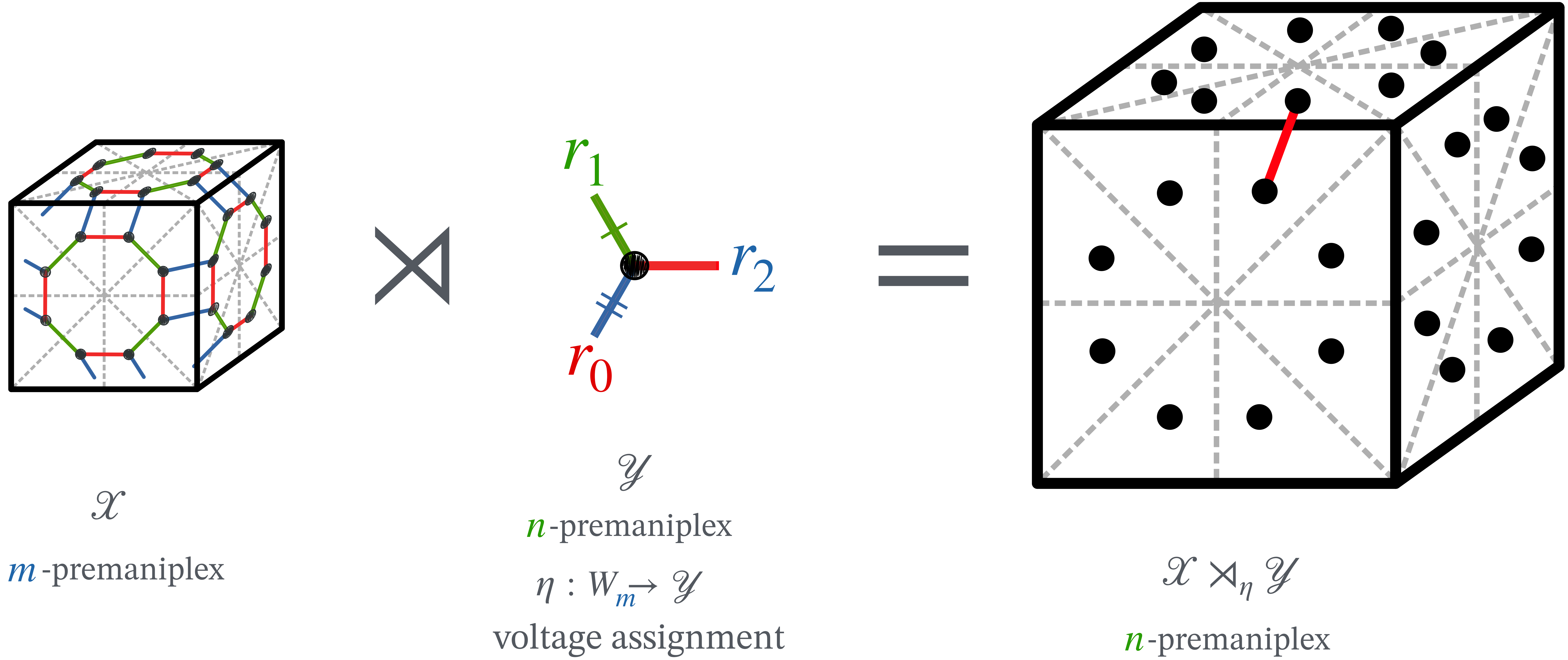

voltage operations

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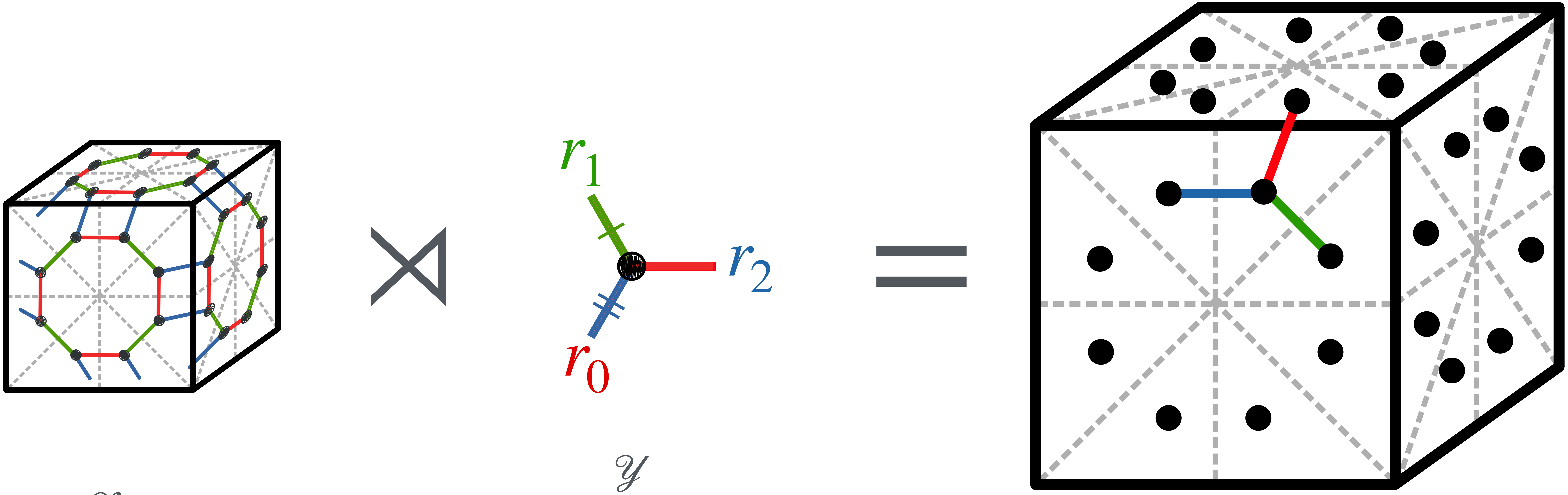

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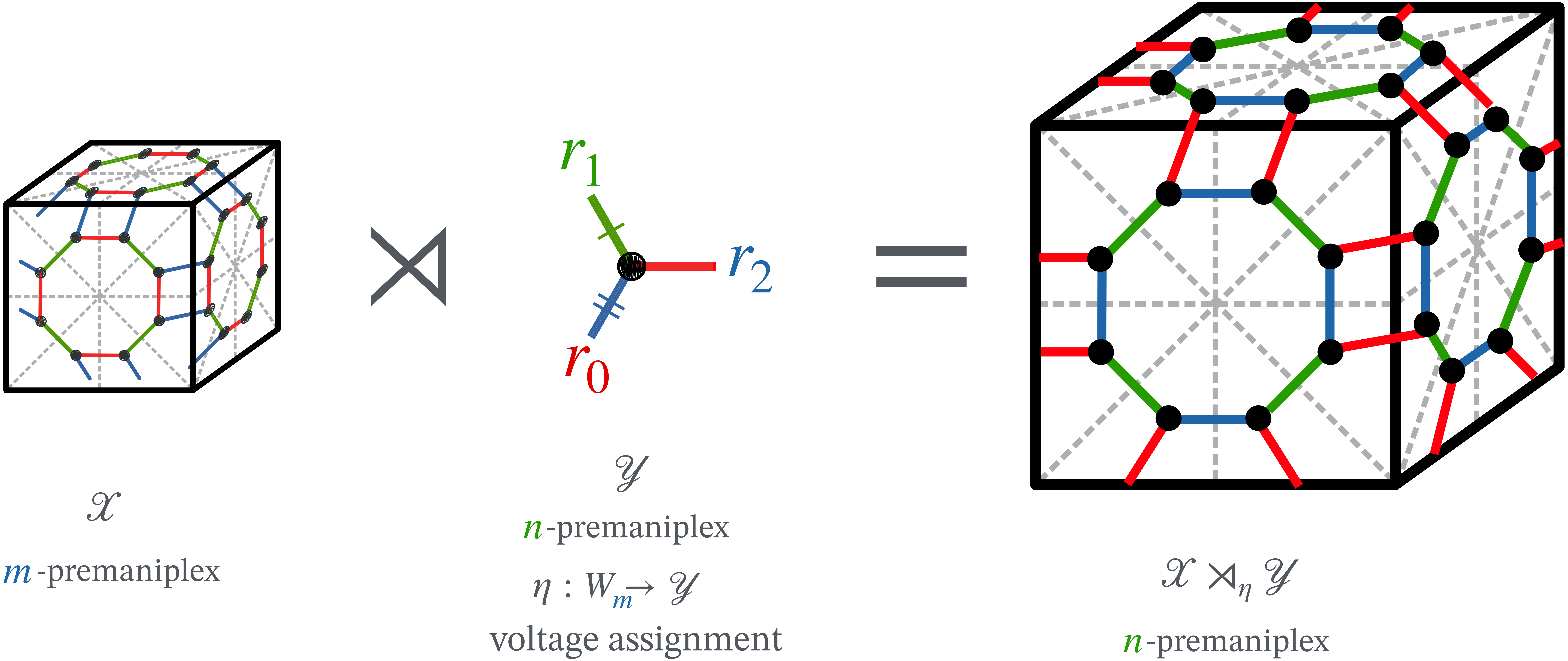

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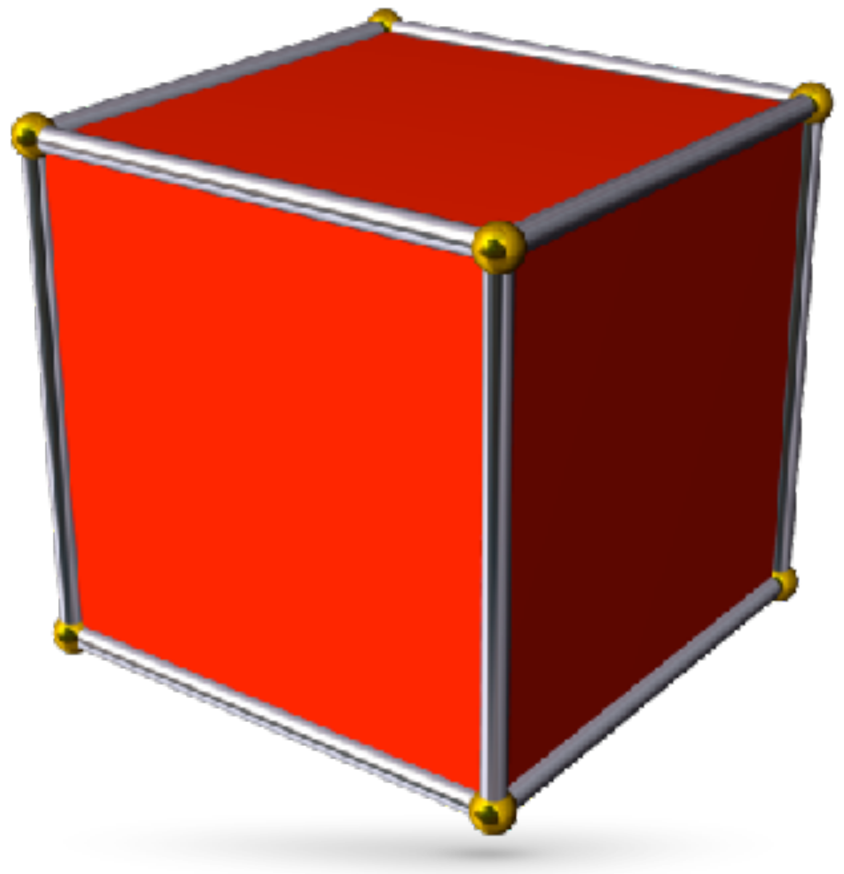
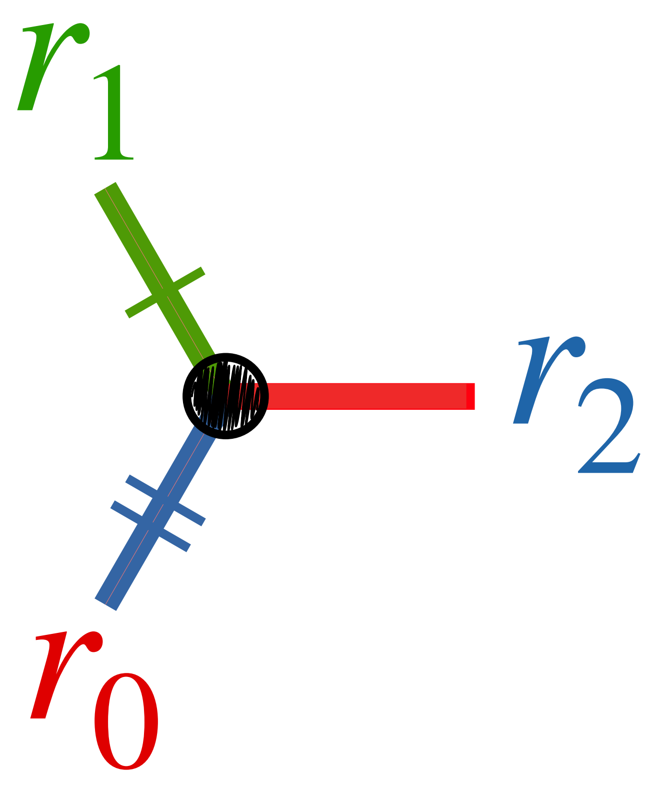


$\rtimes$



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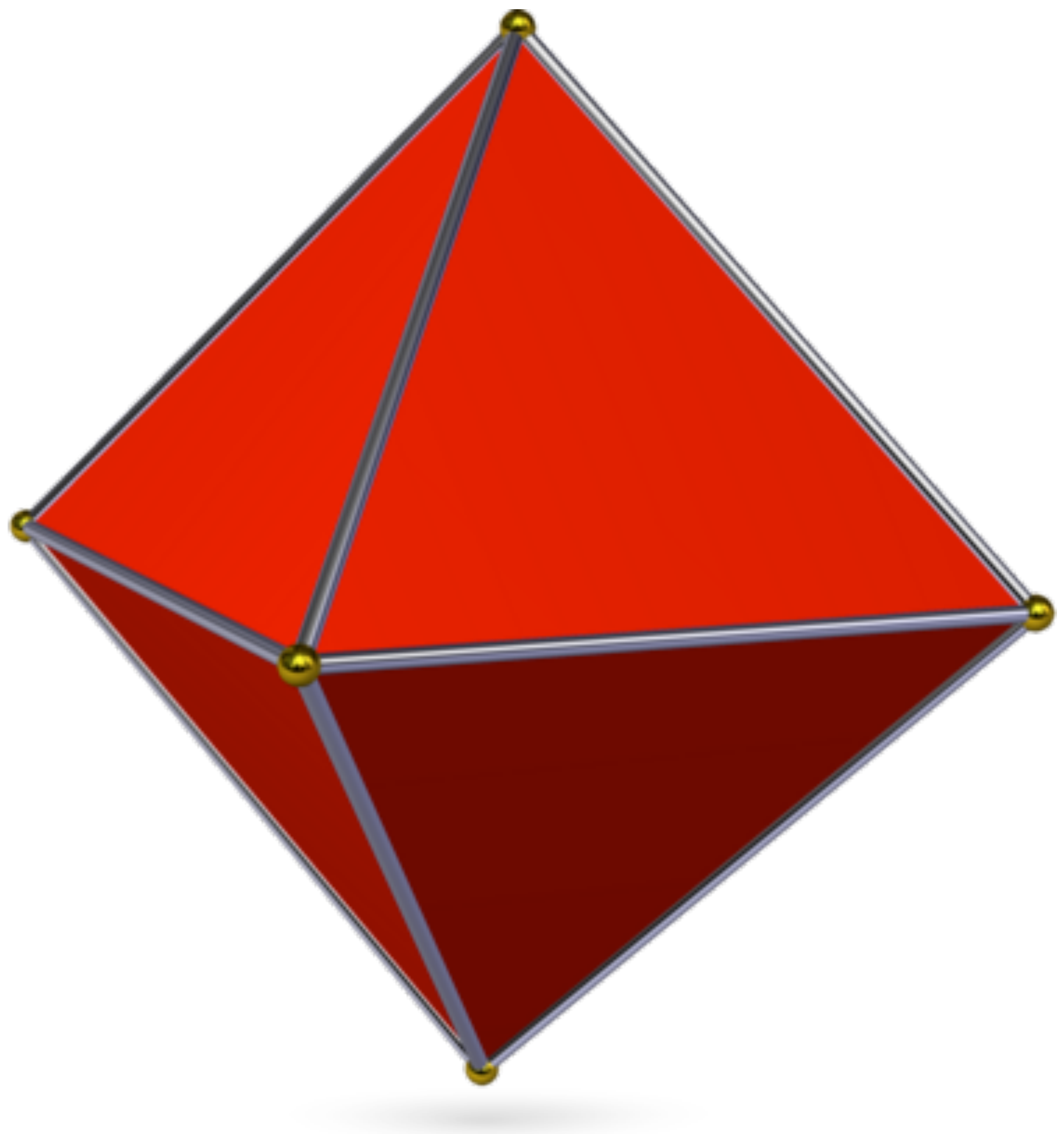

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voltage operations

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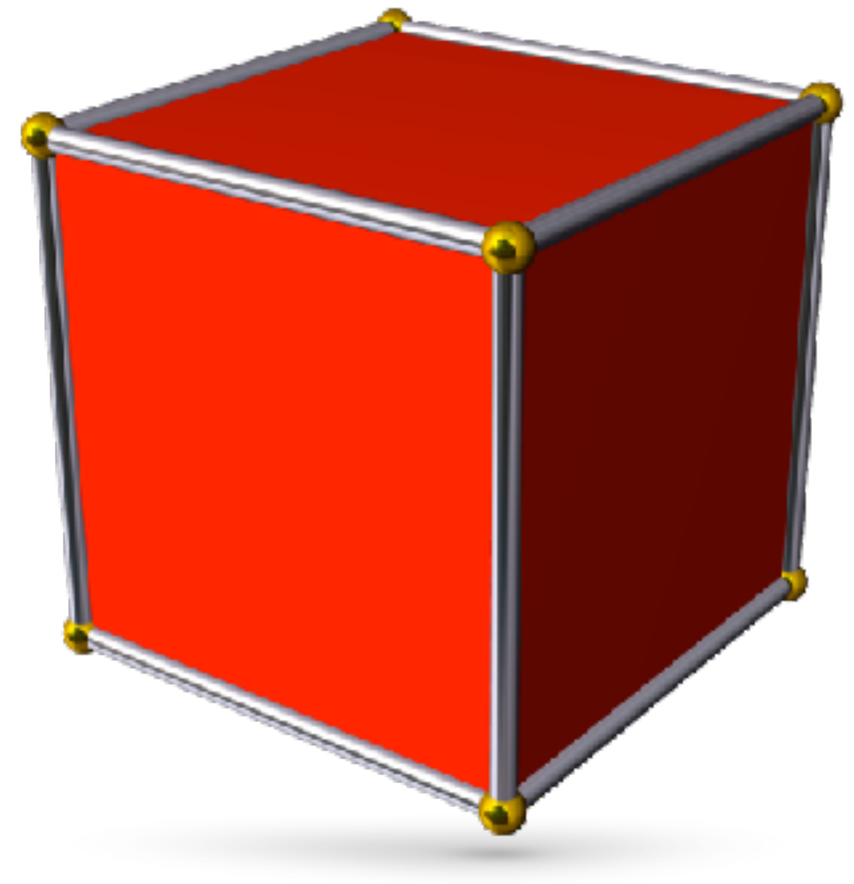
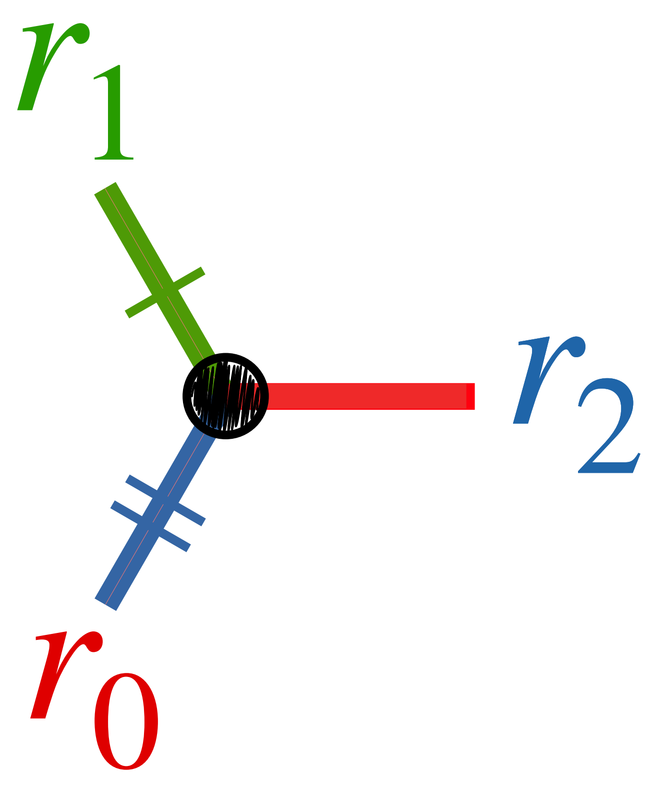


$\mathcal{X}$

m-premaniplex

$\rtimes$



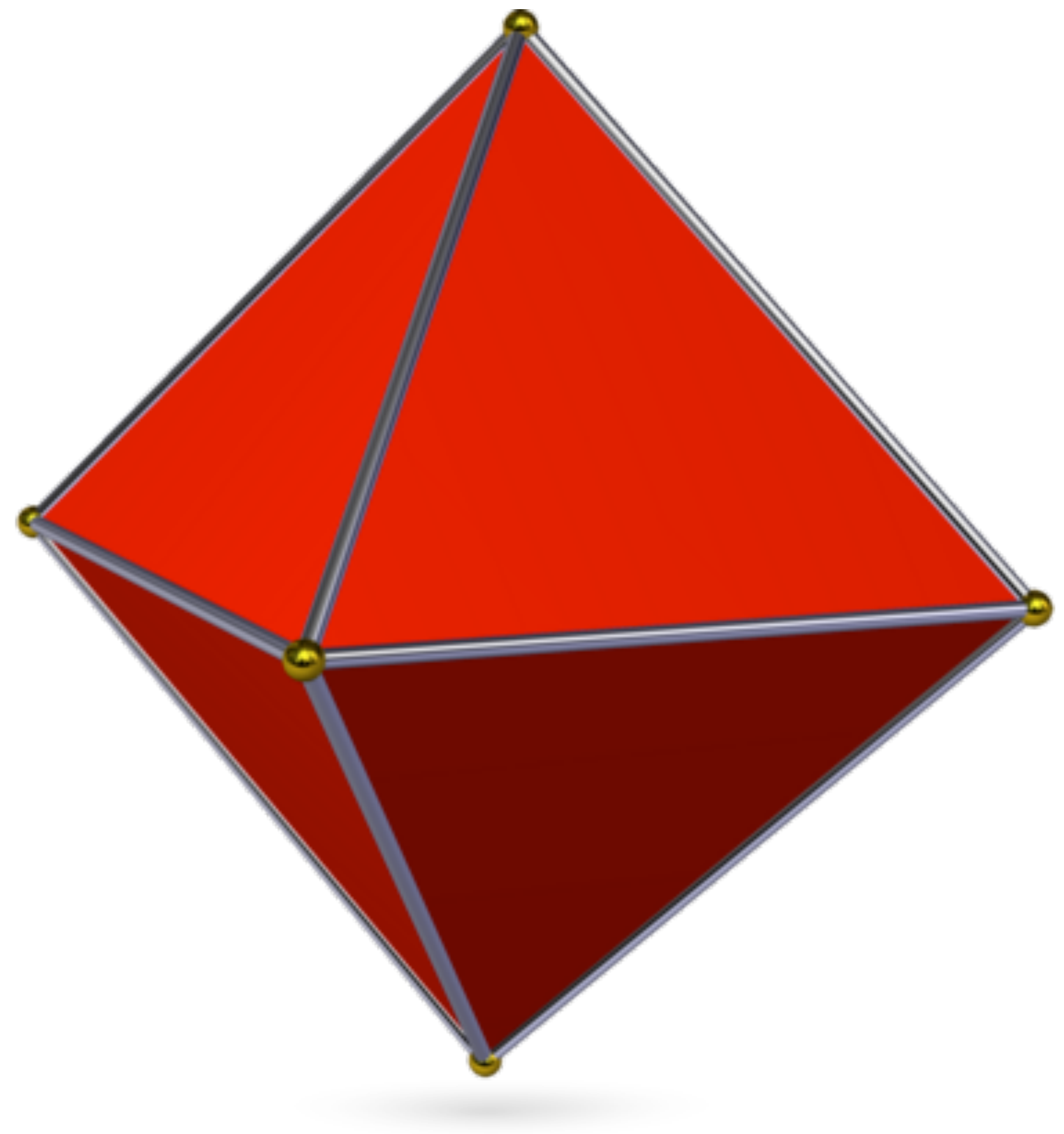
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

voltage assignment

$=$

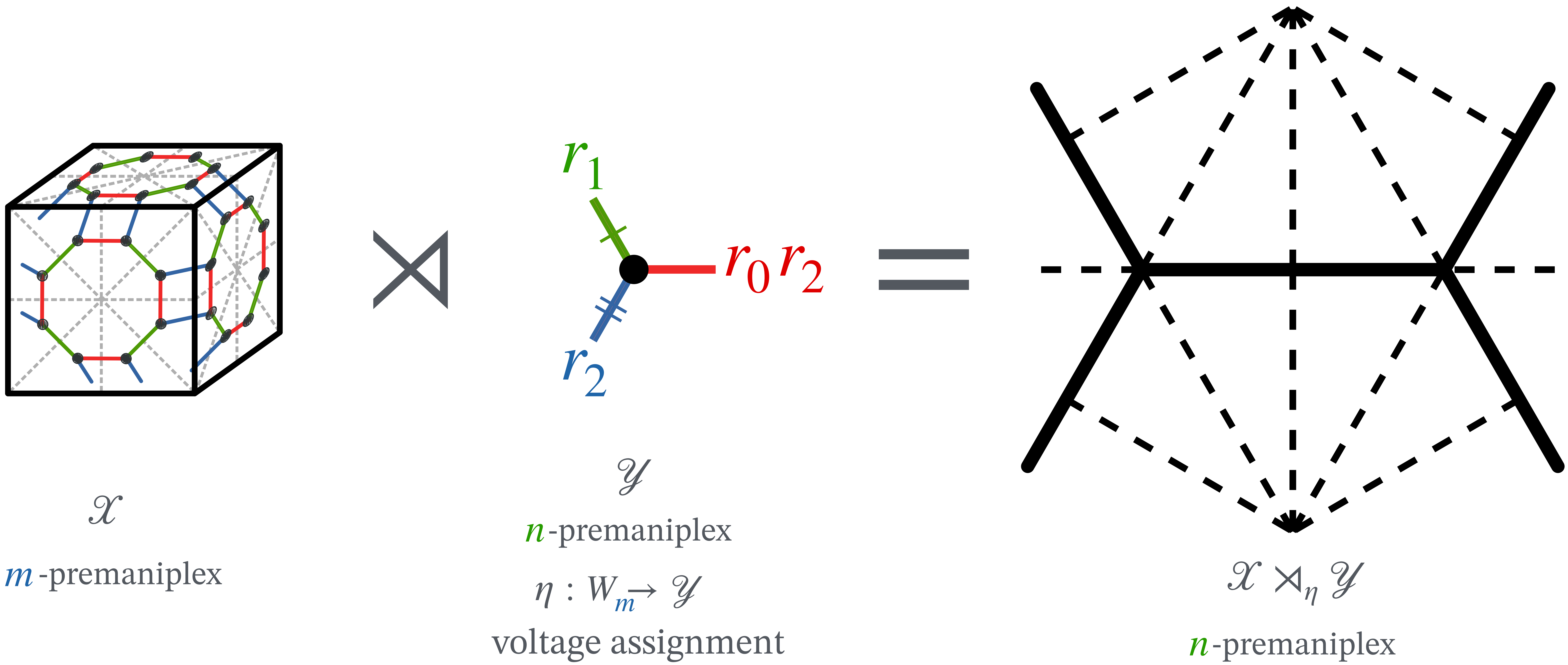


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

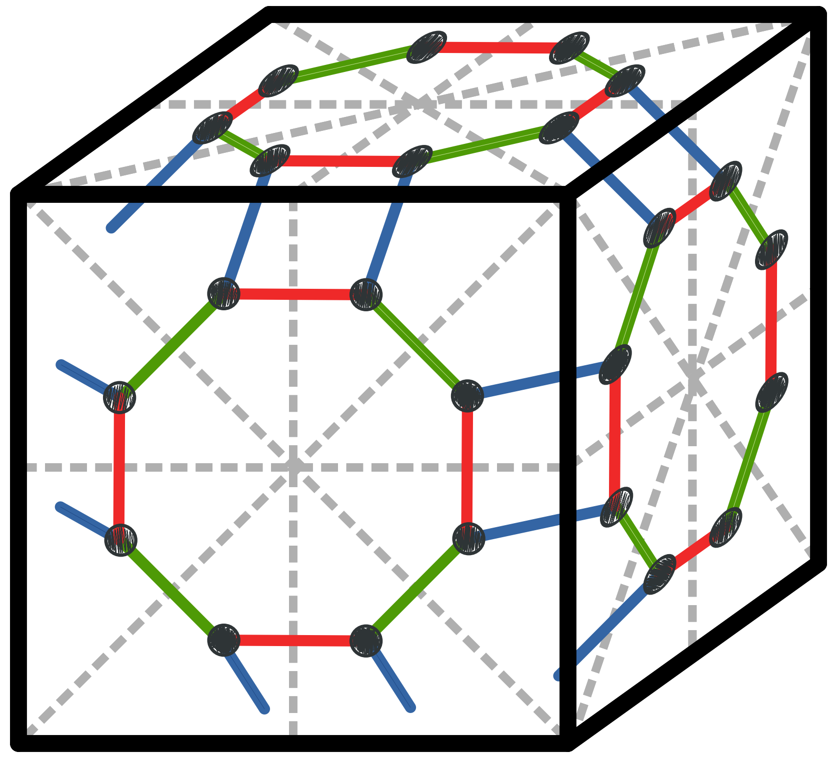
voltage operations

- An (m, n)- voltage operation:



voltage operations

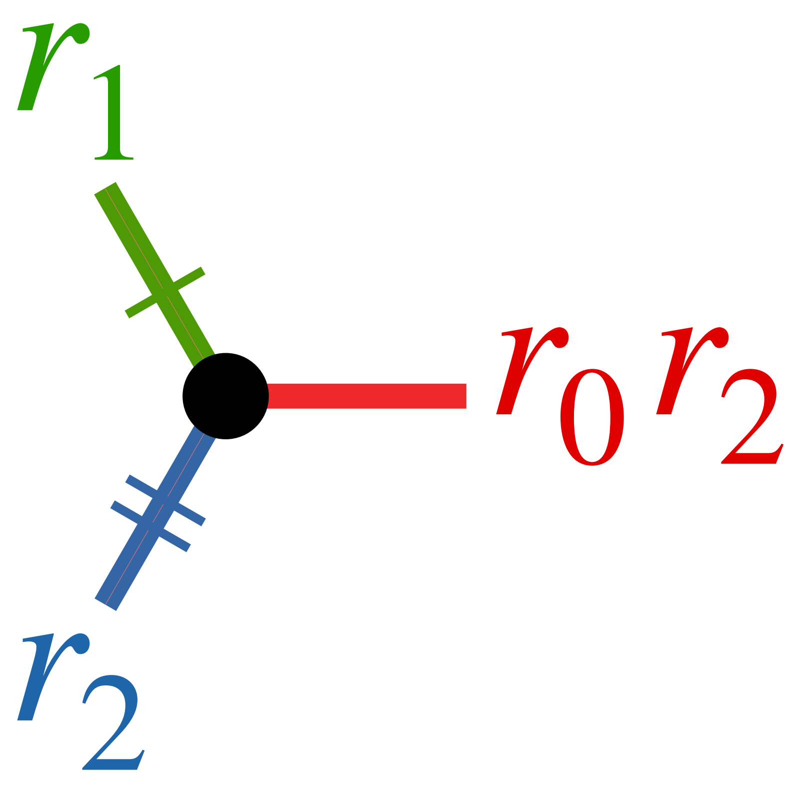
- An (m, n)- voltage operation:



$\mathscr{X}$

m-premaniplex

$\rtimes$



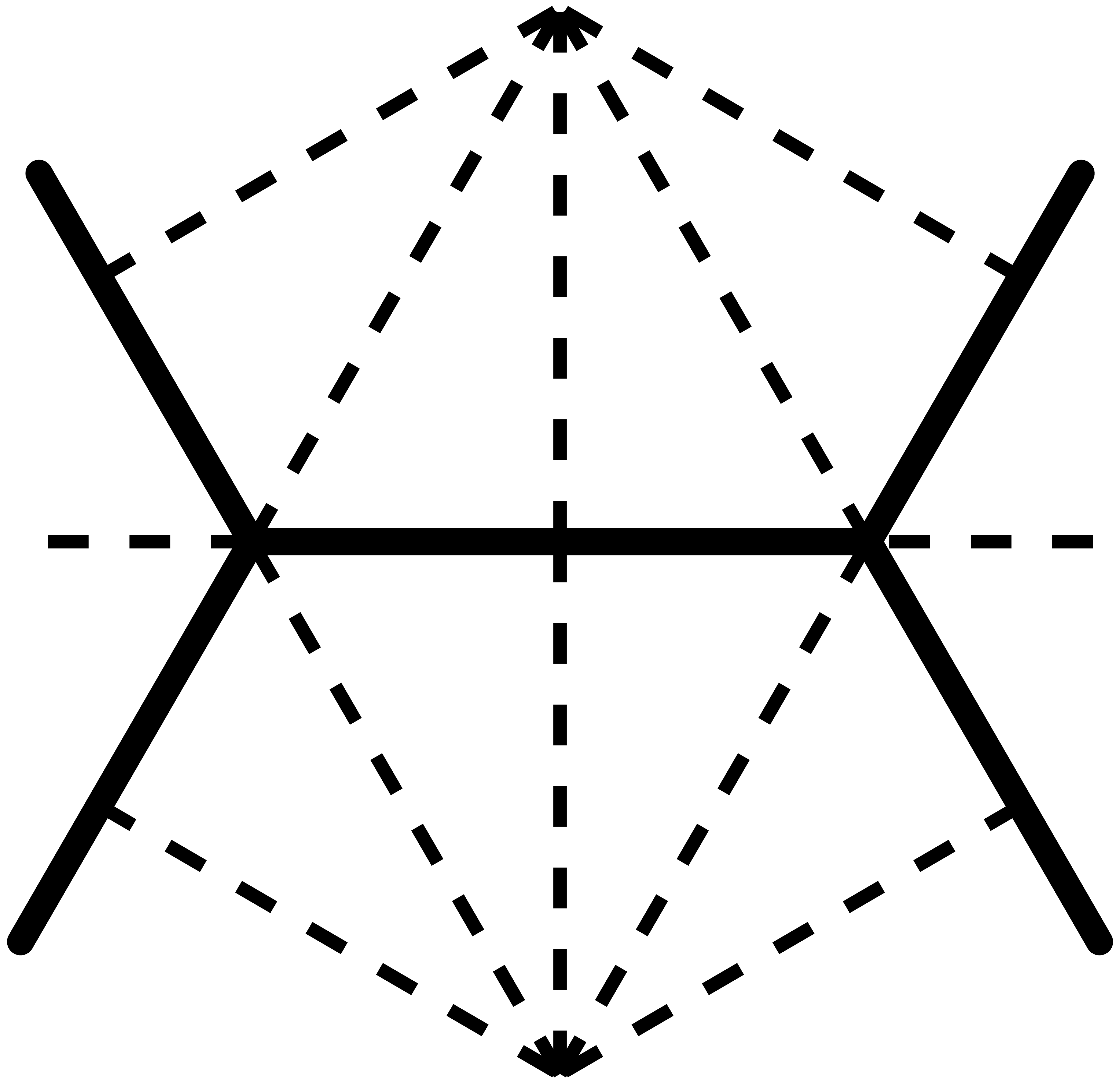
$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

voltage assignment

$=$

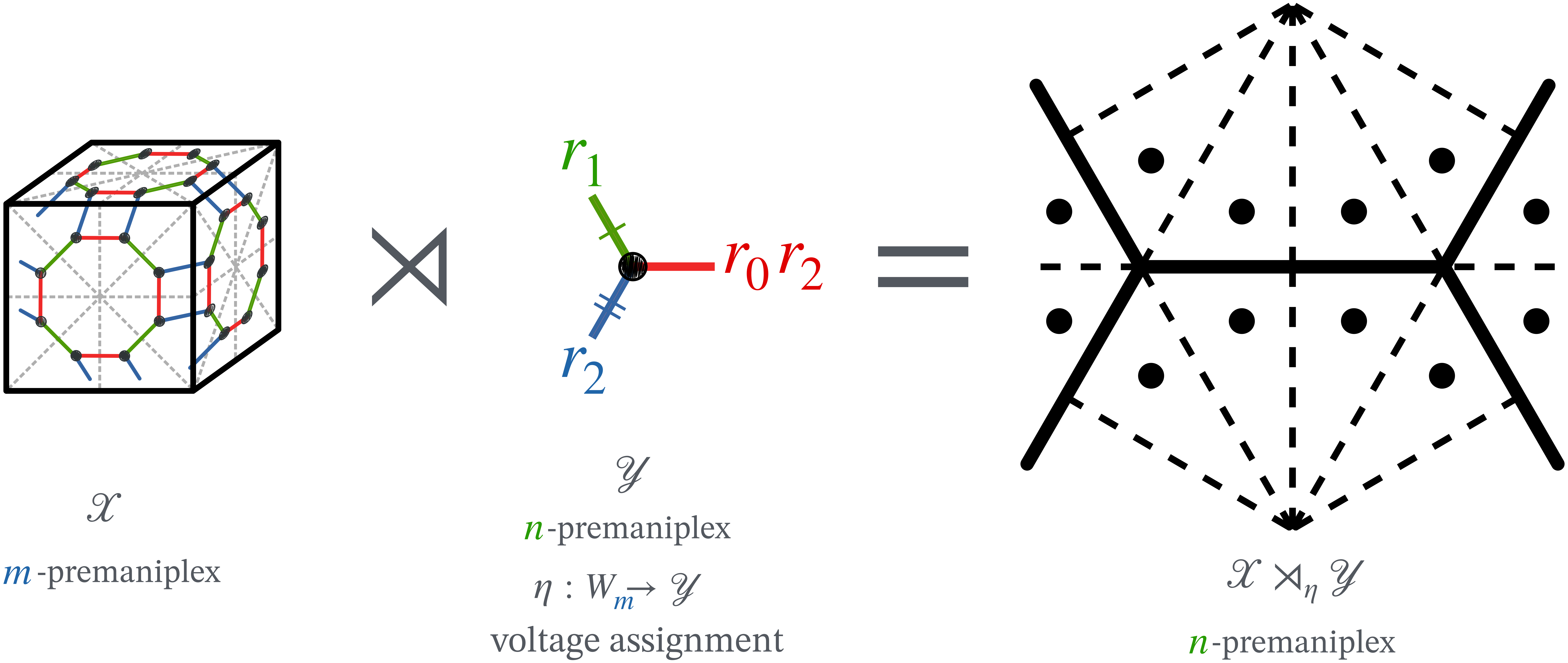


$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

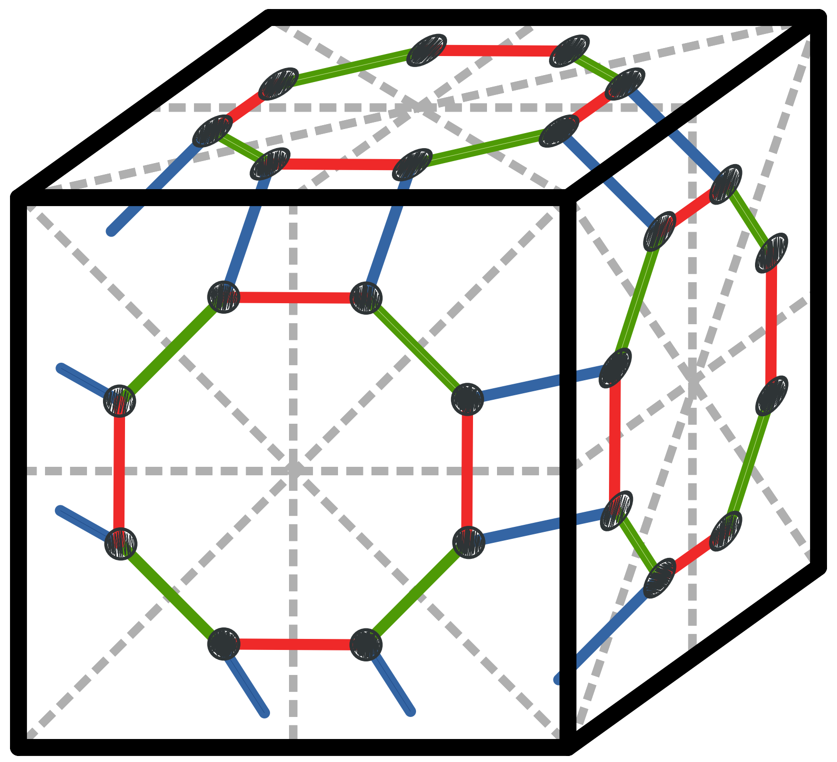
voltage operations

- An (m, n)- voltage operation:



voltage operations

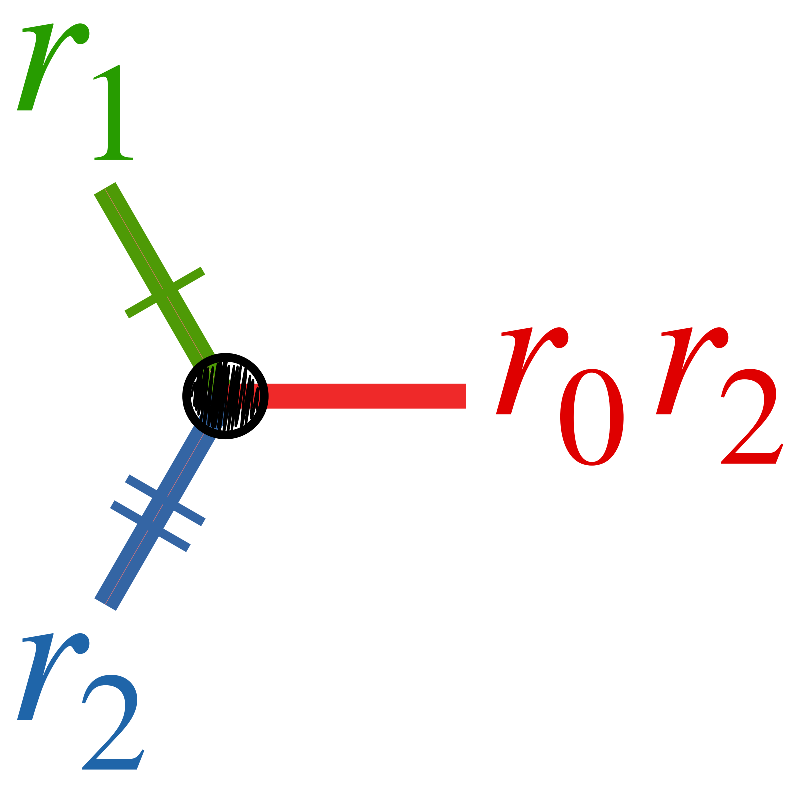
- An (m, n)- voltage operation:



$\mathscr{X}$

m-premaniplex

$\rtimes$



$\mathscr{Y}$

n-premaniplex

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voltage assignment

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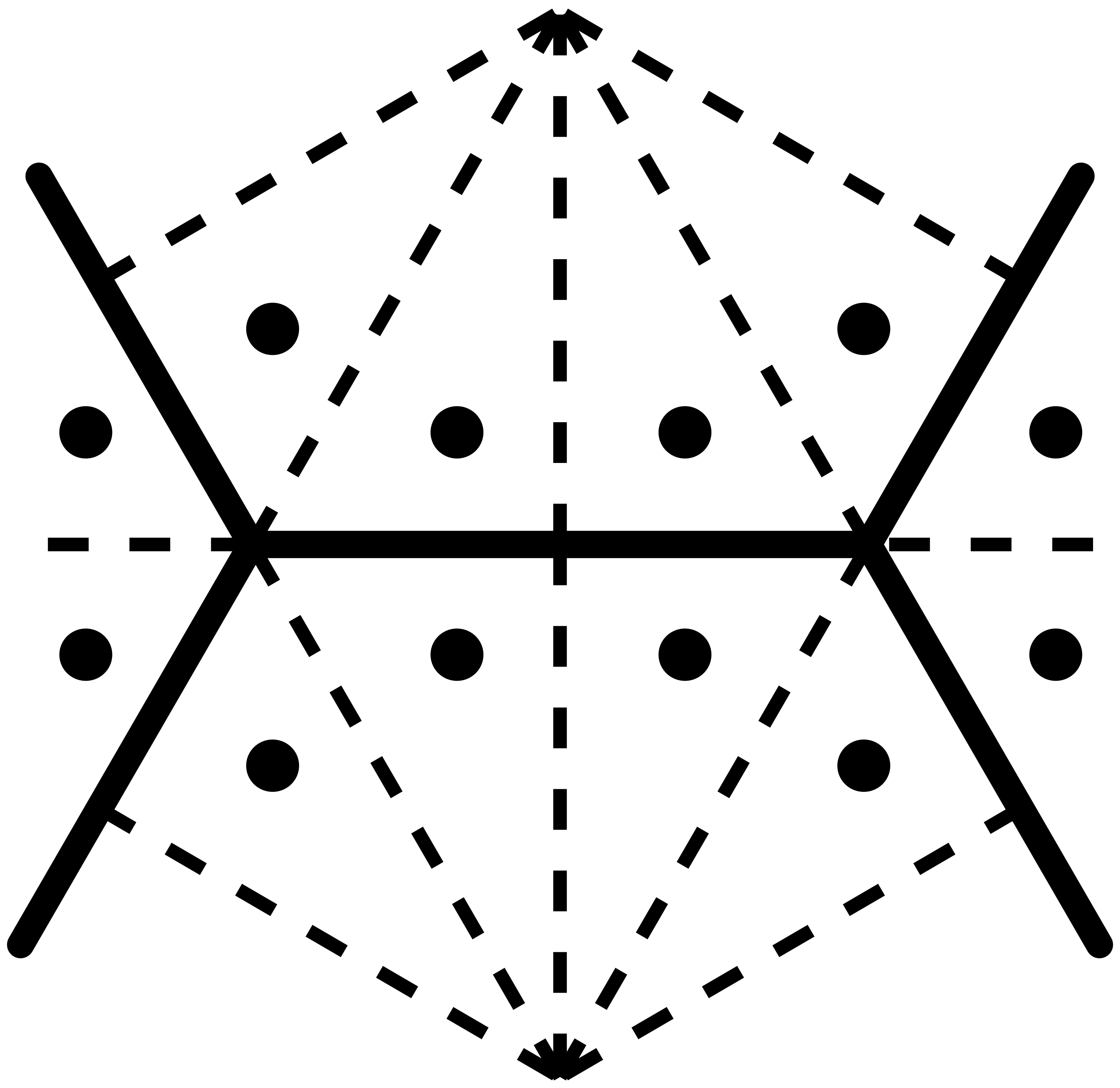
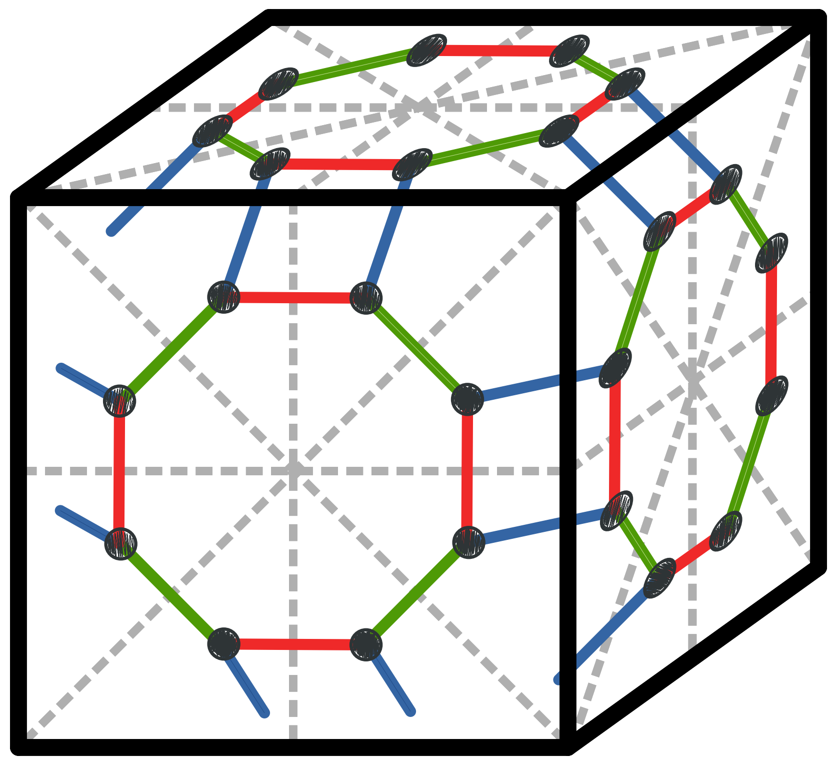


$\mathscr{X} \rtimes_\eta \mathscr{Y}$

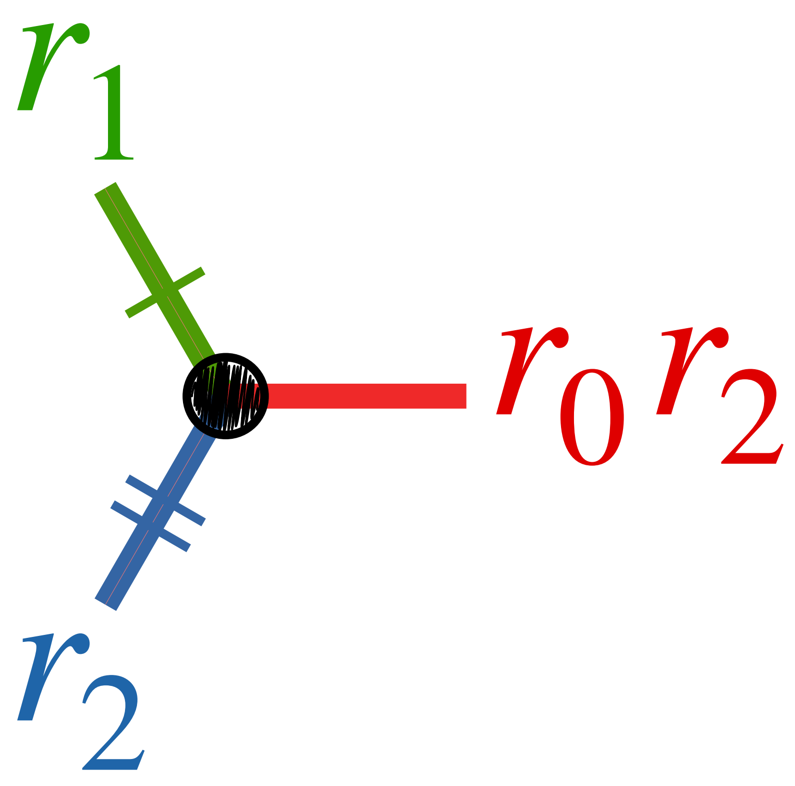
n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\rtimes$



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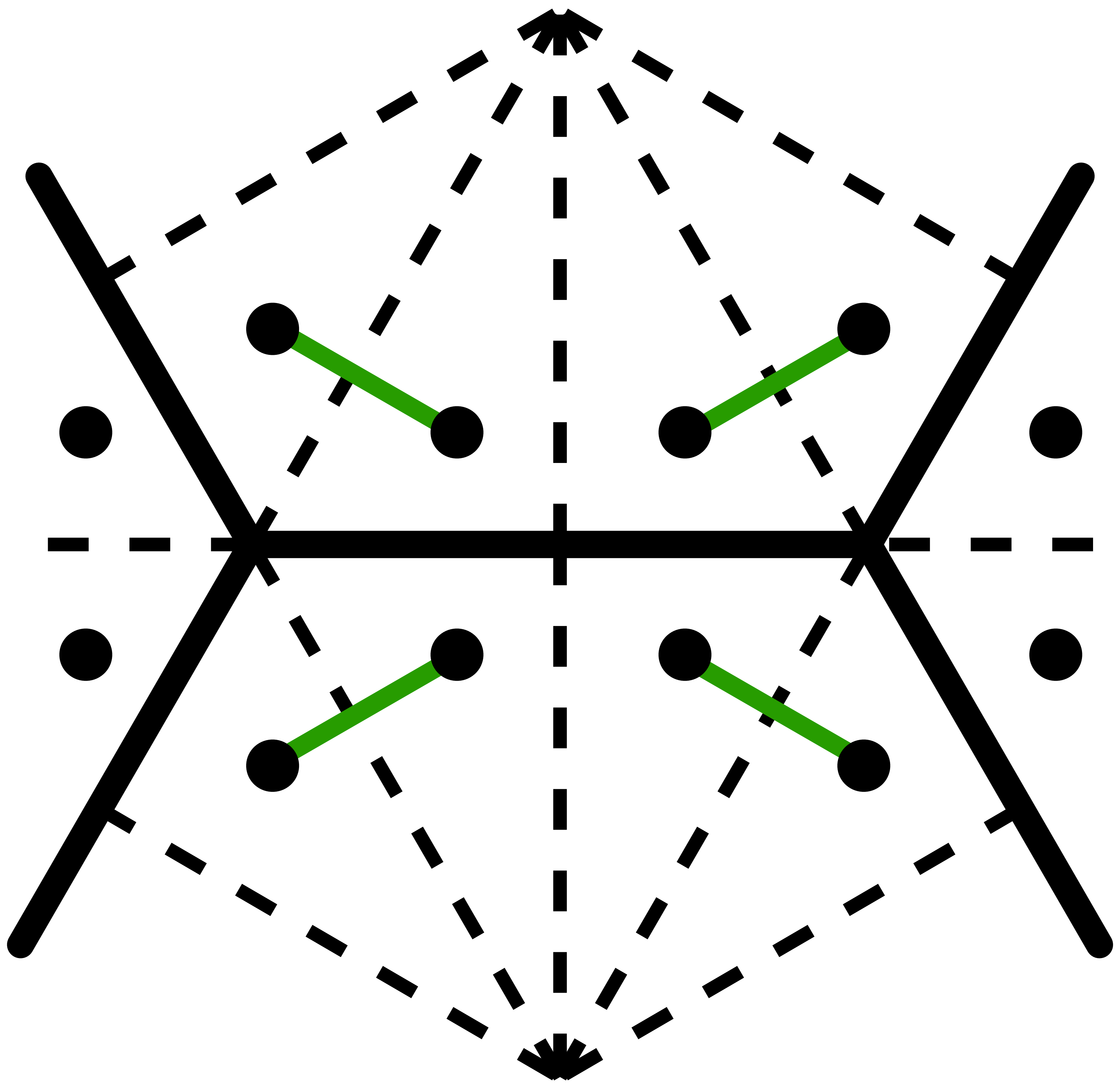


$\mathcal{X}$

m-premaniplex

$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

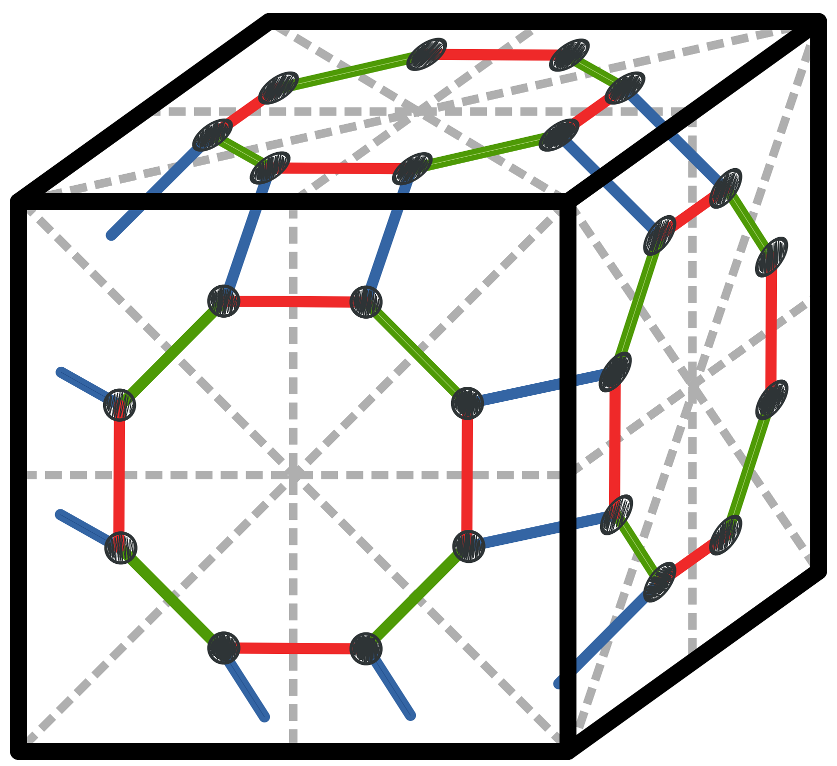
voltage assignment

$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

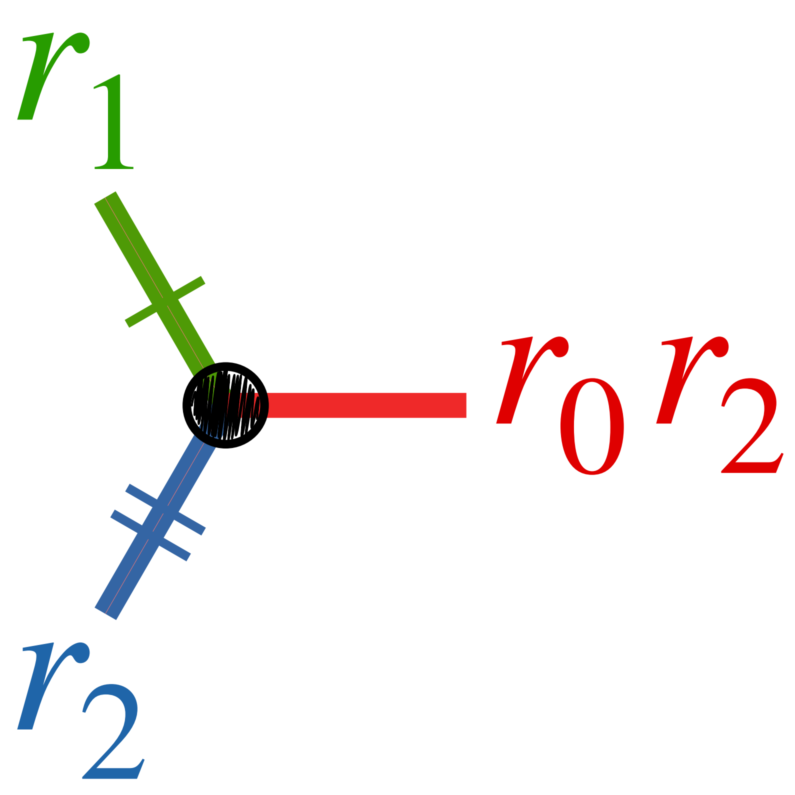
- An (m, n)- voltage operation:



$\mathcal{X}$

m-premaniplex

$\rtimes$



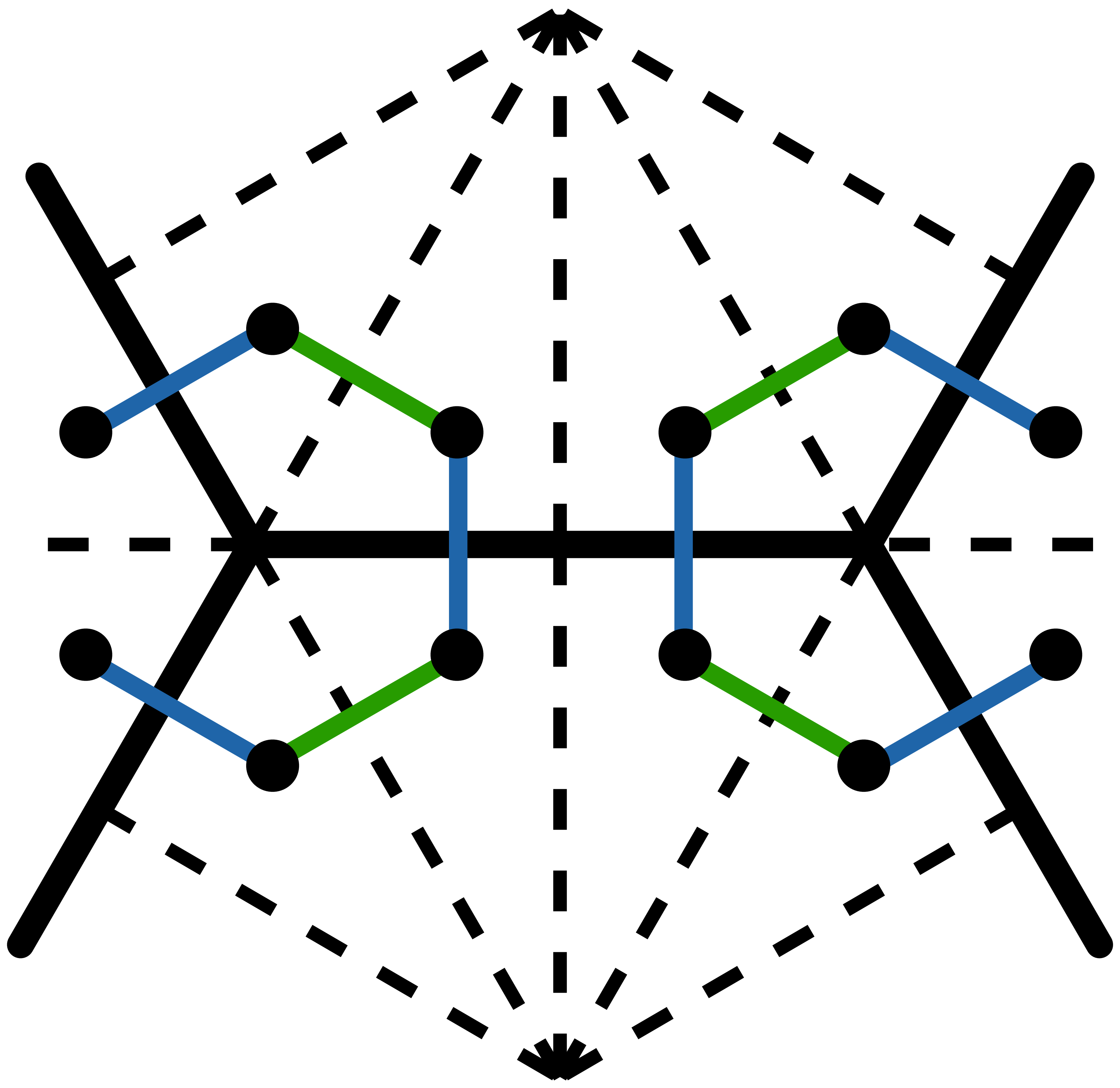
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

voltage assignment

$=$

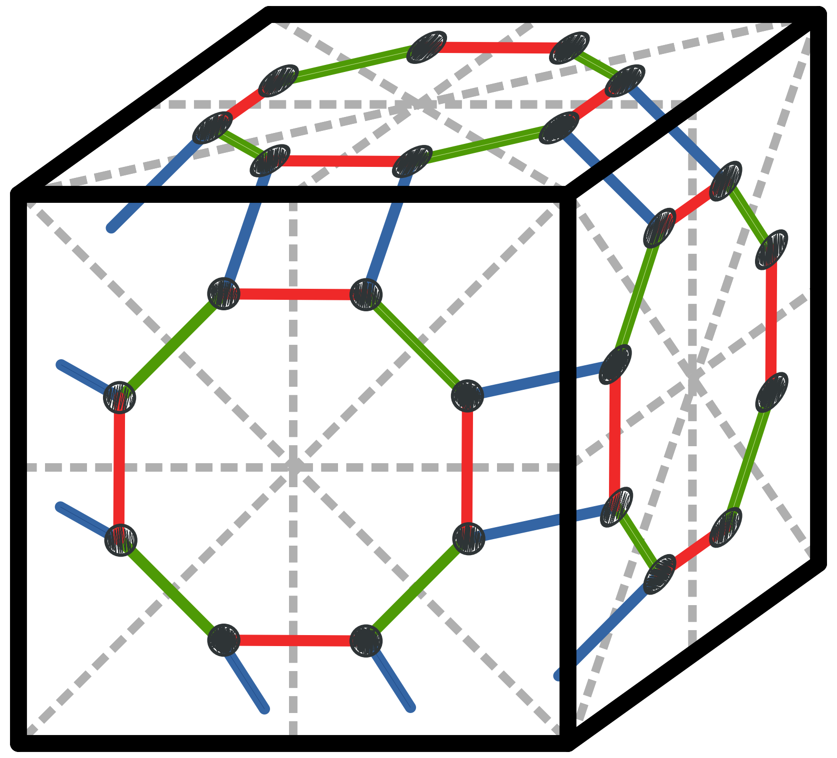


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

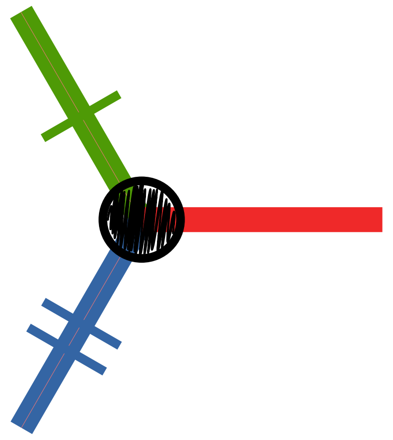
n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\rtimes$



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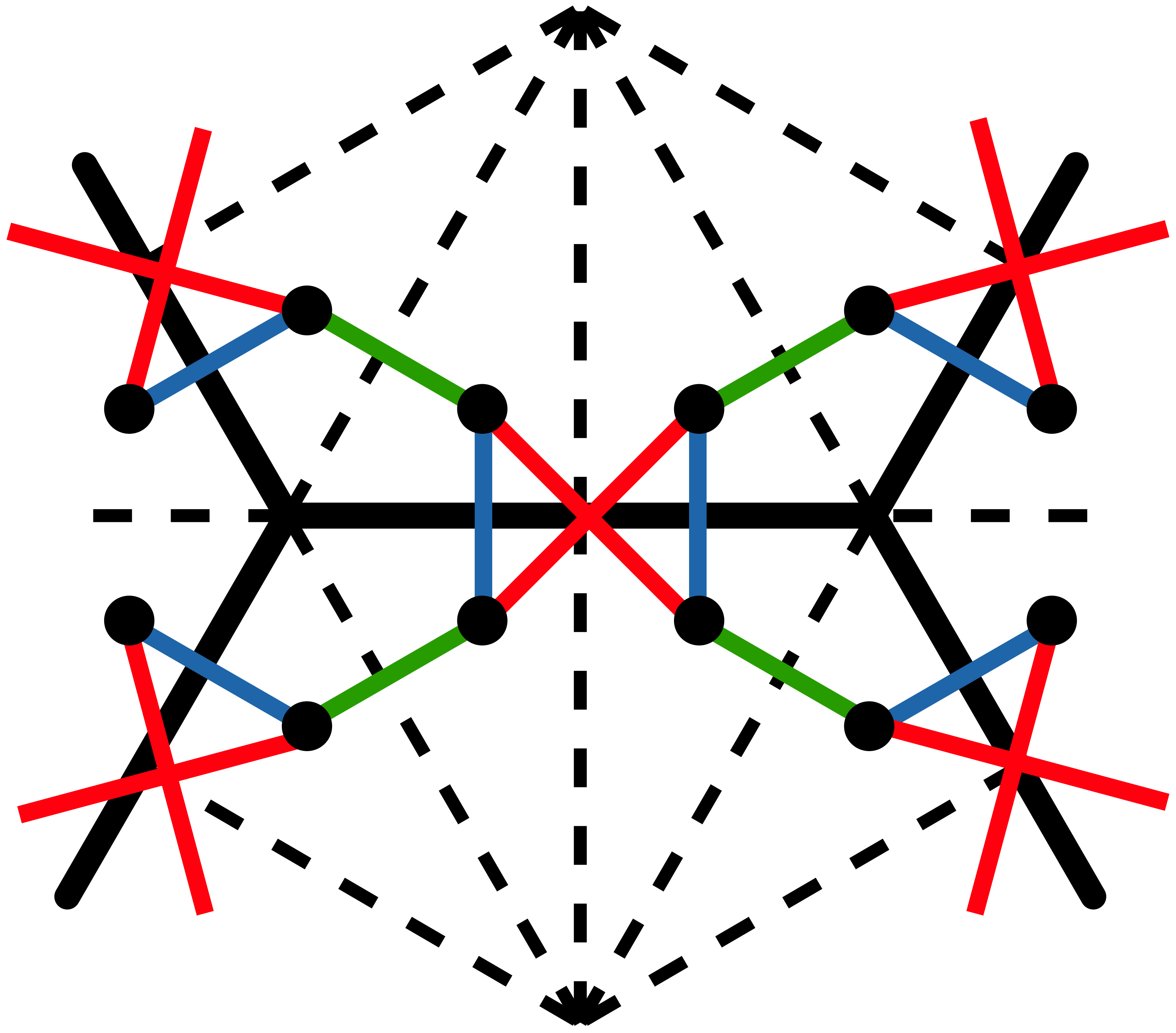


$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

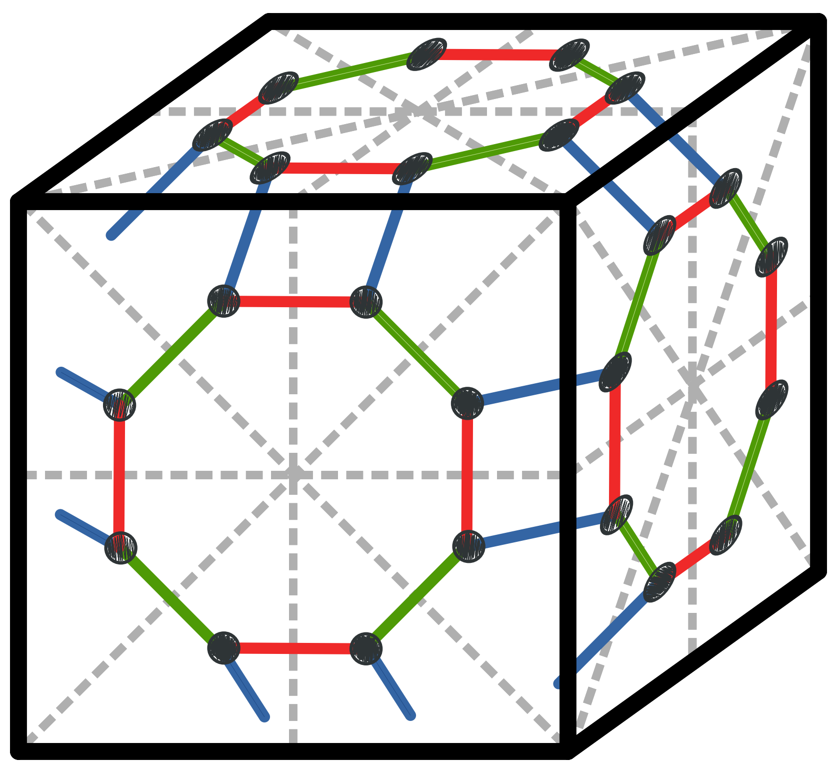
voltage assignment

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

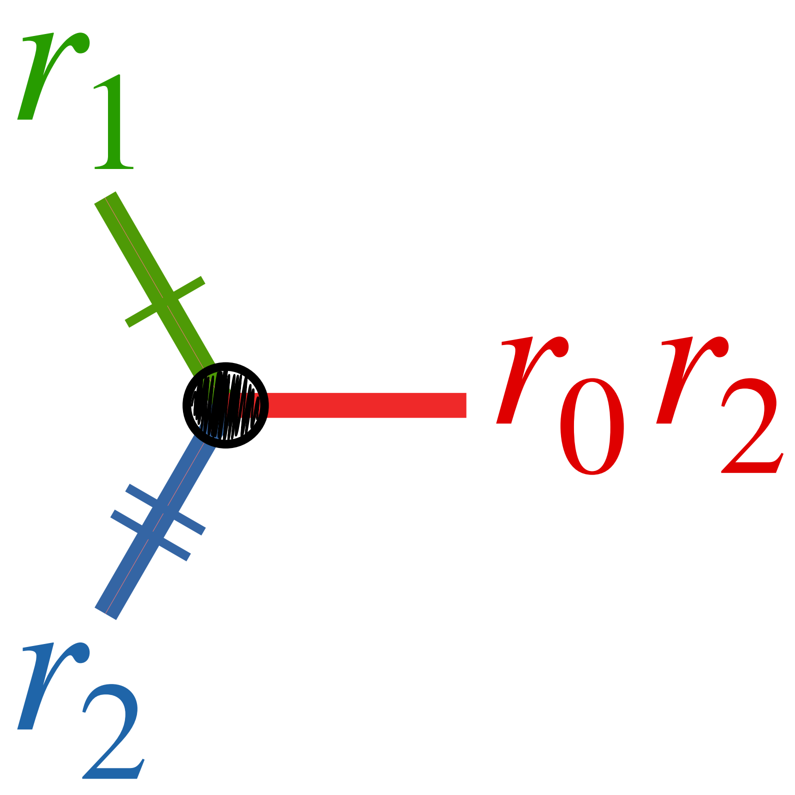
n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\rtimes$



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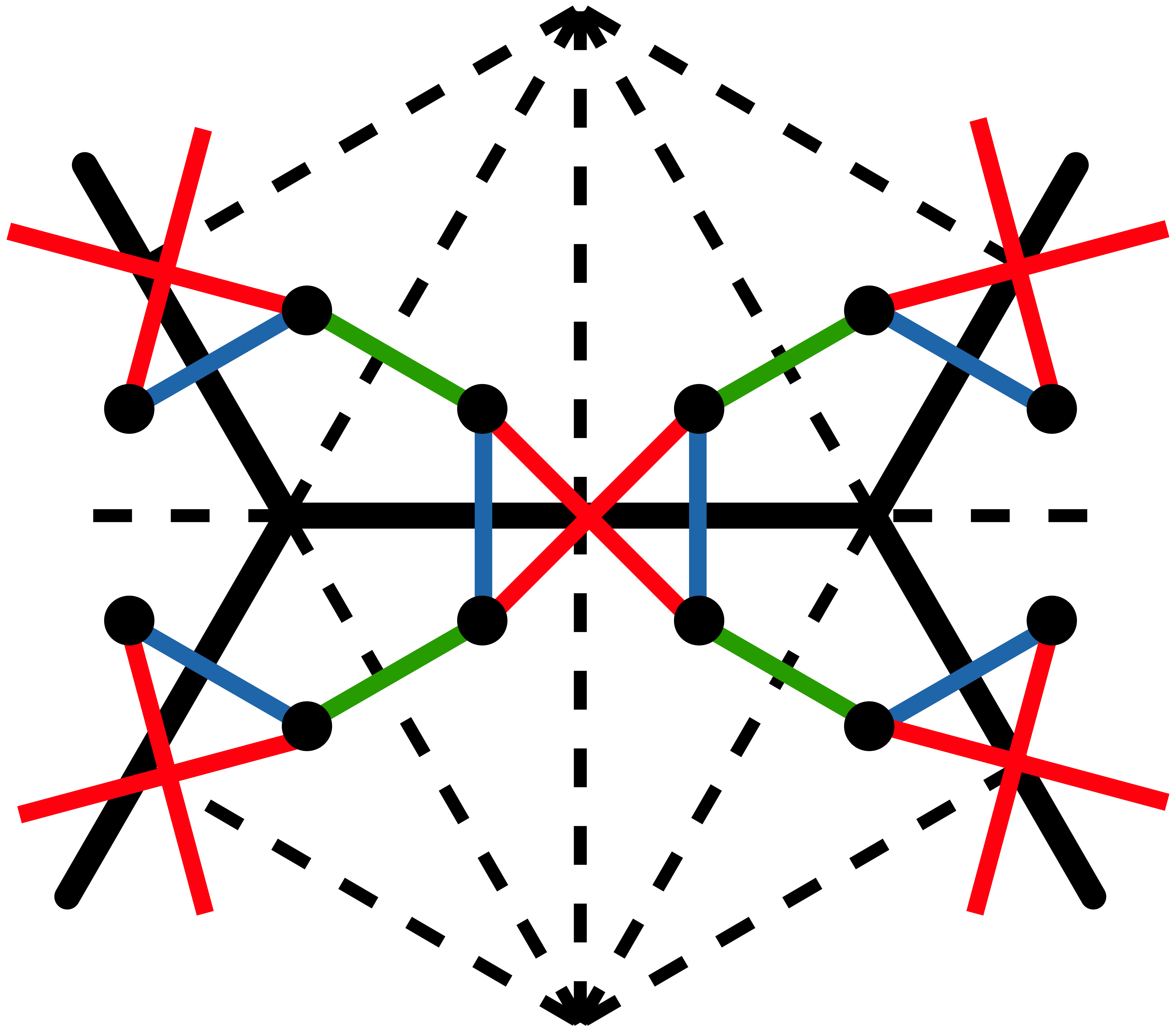


$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

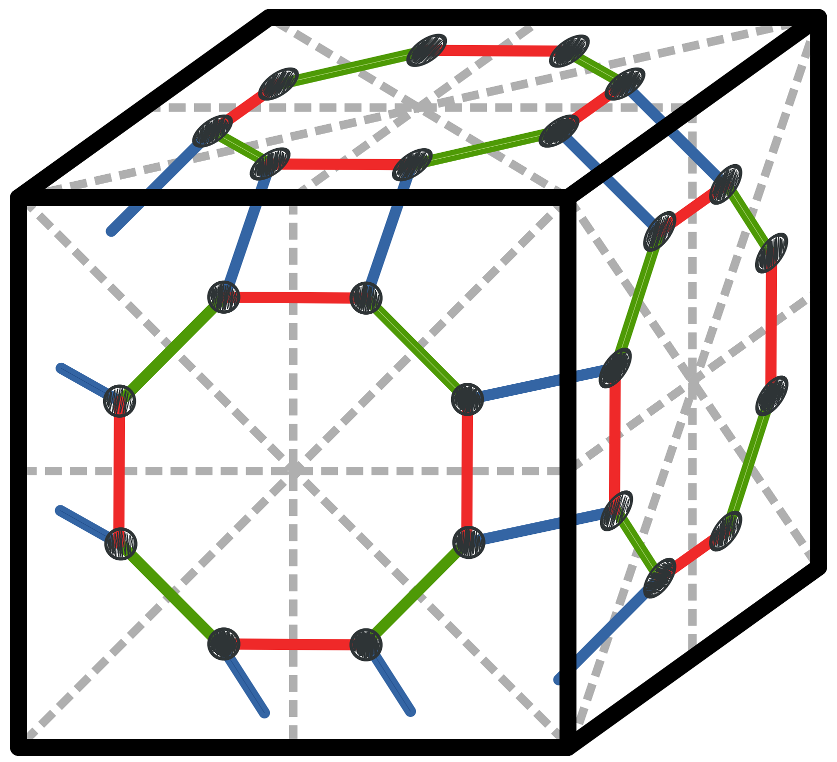
voltage assignment

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

voltage operations

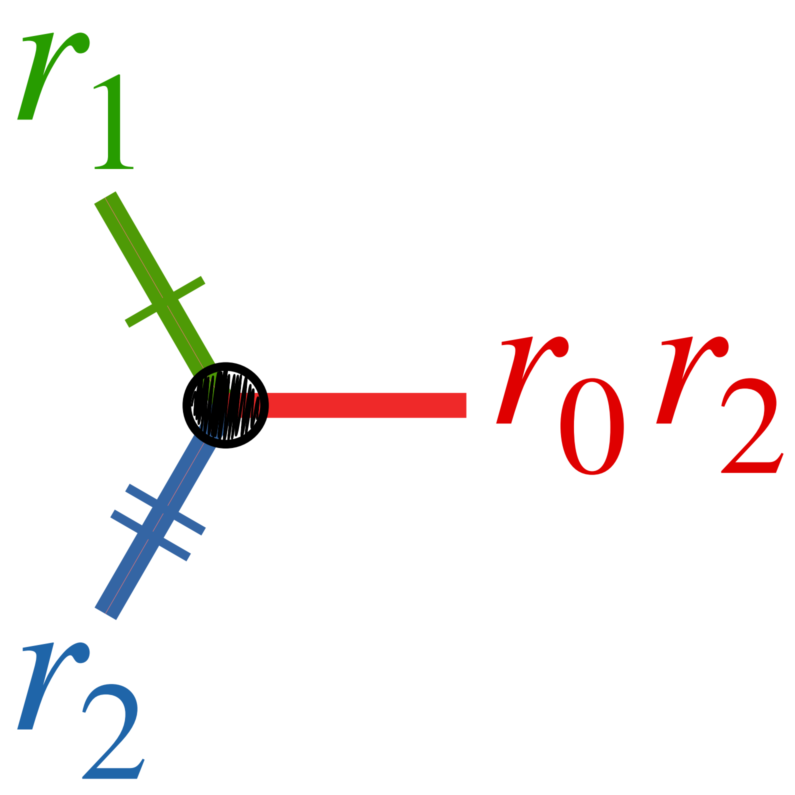
- An (m, n)- voltage operation:



$\mathscr{X}$

m-premaniplex

$\rtimes$



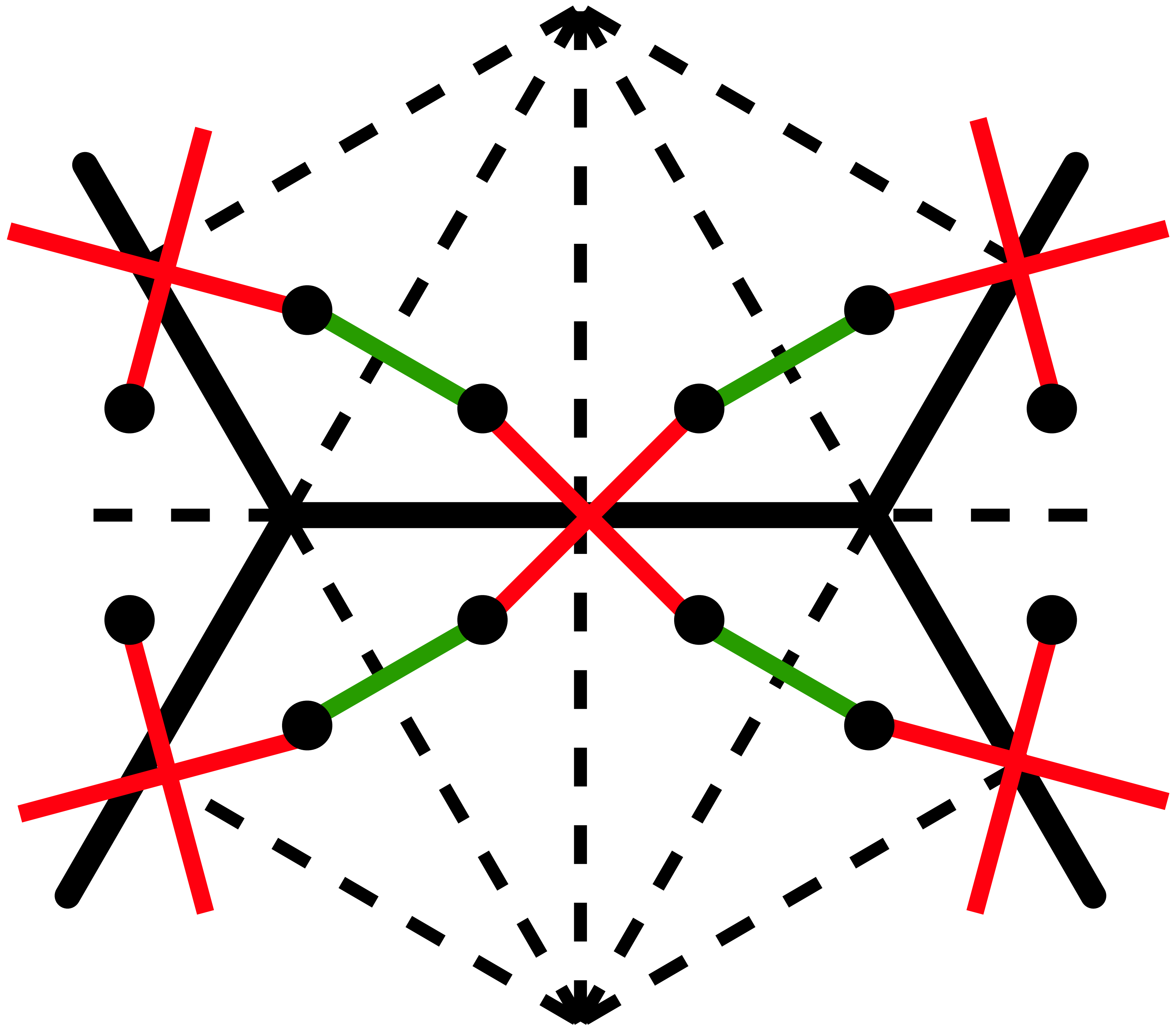
$\mathscr{Y}$

n-premaniplex

$\eta : W_m \to \mathscr{Y}$

voltage assignment

$=$

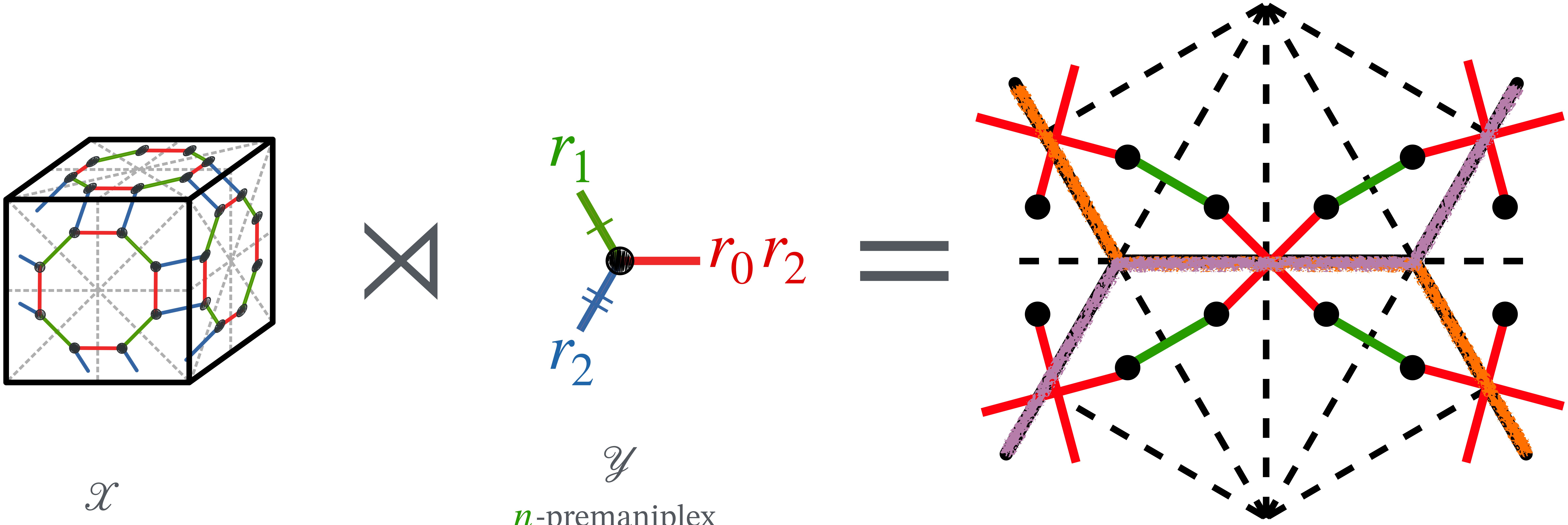


$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\mathscr{X}$

m-premaniplex

$\mathscr{Y}$

n-premaniplex

$\eta : W_m \rightarrow \mathscr{Y}$

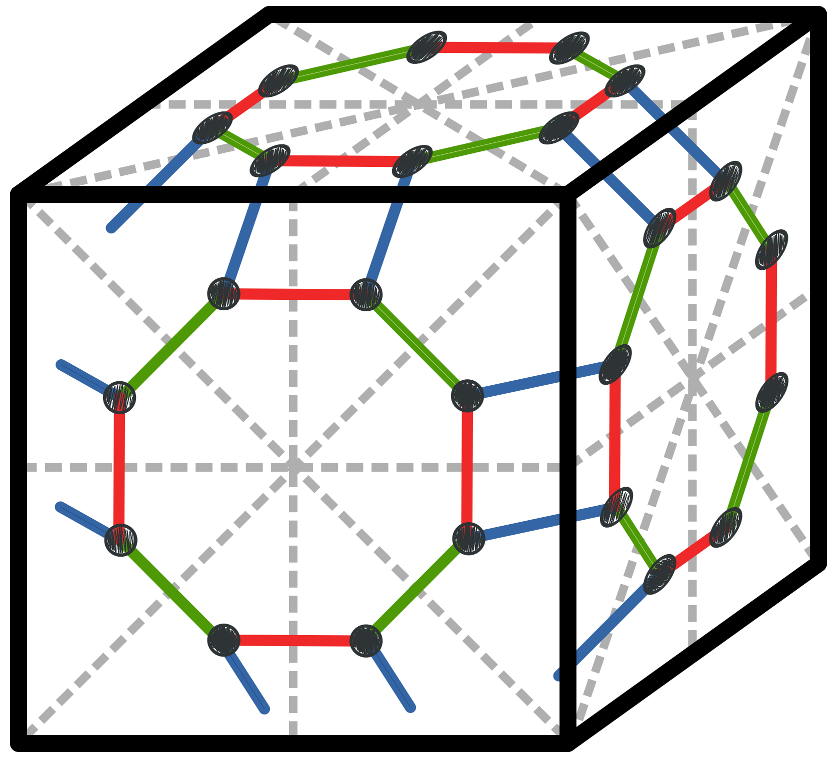
voltage assignment

$\mathscr{X} \rtimes_\eta \mathscr{Y}$

n-premaniplex

voltage operations

- An (m, n)- voltage operation:



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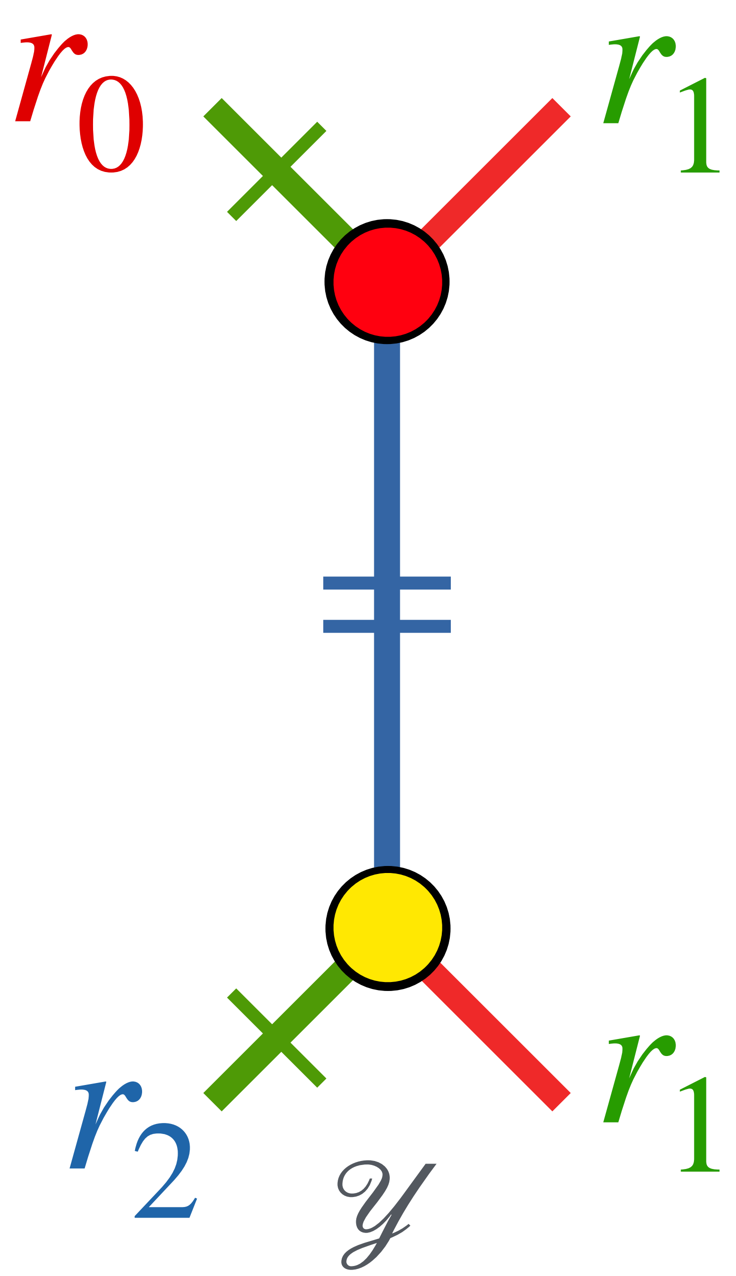

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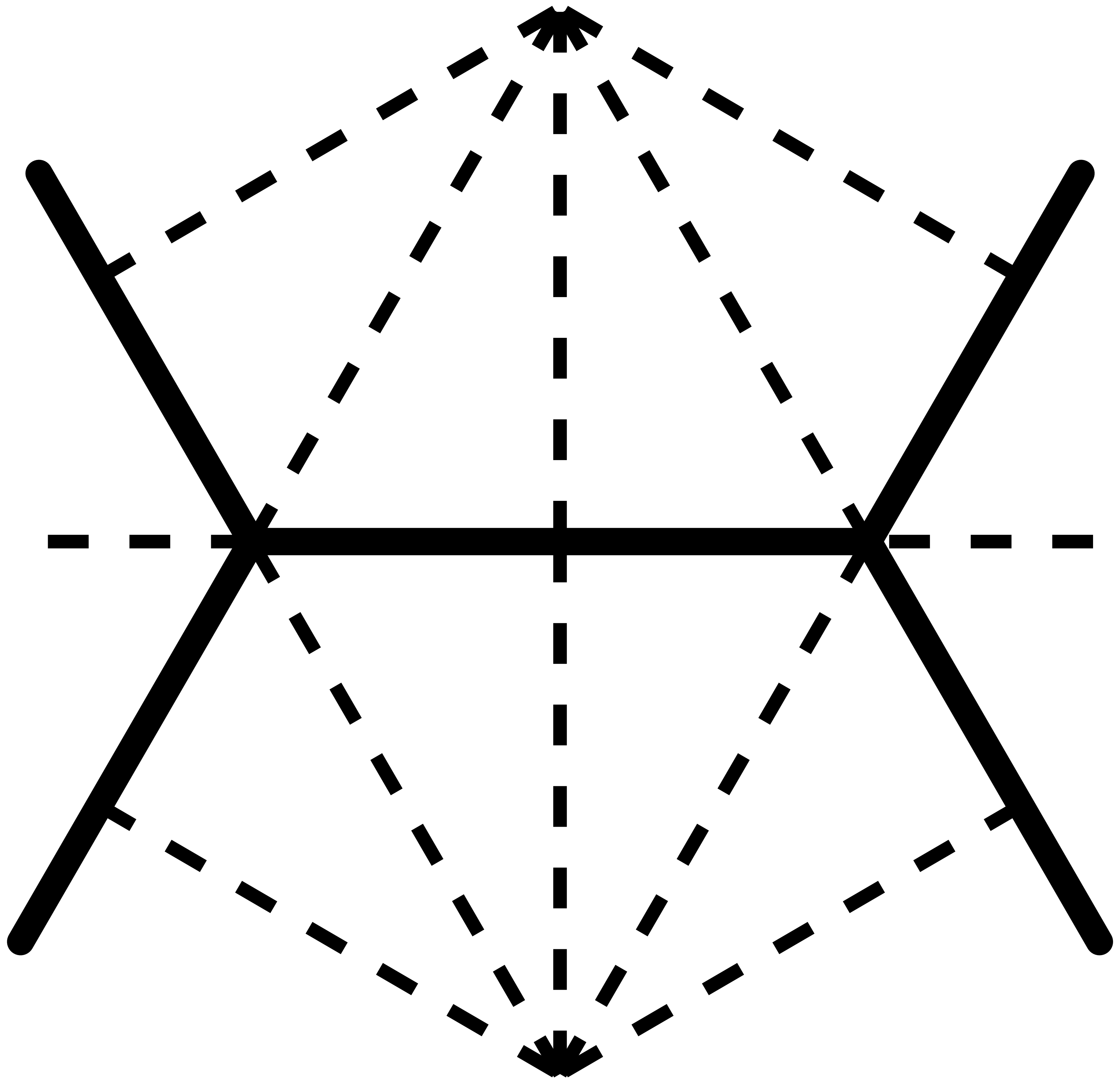
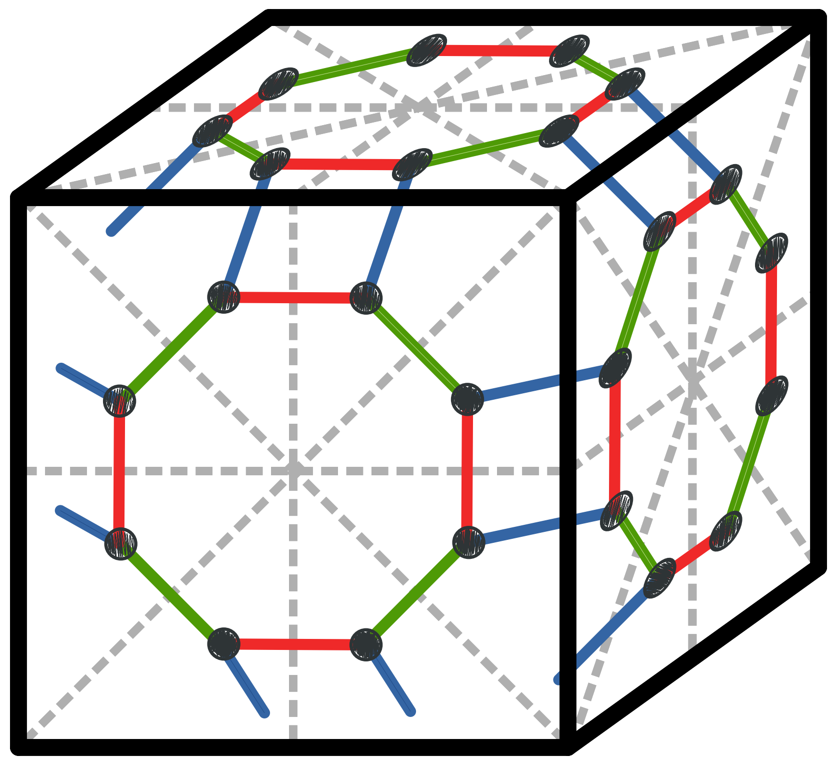


voltage operations

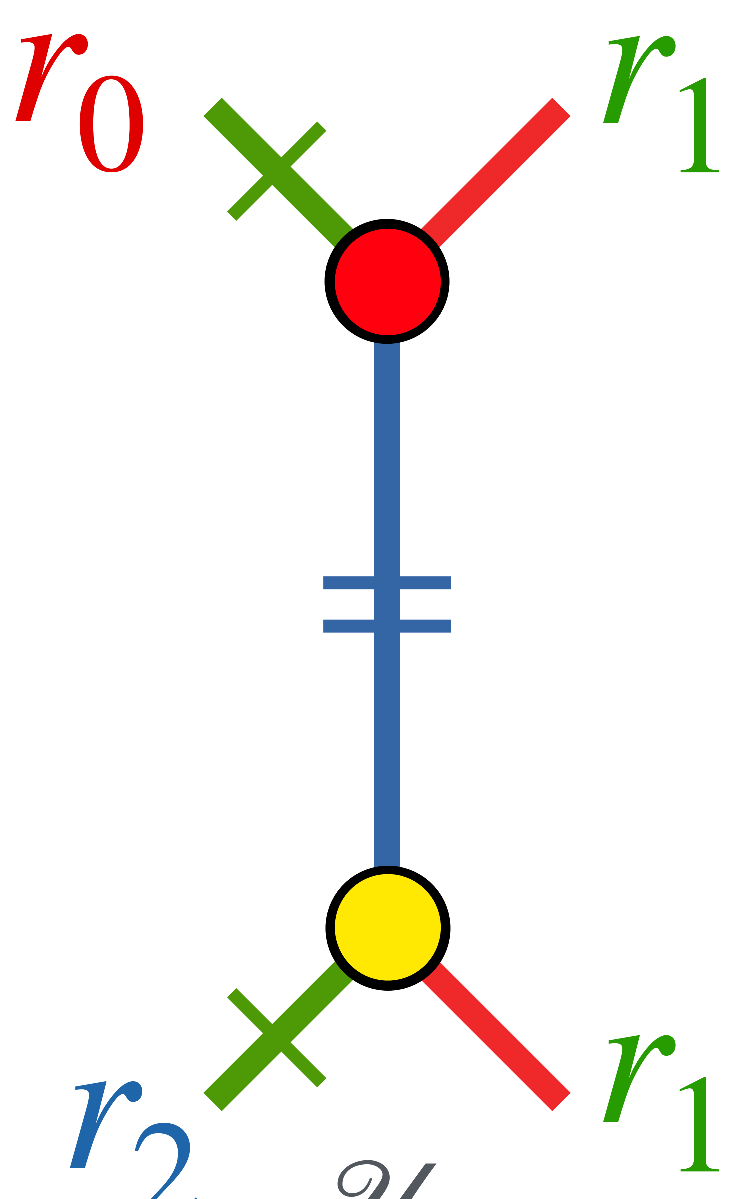
- An (m, n)- voltage operation:



$\mathcal{X}$

m-premaniplex

$\rtimes$



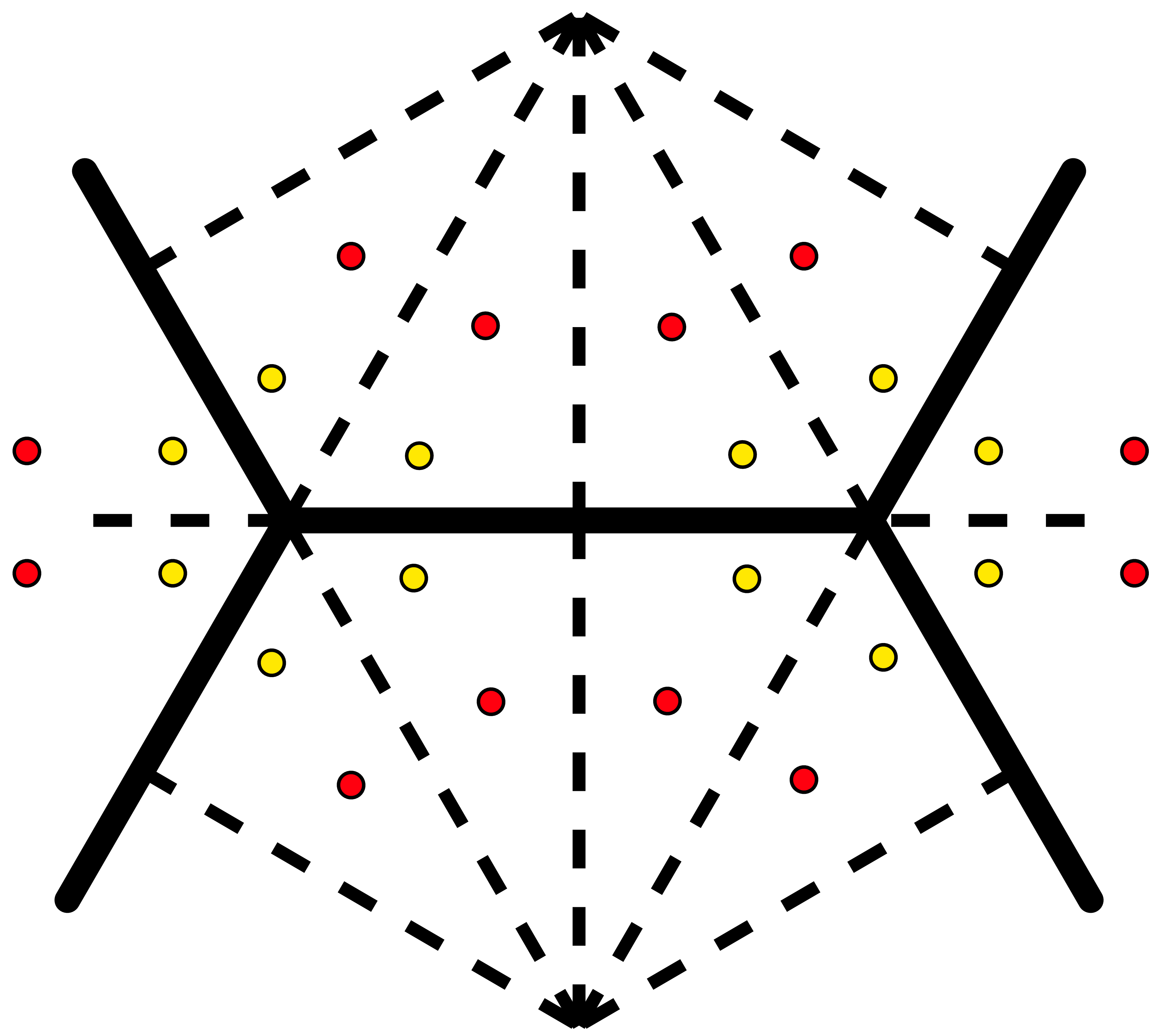
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

voltage assignment

$=$

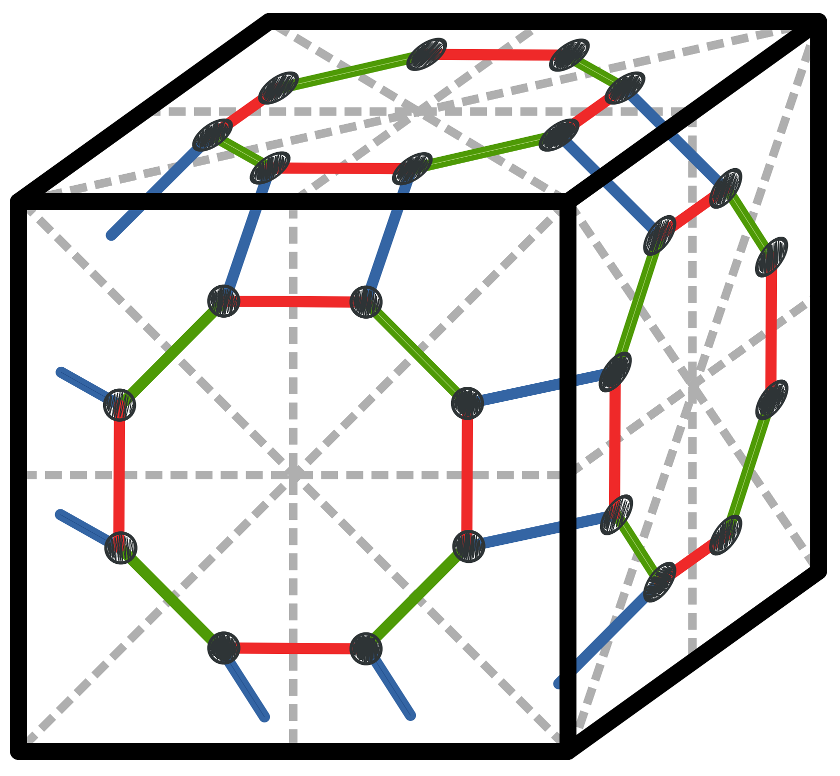


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

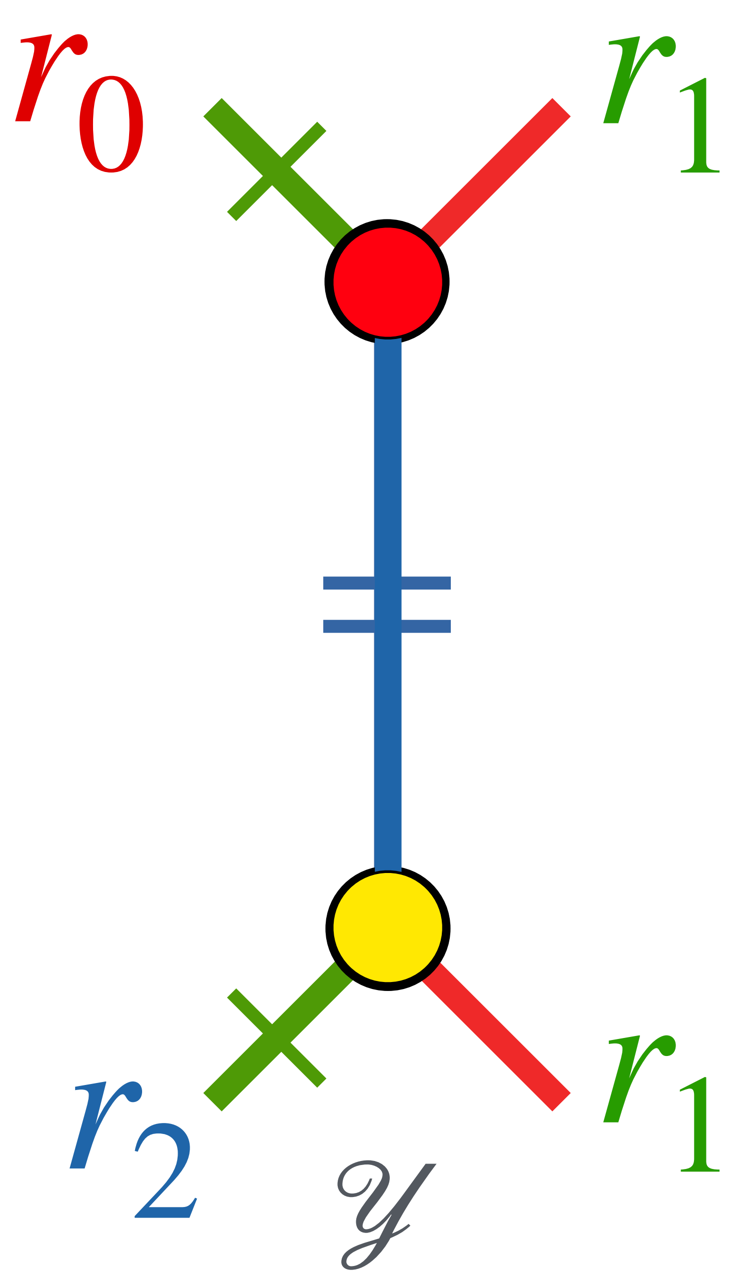
n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\rtimes$



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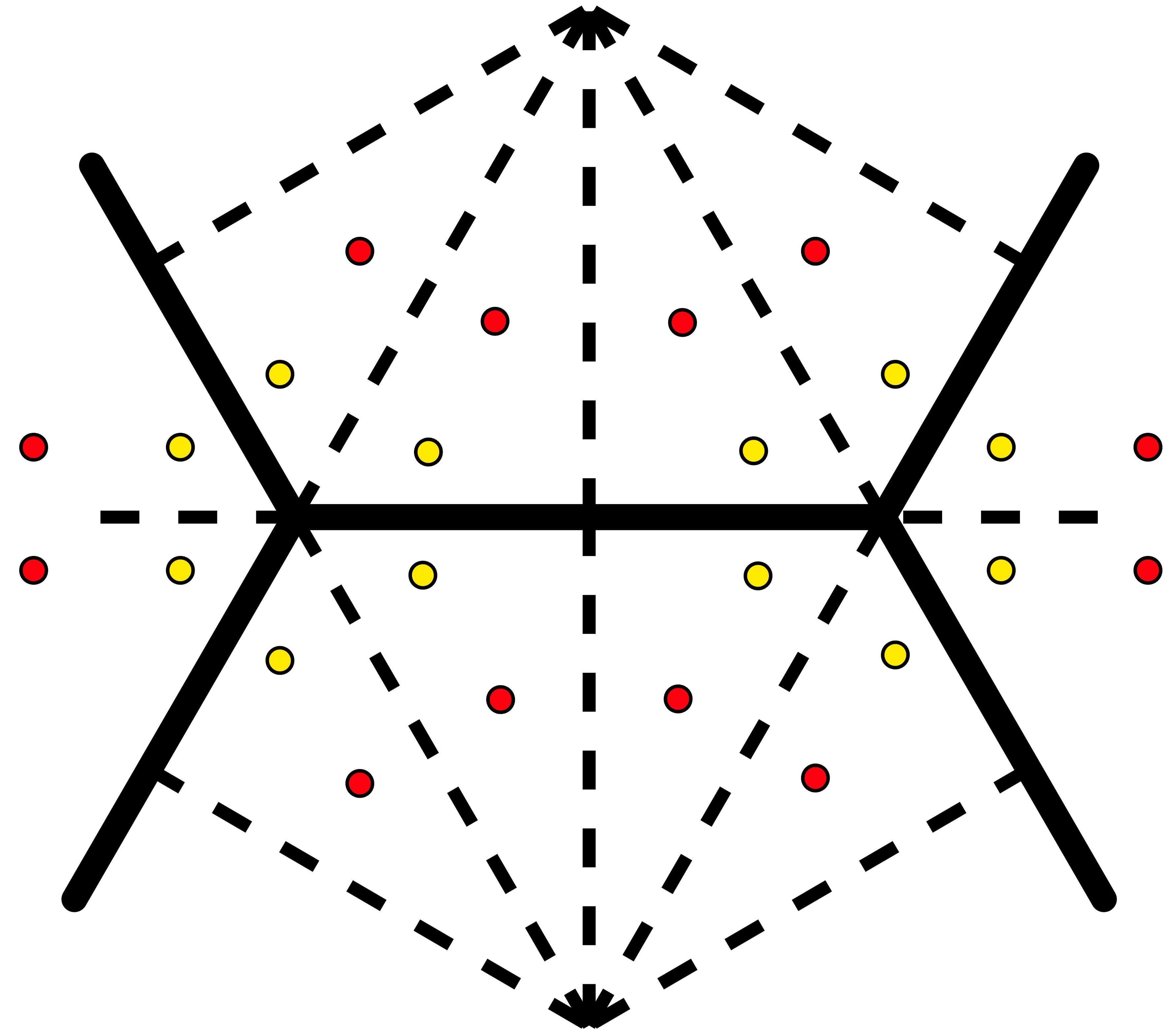
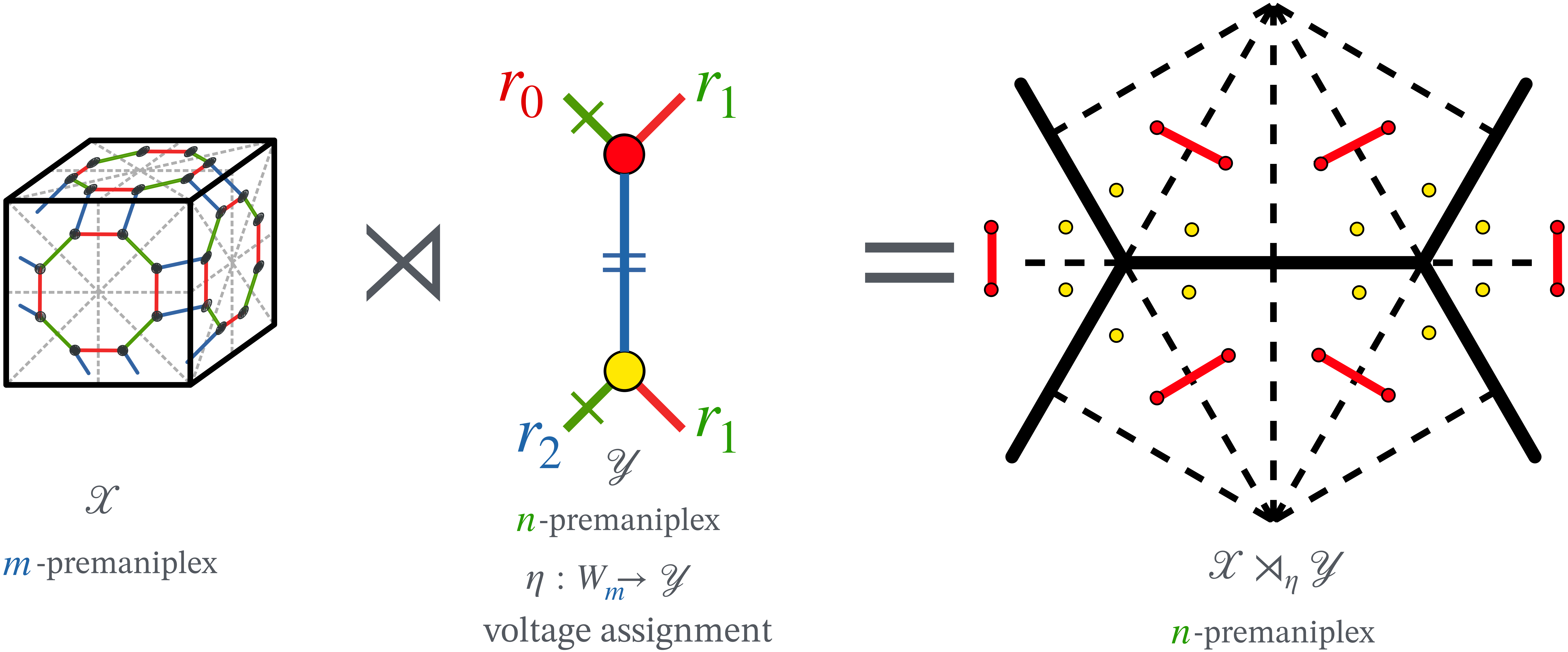


voltage operations

- An (m, n)- voltage operation:



voltage operations

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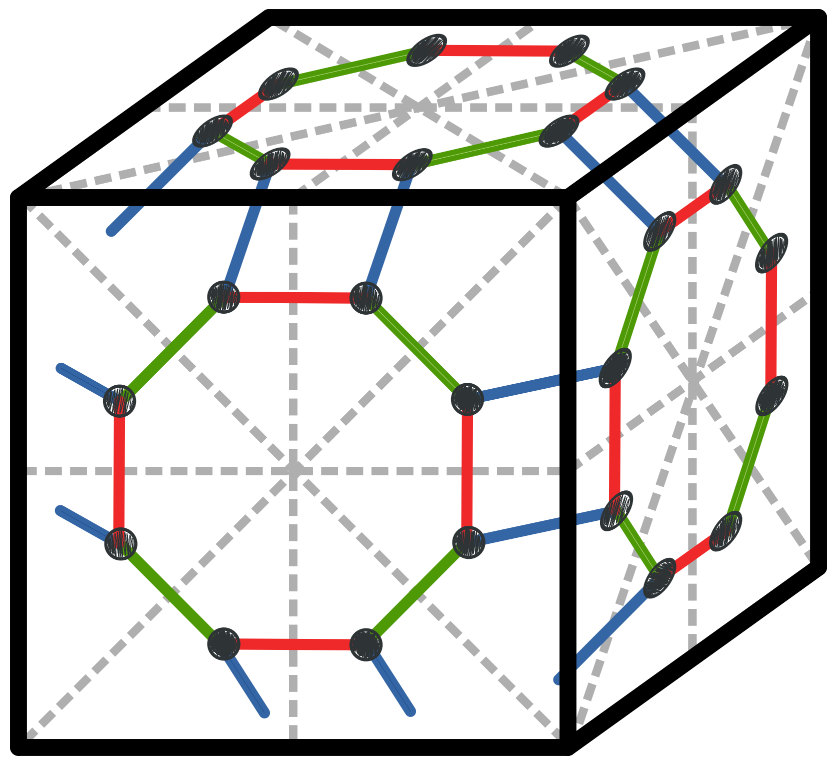
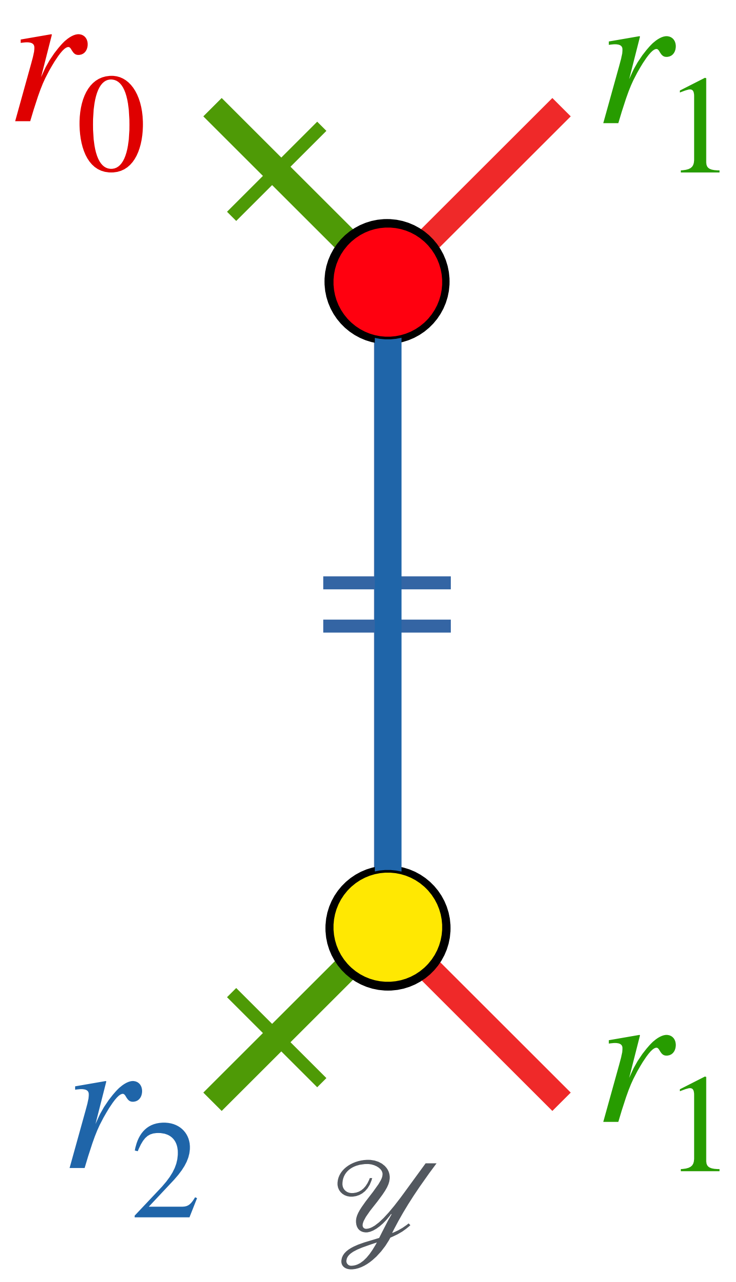


$\mathcal{X}$

m-premaniplex

$\rtimes$



n-premaniplex

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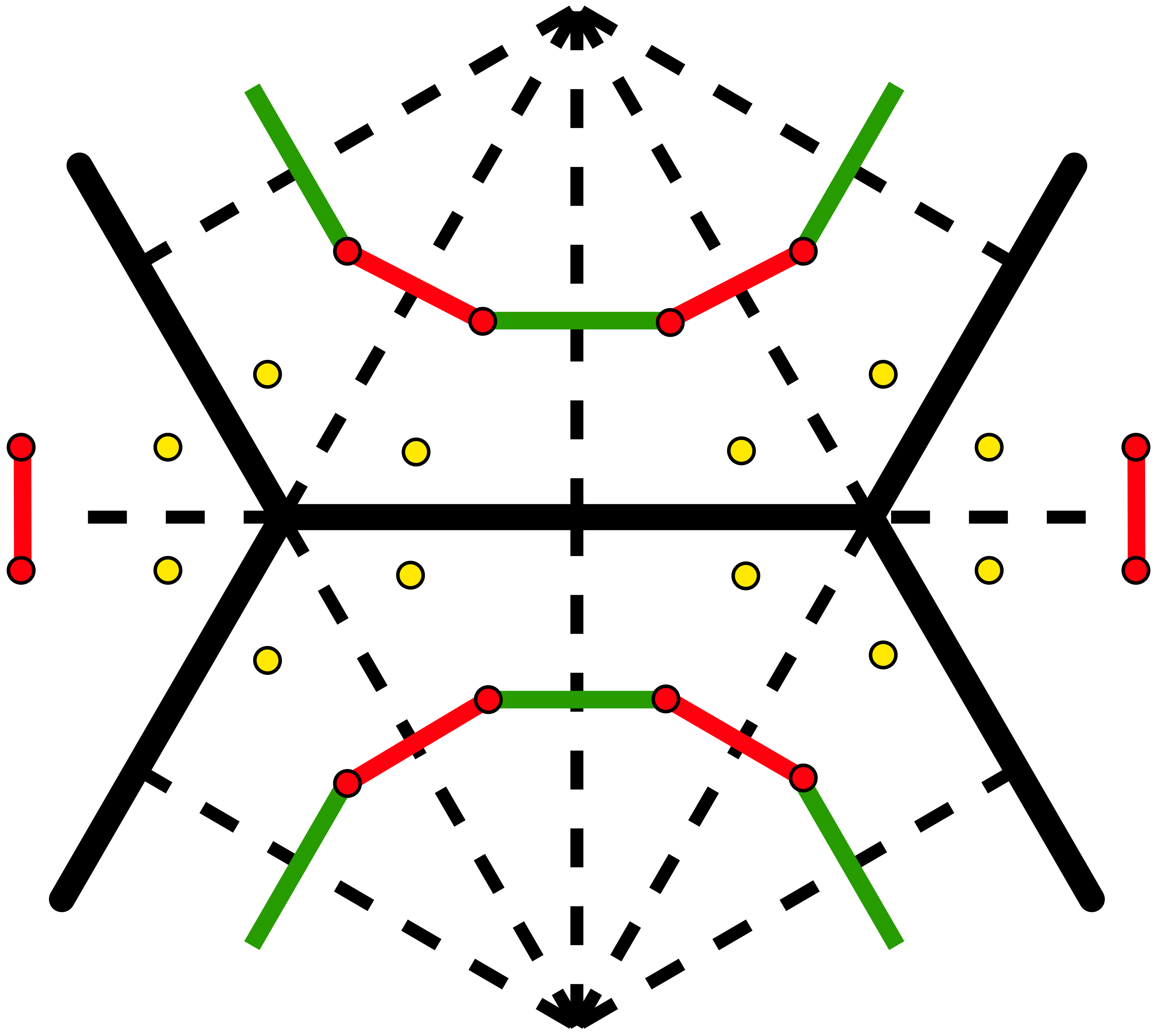
voltage assignment

$=$

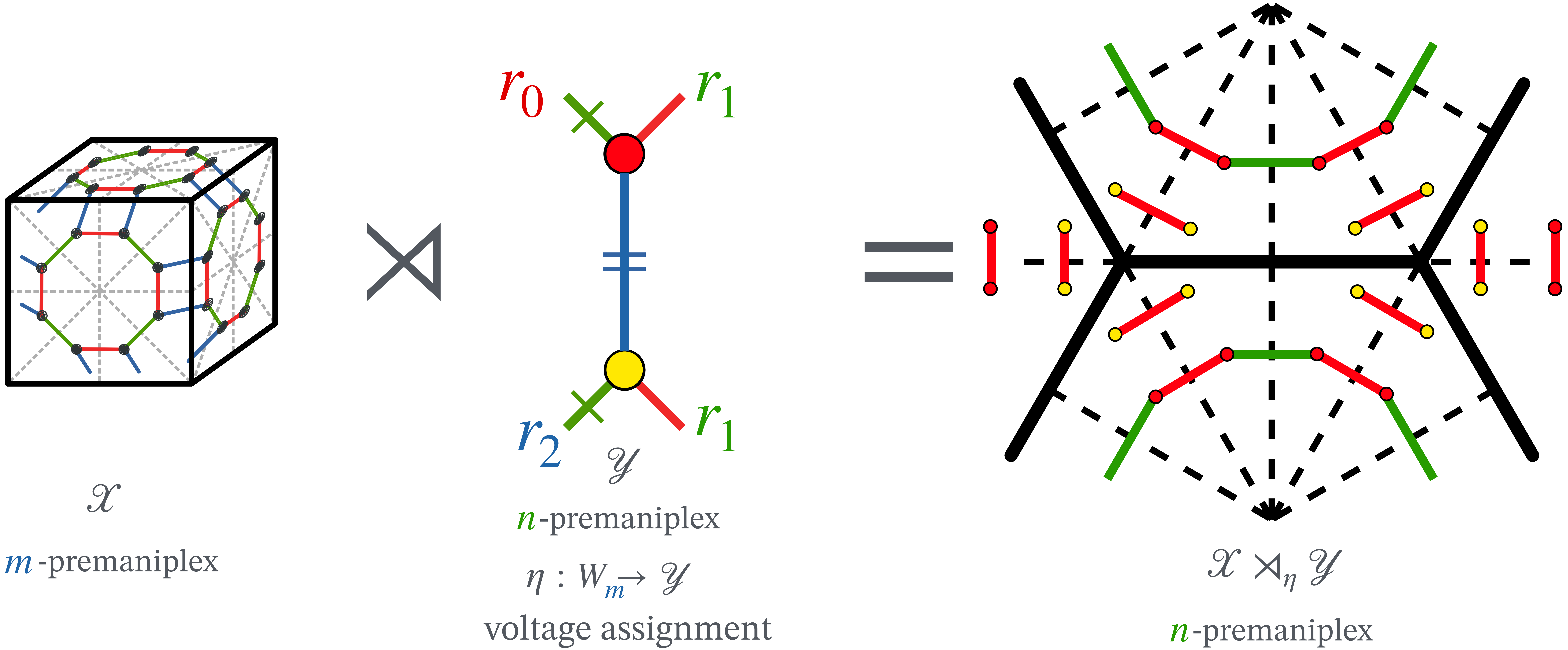


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

- An (m, n)- voltage operation:



voltage operations

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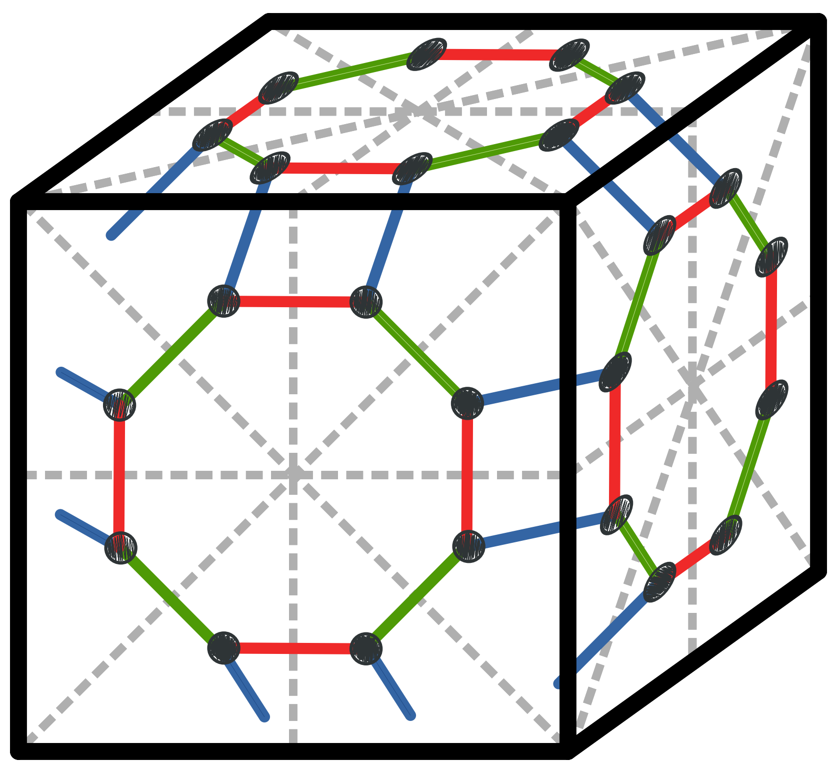

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m-premaniplex

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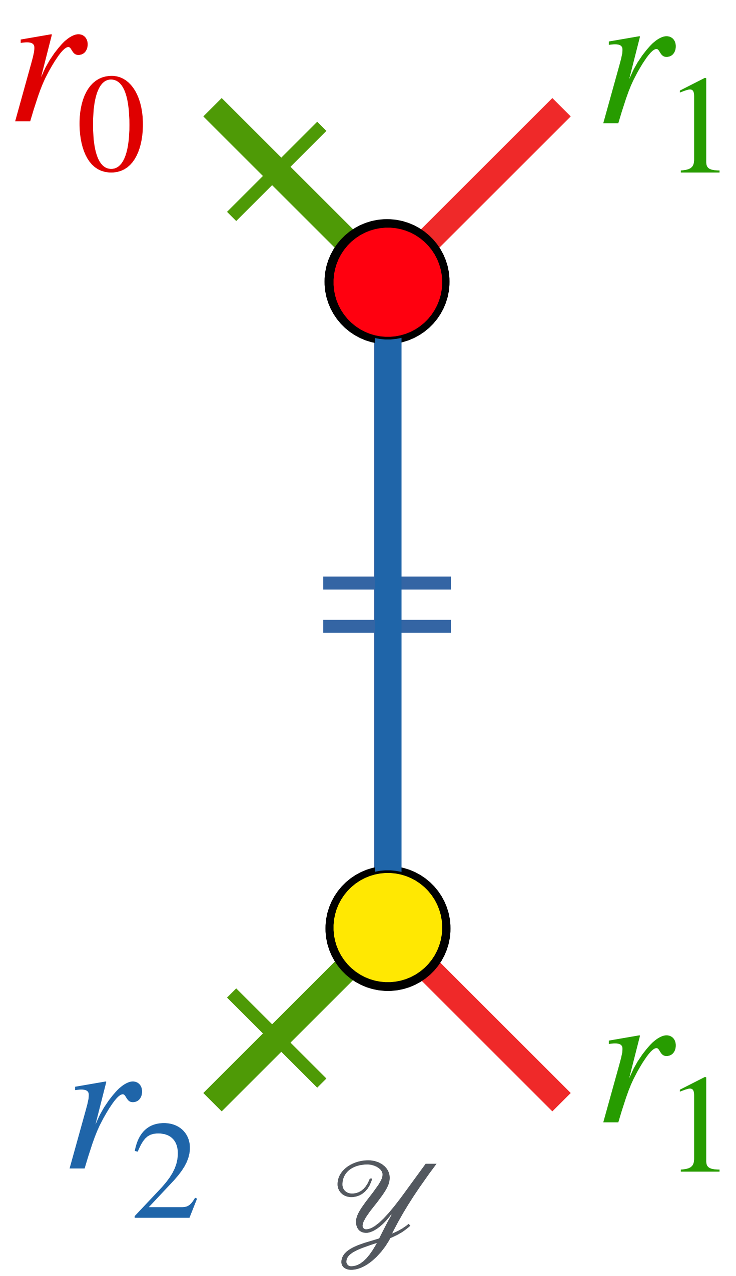

n-premaniplex

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voltage assignment

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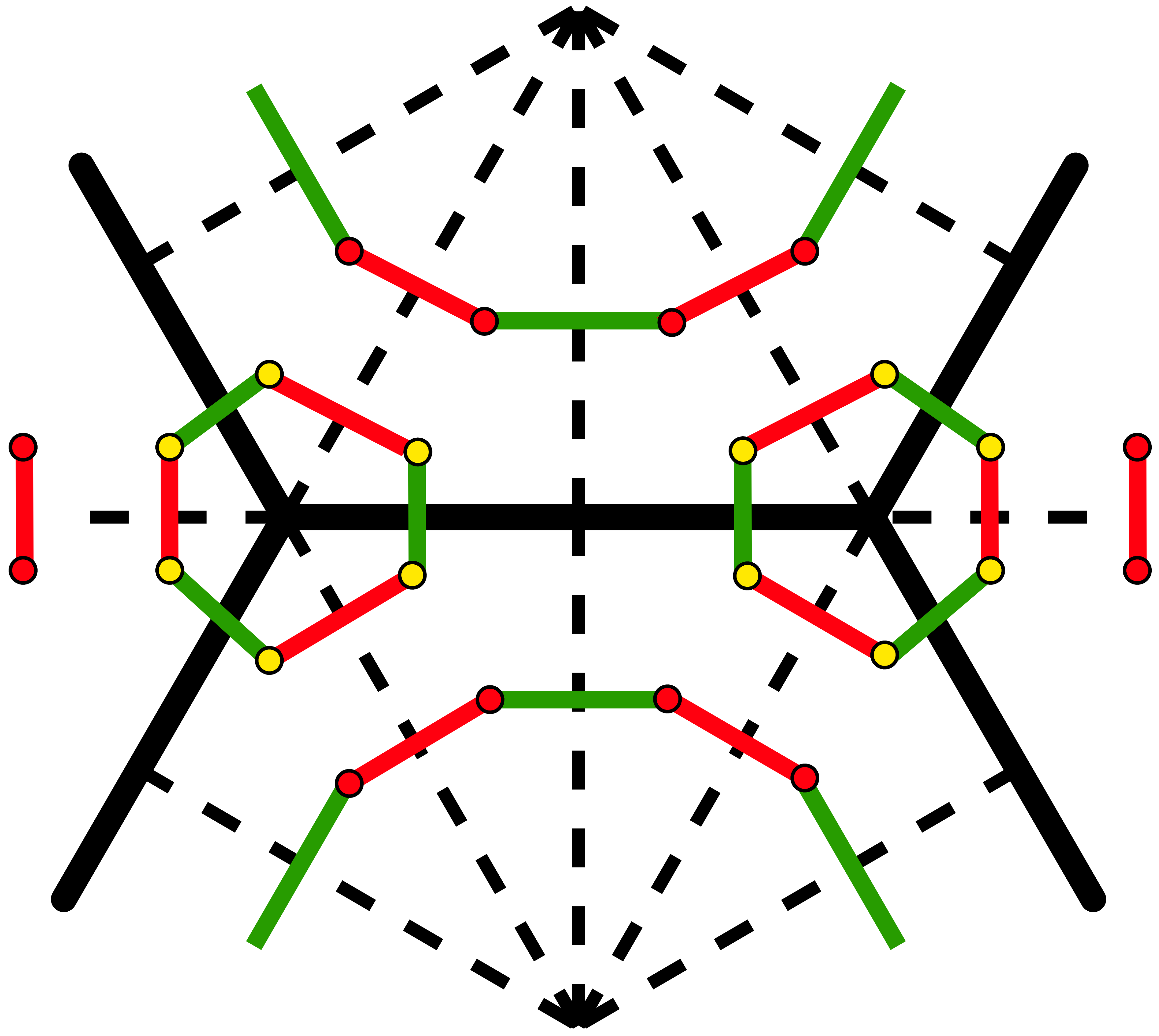

$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

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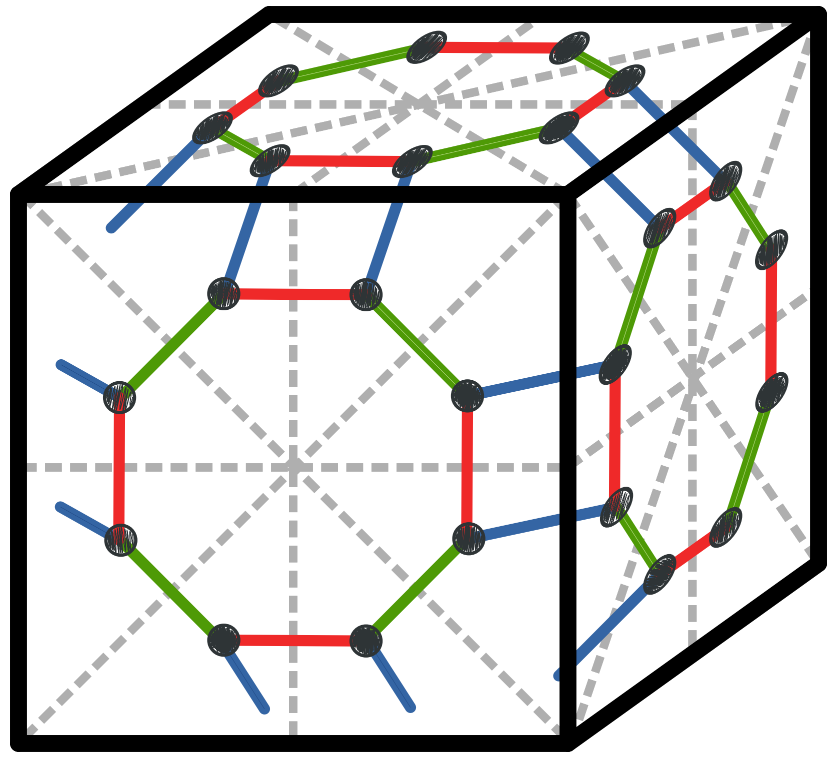
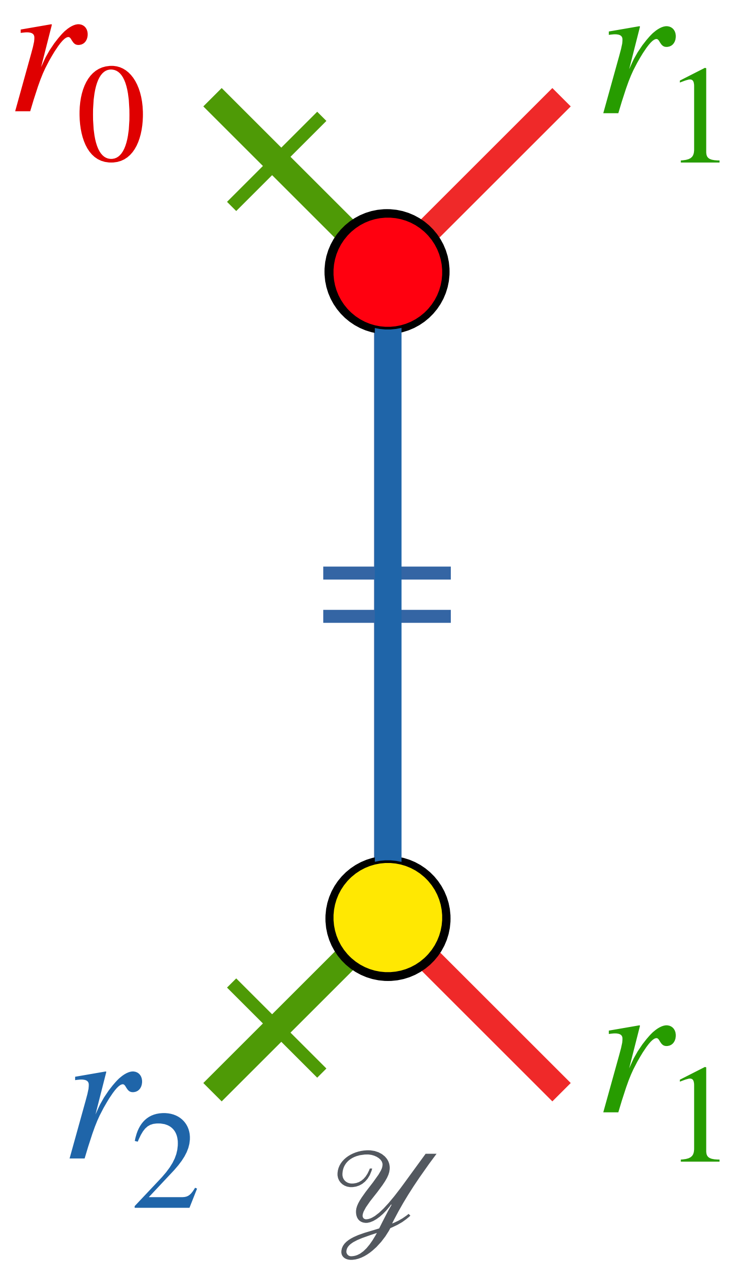


$\mathcal{X}$

m-premaniplex

$\rtimes$



n-premaniplex

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voltage assignment

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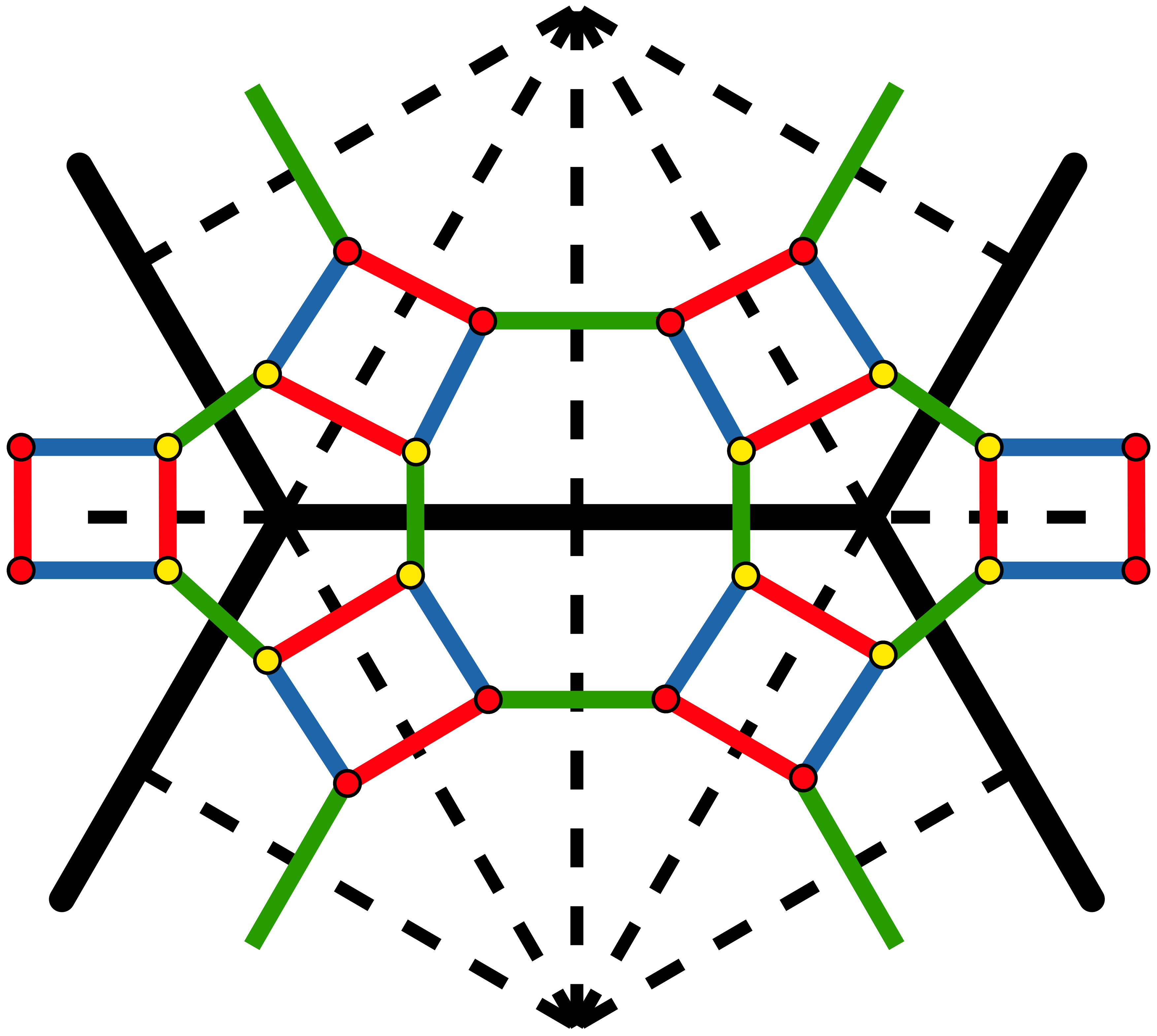

$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

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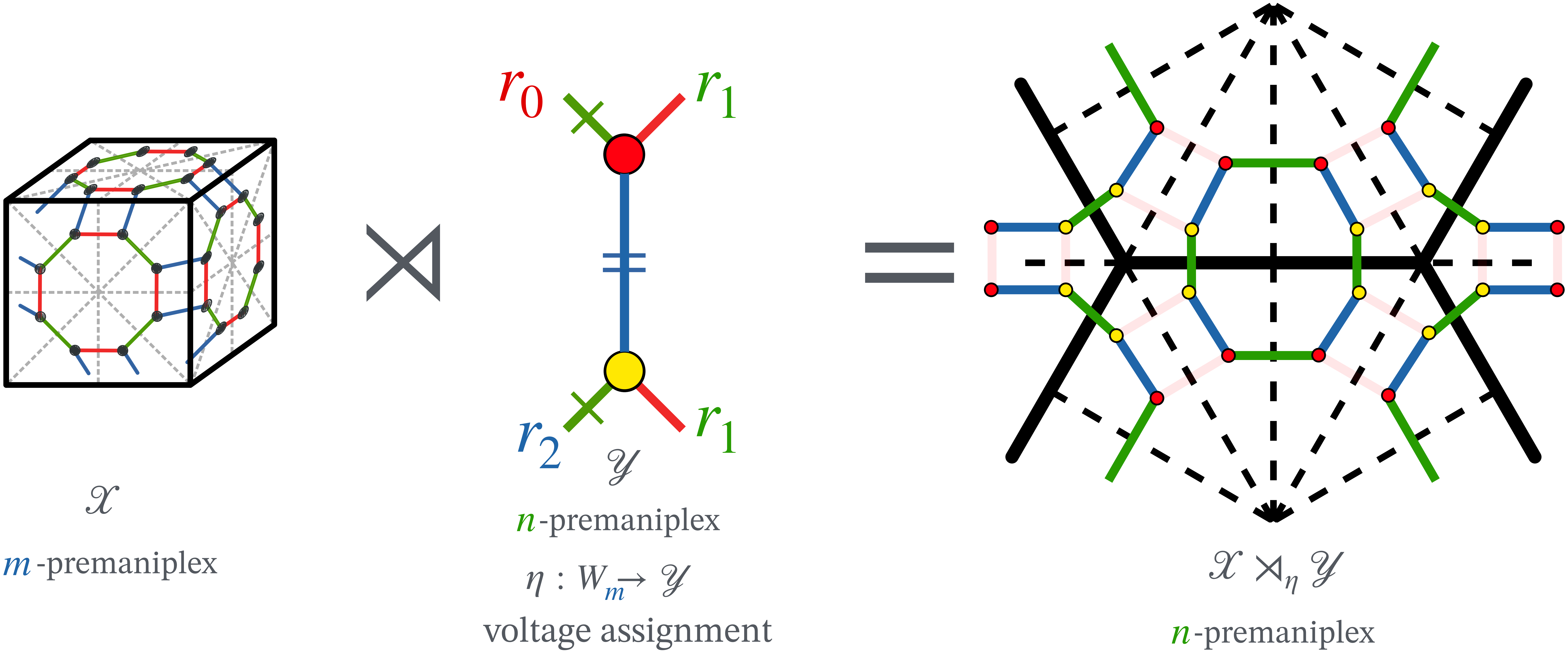

voltage operations

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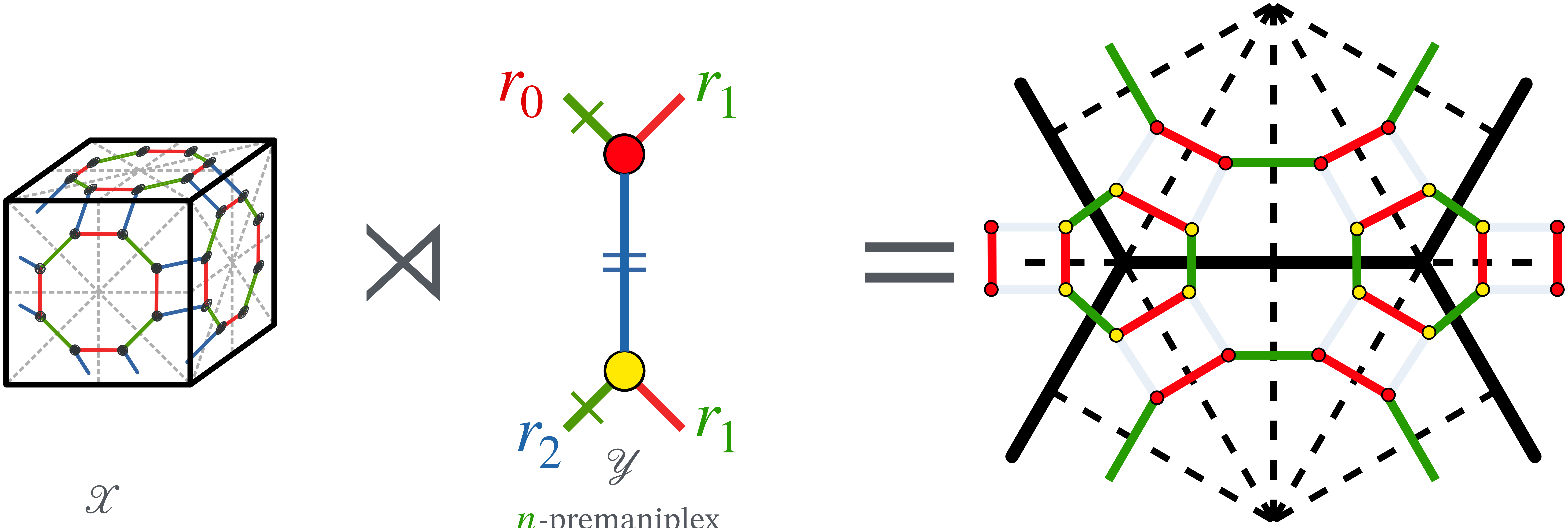

$\mathcal{X}$

m-premaniplex

n-premaniplex

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voltage assignment

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n-premaniplex

voltage operations

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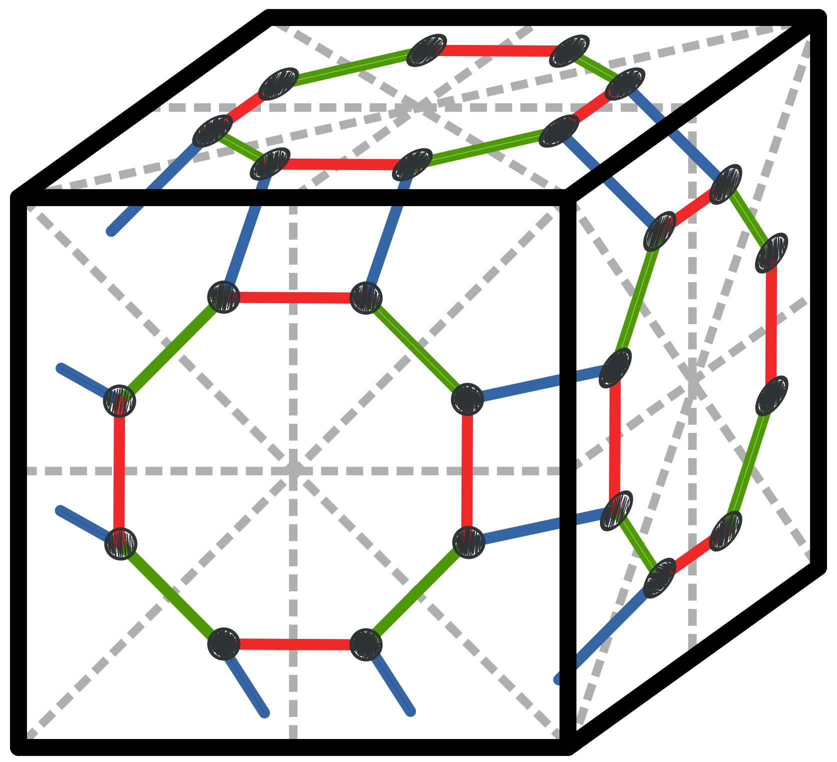
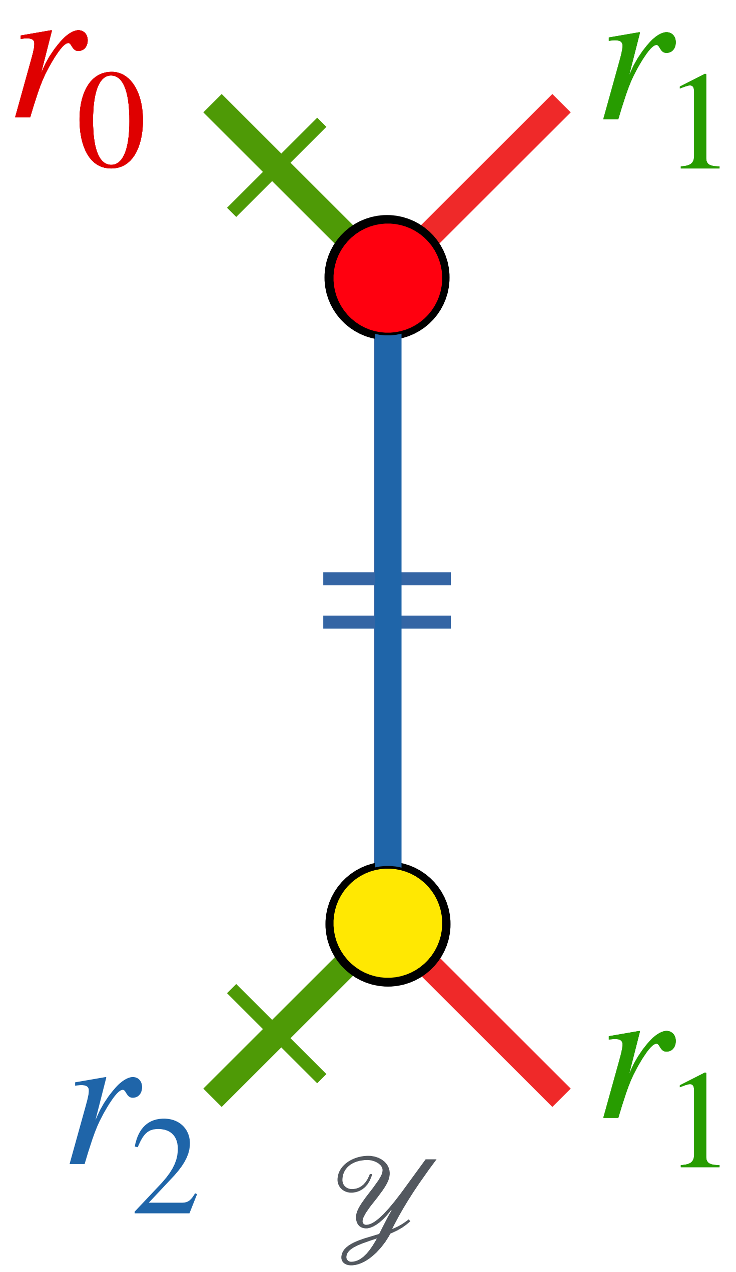


$\mathscr{X}$

m-premaniplex

$\rtimes$



n-premaniplex

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voltage assignment

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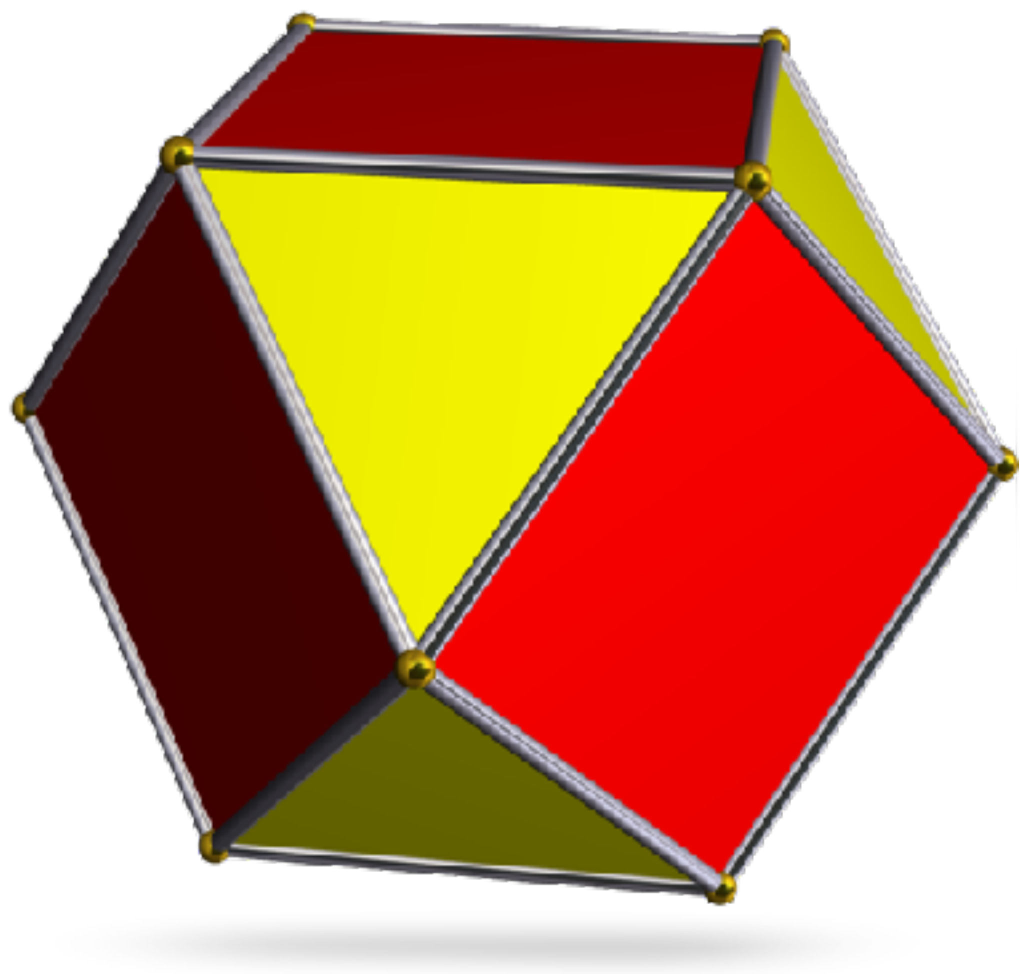
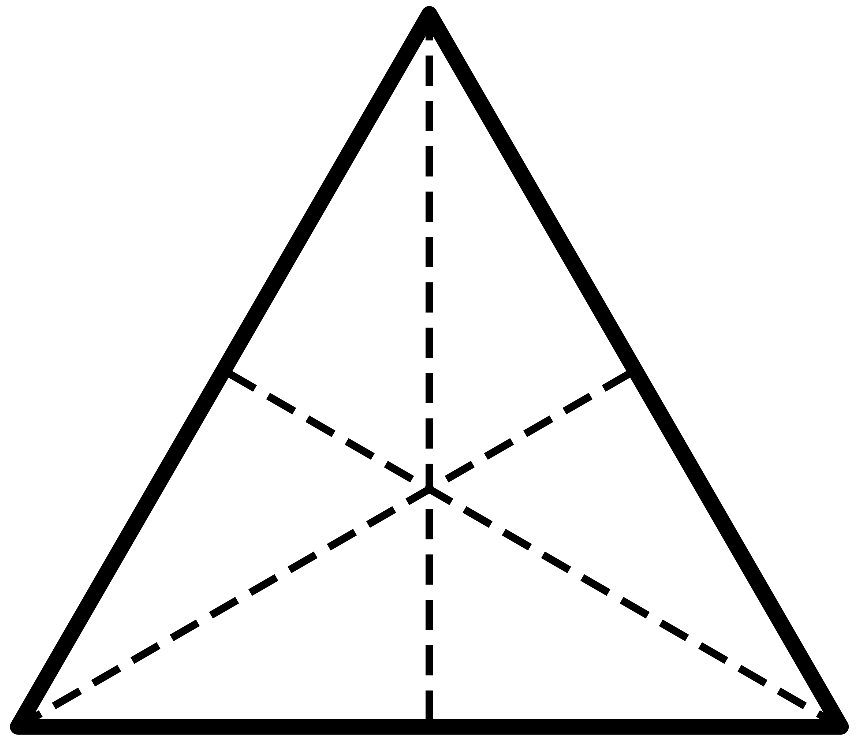


$\mathscr{X} \rtimes_\eta \mathscr{Y}$

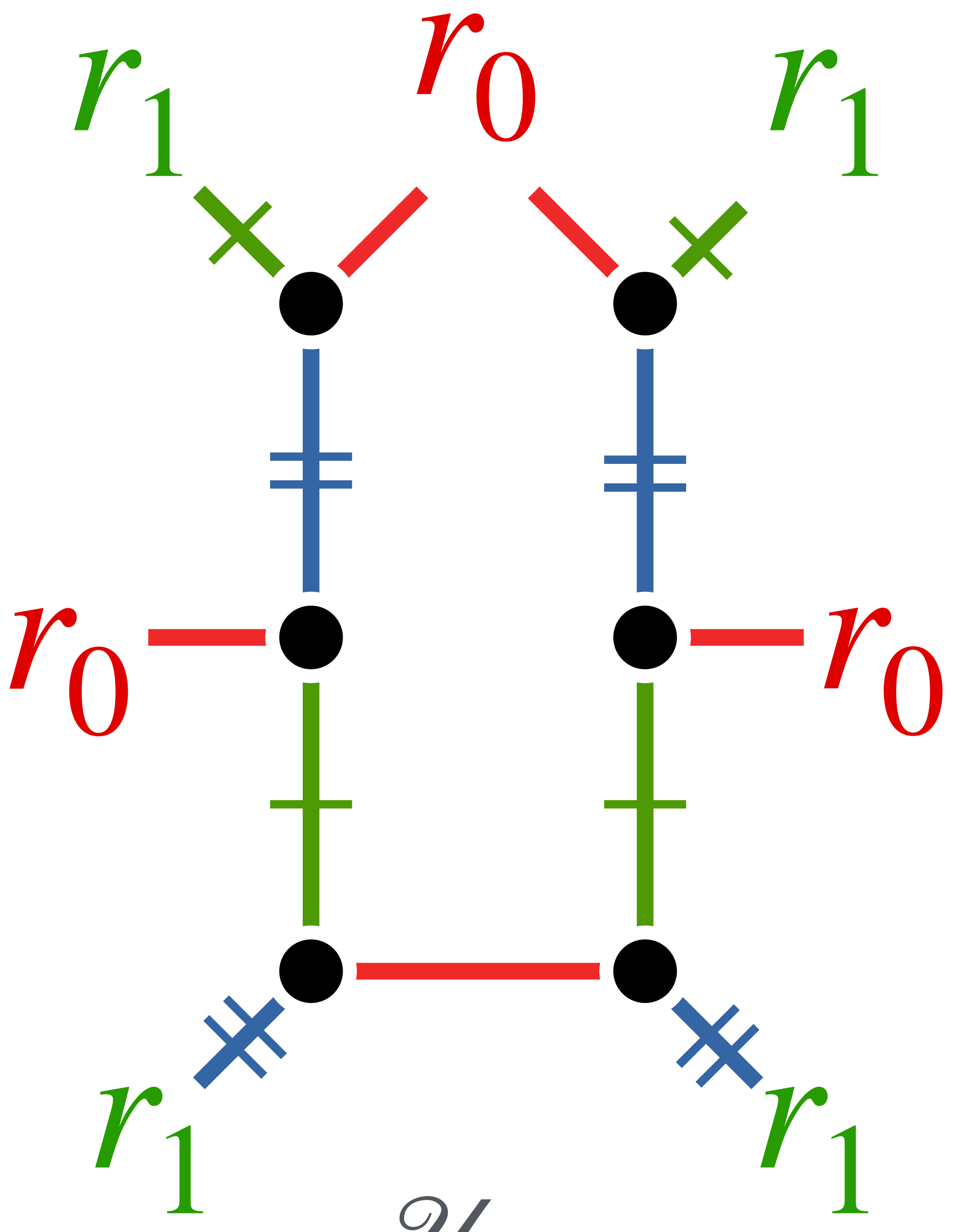
n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\rtimes$



$=$

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m-premaniplex

$\mathscr{Y}$

n-premaniplex

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voltage assignment

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n-premaniplex

voltage operations

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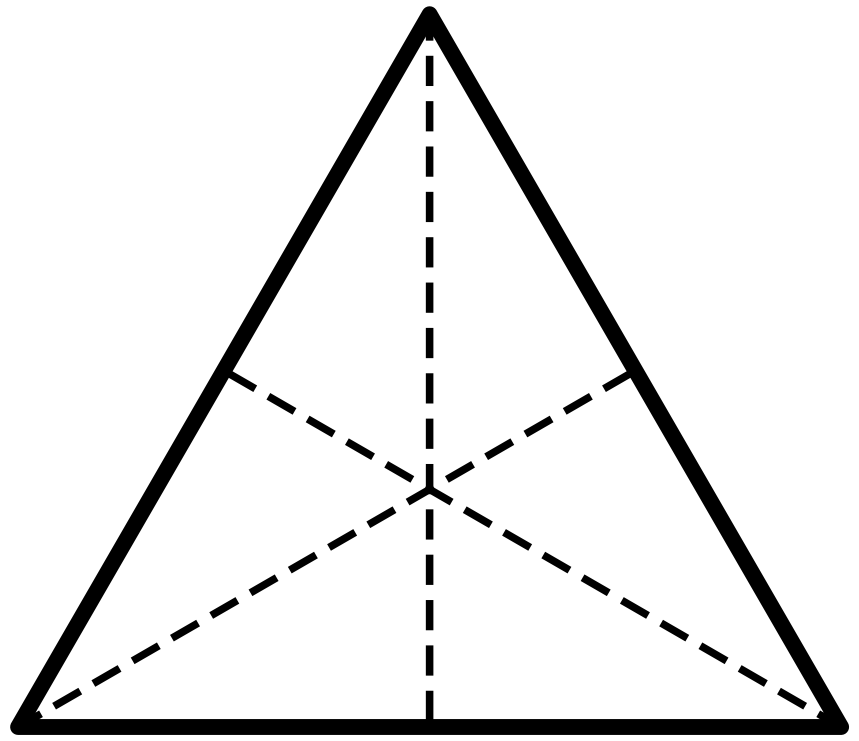
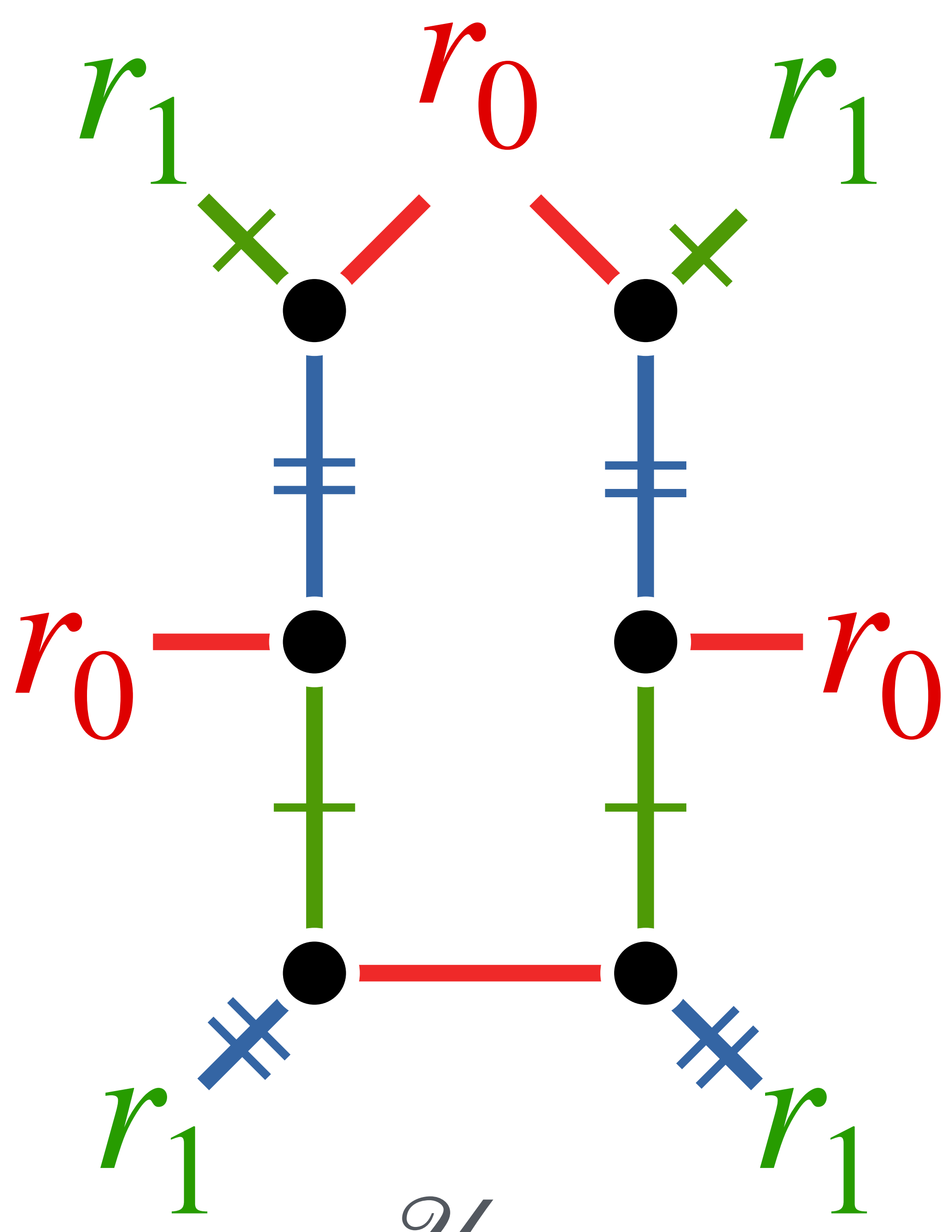


$\mathcal{X}$

m-premaniplex

$\rtimes$



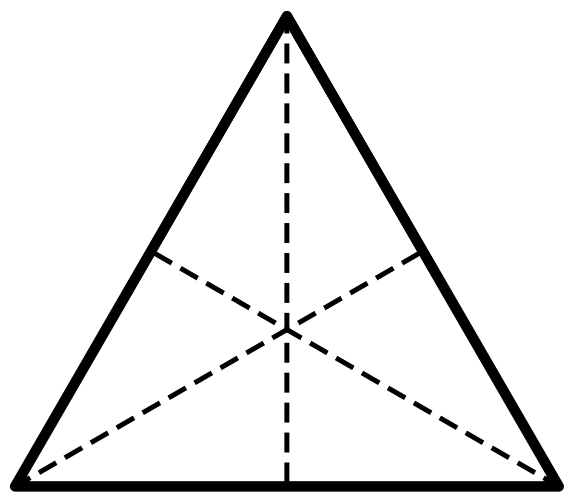
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

voltage assignment

$=$

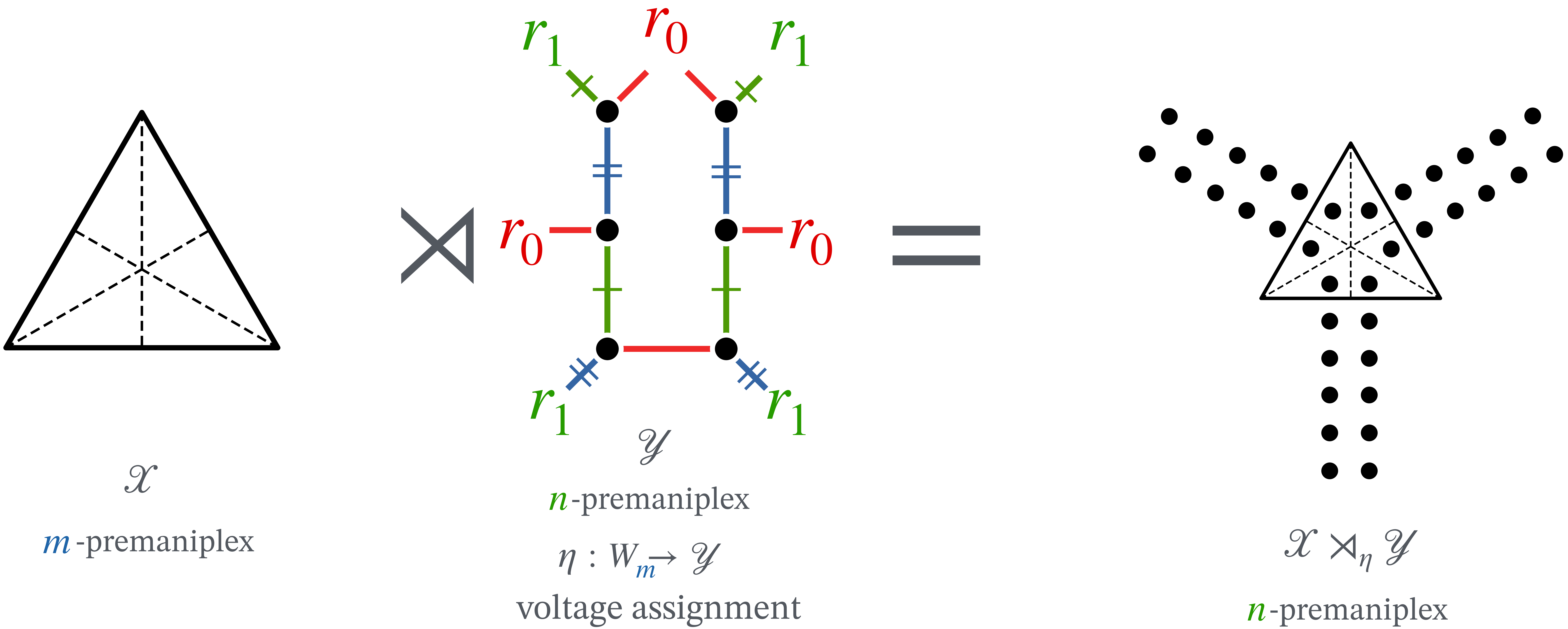


$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

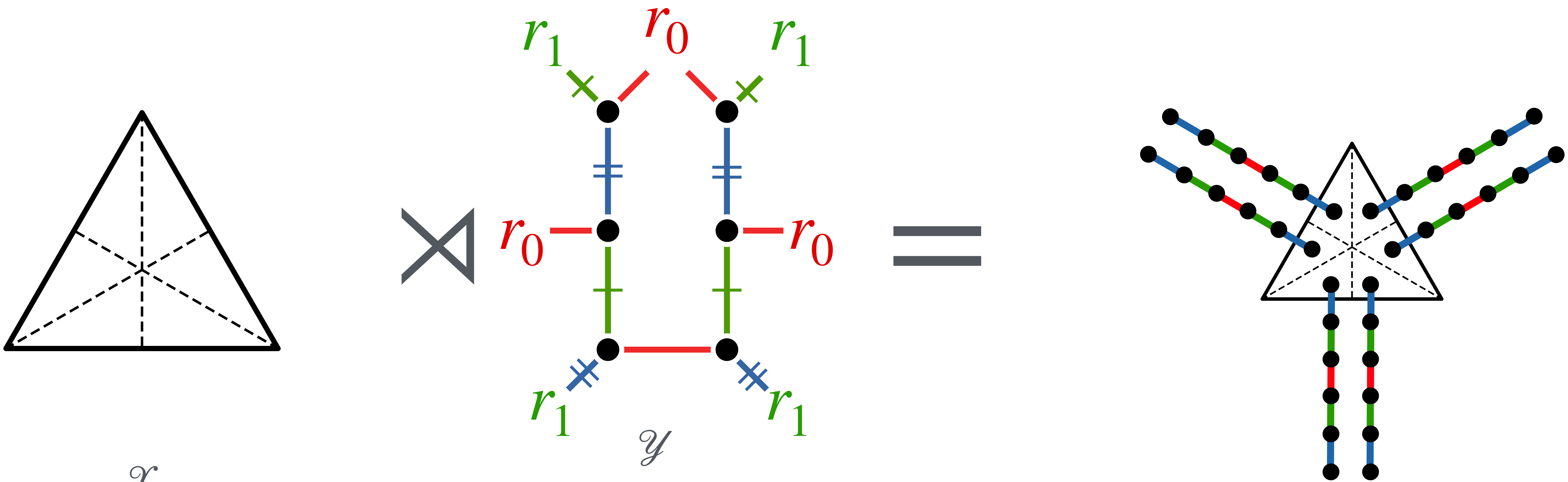
voltage operations

- An (m, n)- voltage operation:



voltage operations

- An (m, n)- voltage operation:



$\mathcal{X}$

m-premaniplex

n-premaniplex

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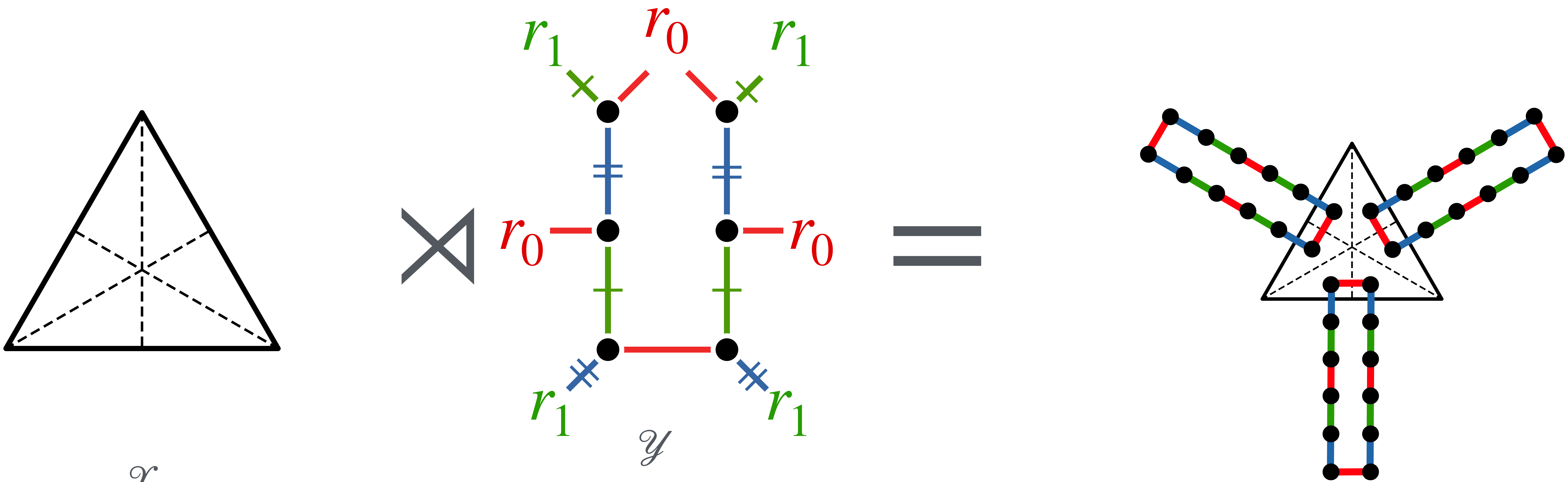
voltage assignment

$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

- An (m, n)- voltage operation:



$\mathcal{X}$

m-premaniplex

$\mathcal{Y}$

n-premaniplex

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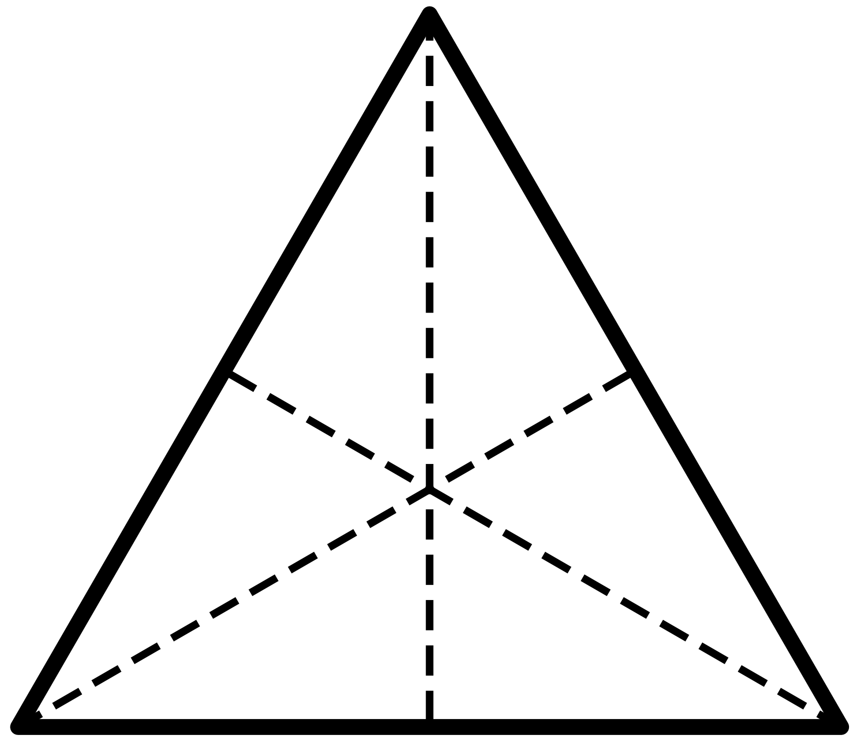
voltage assignment

$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

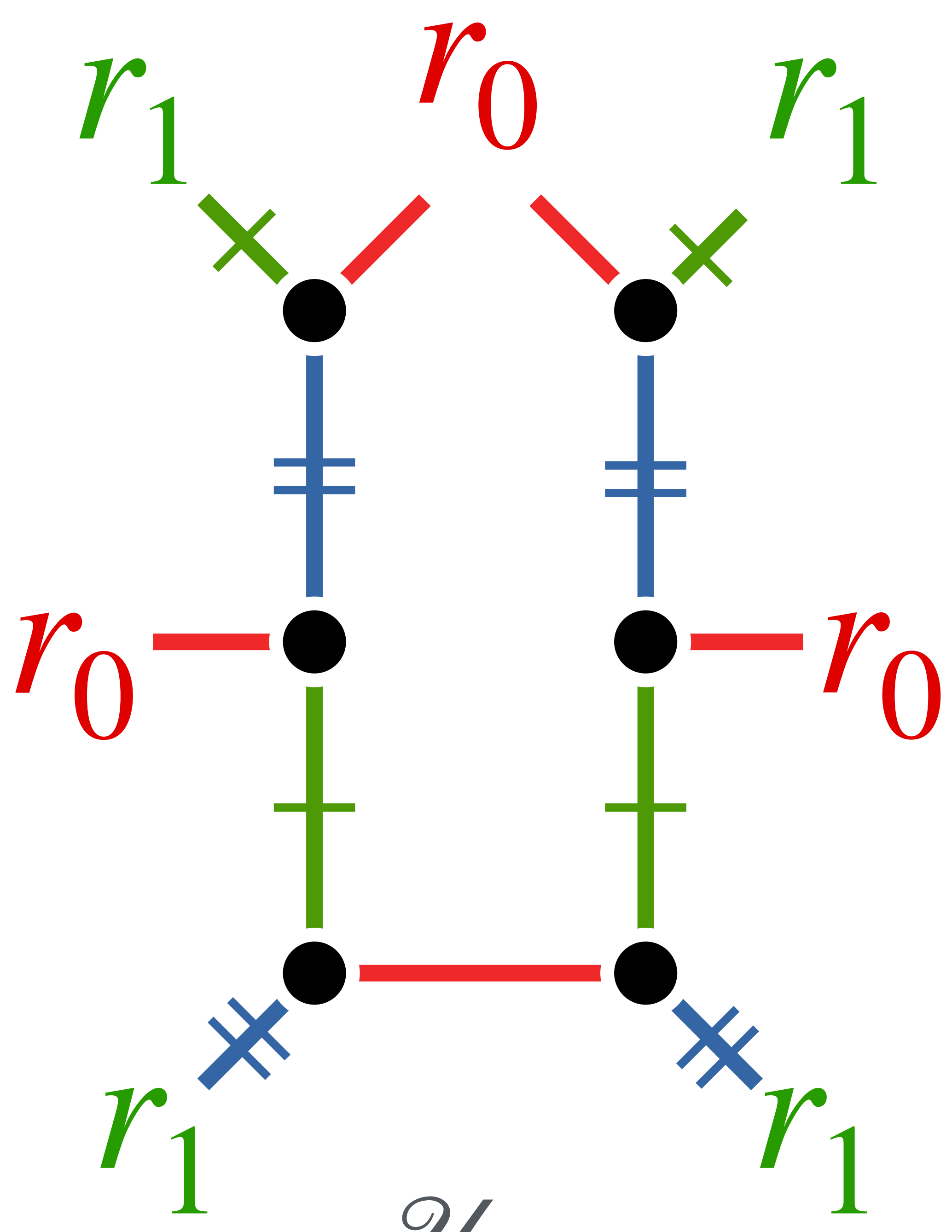
- An (m, n)- voltage operation:



$\mathcal{X}$

m-premaniplex

$\rtimes$



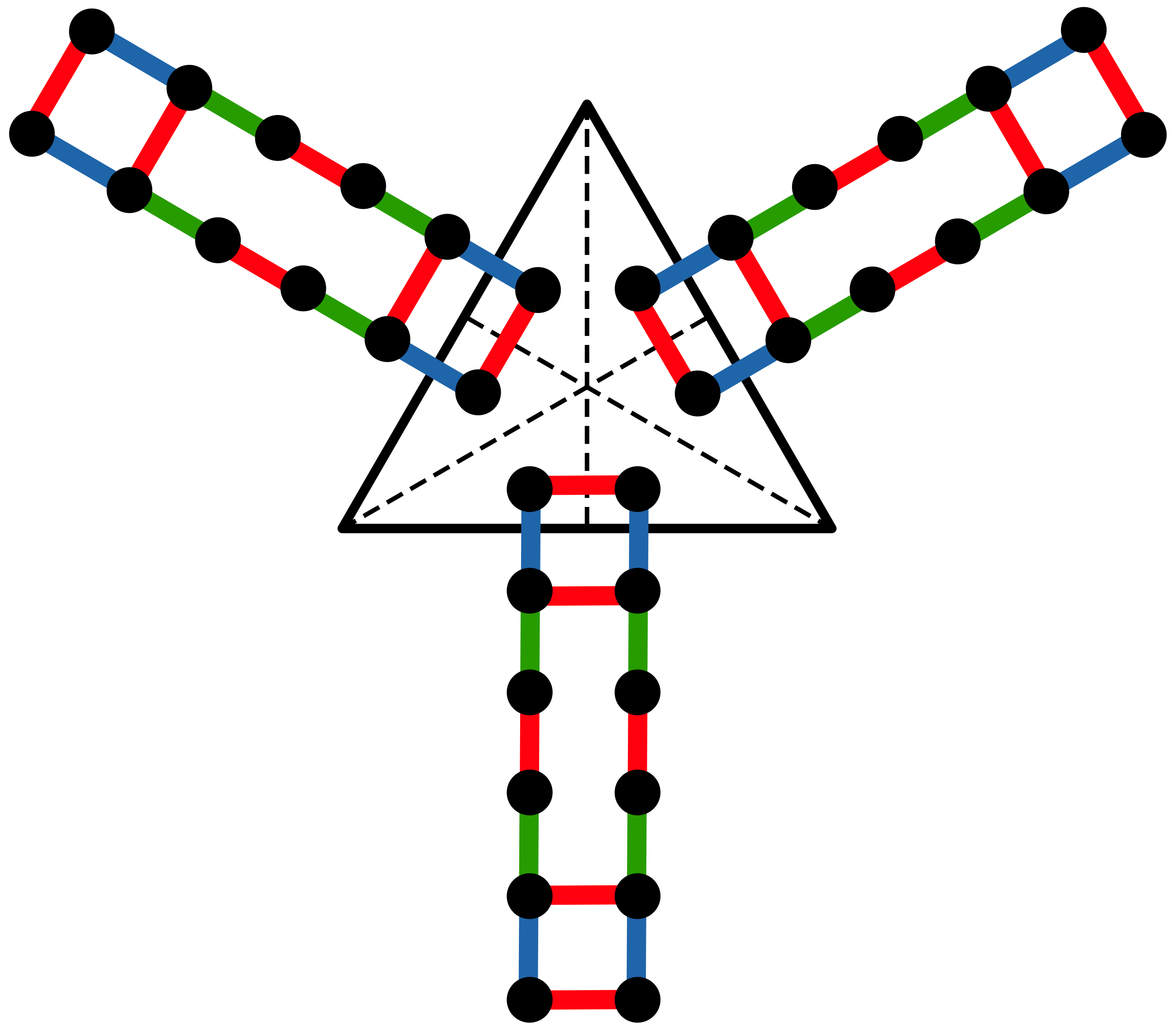
$\mathcal{Y}$

n-premaniplex

$\eta : W_m \to \mathcal{Y}$

voltage assignment

$=$



$\mathcal{X} \rtimes_\eta \mathcal{Y}$

n-premaniplex

voltage operations

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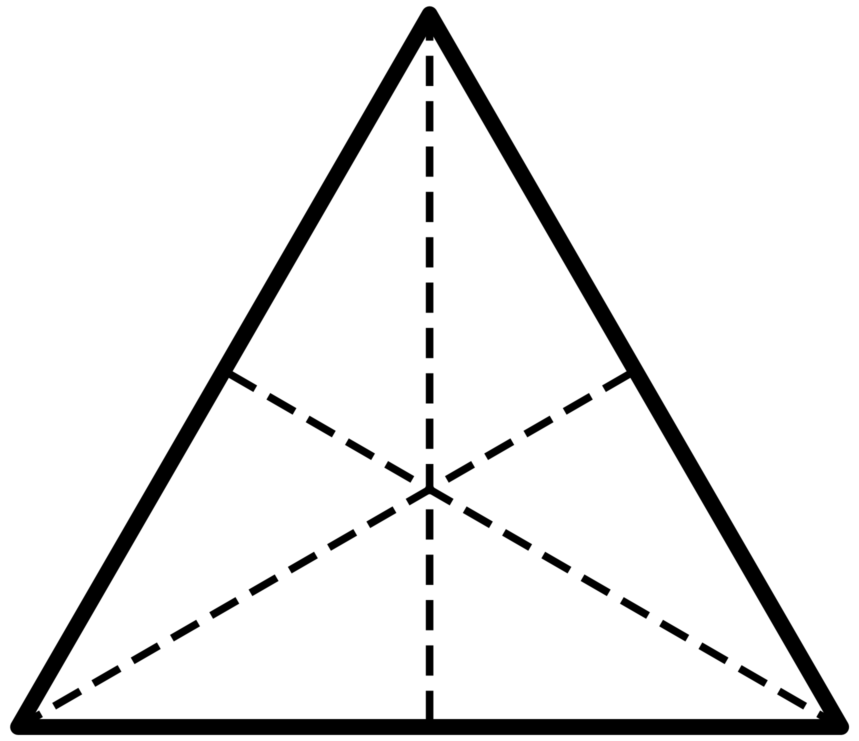

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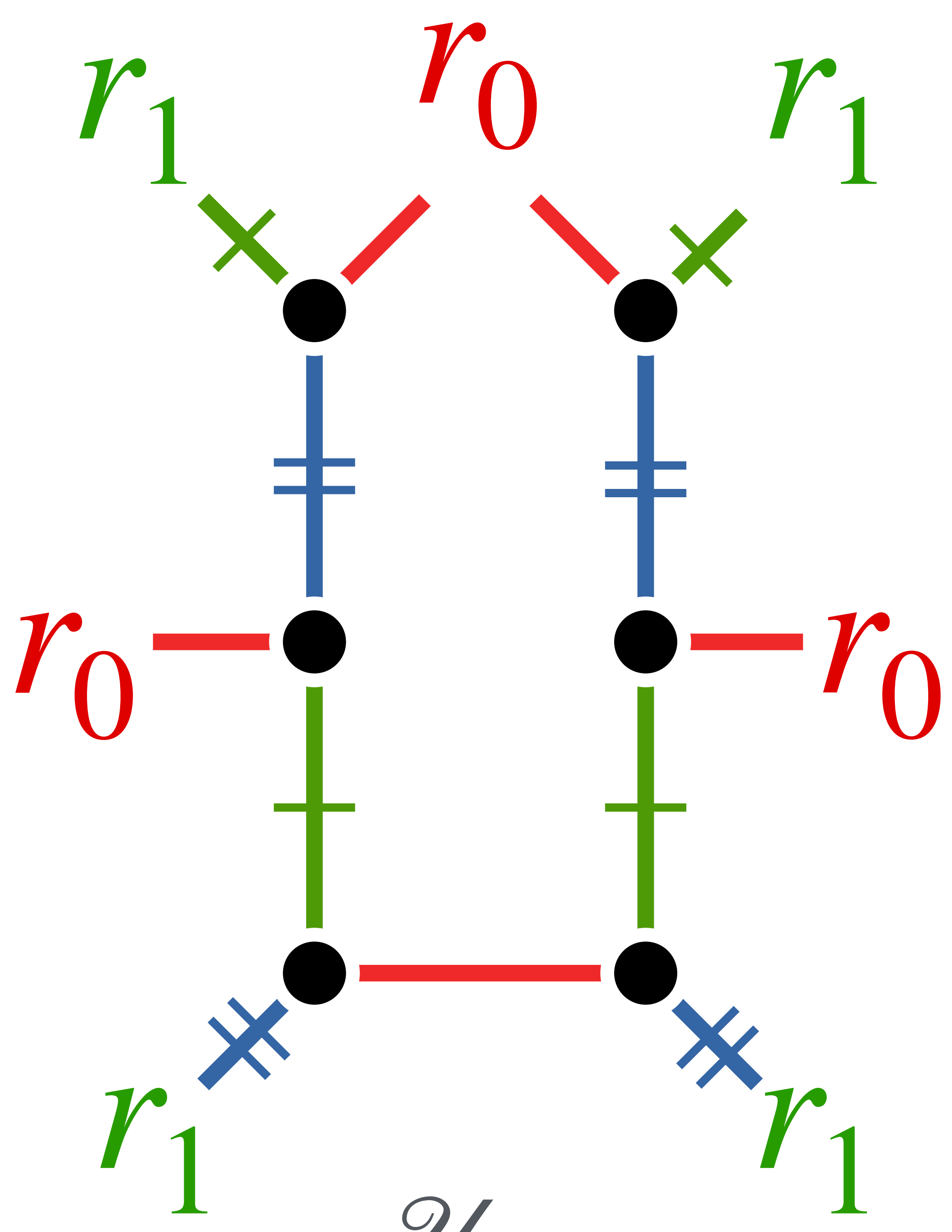

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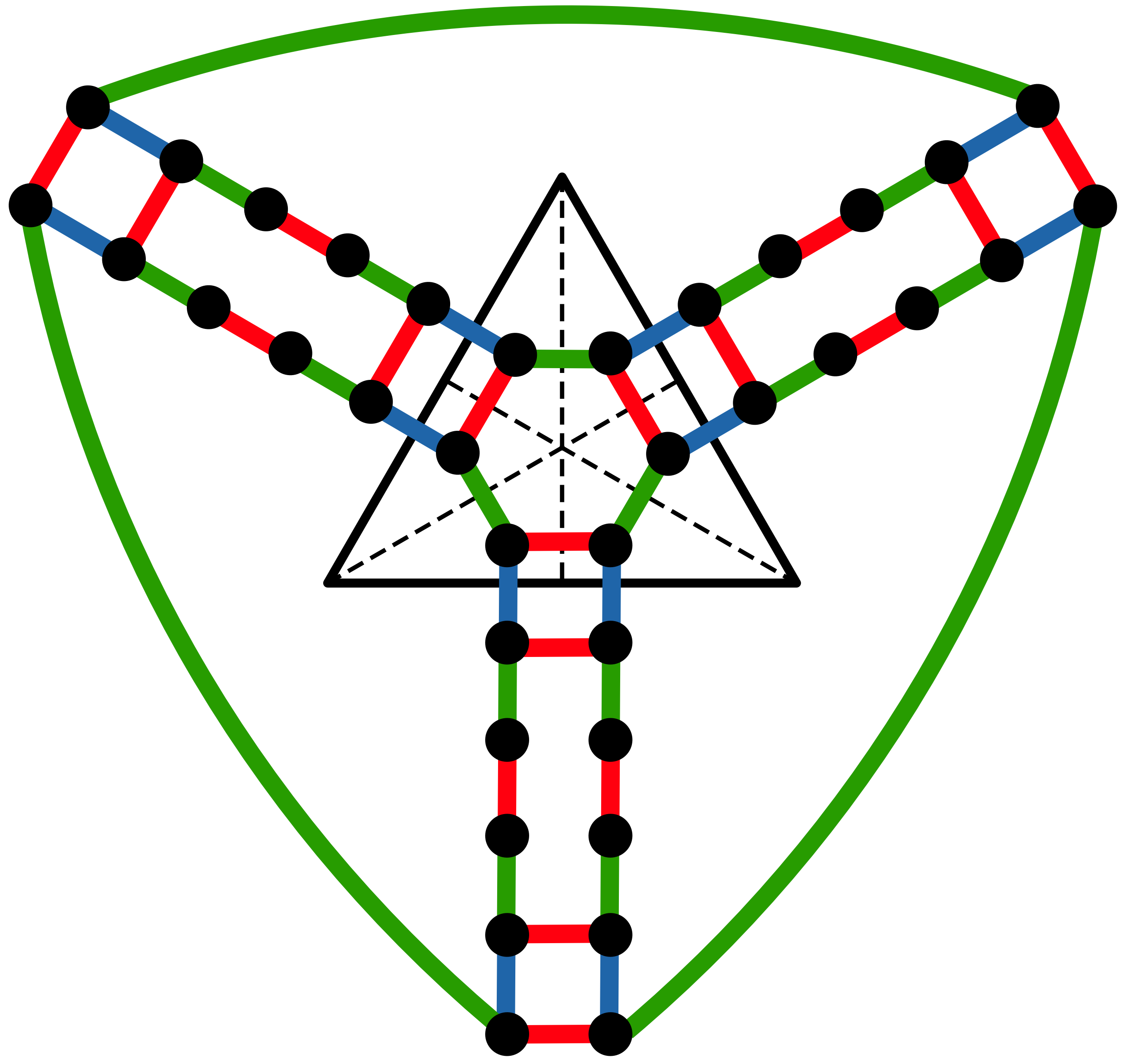

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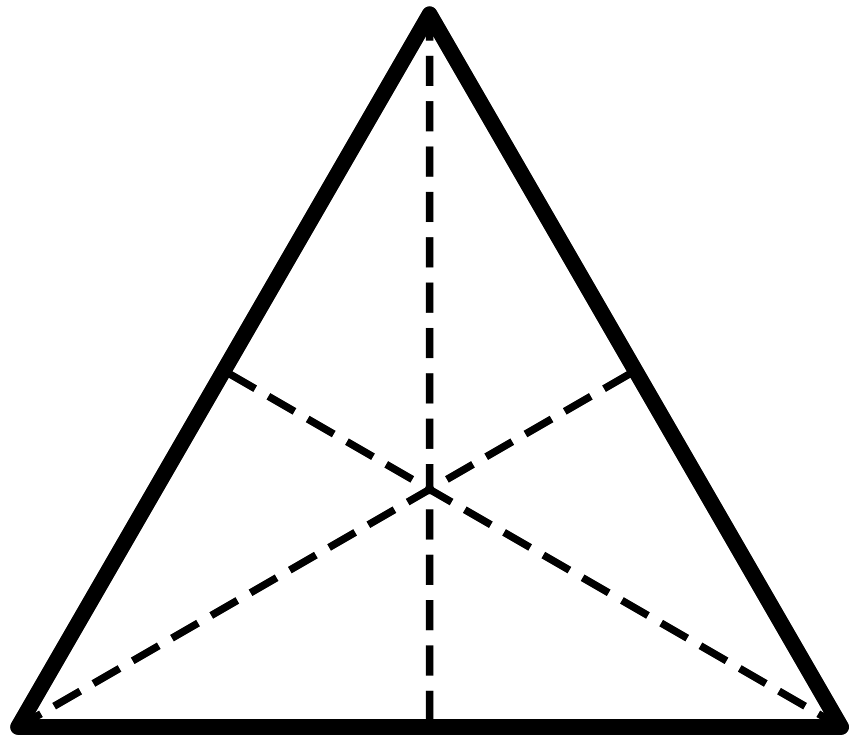

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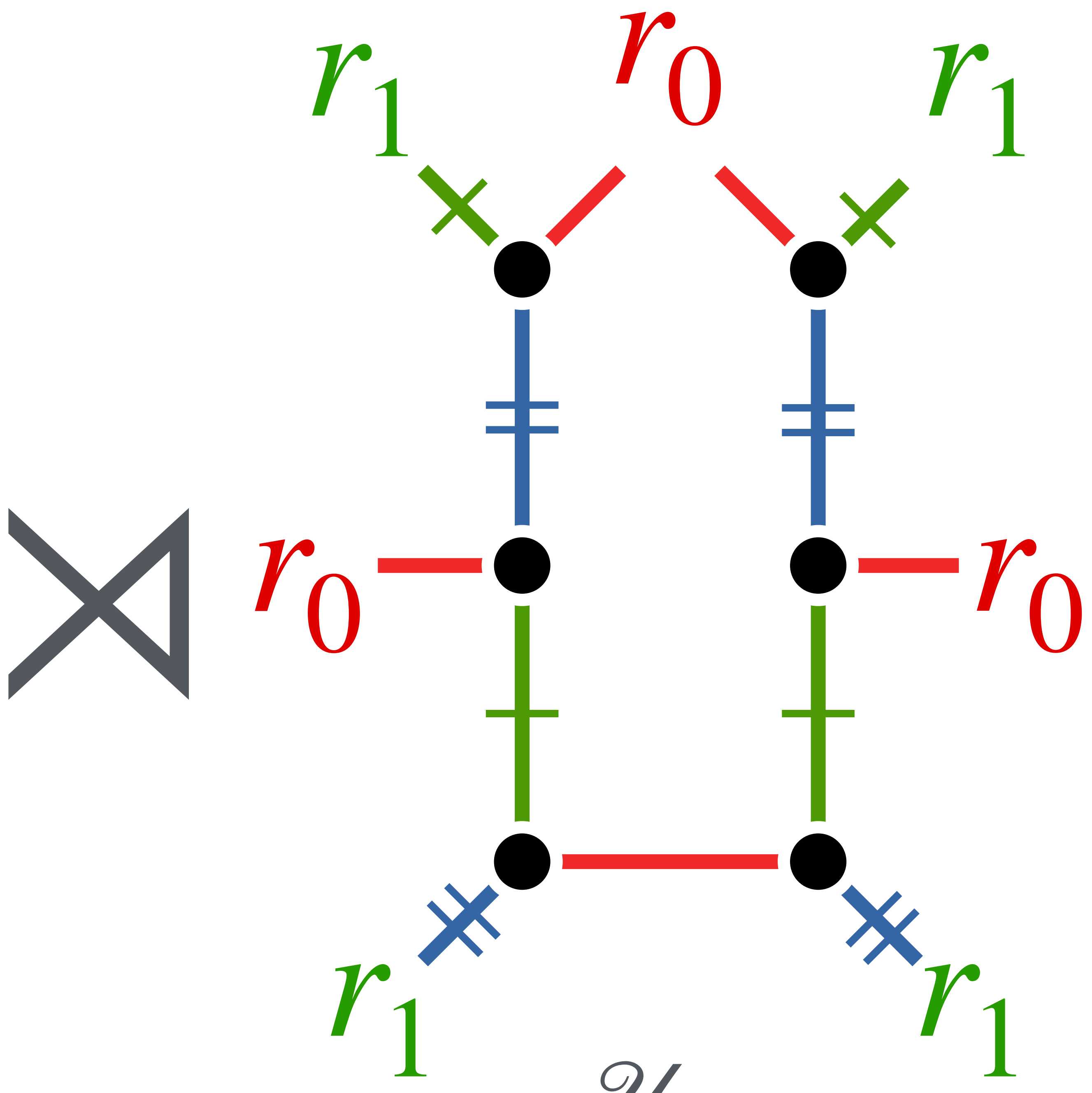

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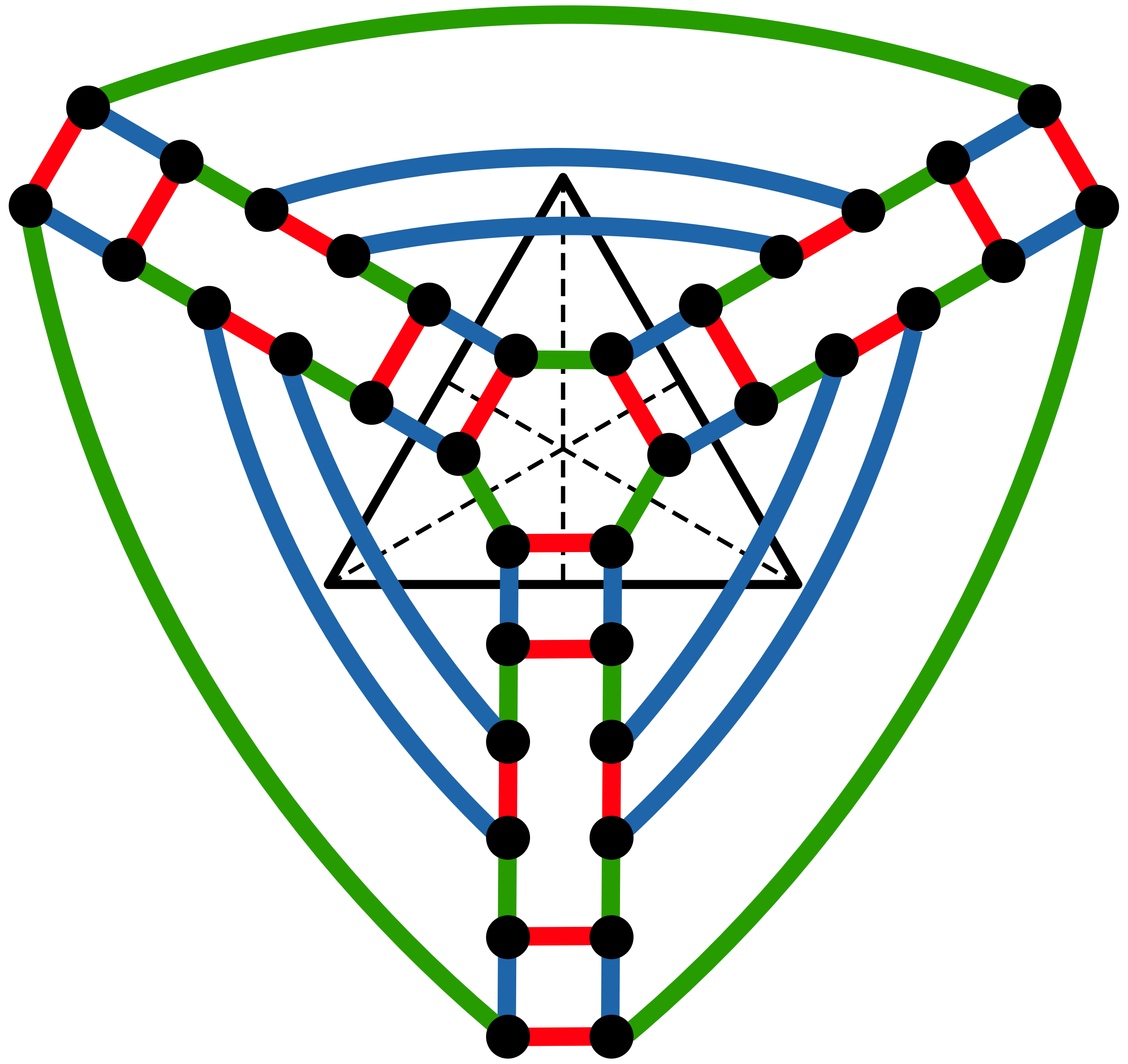

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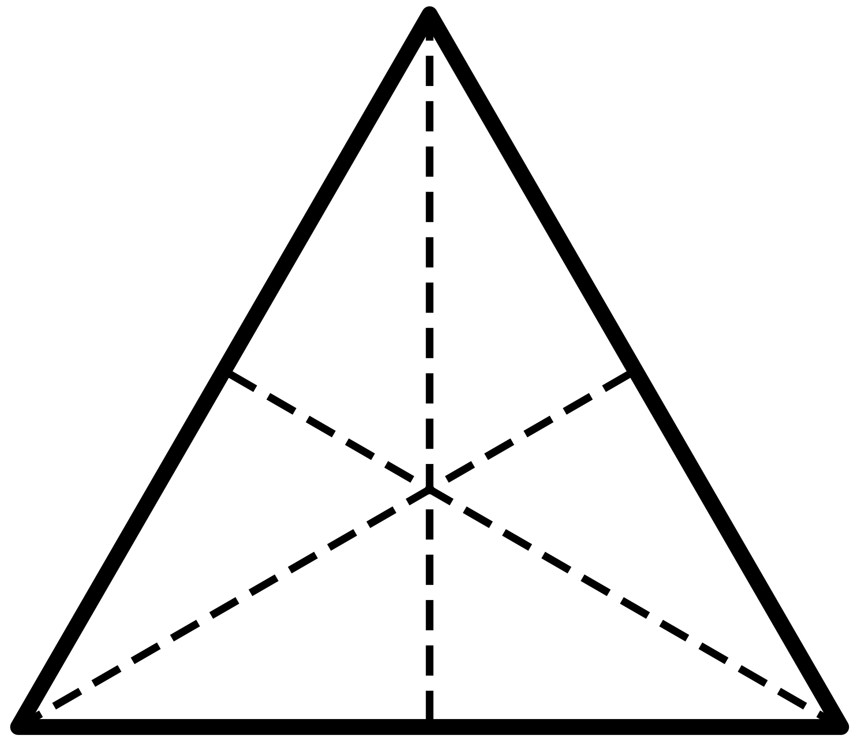

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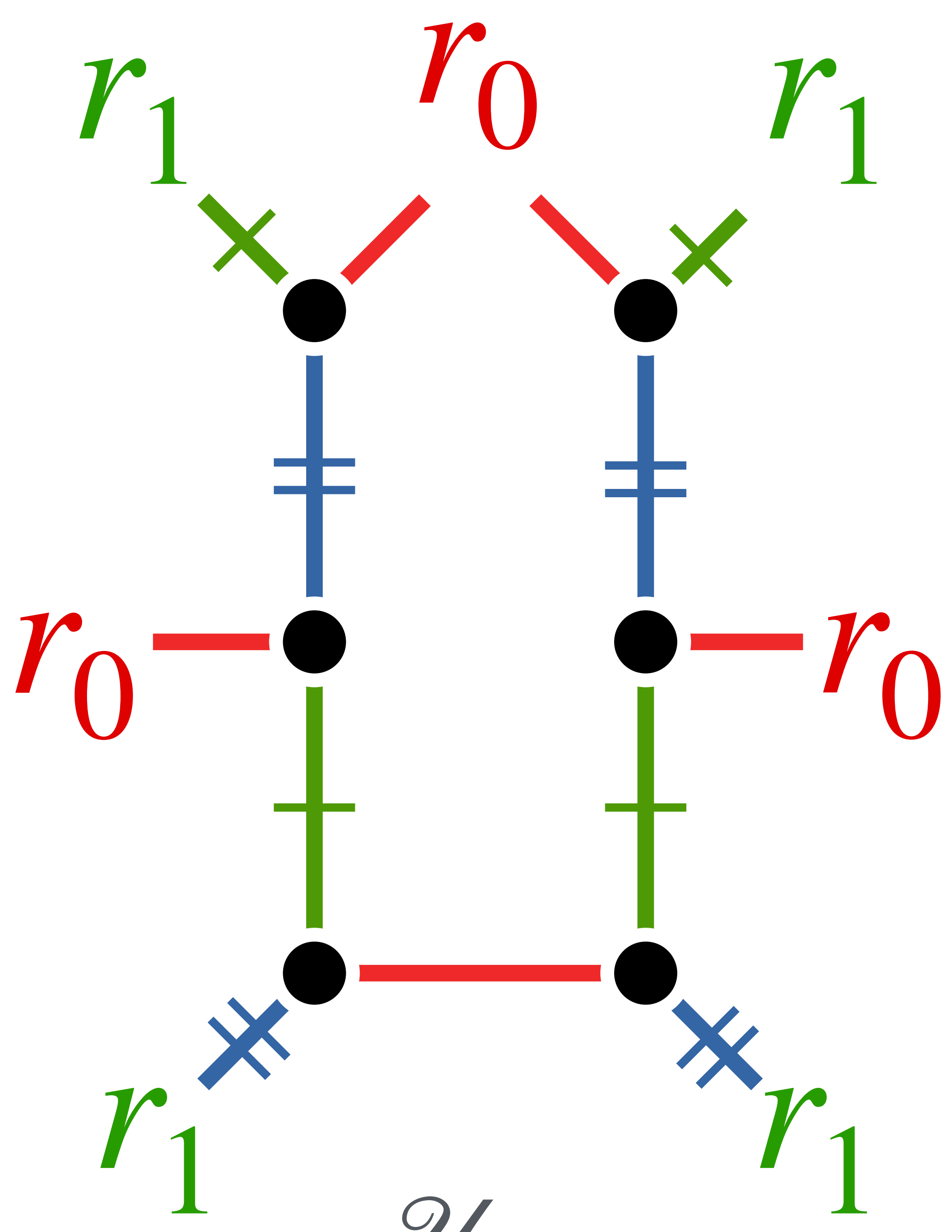

$\mathcal{Y}$

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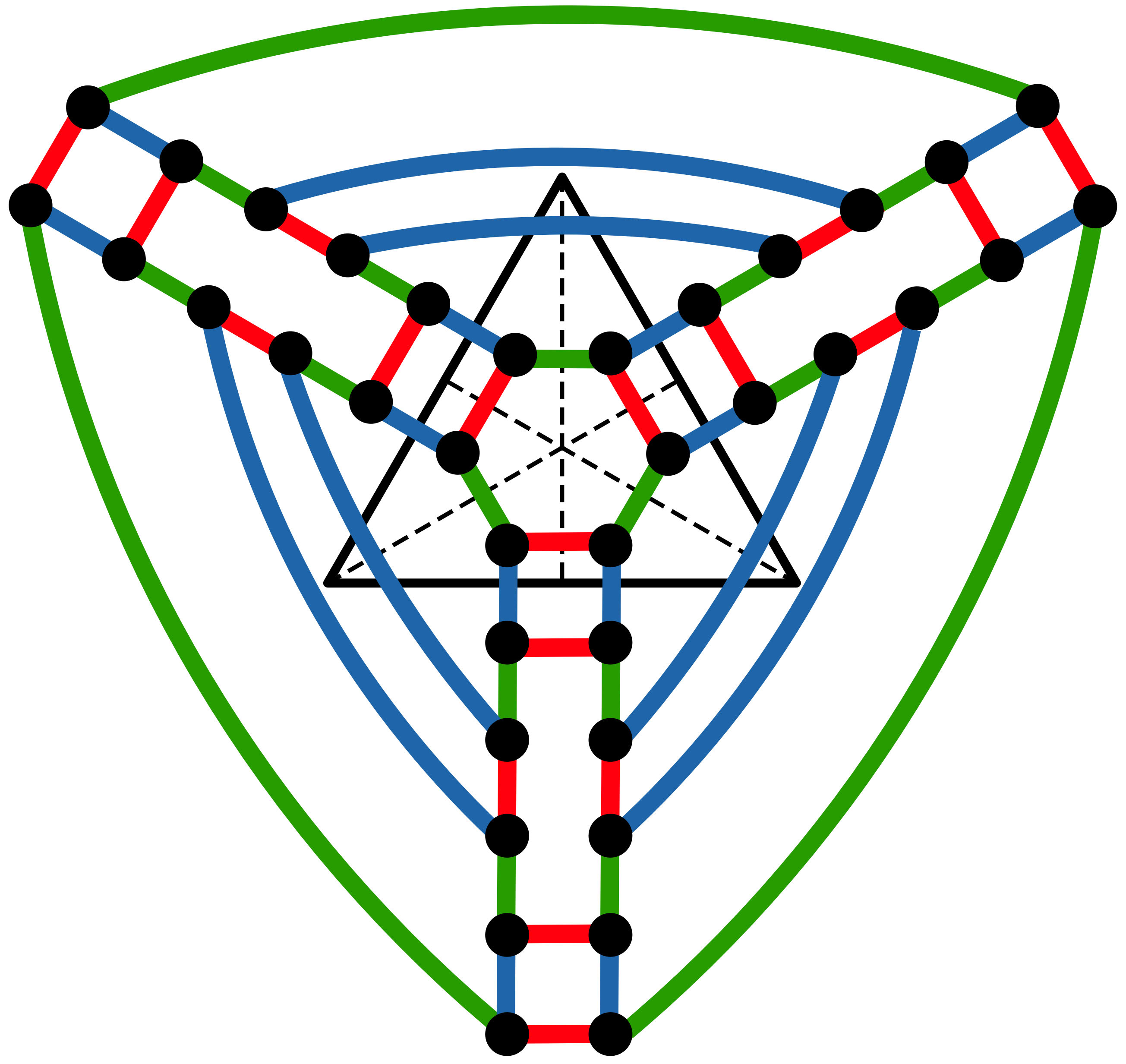

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n-premaniplex

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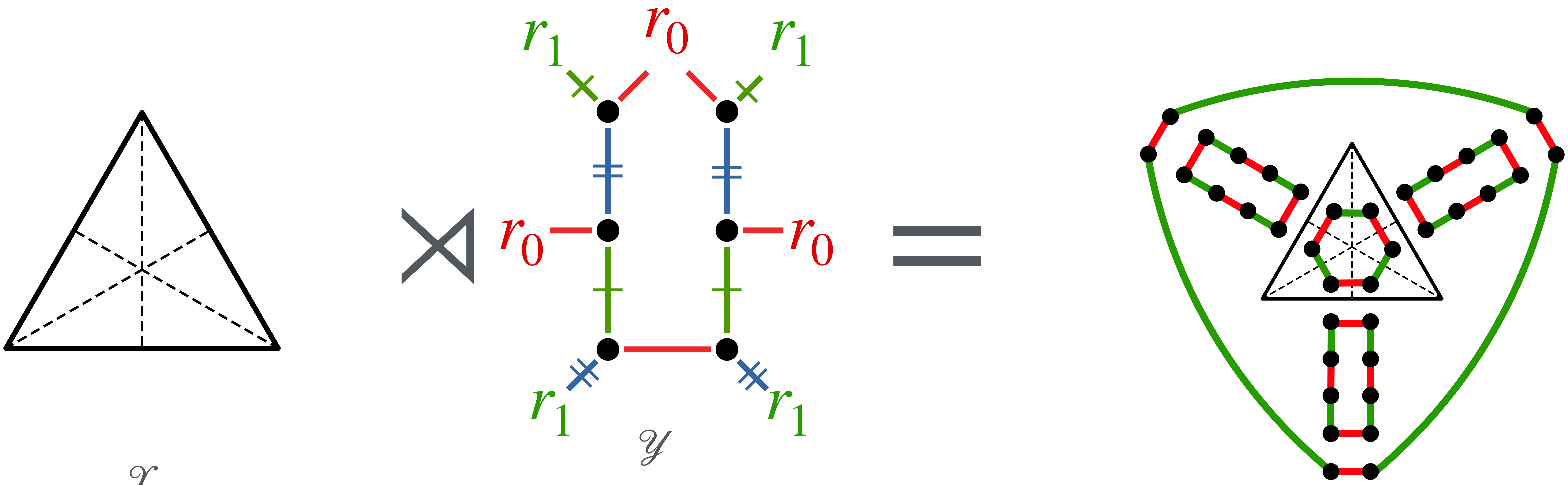

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voltage assignment

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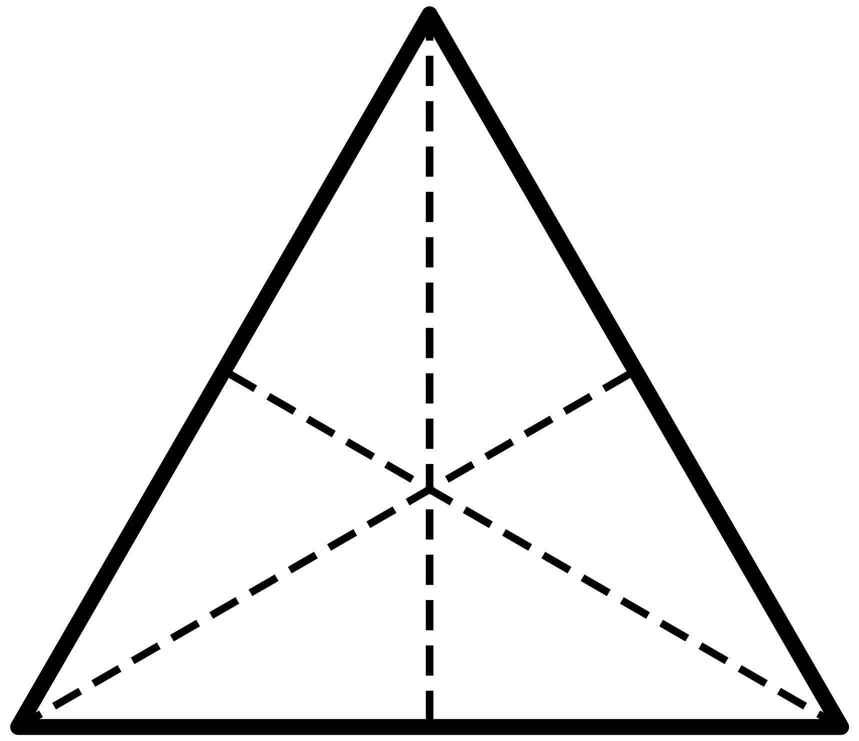

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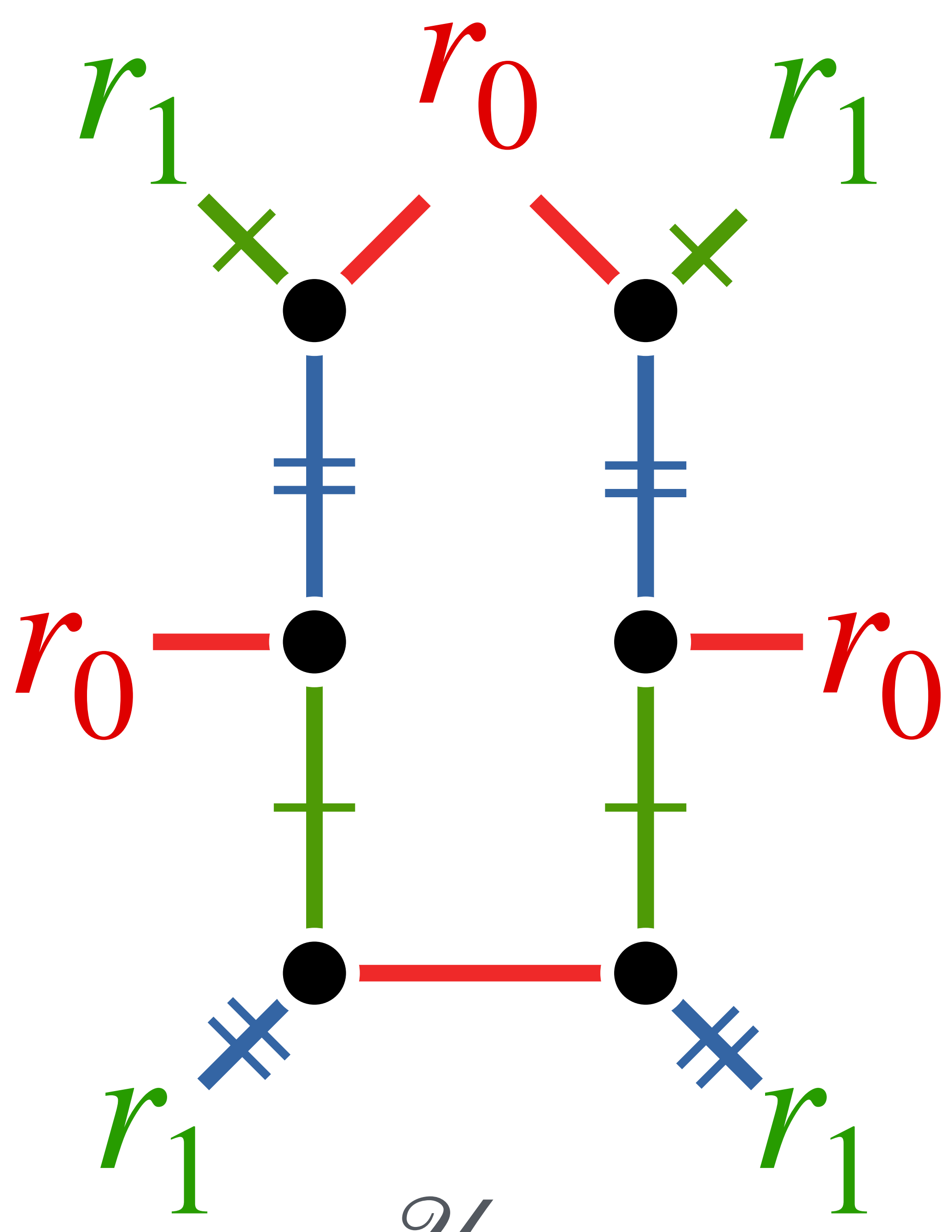

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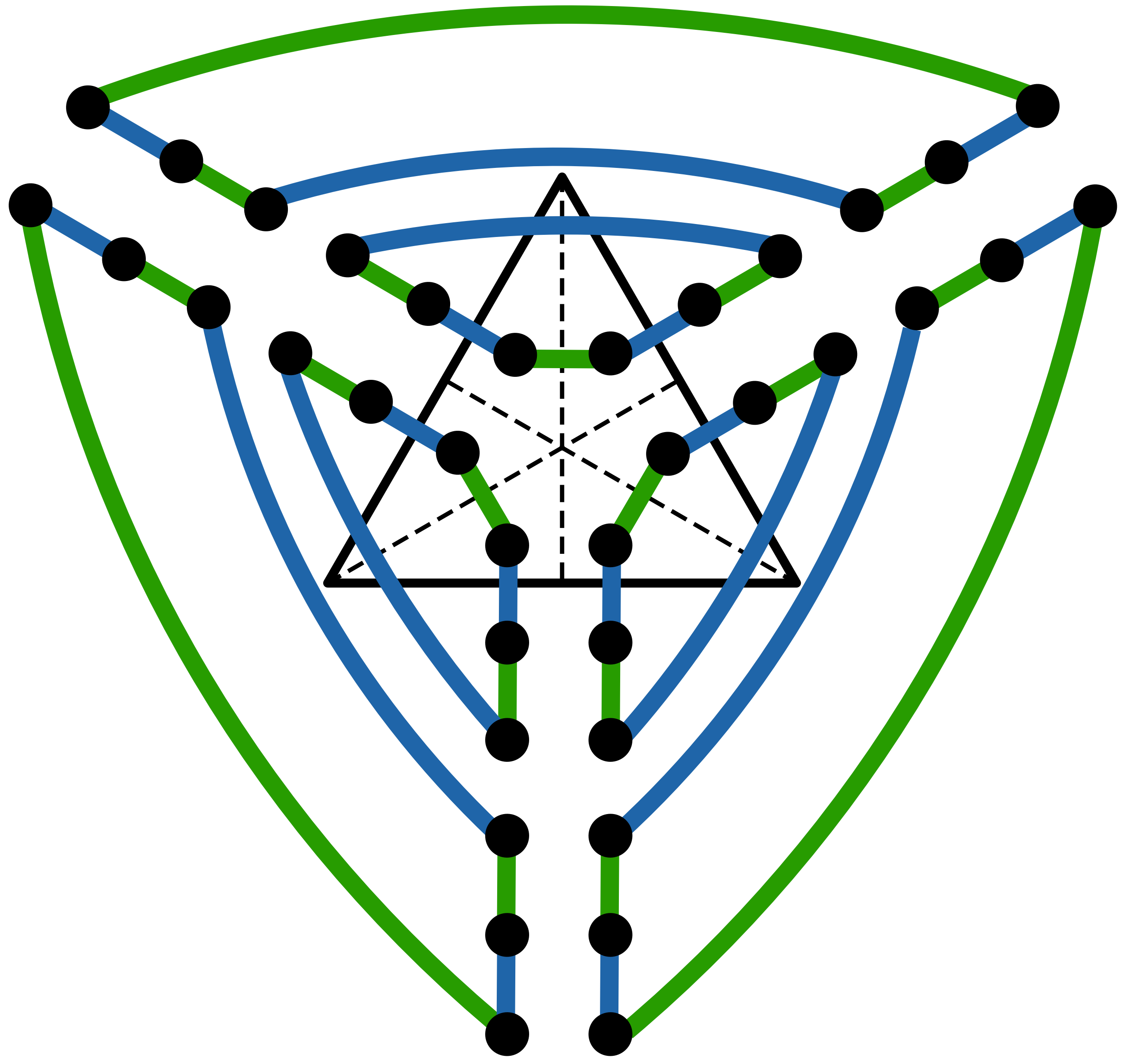

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n-premaniplex

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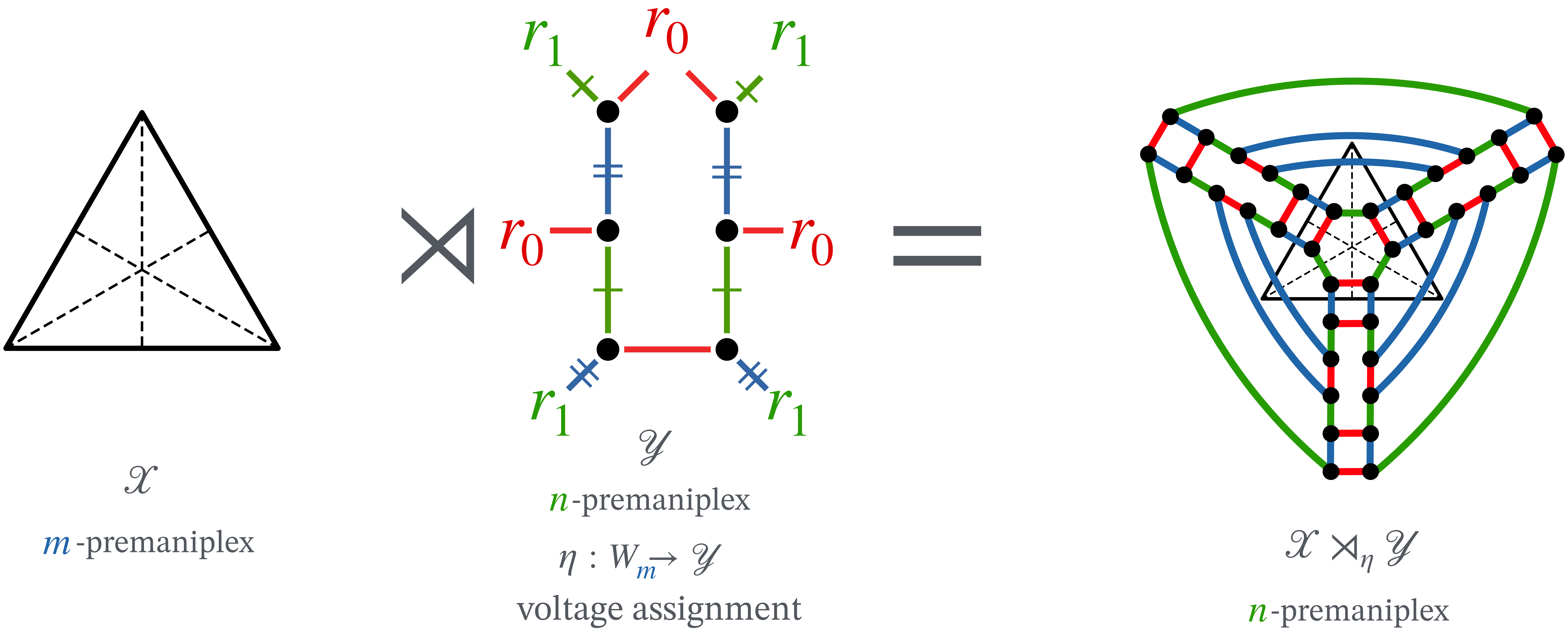

voltage operations

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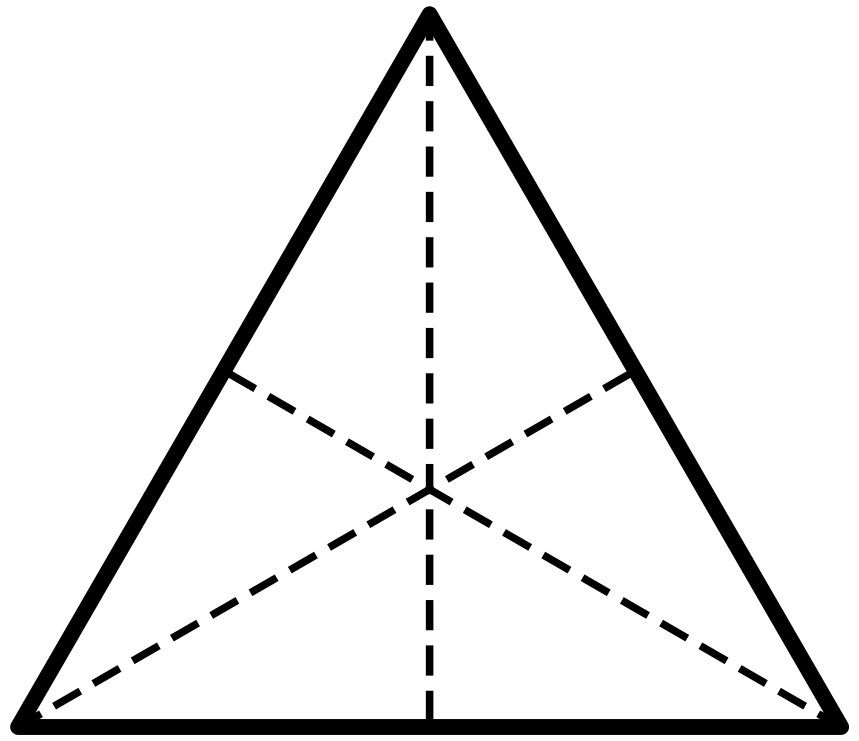

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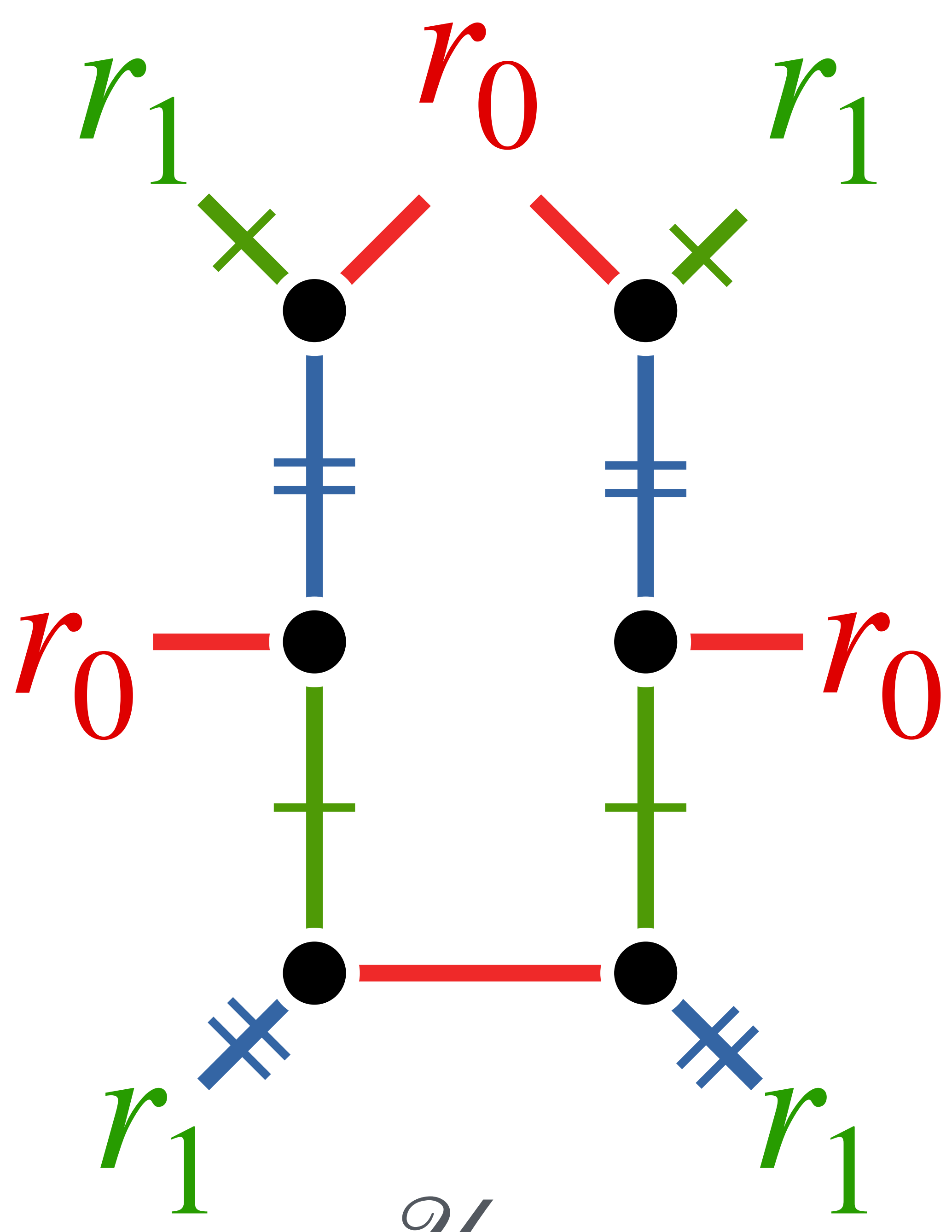

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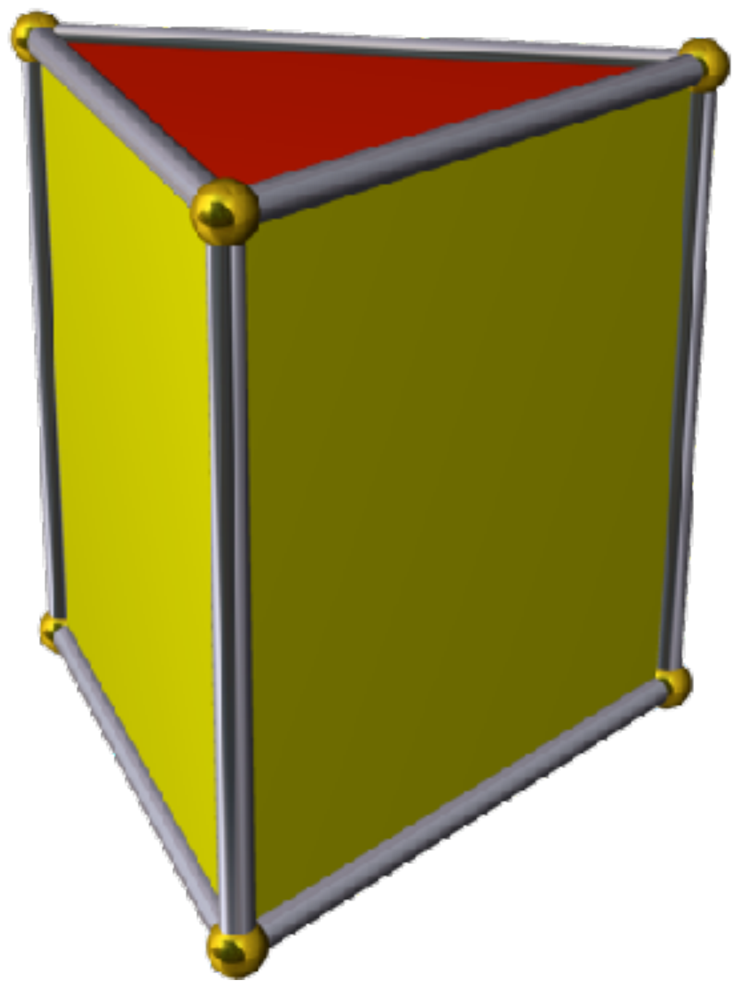
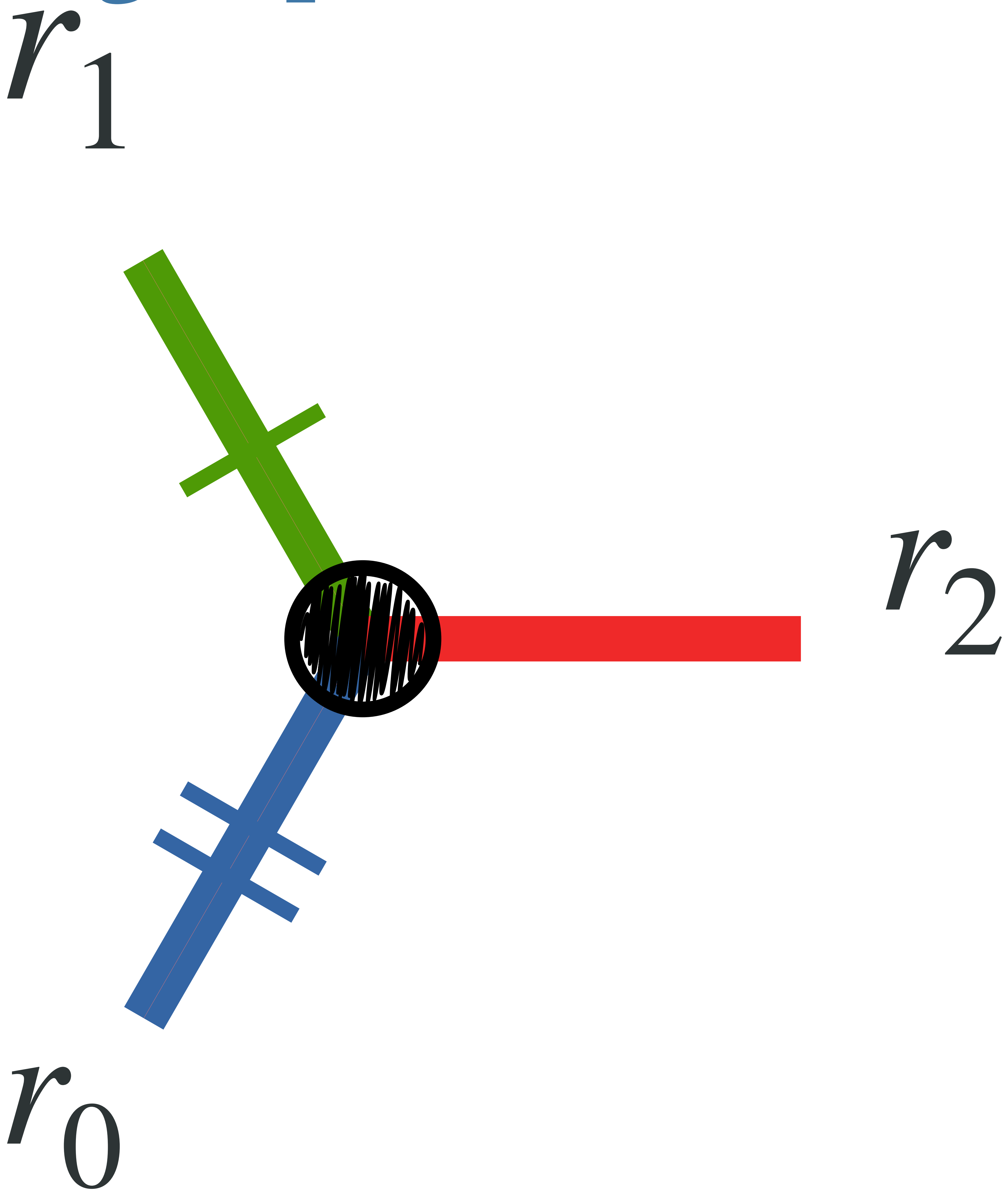


$\mathscr{X} \rtimes_\eta \mathscr{Y}$

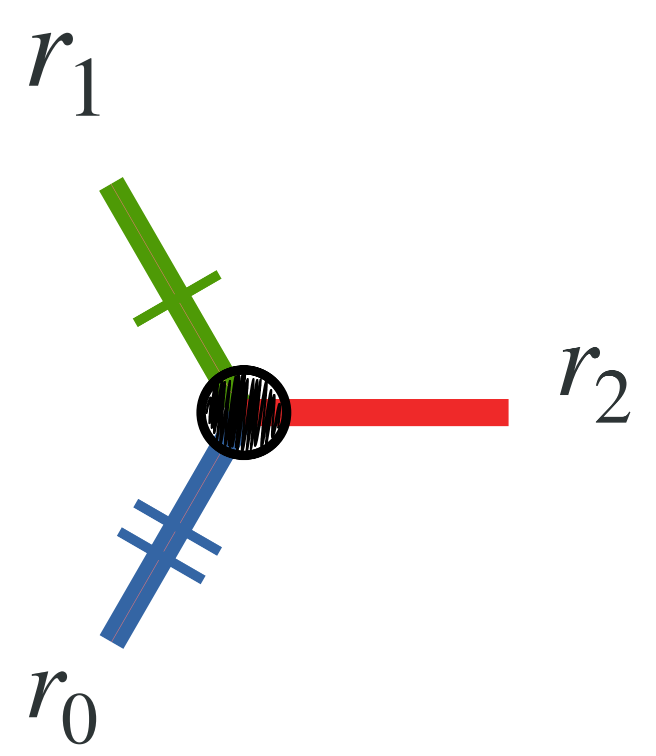
n-premaniplex

voltage operations

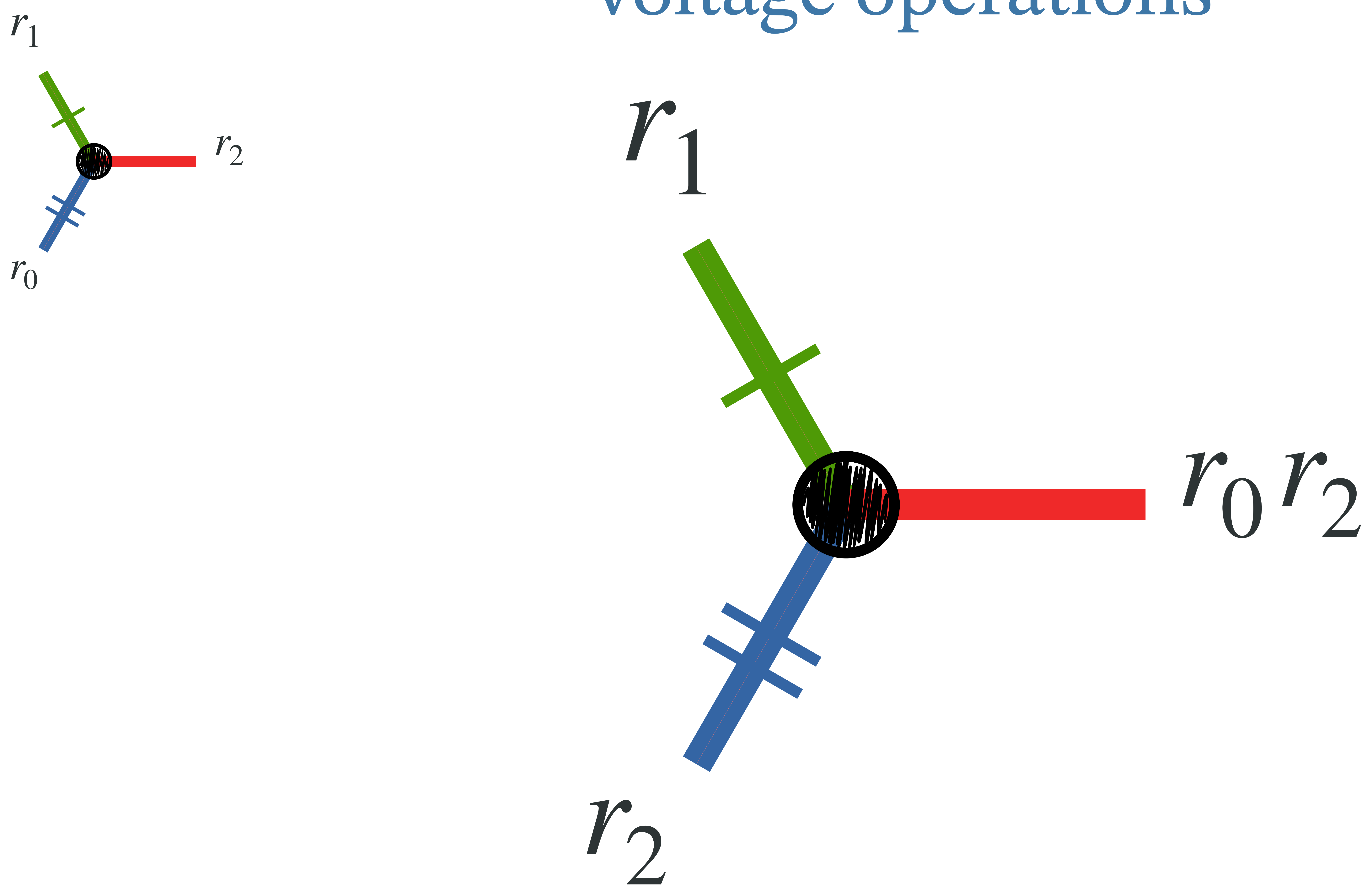
voltage operations



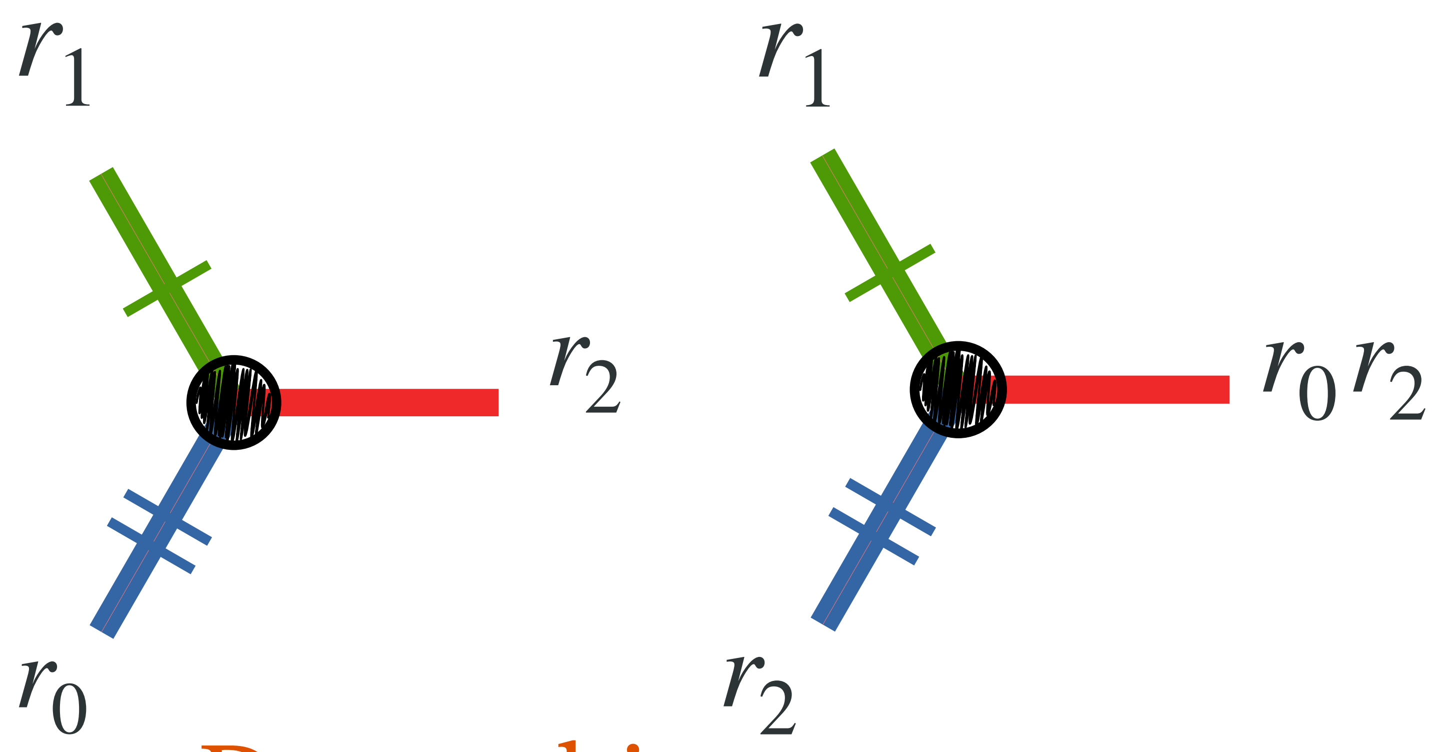
voltage operations



voltage operations

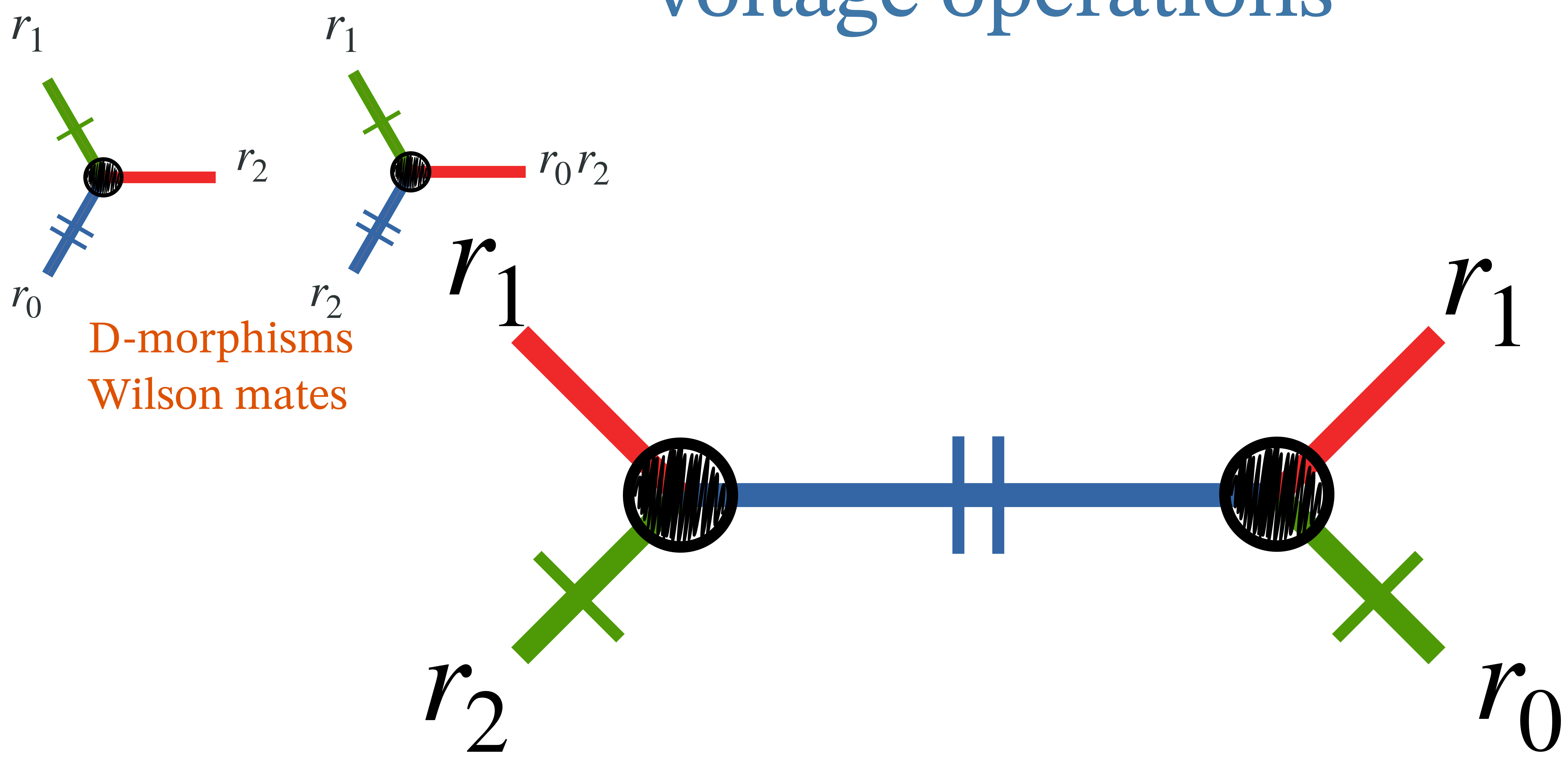


voltage operations



D-morphisms
Wilson mates

voltage operations



D-morphisms
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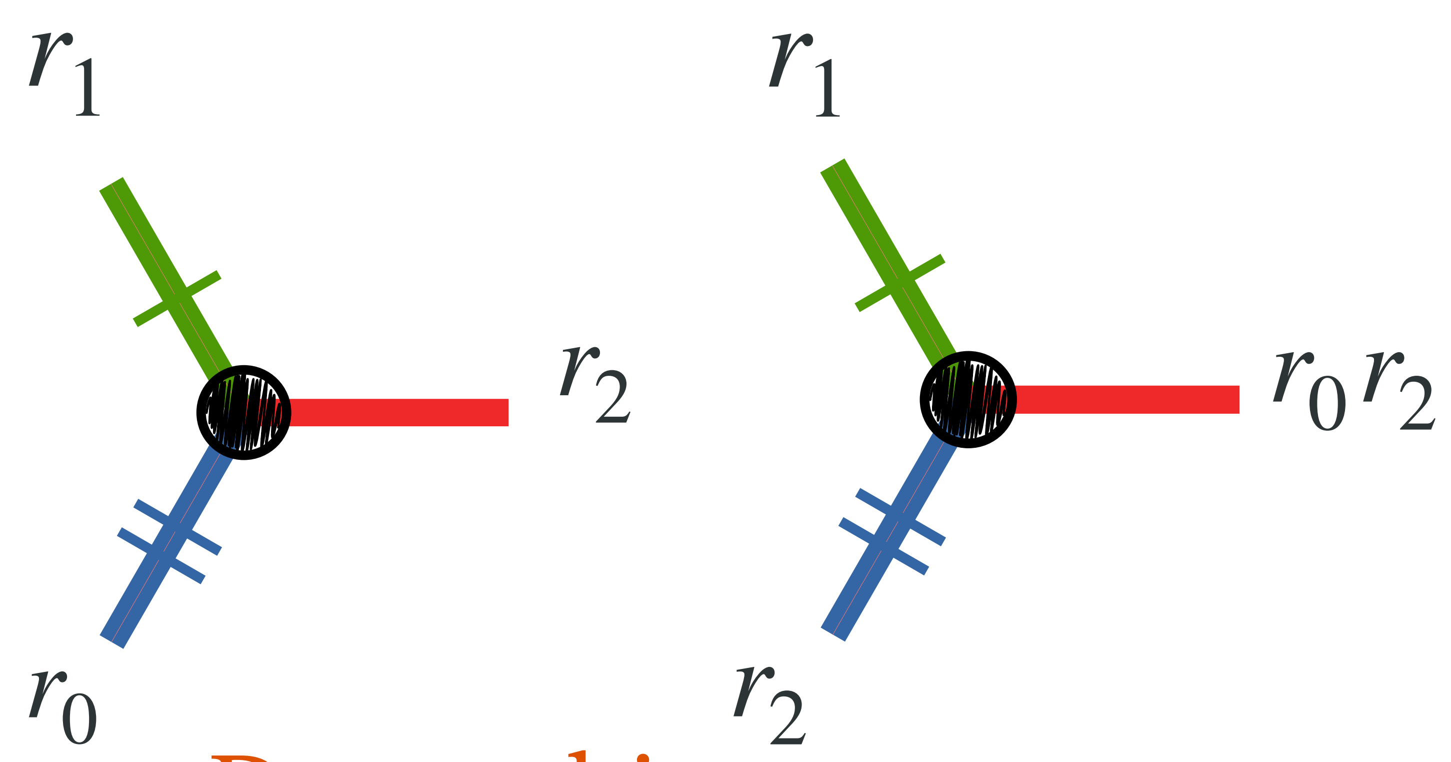
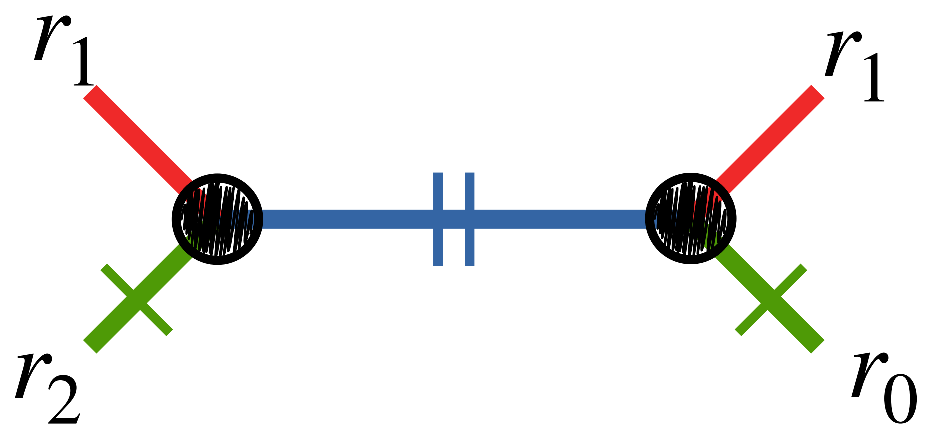
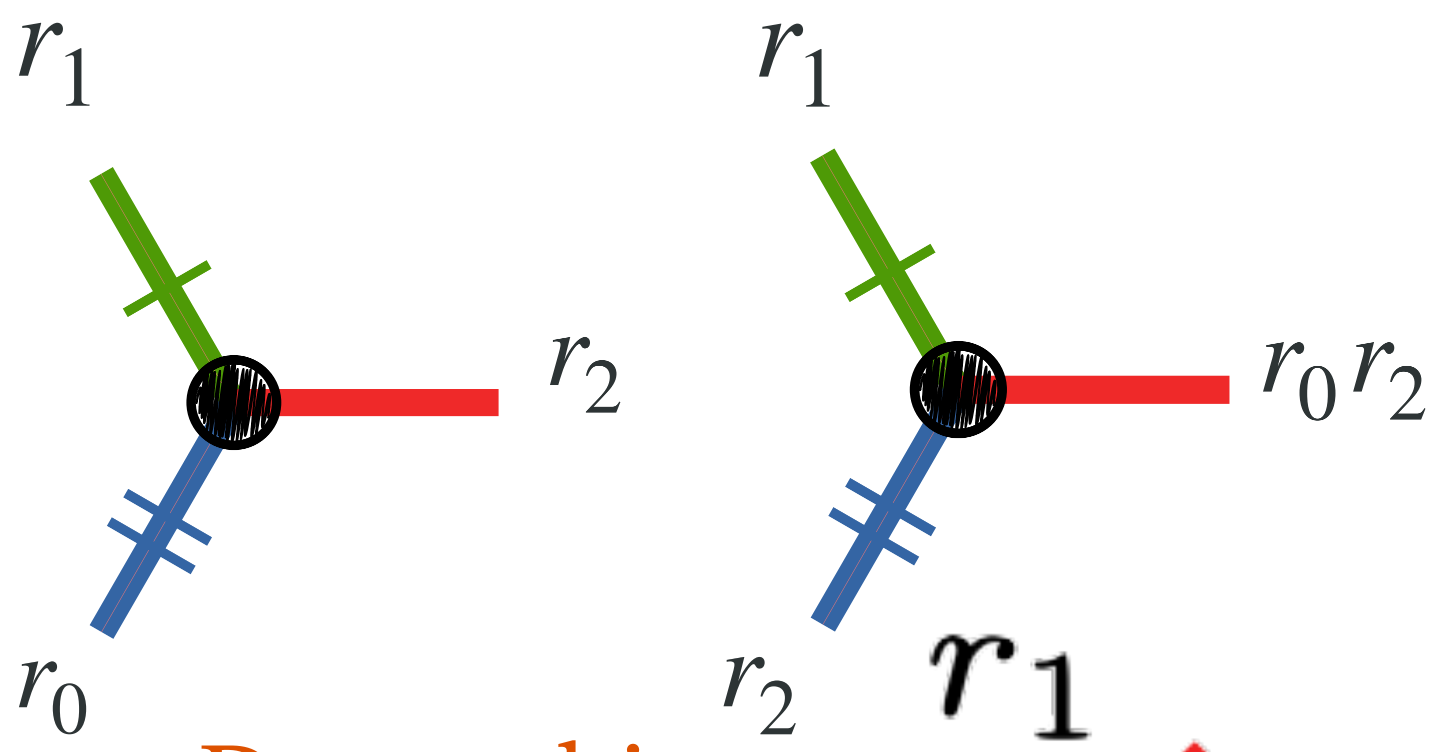
voltage operations



D-morphisms
Wilson mates



voltage operations



D-morphisms
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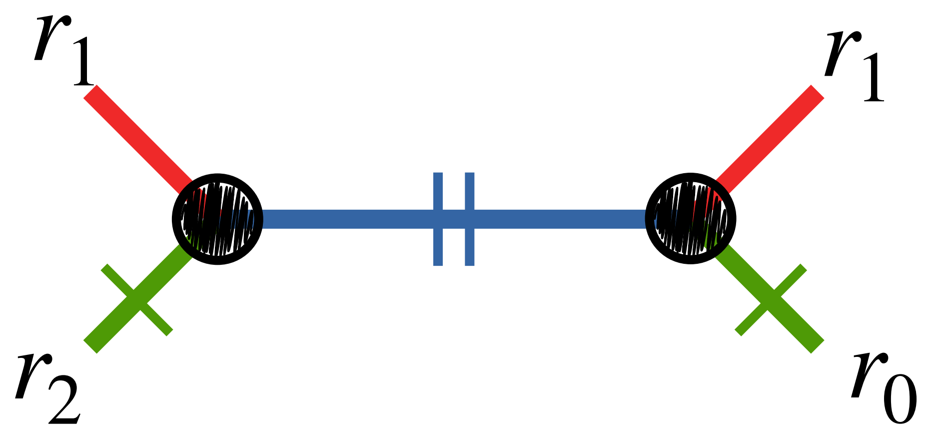
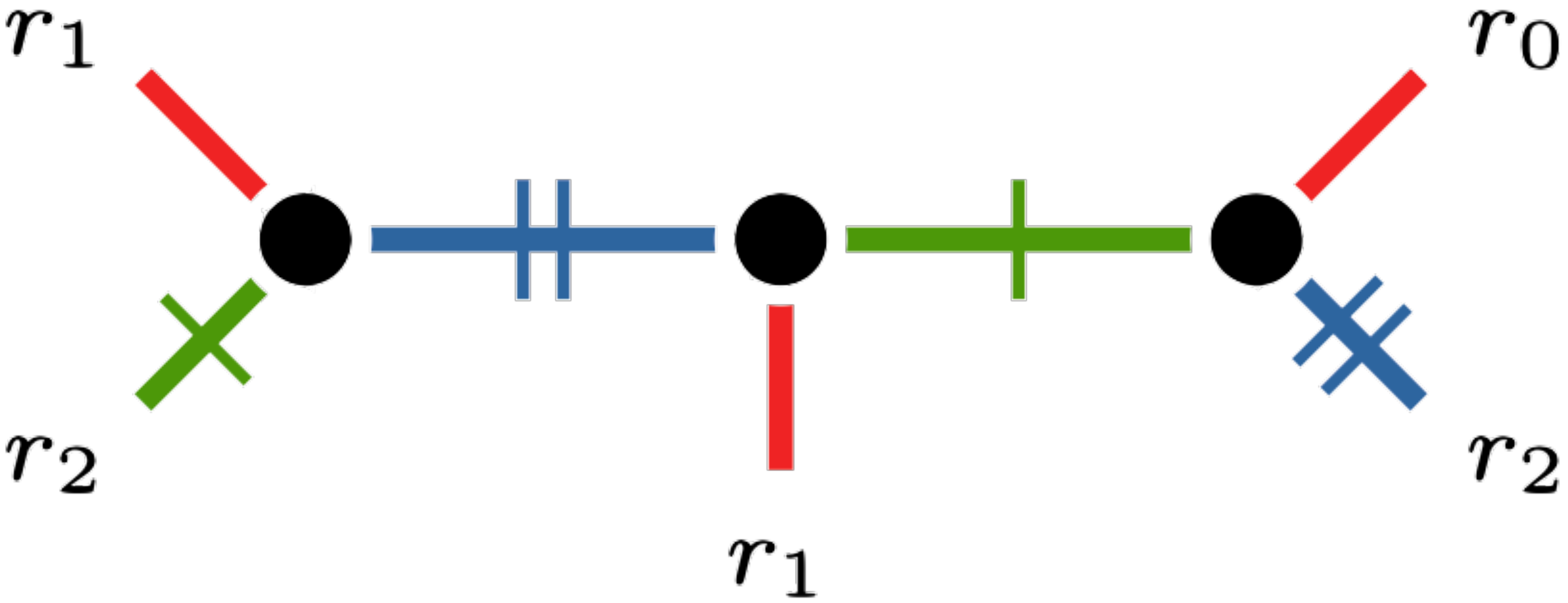
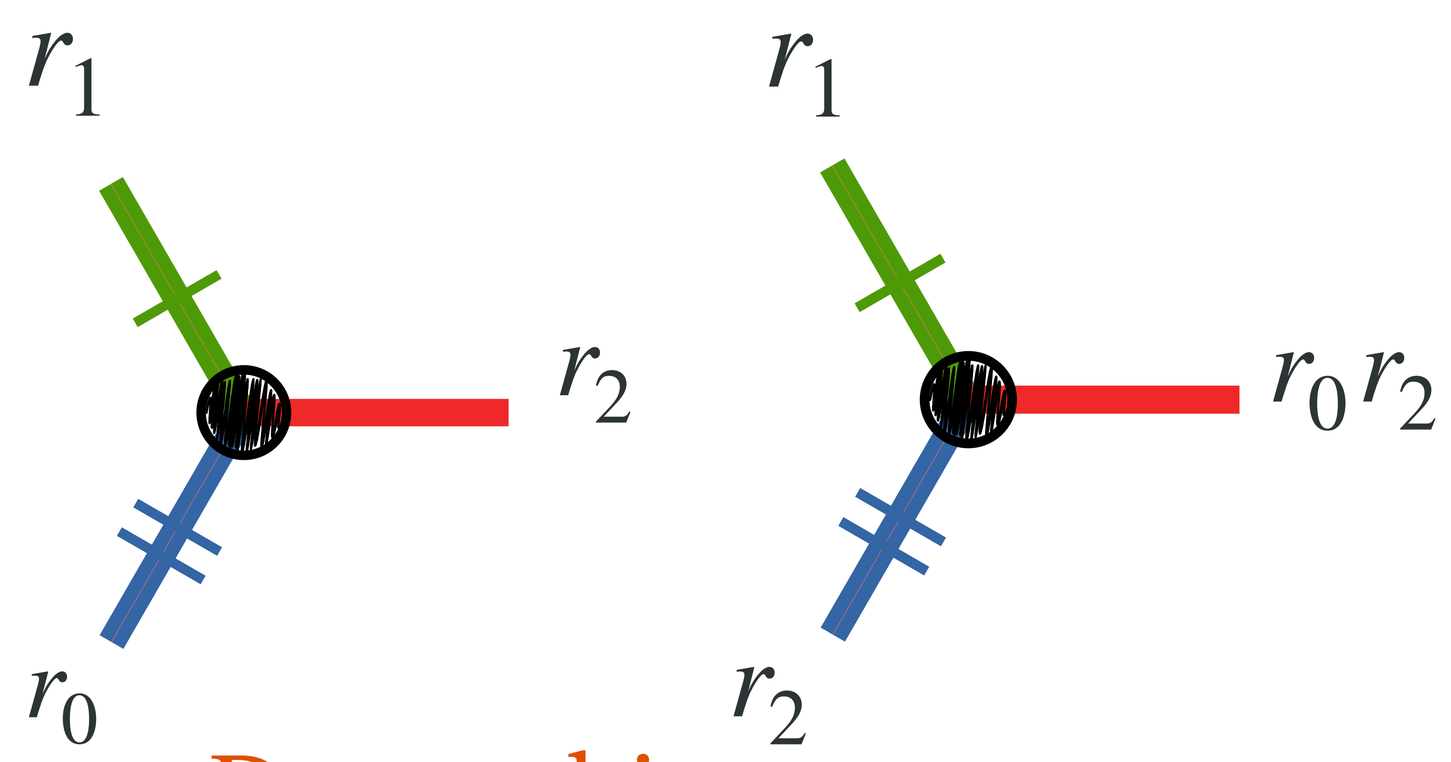




voltage operations



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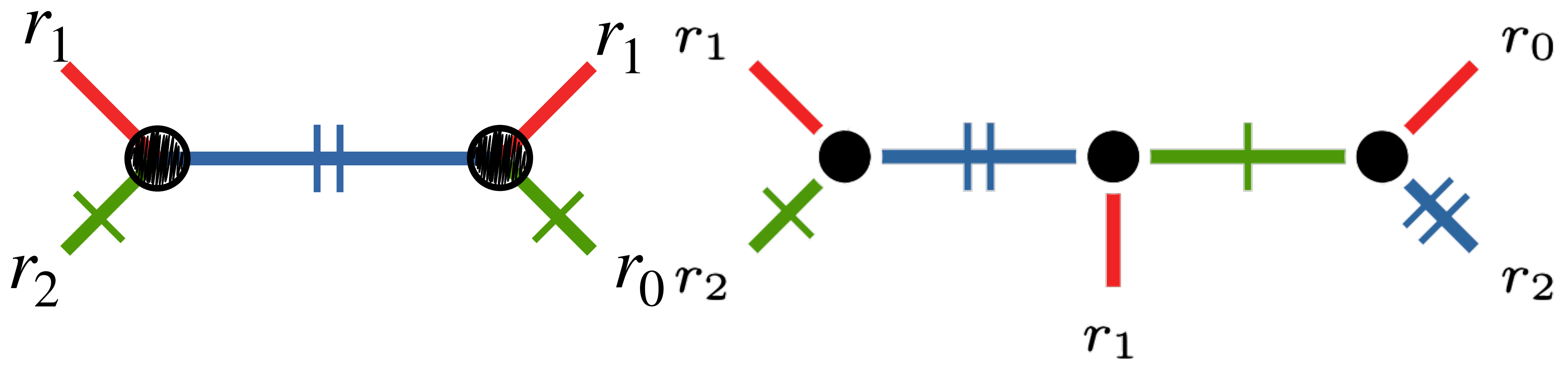
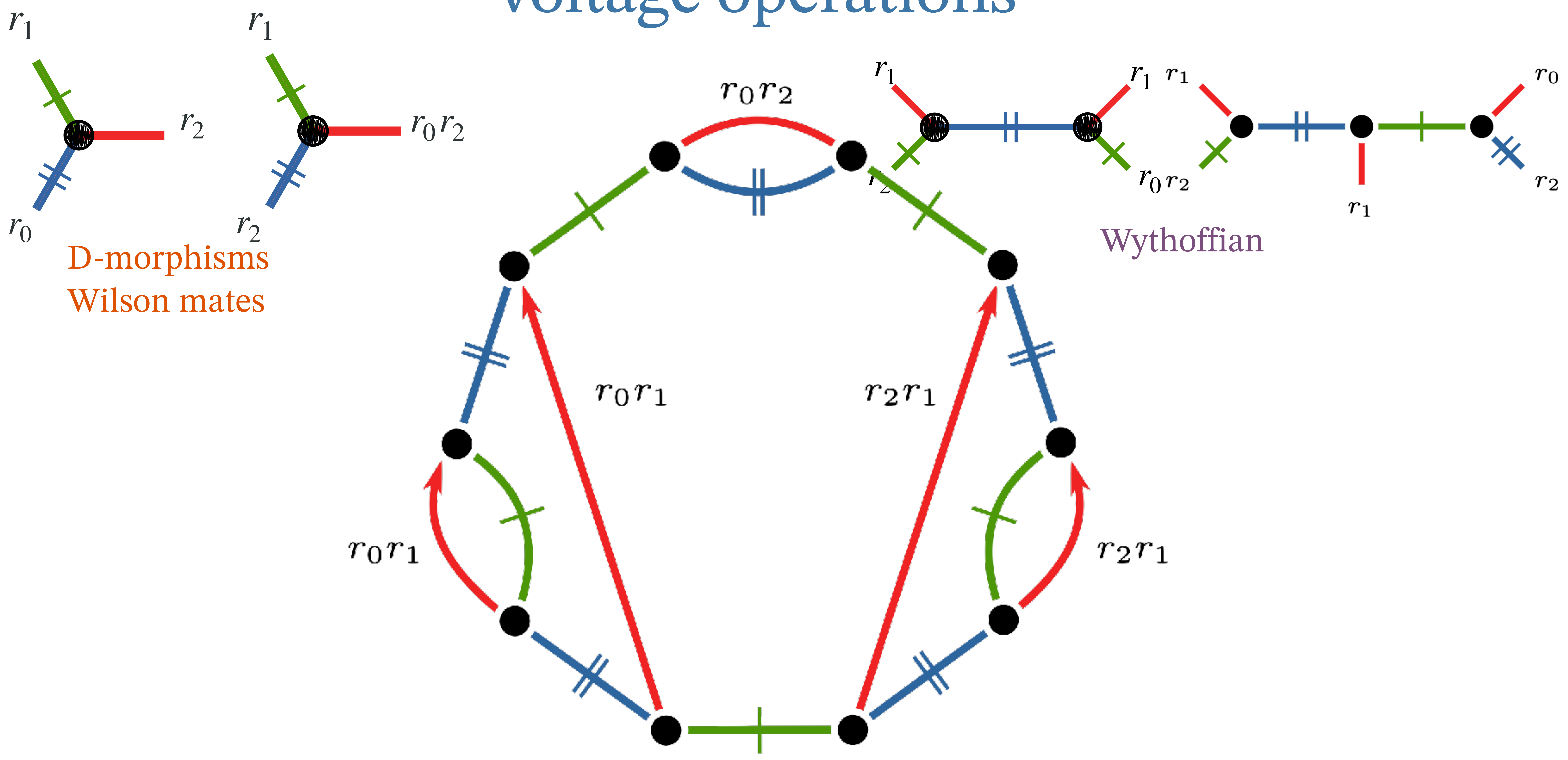
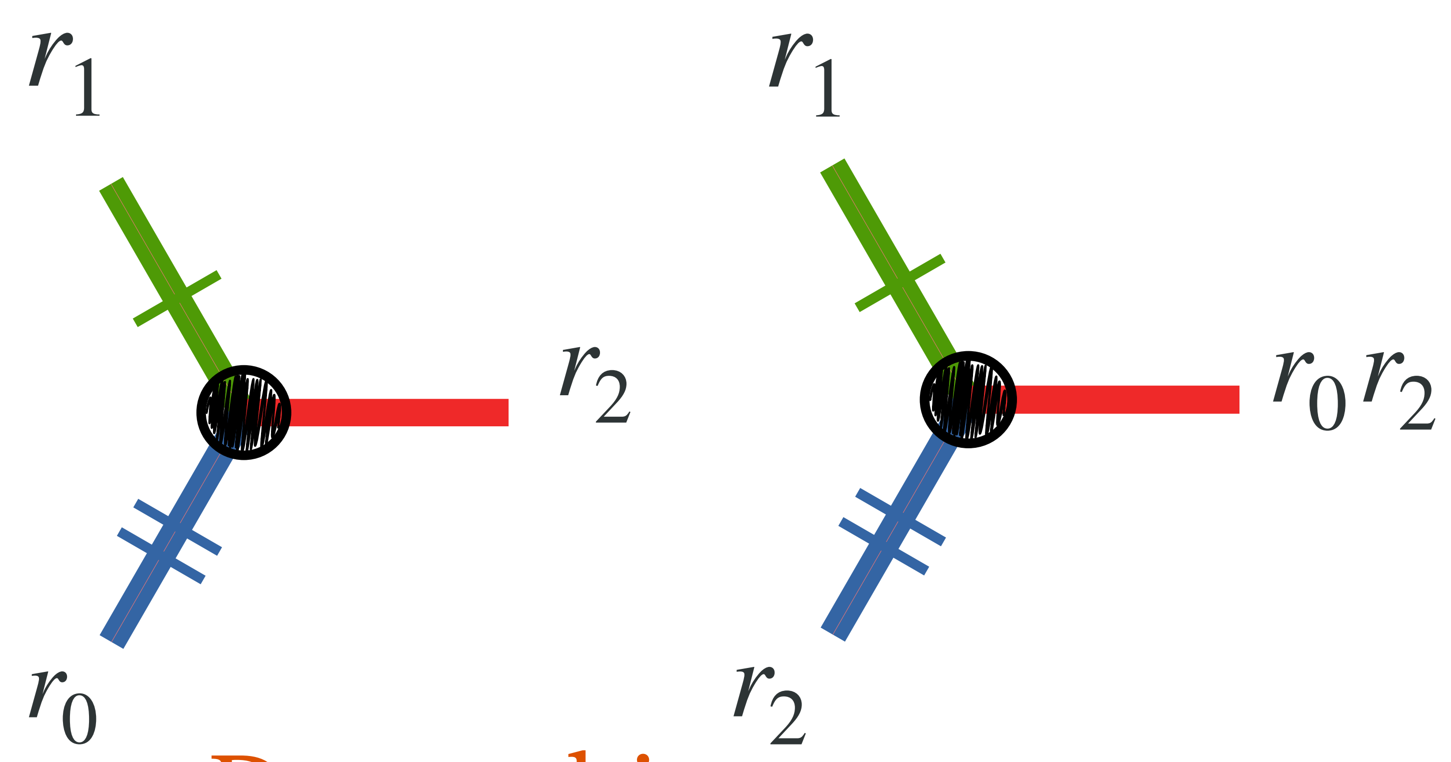


Wythoffian

voltage operations



voltage operations



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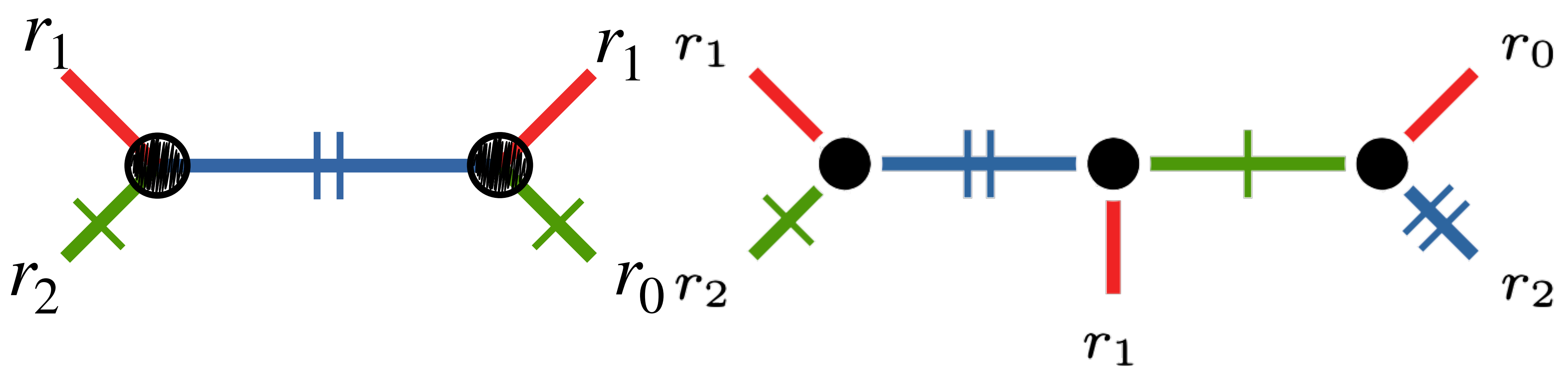

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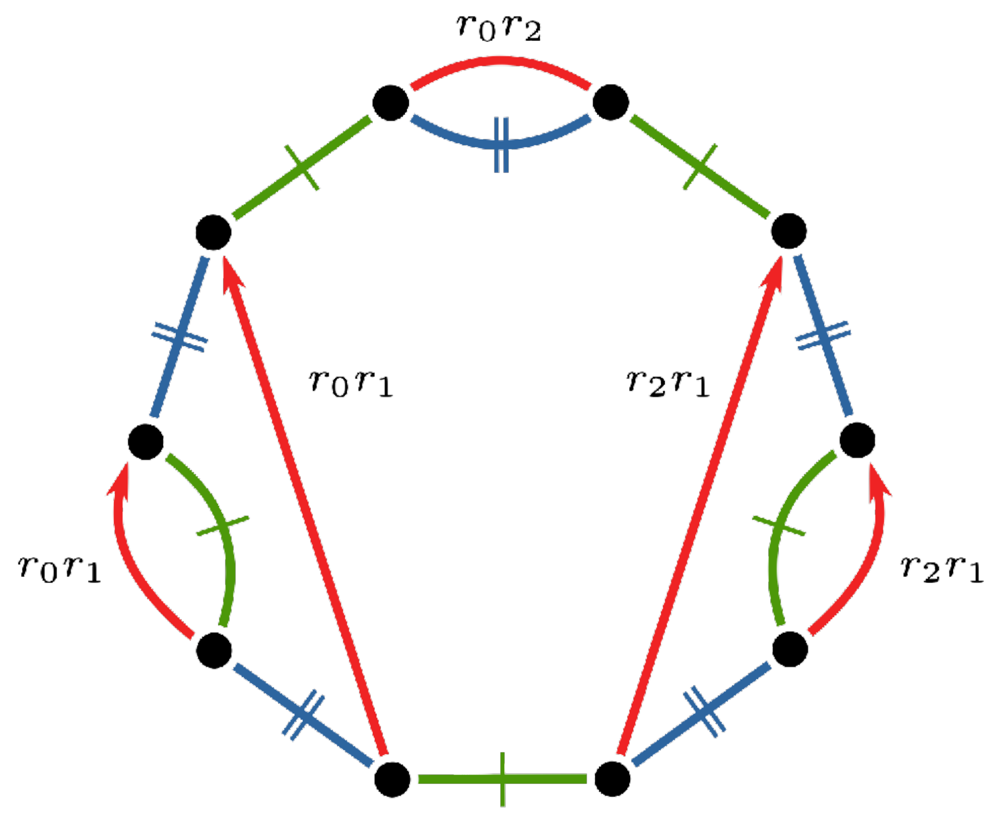
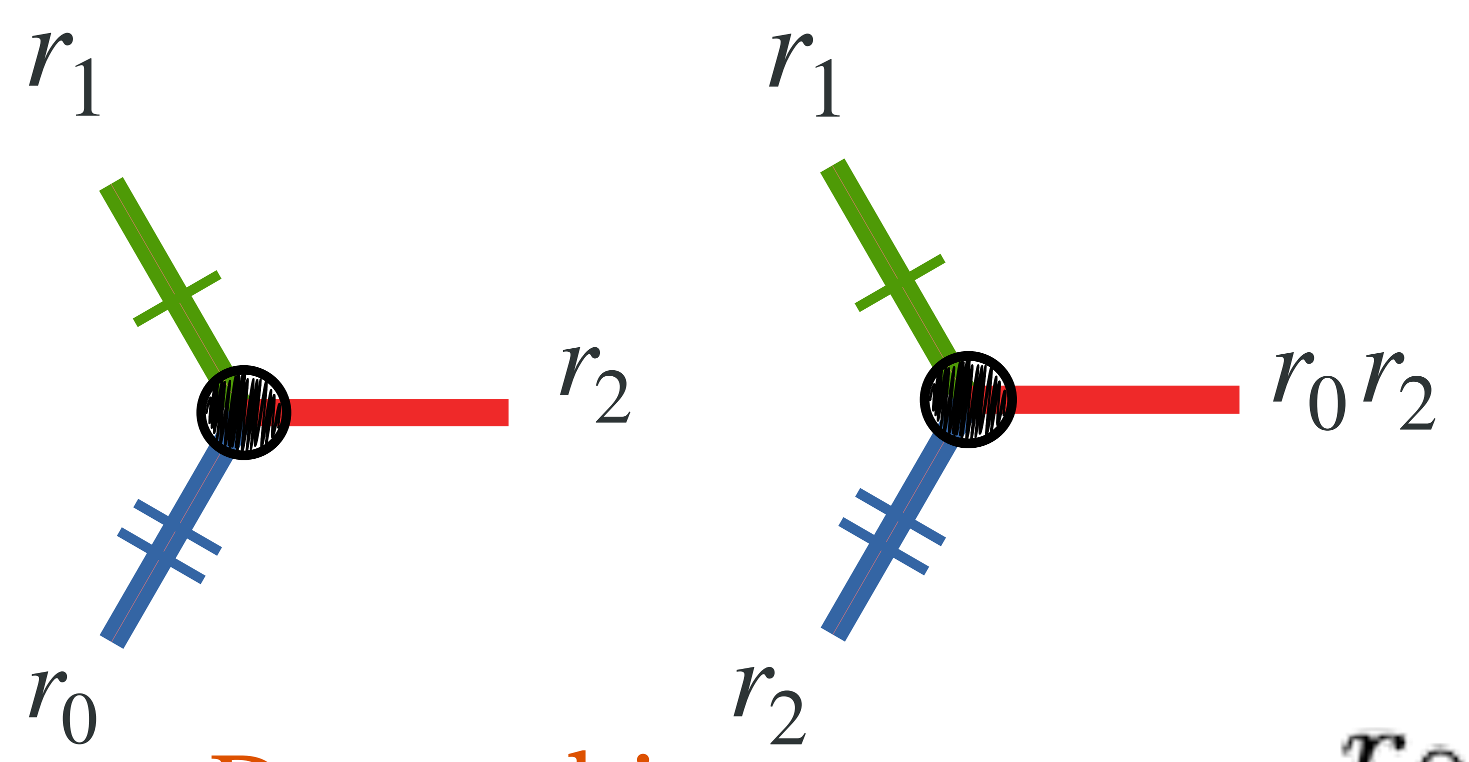


Snub

voltage operations



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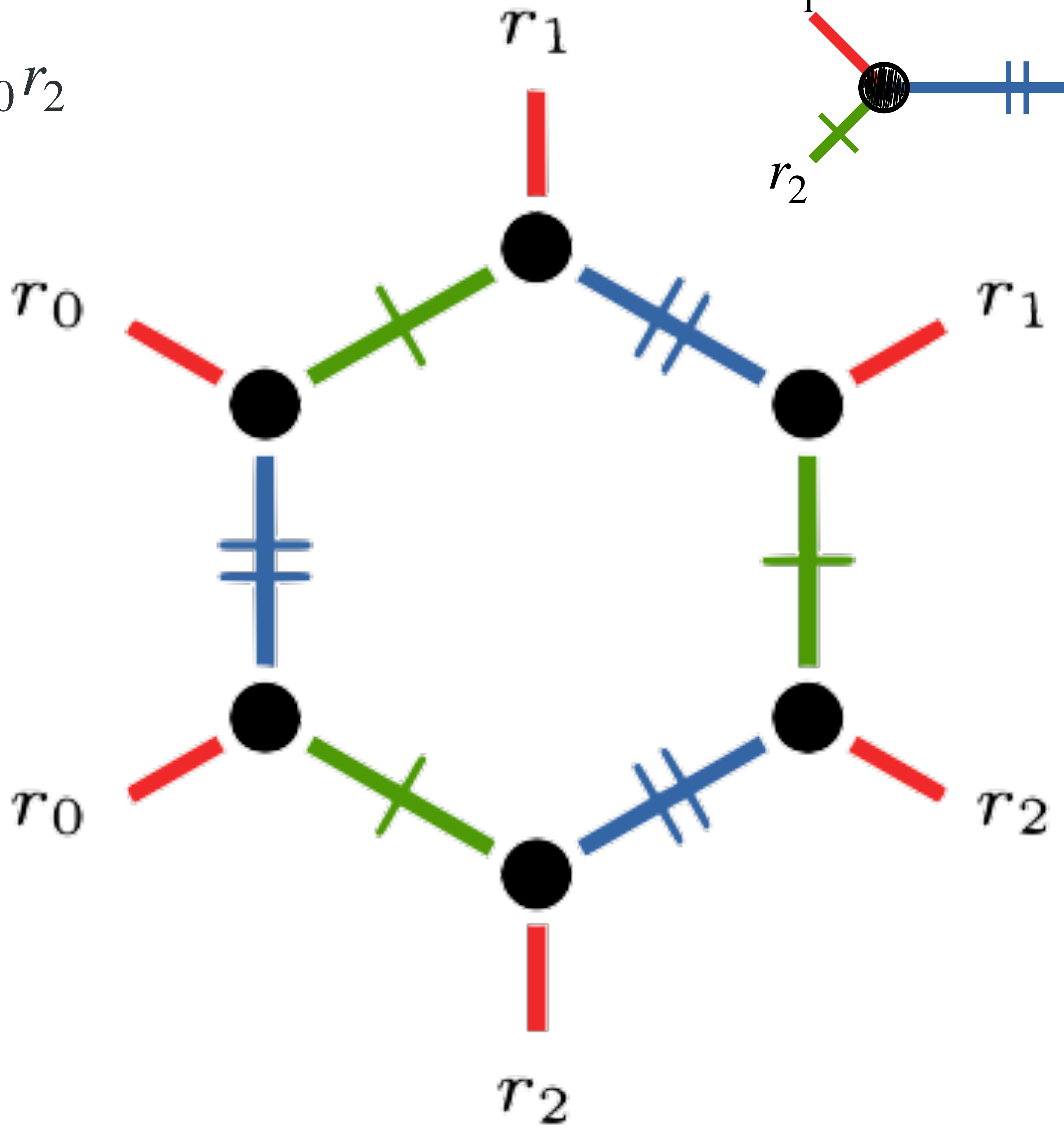

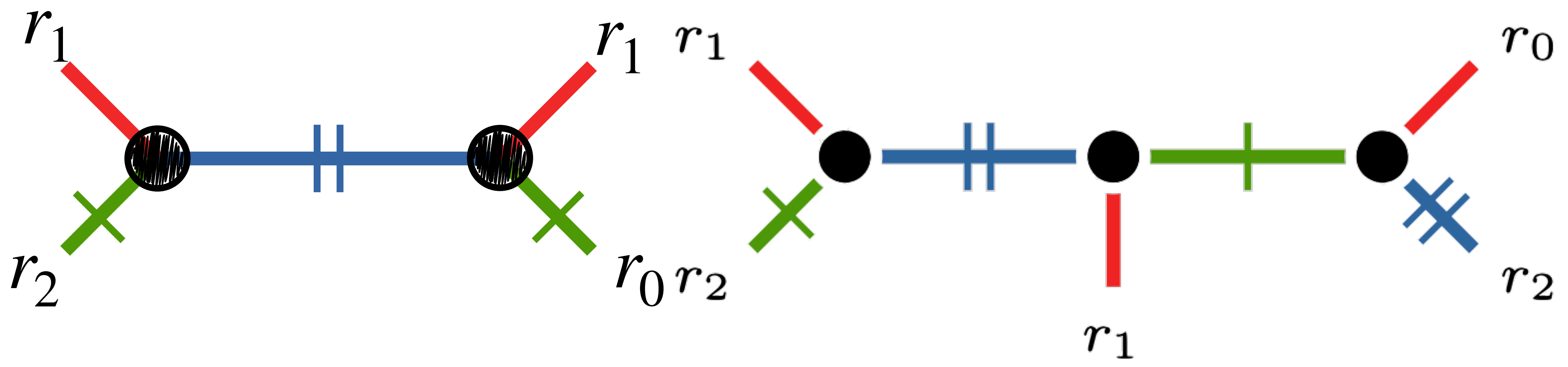

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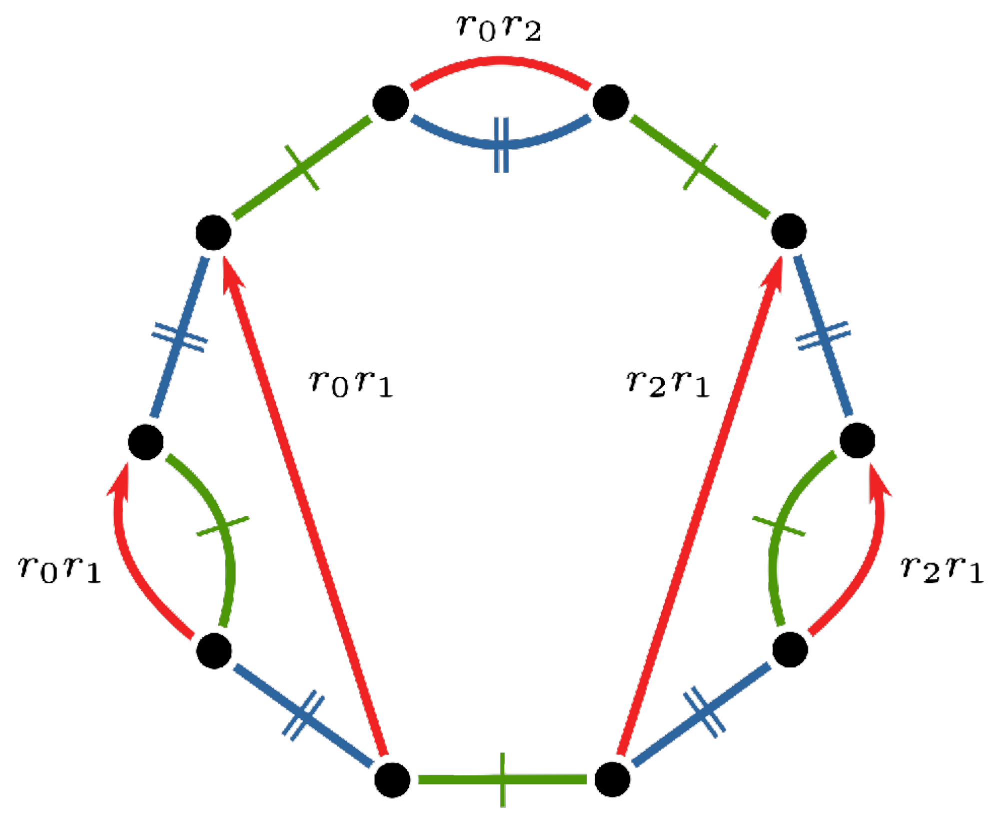

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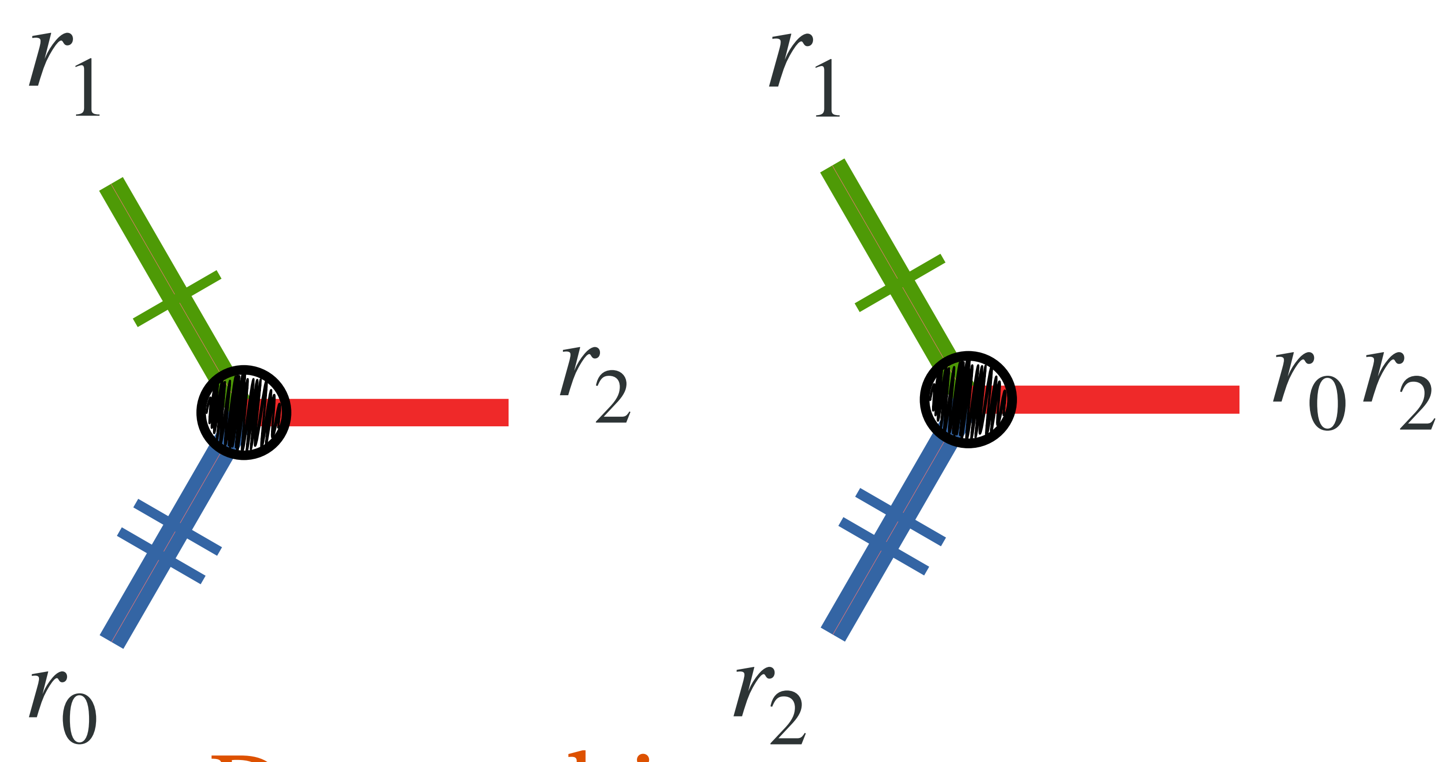
voltage operations



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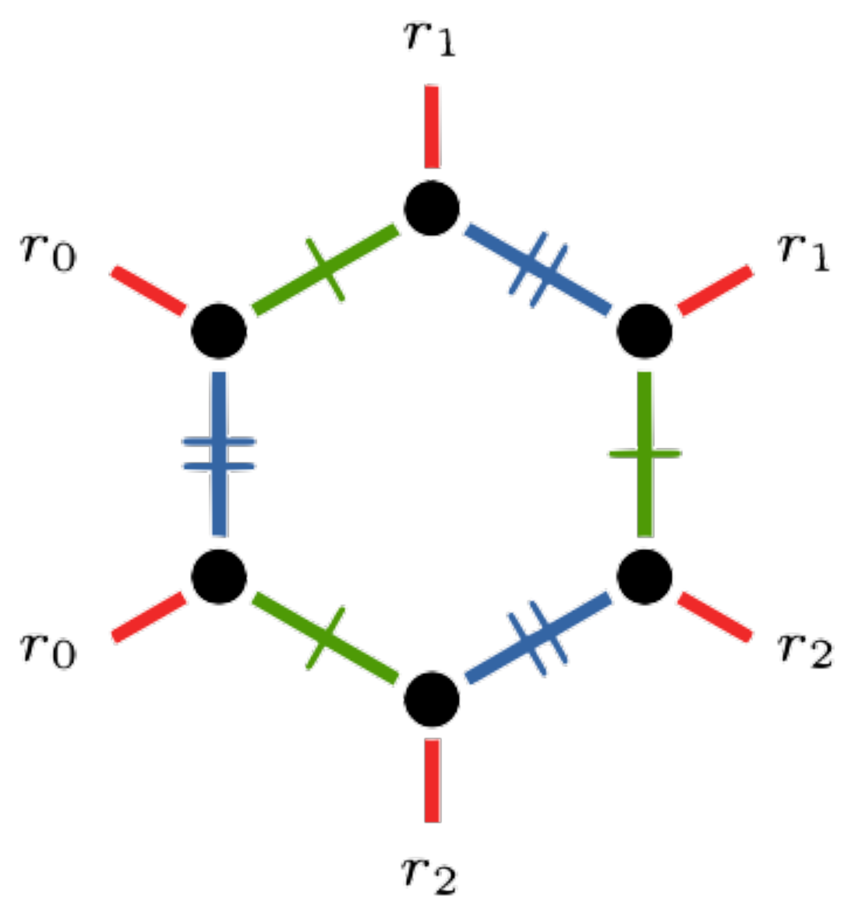

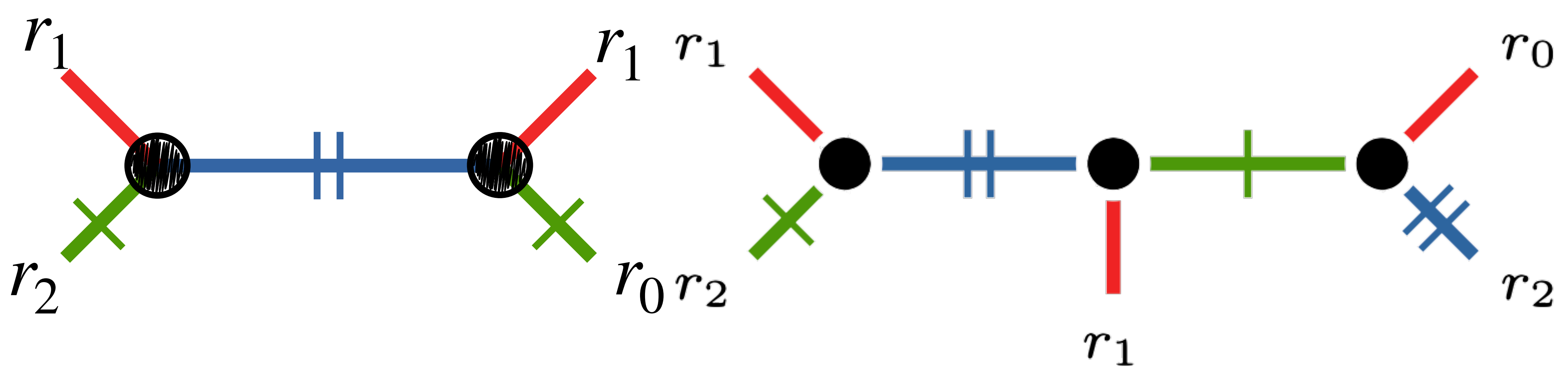

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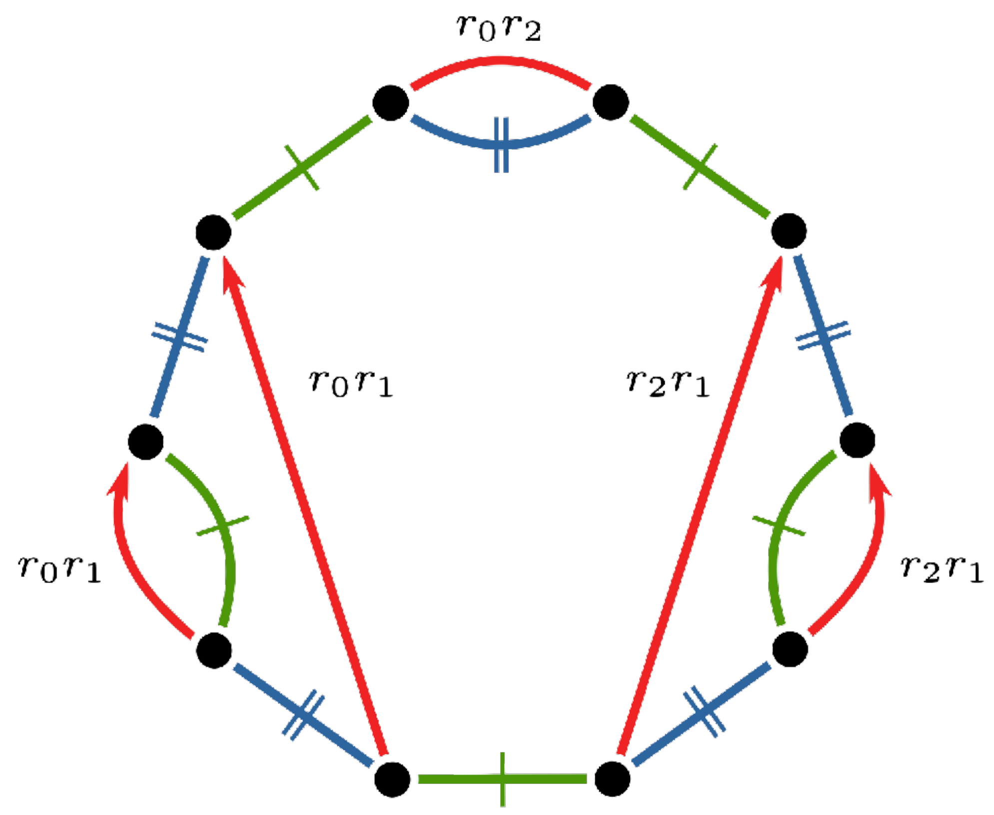

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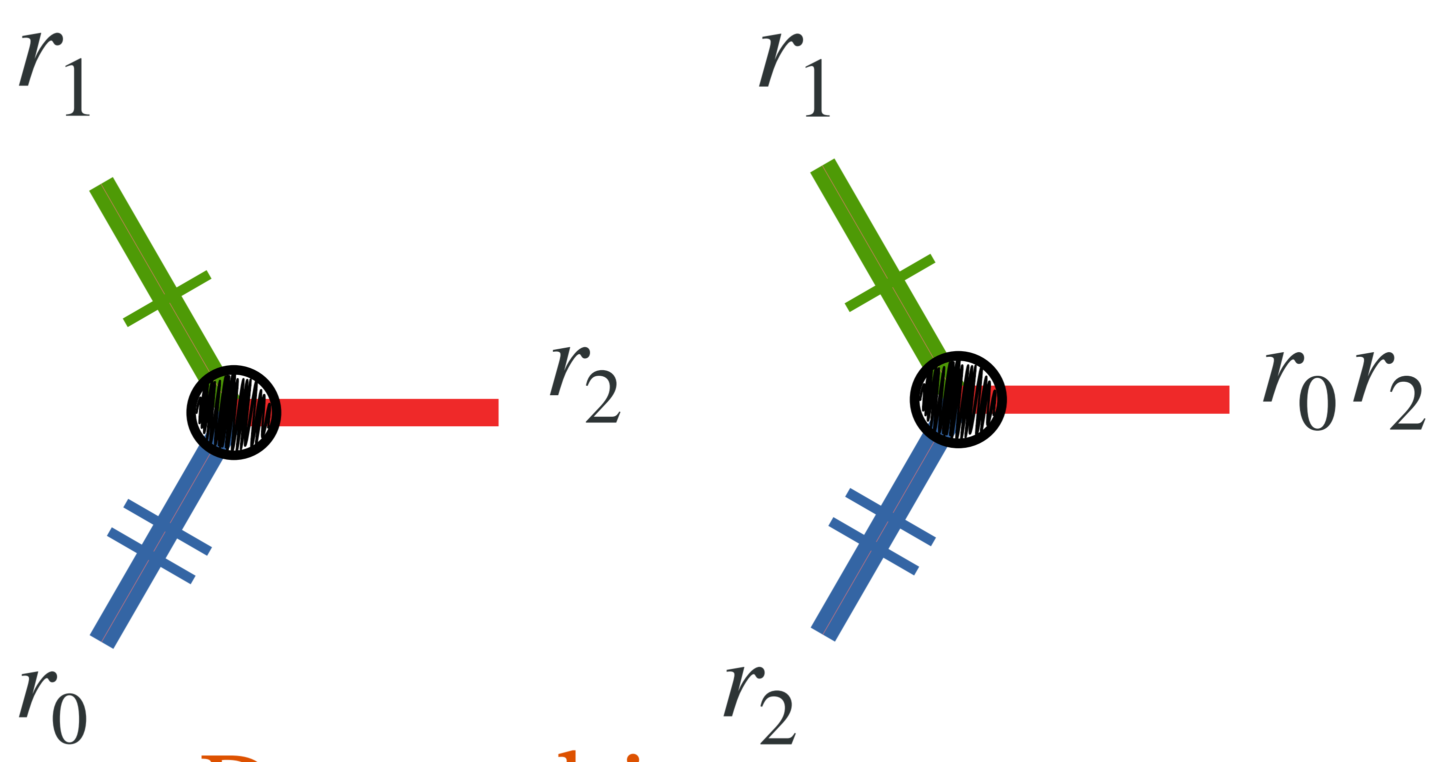

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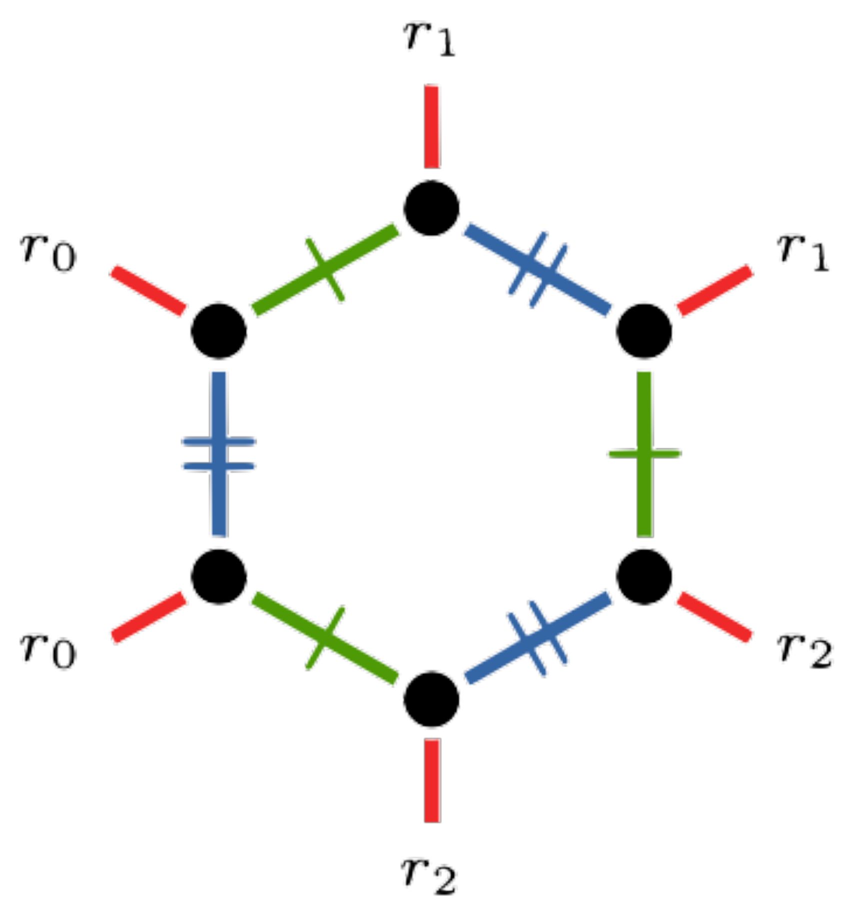
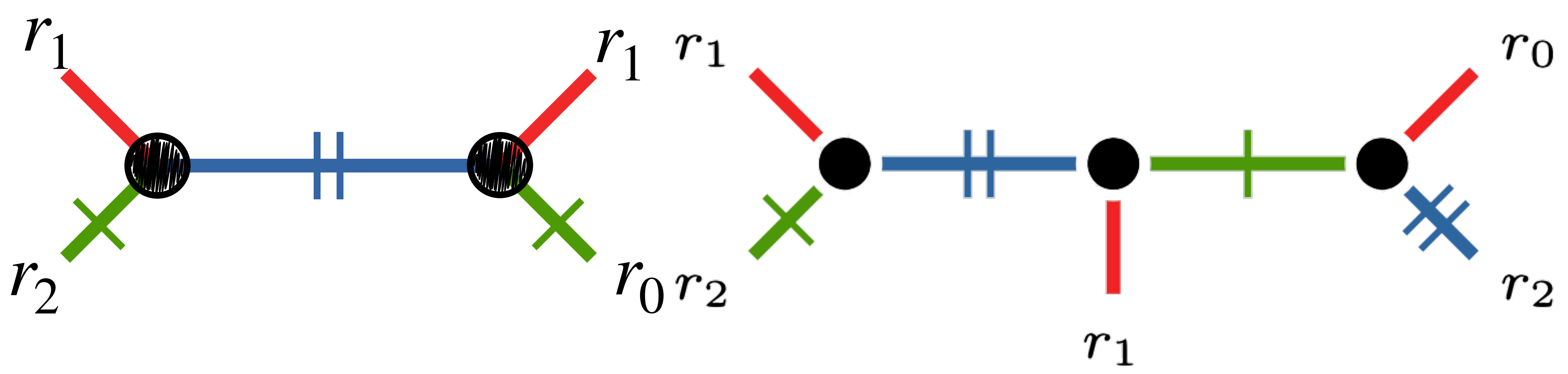


Colourful polytopes



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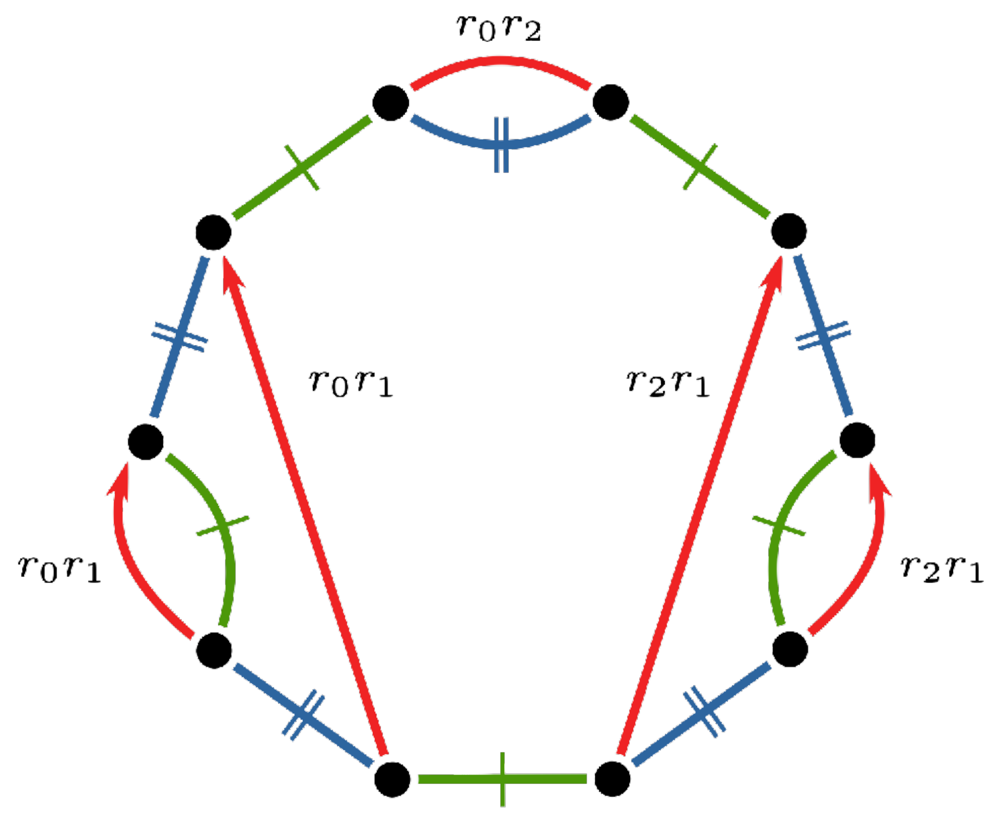
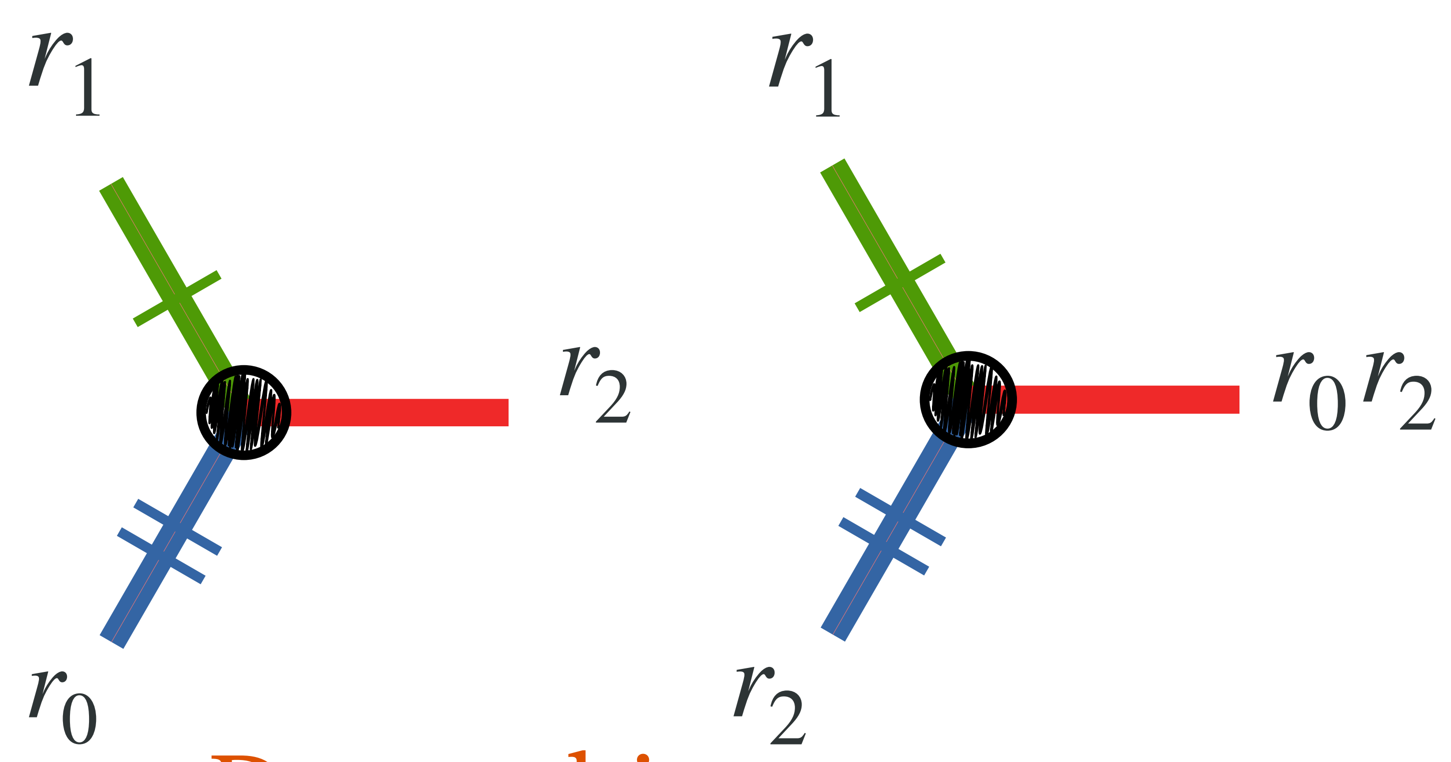


Snub

voltage operations



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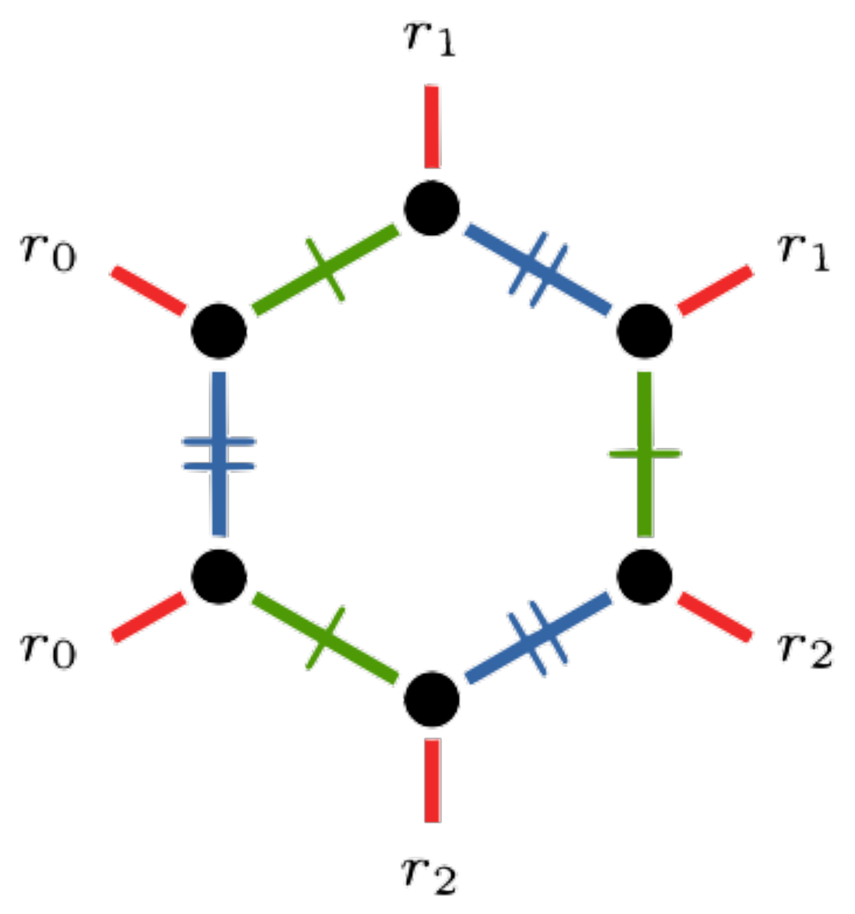

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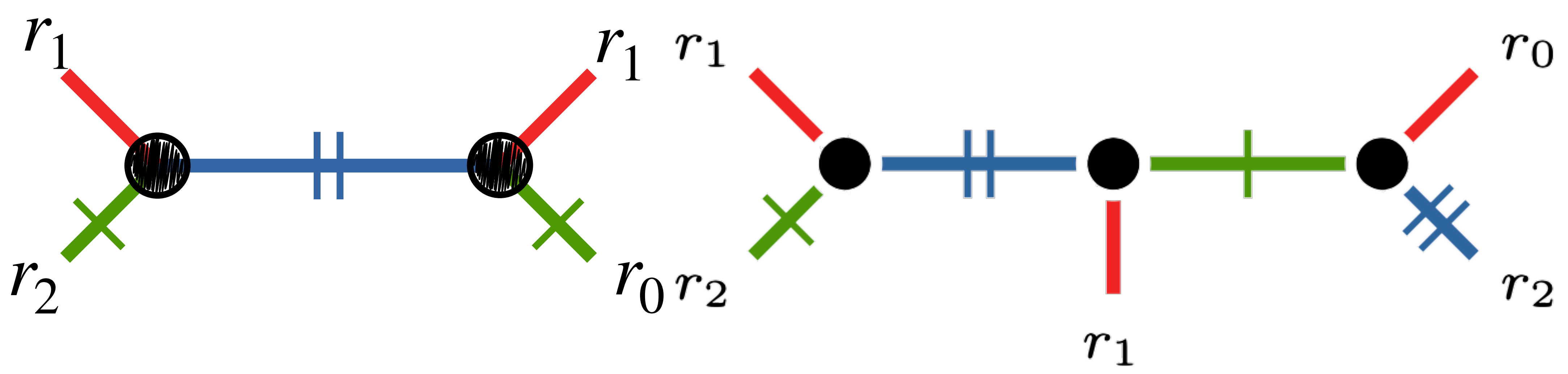

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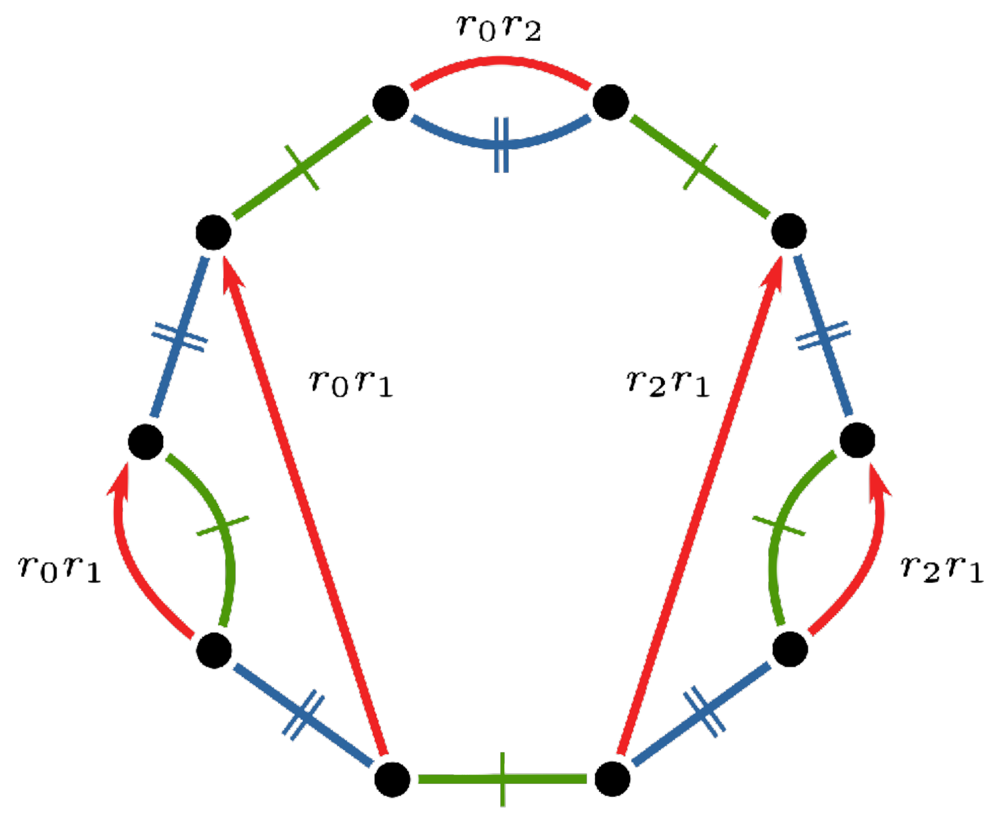
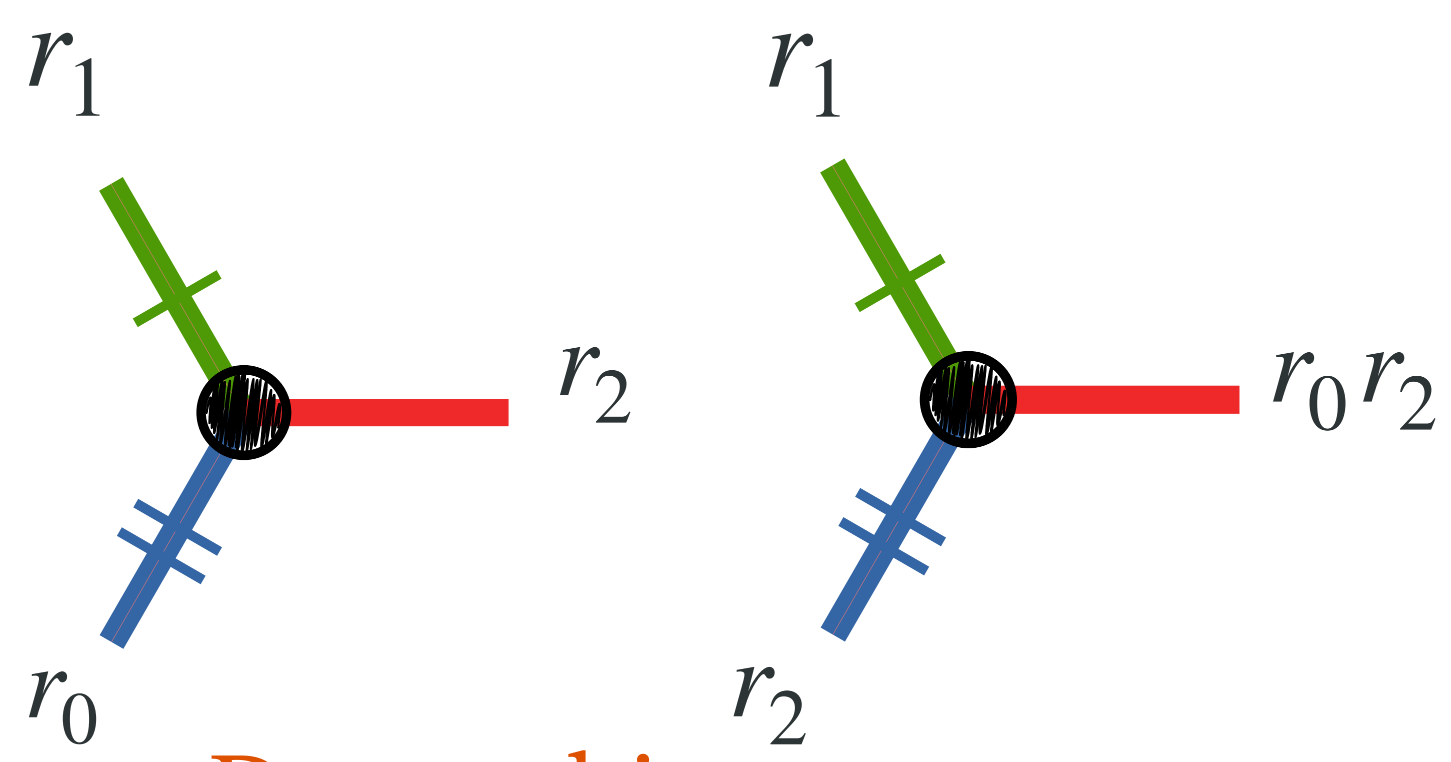


Snub

voltage operations



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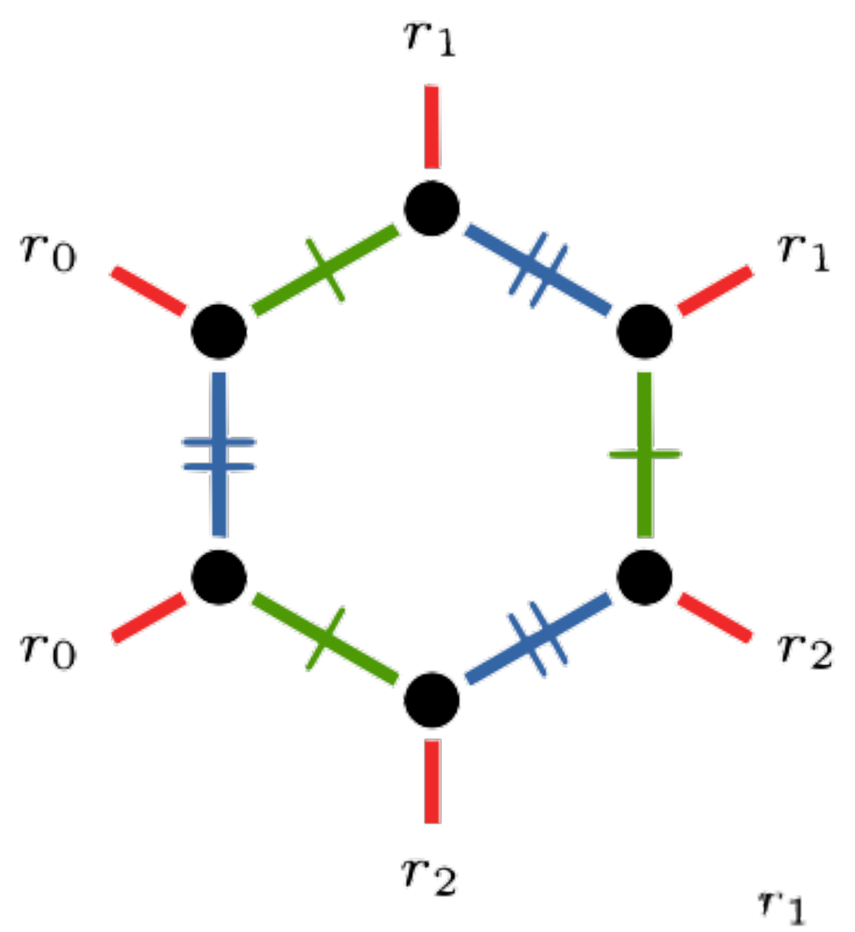

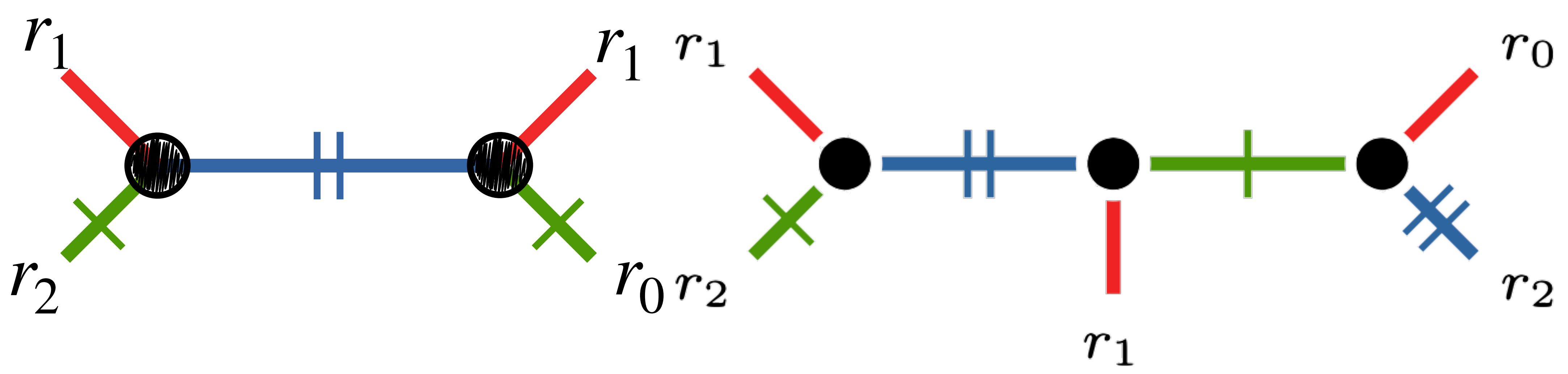

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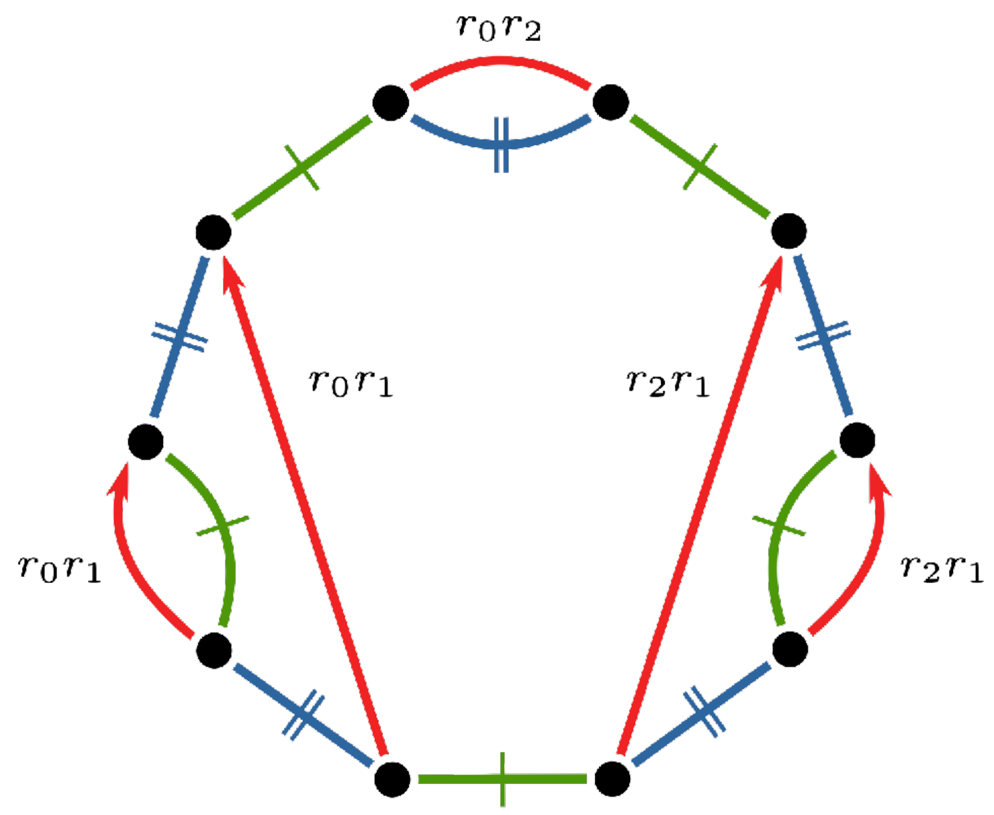

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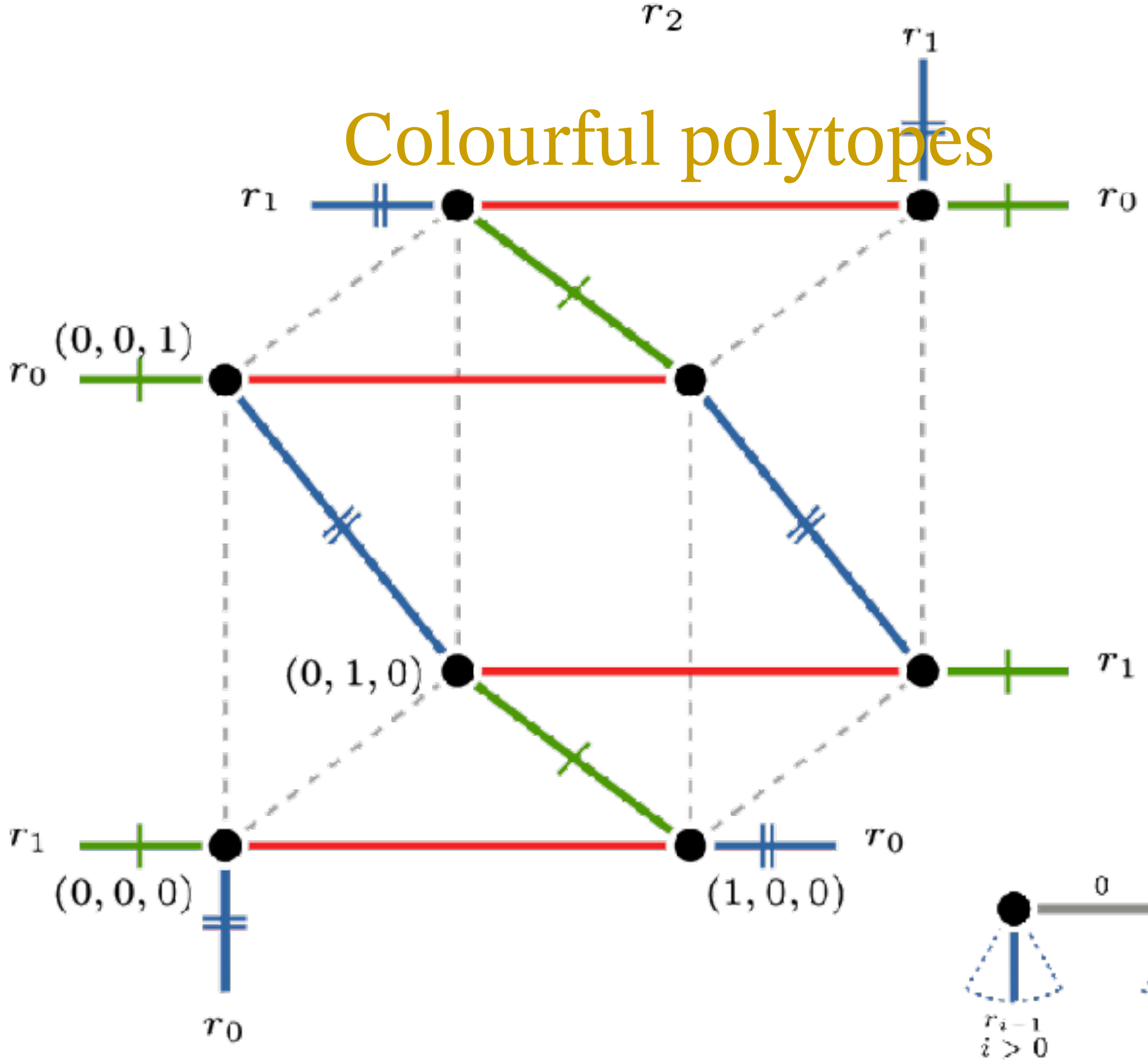
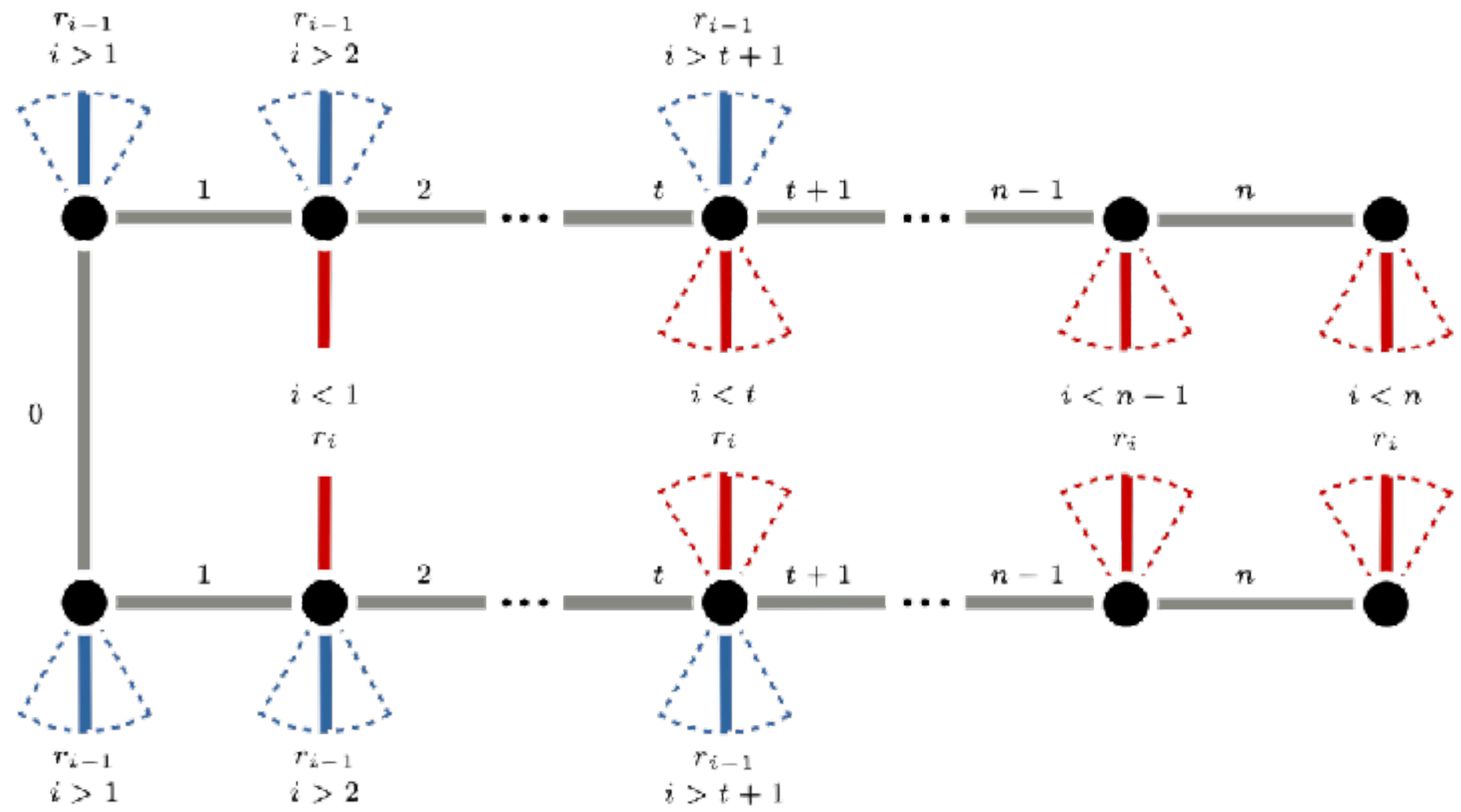
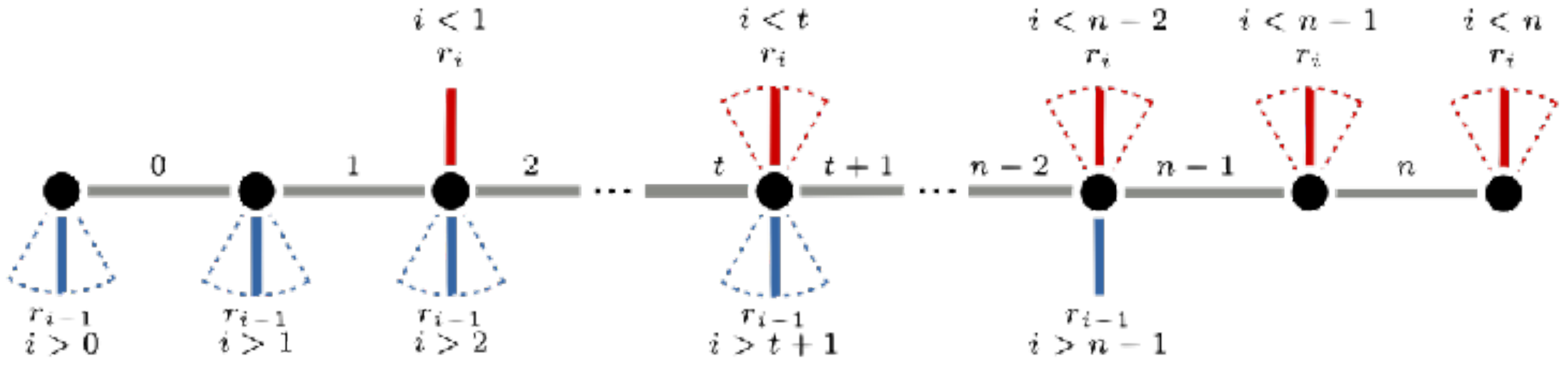
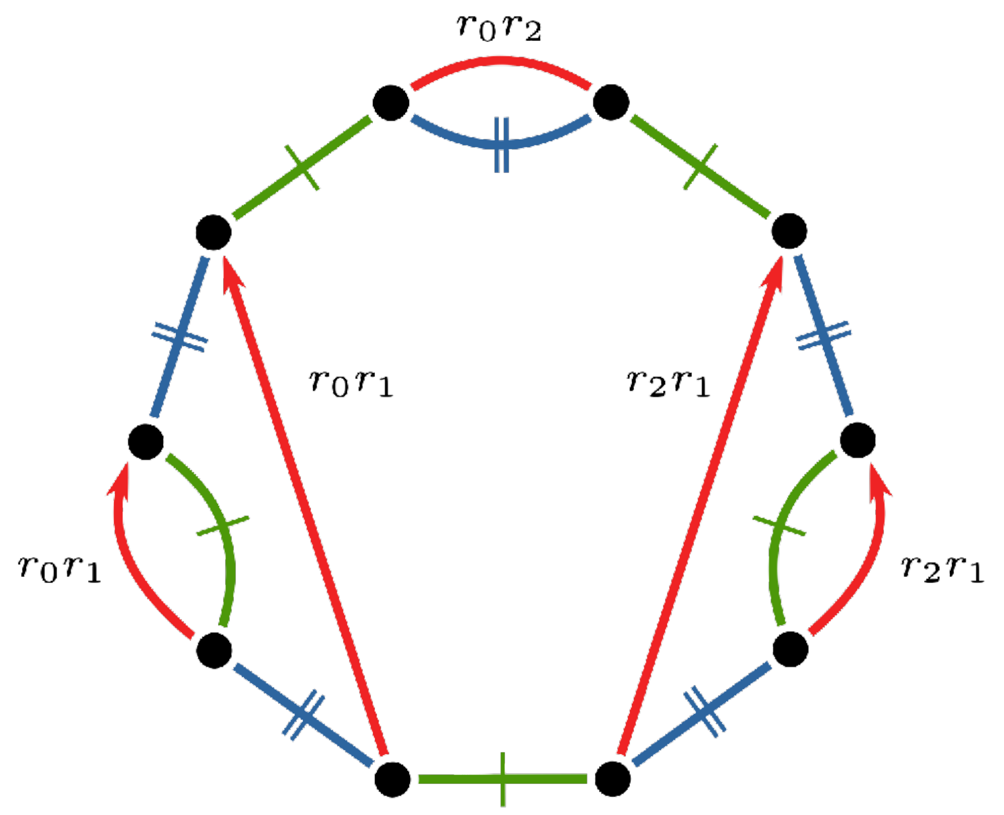






voltage operations



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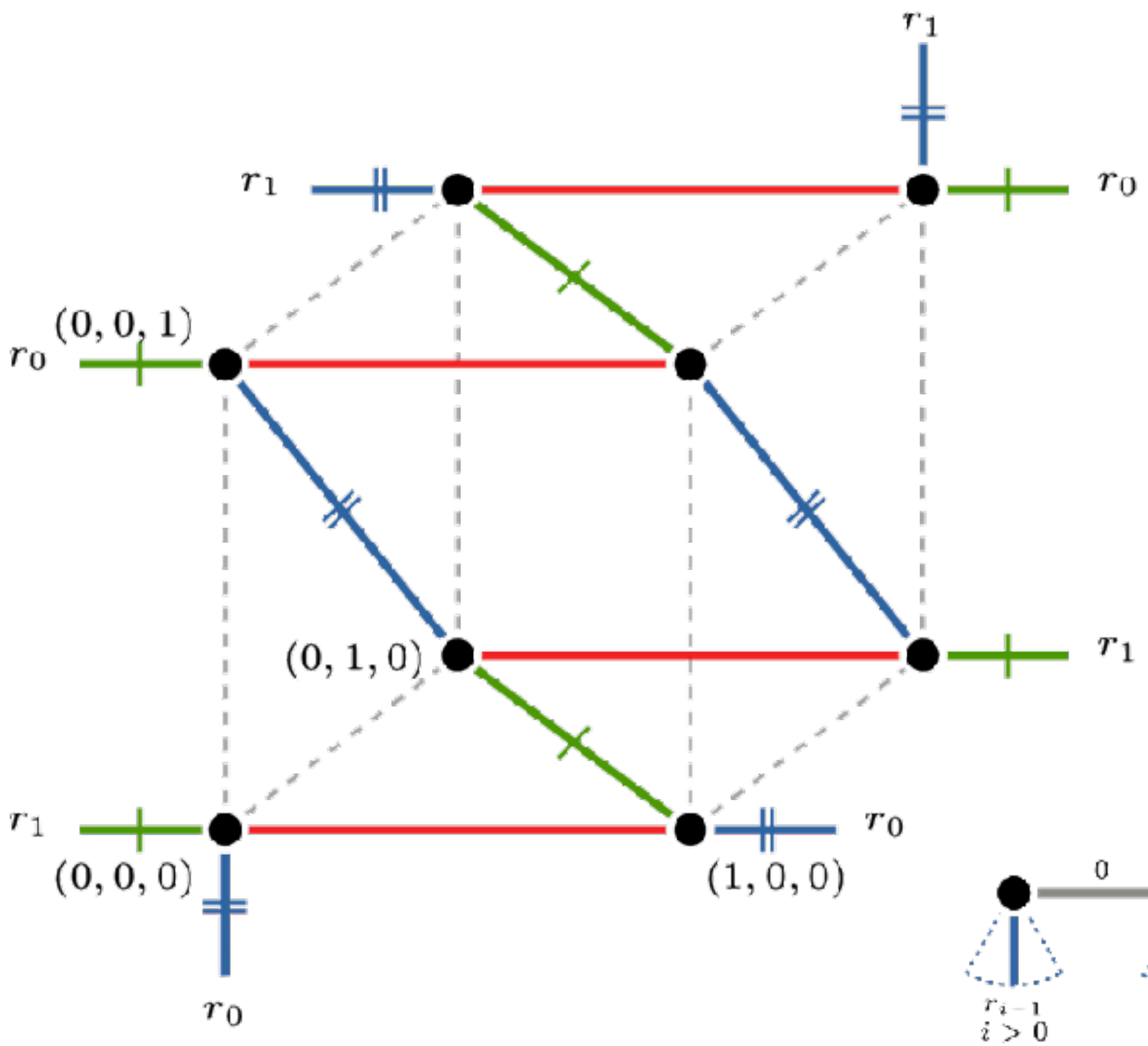
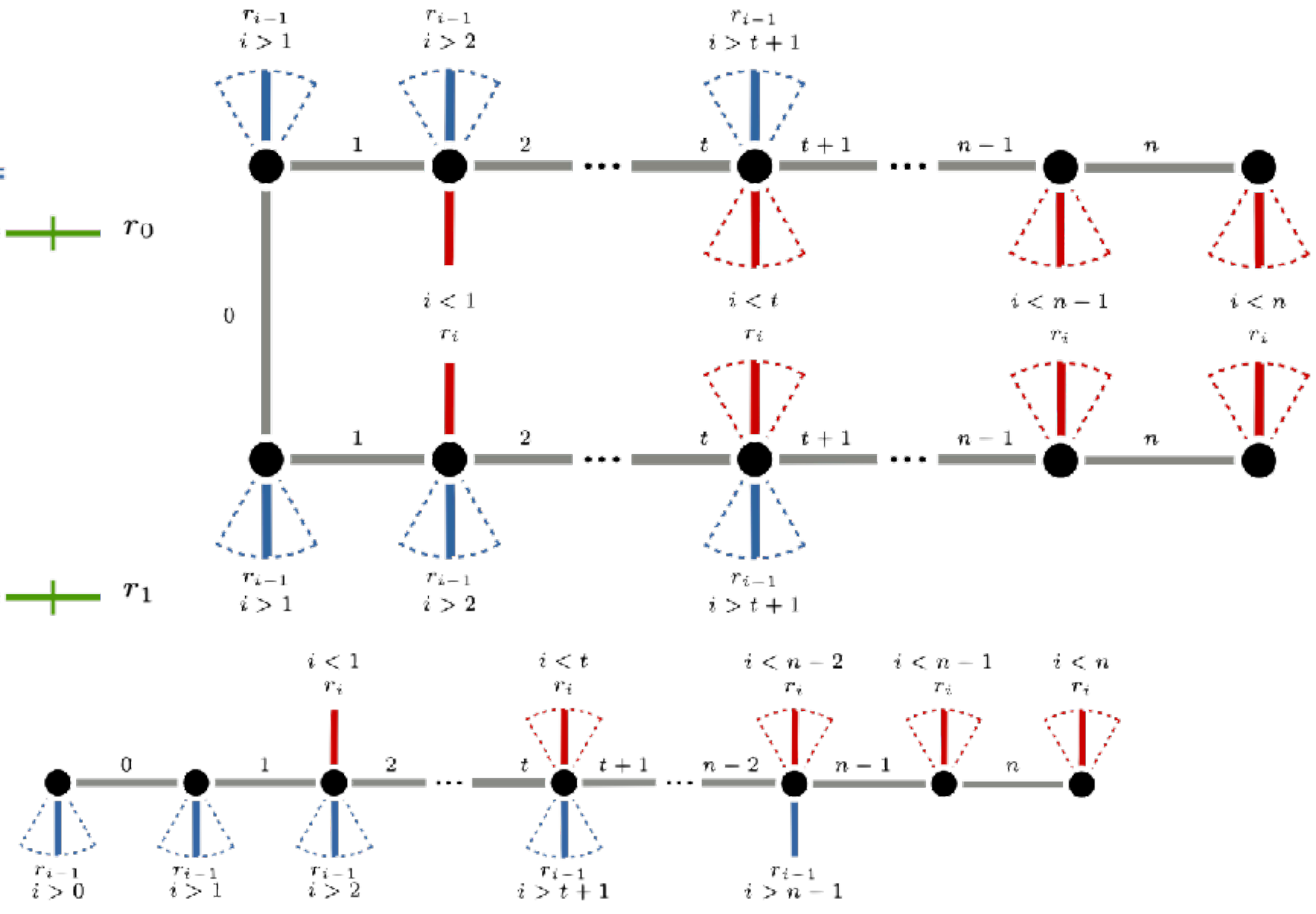
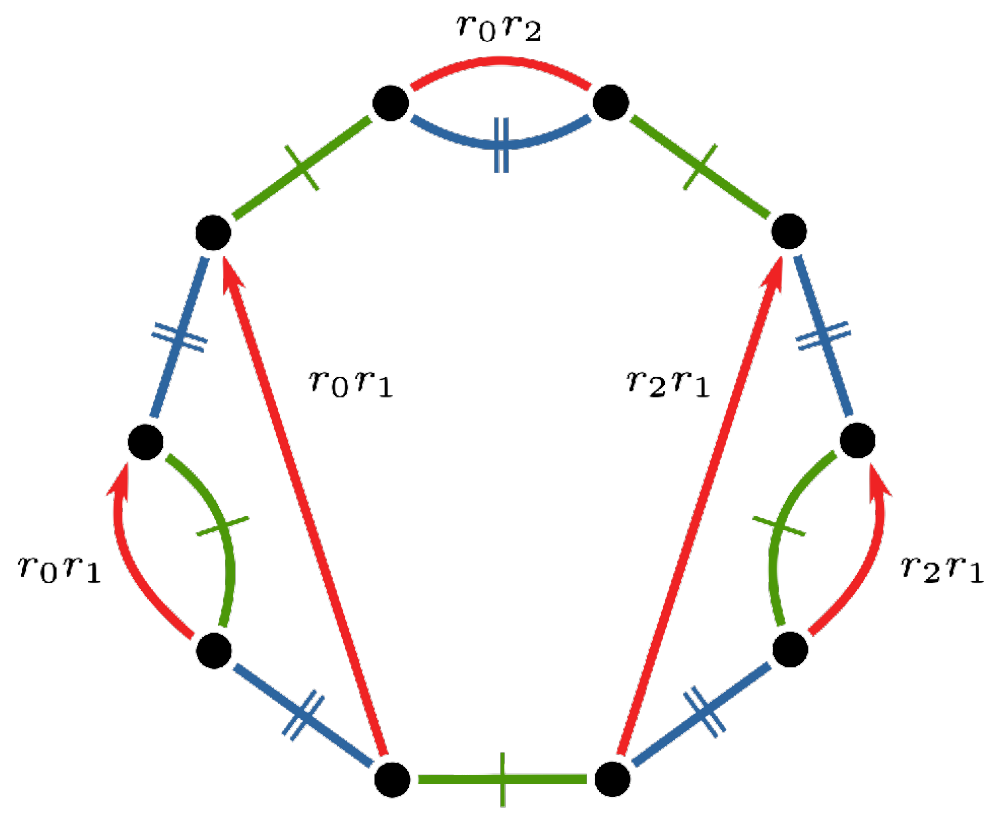


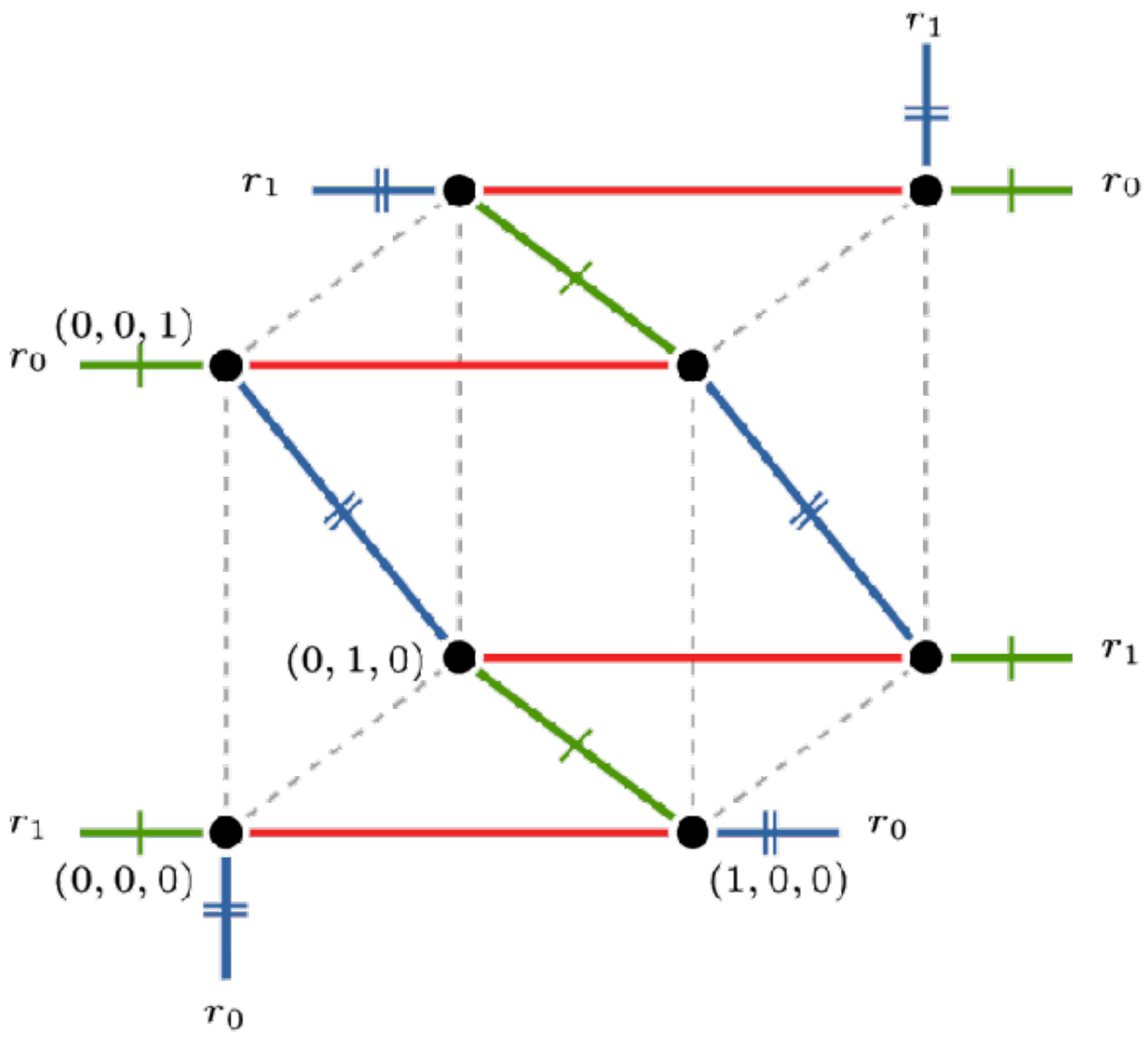


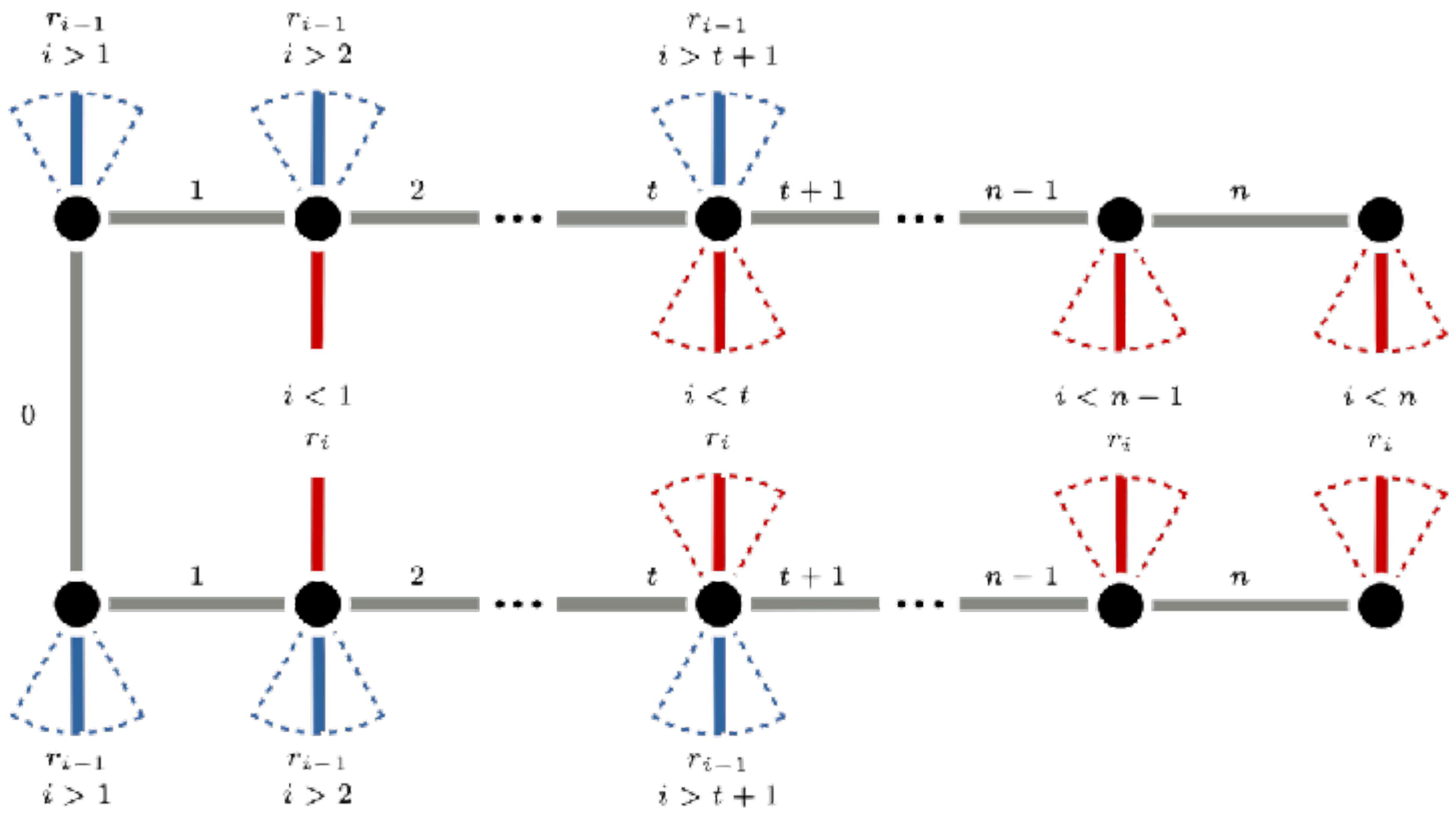
voltage operations

$\mathcal{M} \diamond \mathcal{N}$



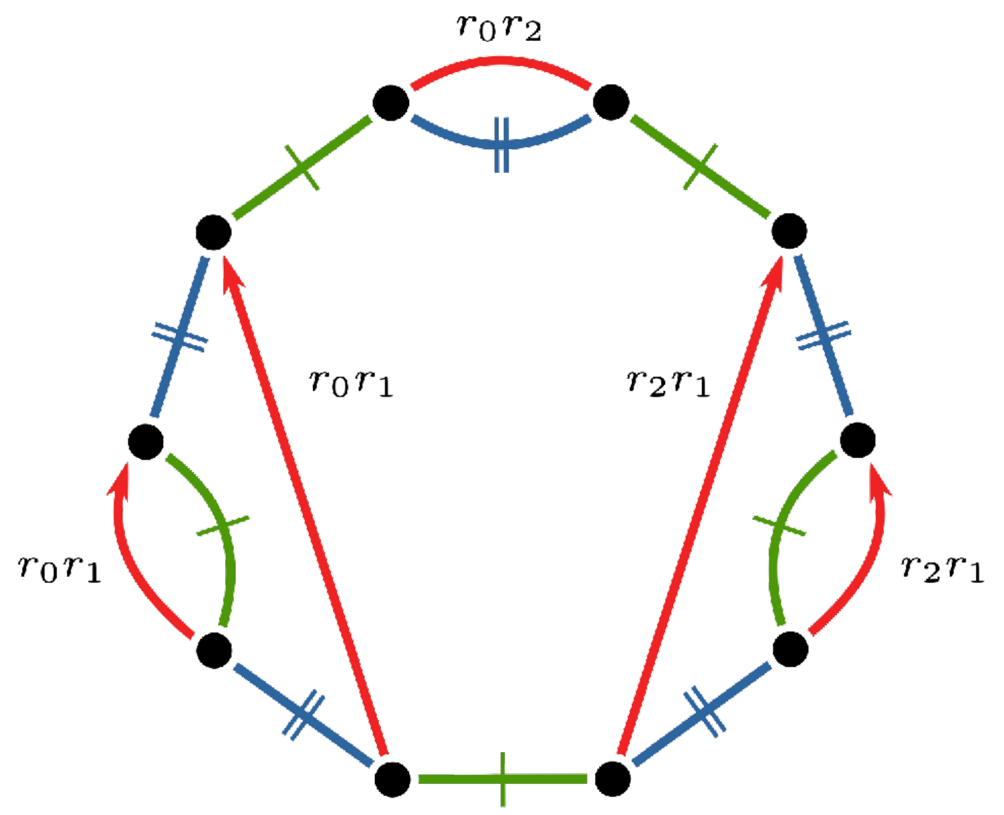
Snub





voltage operations

$\mathcal{M} \diamond \mathcal{N}$



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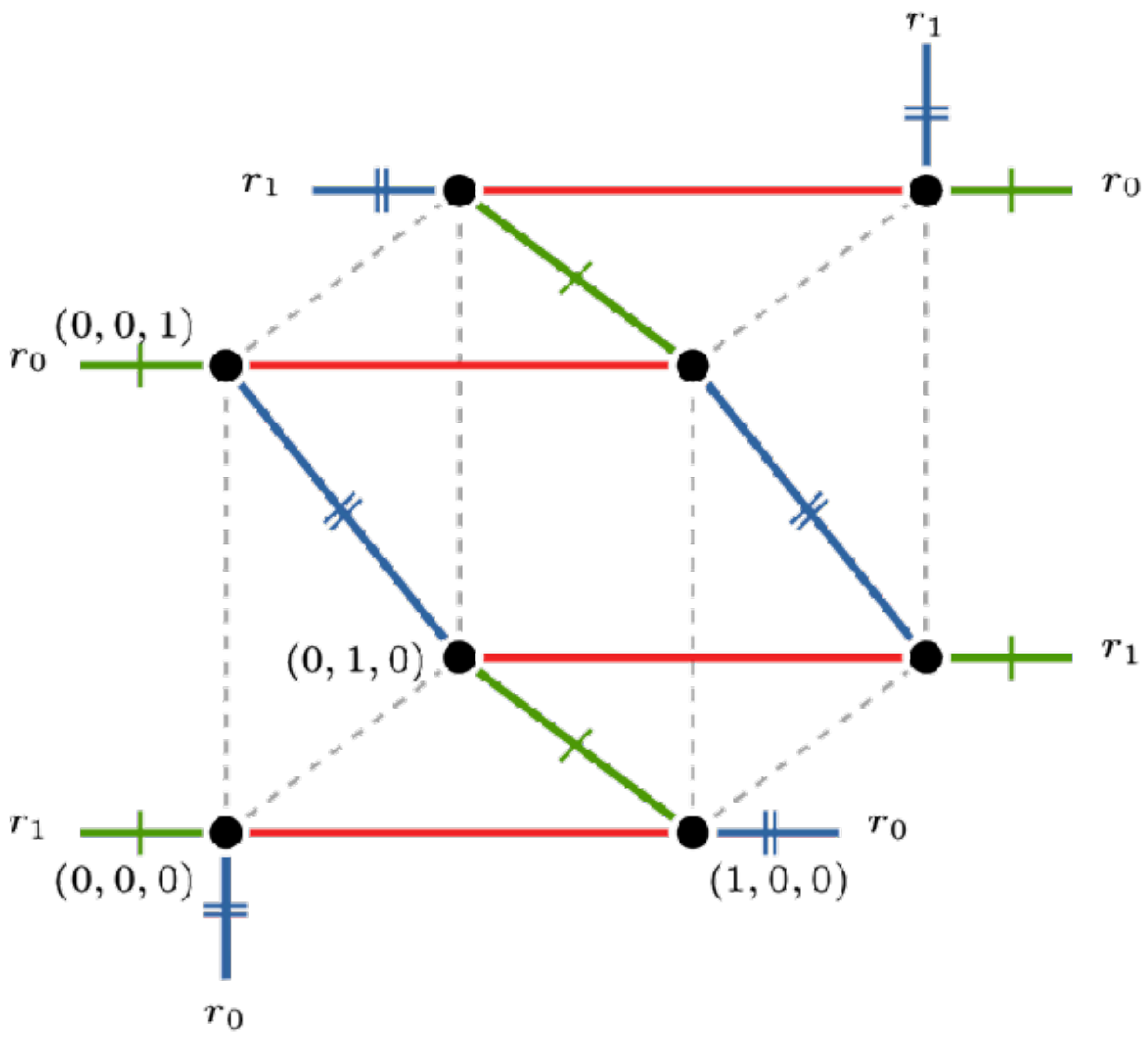

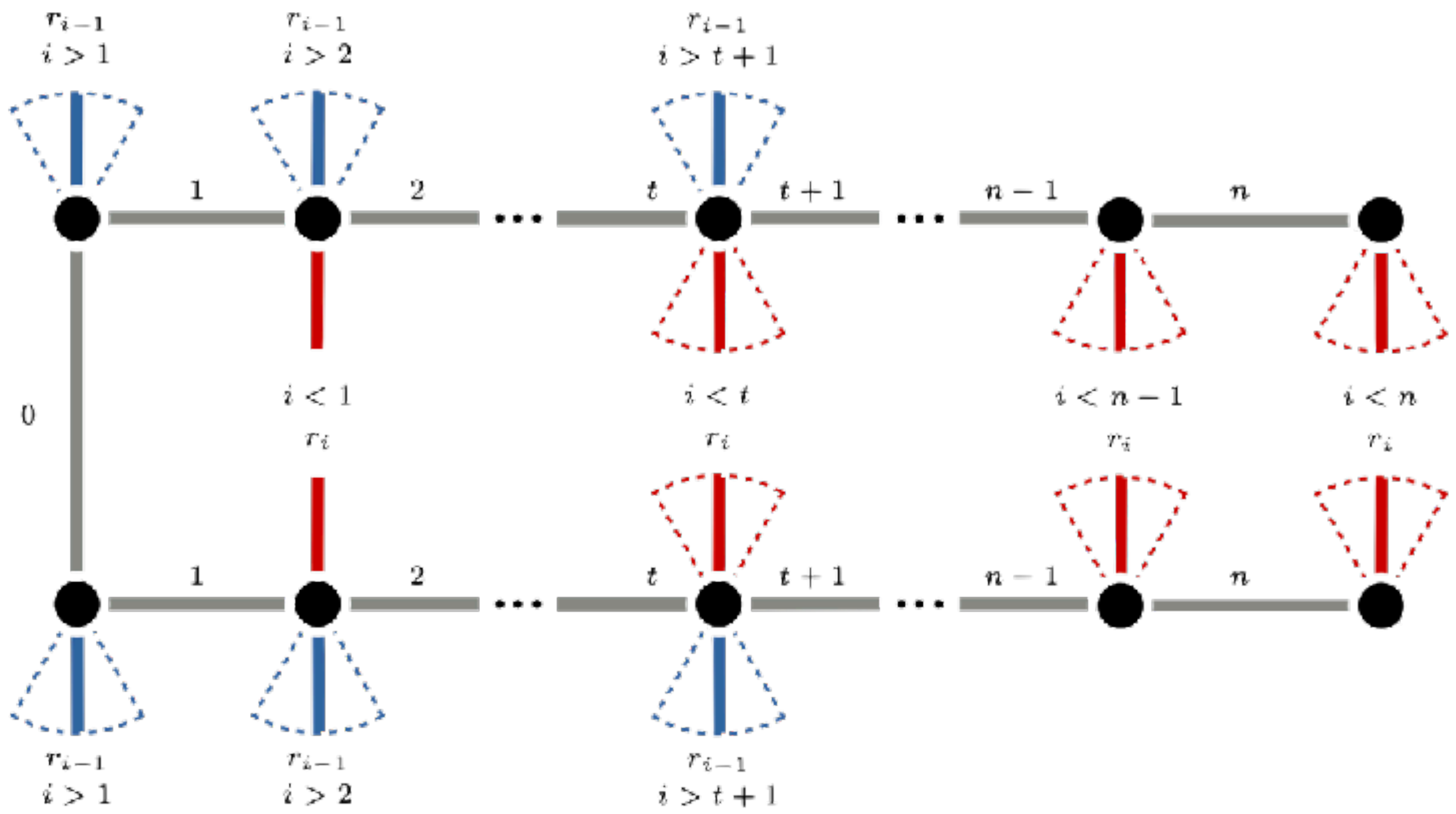

voltage operations

voltage operations

Hubard, Mochán, M. (2023)

$(\mathscr{Y}, \eta)$ a voltage operator

$\mathscr{X}$ a premaniplex

$\Gamma \leq \mathrm{Aut}(\mathscr{X})$

$\Gamma \leq \mathrm{Aut}(\mathscr{X} \rtimes_\eta \mathscr{Y})$

then

$$(\mathscr{X} \rtimes_\eta \mathscr{Y})/\Gamma \cong (\mathscr{X}/\Gamma) \rtimes_\eta \mathscr{Y}$$

voltage operations

Hubard, Mochán, M. (2023)

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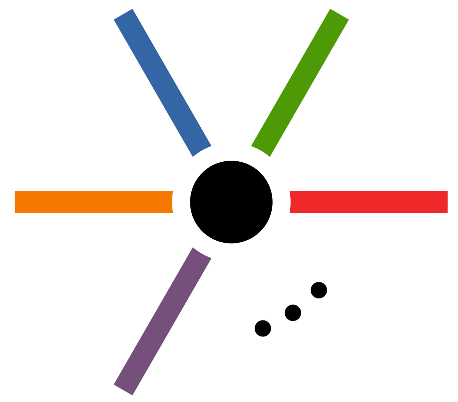
$\mathrm{STG}(\mathscr{X}) =$ 

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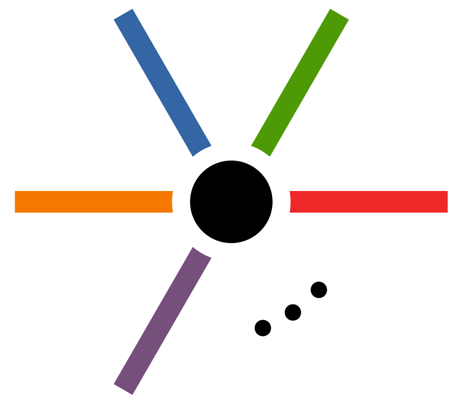
$\mathrm{STG}(\mathscr{X}) =$ 

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voltage operations

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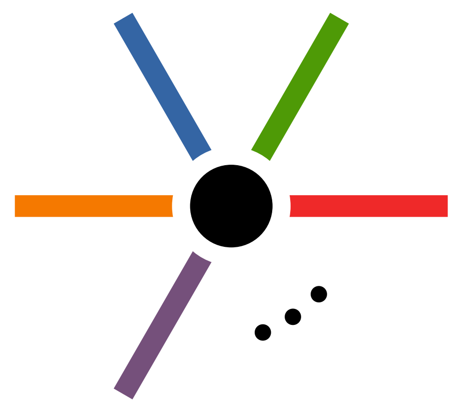
$\mathrm{STG}(\mathscr{X}) =$ 

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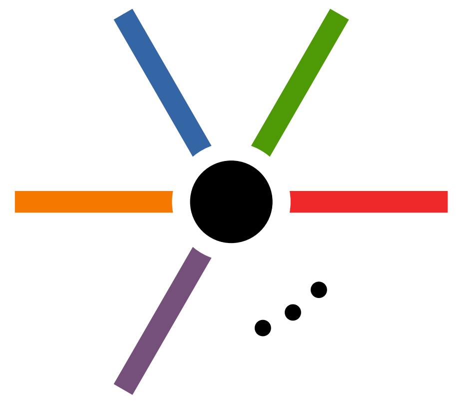
$\mathrm{STG}(\mathscr{X}) =$ 

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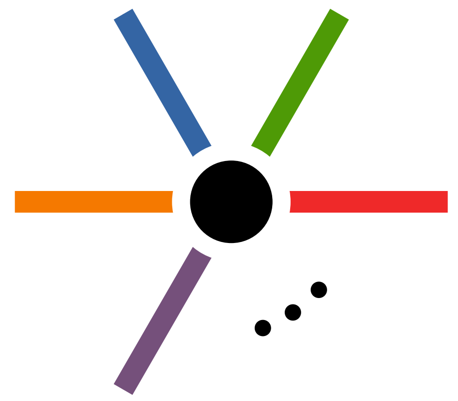
$\mathrm{STG}(\mathcal{X}) =$ 

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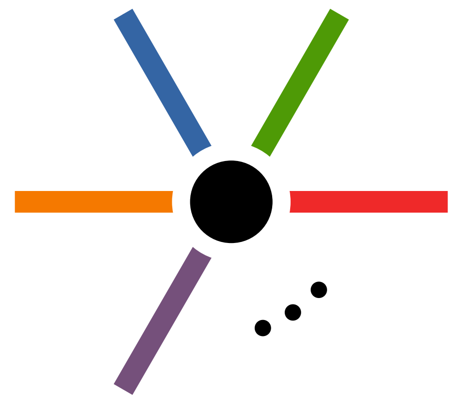
$\mathrm{STG}(\mathcal{X}) =$ 

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Symmetries of voltage operations on maniplexes and polytopes

Symmetries of voltage operations

on maniplexes and polytopes

Symmetries of voltage operations

Map operations and k-orbit maps

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ABSTRACT

A k-orbit map is a map with k flag-orbits under the action of its automorphism group. We give a basic theory of k-orbit maps and classify them up to $k \leqslant 4$. "Hurwitz-like" upper bounds for the cardinality of the automorphism groups of 2-orbit and 3-orbit maps on surfaces are given. Furthermore, we consider effects of

Symmetries of voltage operations

Proposition 4.3. *Let $M = (\mathcal{C}/N, \mathcal{C})$ be a k-orbit map. Then* $\mathrm{Tr}(M)$ *is either a k-orbit map, a $\frac{3k}{2}$-orbit map or a 3k-orbit map.*

Proof. There are three cosets in $\mathcal{C}_3/3_3^0$, namely $3_3^0 a$, $a \in A = \{id, s_1, s_{121}\}$. Therefore, every element $x \in \mathcal{C}_3$ is of the form $x = ta$, $t \in 3_3^0$, $a \in A$.

Let $N' = \varphi(N)$ (see diagram in Fig. 5). For any $a \in A$ let $x, y \in 3_3^0 a$, with $x = ta$, $y = sa$, for some $t, s \in 3_3^0$. If both x and y normalize N', it follows that $t^{-1}N't = aN'a^{-1} = s^{-1}N's$. Hence t and s must be in the same coset in $3_3^0/\mathcal{N}$, where $\mathcal{N} = \mathrm{Norm}_{3_3^0}(N') \leqslant 3_3^0$. This implies that $\mathrm{Norm}_{\mathcal{C}_3}(N')$ consists of cosets in $\mathcal{C}_3/\mathcal{N}$, where at most one such coset can be contained in any of the cosets $3_3^0 a$, $a \in A$. Since

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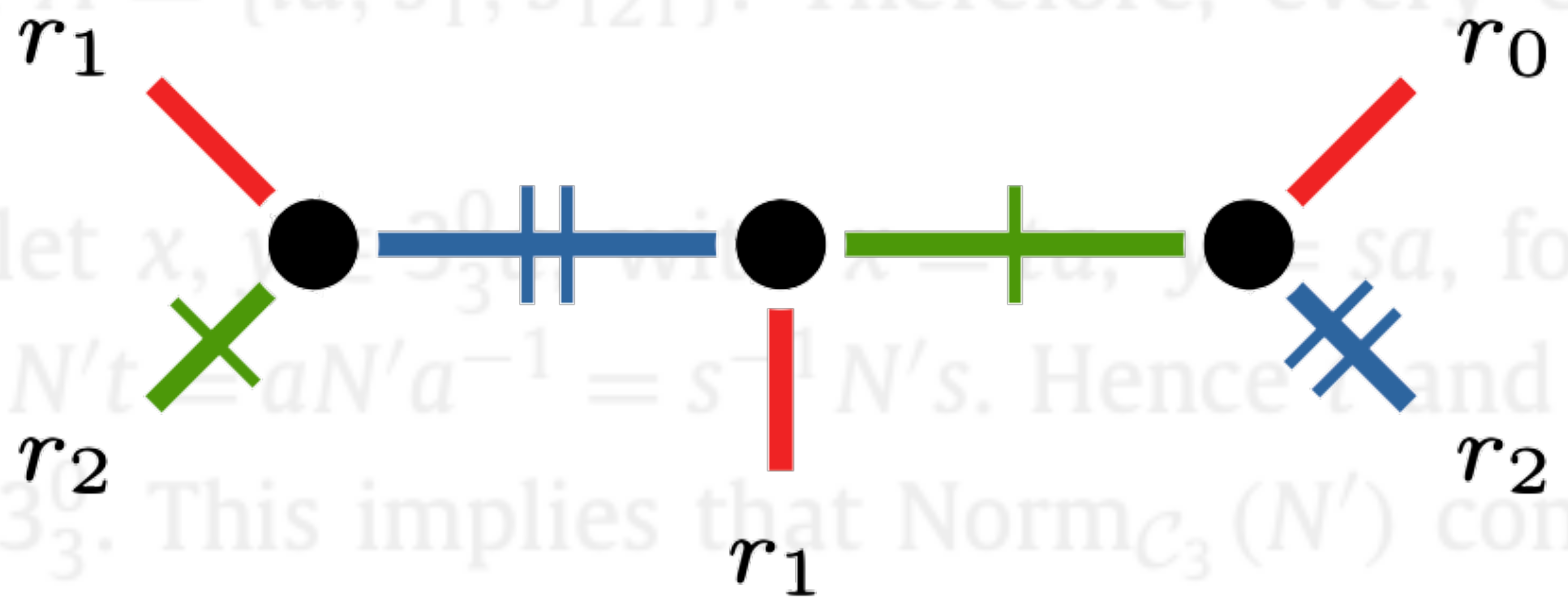

from the fact that $[3^0_3 : \mathcal{N}] = k$. $\square$

We illustrate the proposition with the following examples.

The truncations of the regular maps of Schläfli type $\{4,4\}$ and $\{6,3\}$ are have faces of two different sizes and therefore are 3-orbit maps. The truncation of the regular map $\{3,6\}_{(t,0)}$ is the regular map $\{6,3\}_{(t,t)}$, whereas the truncation of the regular map $\{3,6\}_{(t,t)}$ is the regular map $\{6,3\}_{(3t,0)}$. Finally, the map given in dotted lines in Fig. 6 is a 3-orbit map on an orientable surface of genus 2. As we can see, this map can be obtained by truncating the regular map $\{3,8|\cdot,\cdot,2\}$ (see Fig. 6(a)) or by truncating the 2-orbit map given in Fig. 6(b) (belonging to class 2_{01}).

The core of 3^0_3 is the index two subgroup $K = \langle s_0, s_{101}, s_{21012} \rangle$ of $\mathcal{C}_3$. The group K plays an important role for the truncation operation, as is shown by the following two results.

Proposition 4.4. *Let $M = (\mathcal{C}/N, \mathcal{C})$ be a k-orbit map such that $\mathrm{Tr}(M)$ is $\frac{3k}{2}$-orbit or k-orbit. Then M must necessarily be 2_{01}-compatible.*

Symmetries of voltage operations

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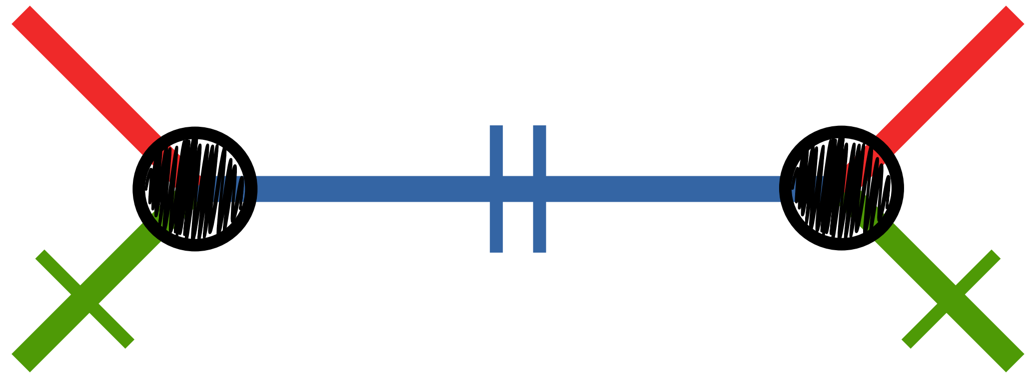
$\mathscr{M}$ has to cover



Symmetries of voltage operations

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From now on, we shall assume that our voltage operations preserve connectivity

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Cannonical (orientable) double cover

You'll need to wait for the next talk for this one

Symmetries of voltage operations

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for some t with $1 \leq t \leq \dfrac{|\mathcal{Y}|}{\ell}$

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The number of flag-orbits of the prism over an k-orbit n-maniplex is $\dfrac{k(n+1)}{t}$ for some $t \leq n+1$

Symmetries of voltage operations

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Symmetries of voltage operations

Hubard, Mochán, M. (2025)

For $(\mathscr{Y}, \eta)$ a voltage operator, there exists a family $\mathscr{F}$ of premaniplexes satisfying:

- $|\mathscr{F}| \leq |\mathscr{Y}/\mathrm{Aut}(\mathscr{Y})|$
- if $\mathscr{X} \rtimes_\eta \mathscr{Y}$ has unexpected symmetry, then $\mathscr{X}$ covers an element of $\mathscr{F}$.

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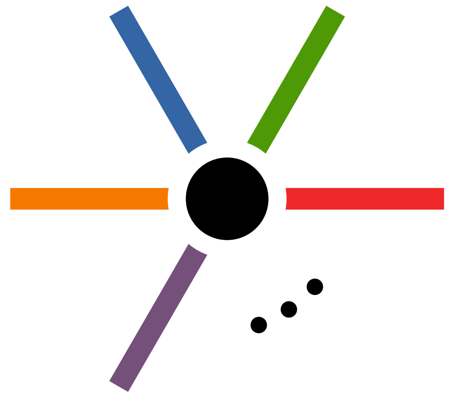
It is not uncommon that  $\in \mathscr{F}$

Every non-trivial automorphism of $\mathscr{Y}$ induces such a case.

Can we understand those?

Symmetries of voltage operations

Symmetries of voltage operations

$$
\begin{array}{ccc}
\mathscr{X} \rtimes_\eta \mathscr{Y} & \xrightarrow{\tau^*} & \mathscr{X} \rtimes_\eta \mathscr{Y} \\
\downarrow & & \downarrow \\
\mathscr{Y} & \xrightarrow{\tau} & \mathscr{Y}
\end{array}
$$

Symmetries of voltage operations

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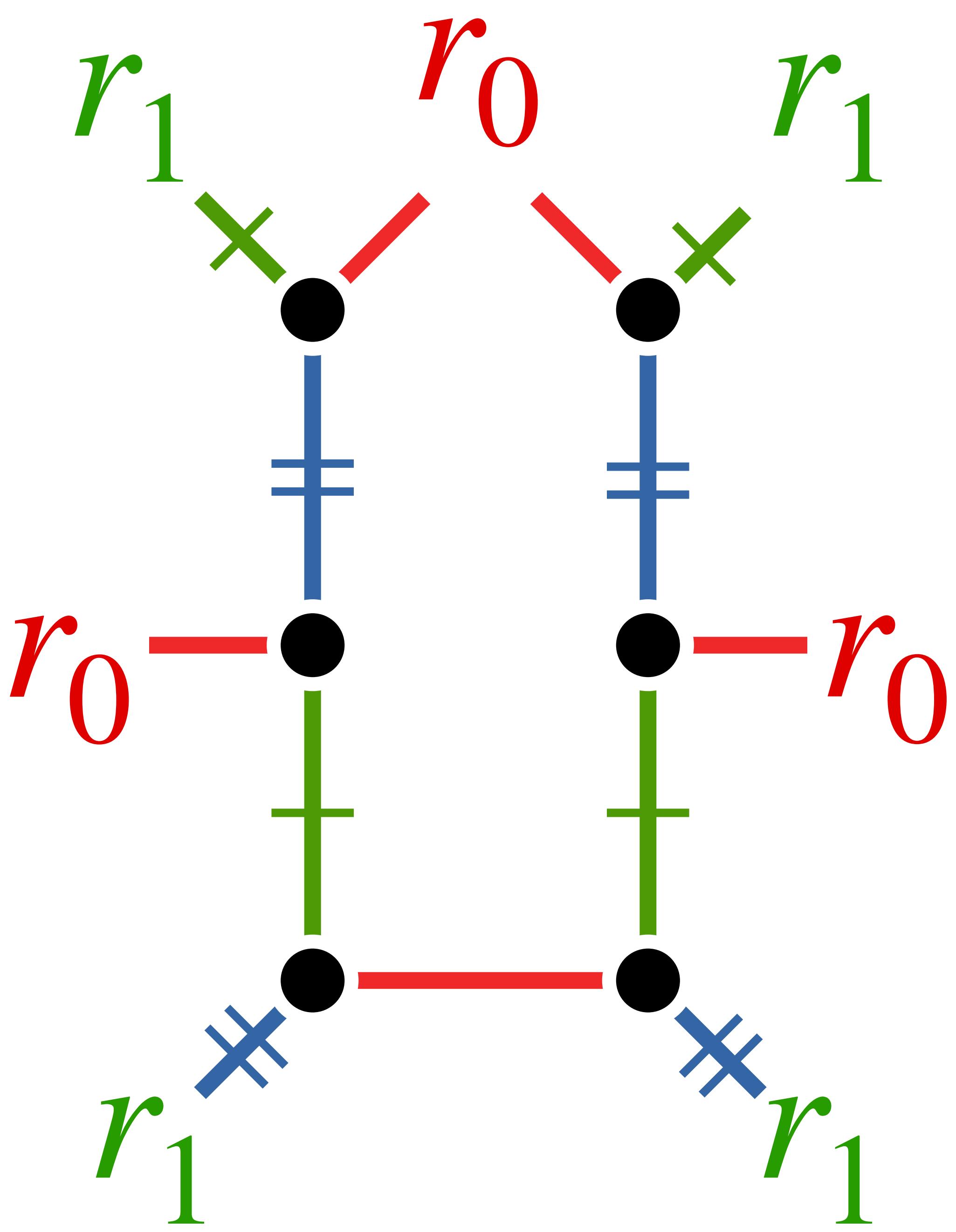
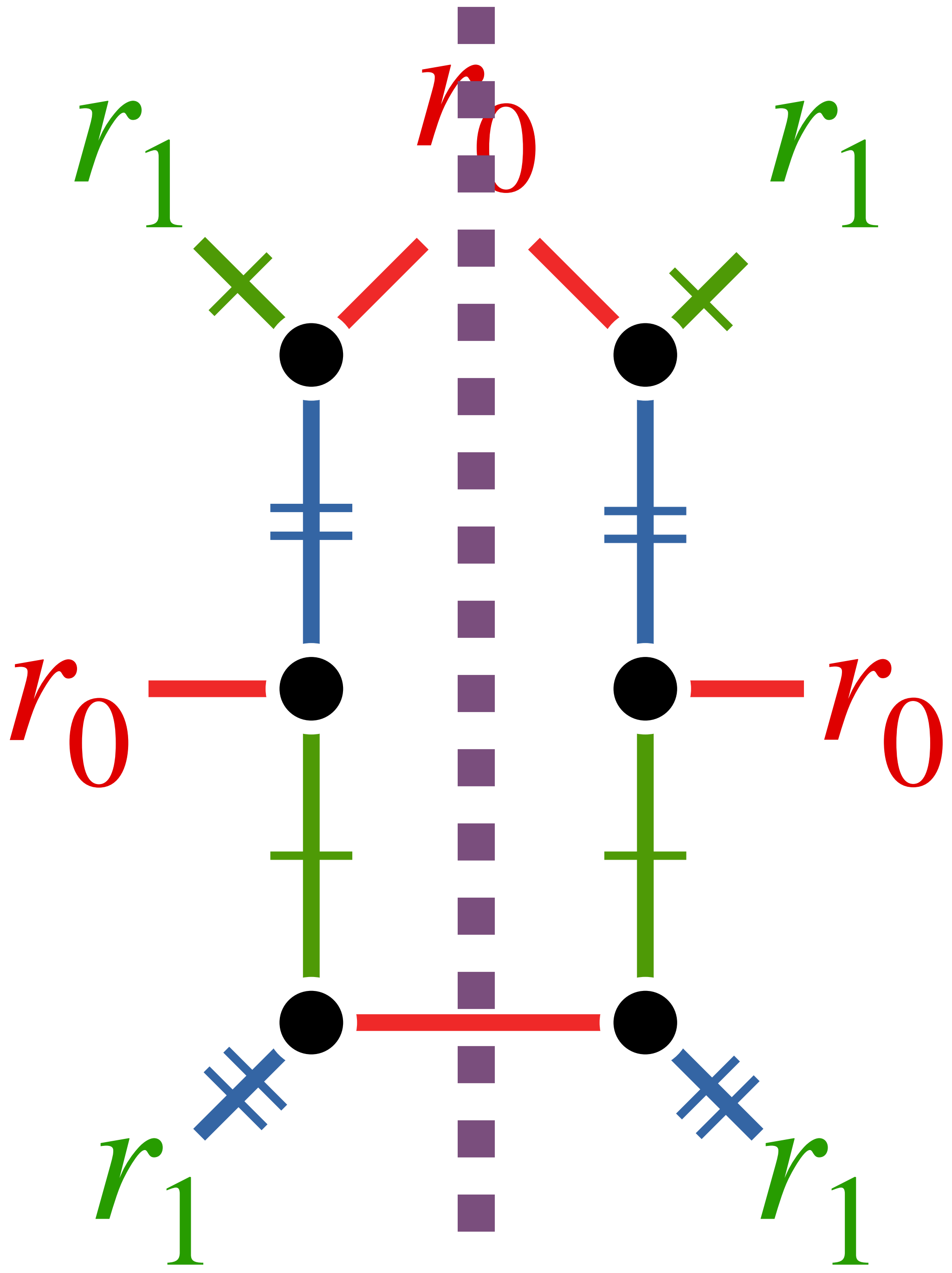
The automorphisms of $\mathscr{X} \rtimes_\eta \mathscr{Y}$ that project to id are exactly those in $\mathrm{Aut}(\mathscr{X})$.

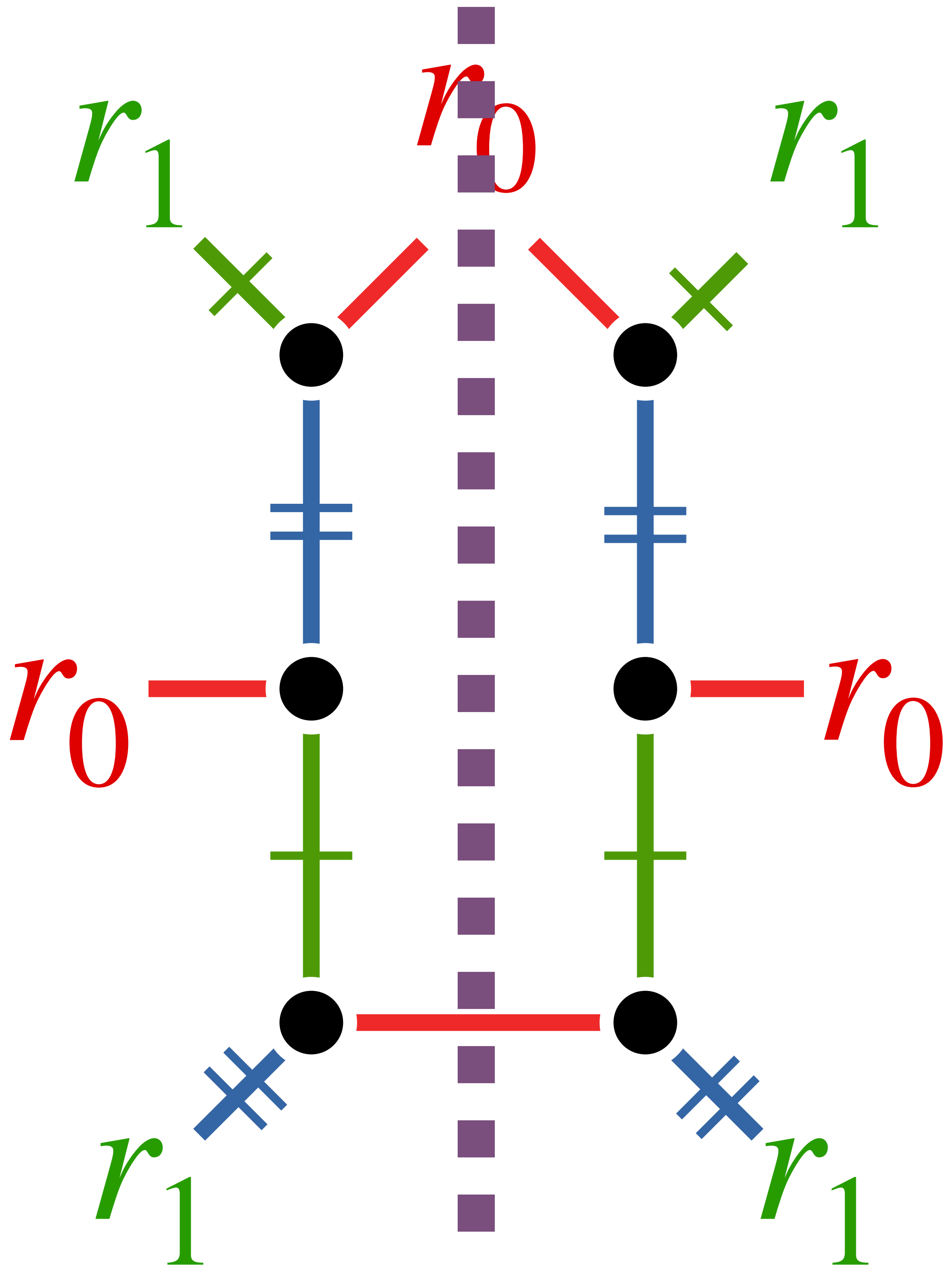
Symmetries of voltage operations



Symmetries of voltage operations



Symmetries of voltage operations



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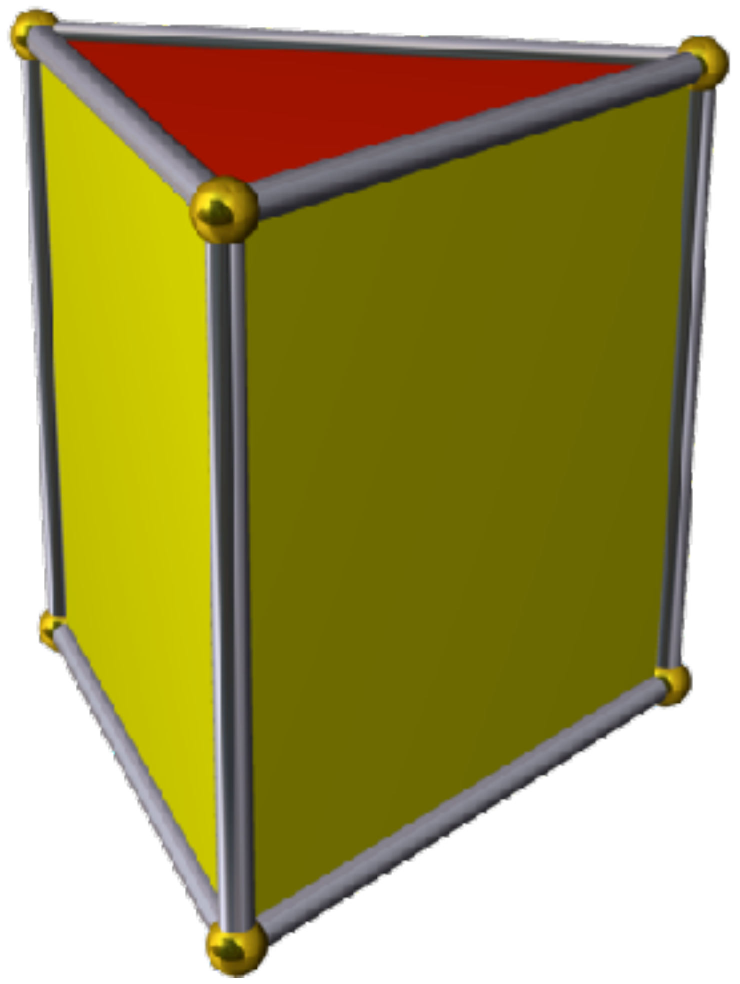
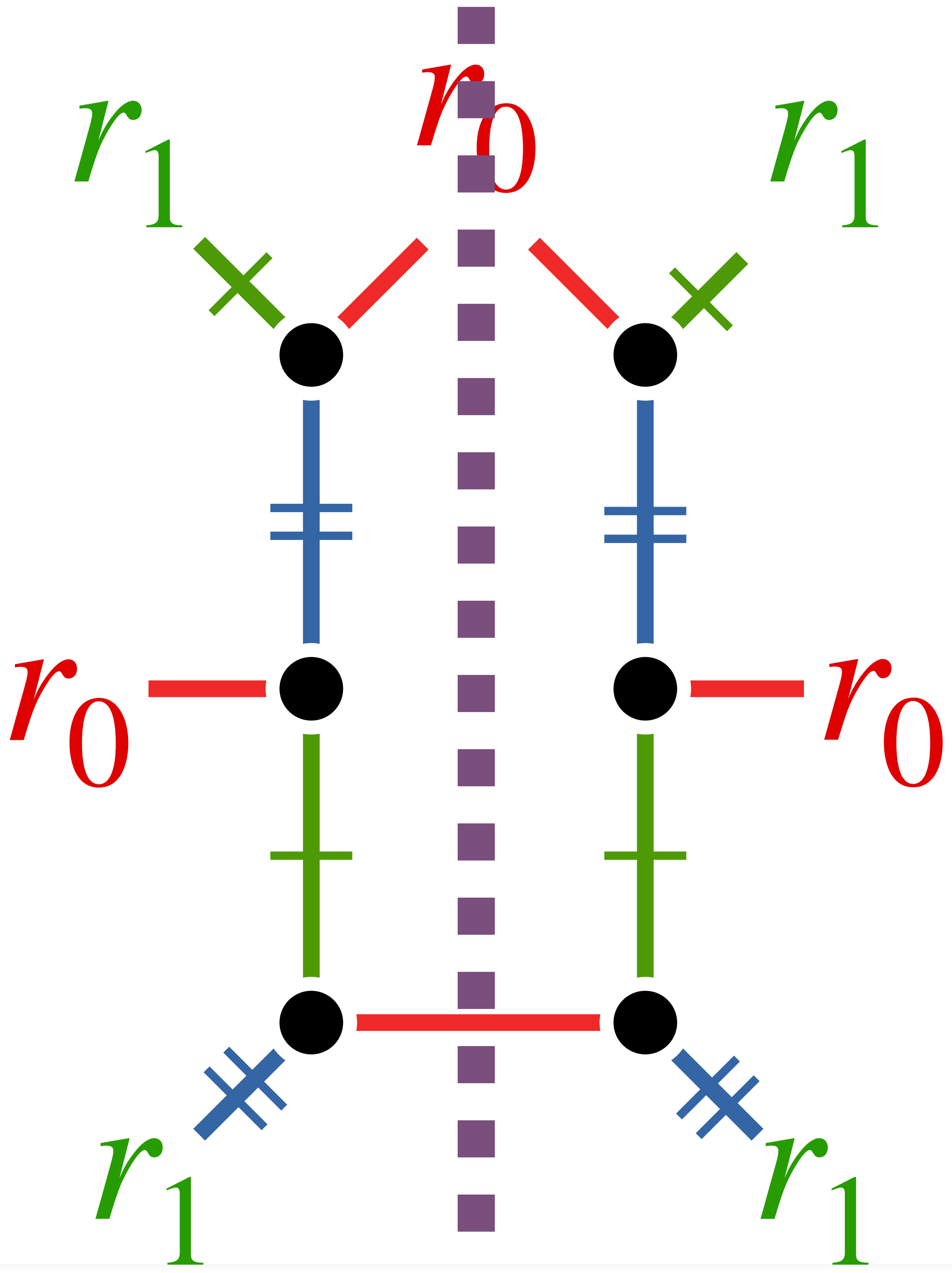
The group $\mathrm{Aut}(\mathscr{Y}, \eta) \leq \mathrm{Aut}(\mathscr{Y})$ of automorphisms of $\mathscr{Y}$ that preserve η always lifts to $\mathscr{X} \rtimes_\eta \mathscr{Y}$ and induces a group isomorphic to $\mathrm{Aut}(\mathscr{X}) \times \mathrm{Aut}(\mathscr{Y}, \eta)$.

Symmetries of voltage operations





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Symmetries of voltage operations

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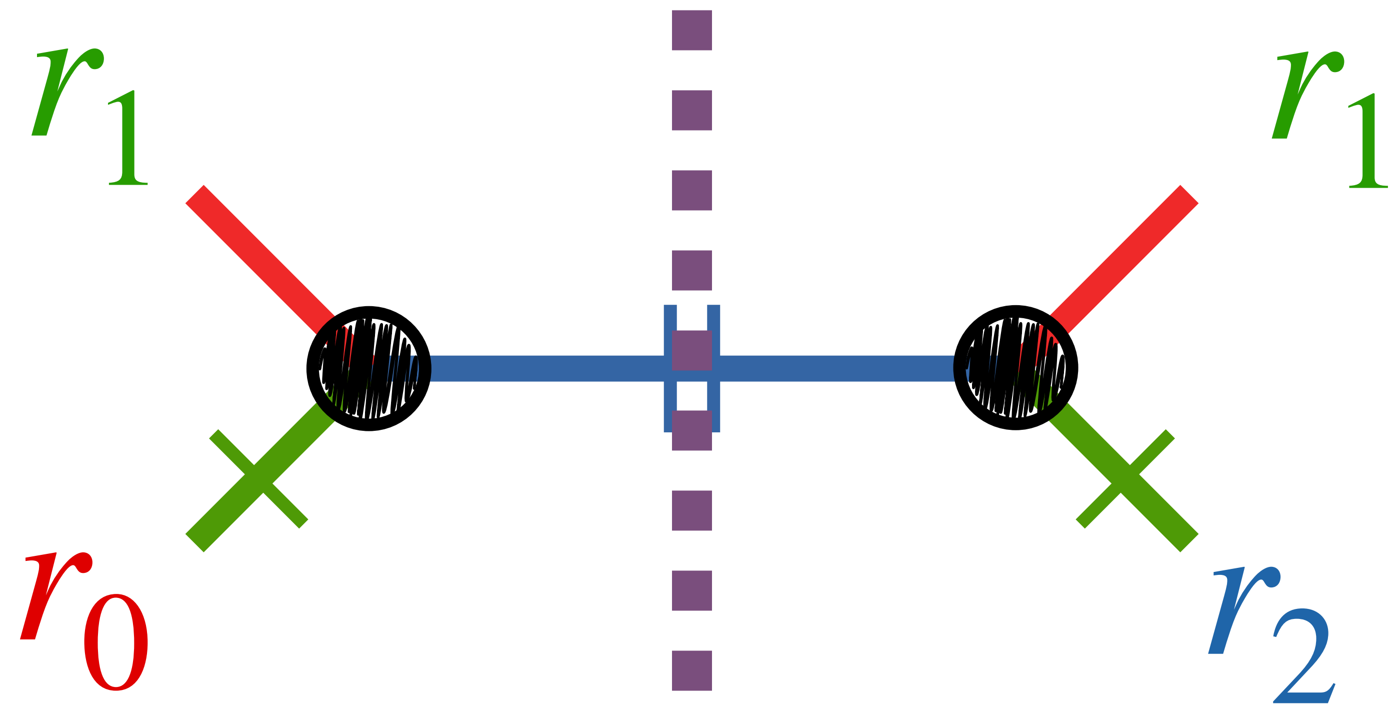
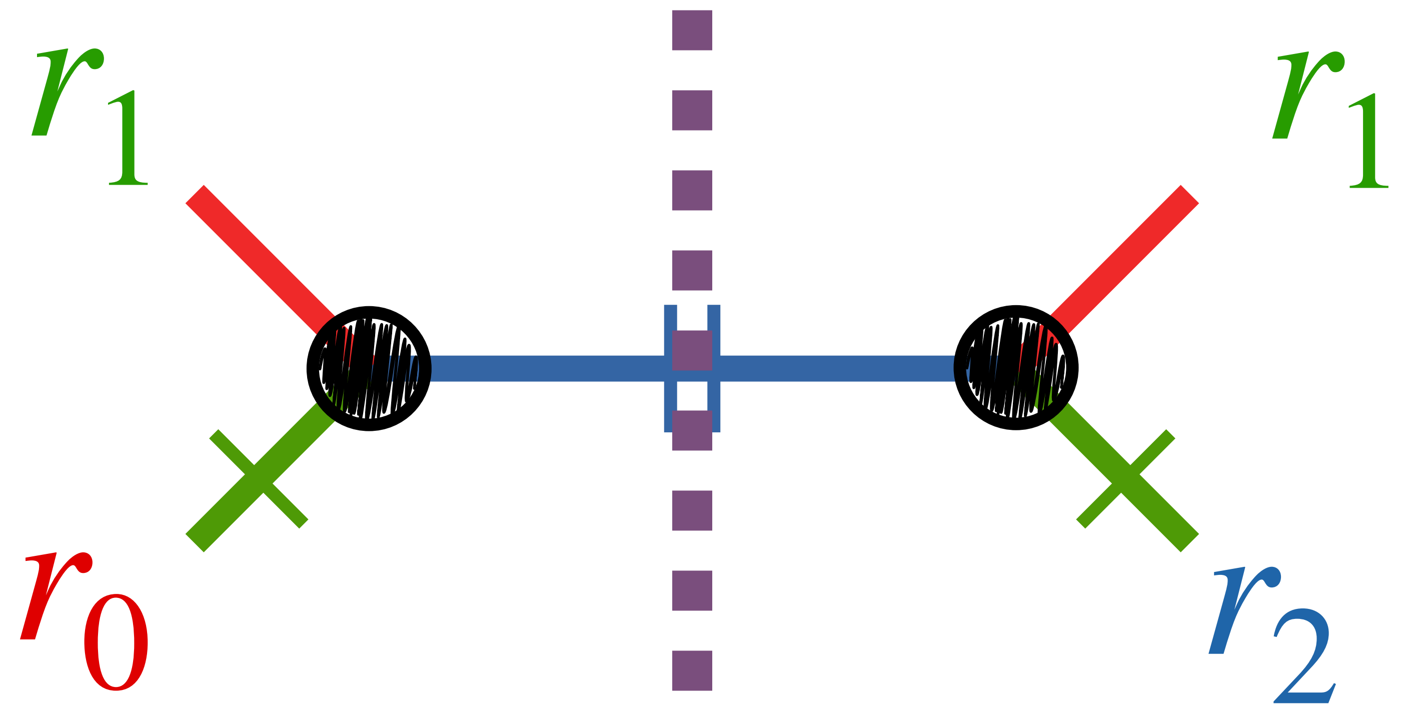


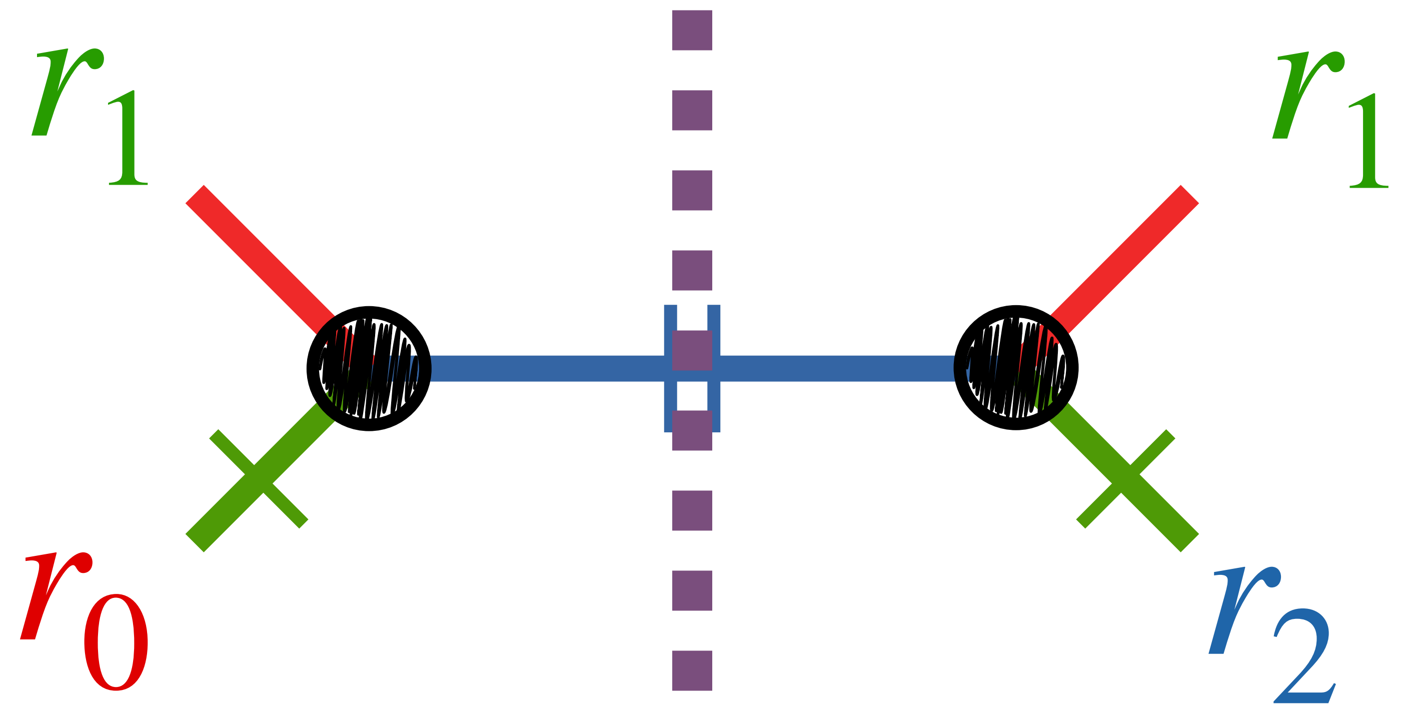
Symmetries of voltage operations



Hubard, Mochán, M. (2025)

- Such an automorphism τ induces a d-morphism $\tau^{\#}$.

Symmetries of voltage operations



Hubard, Mochán, M. (2025)

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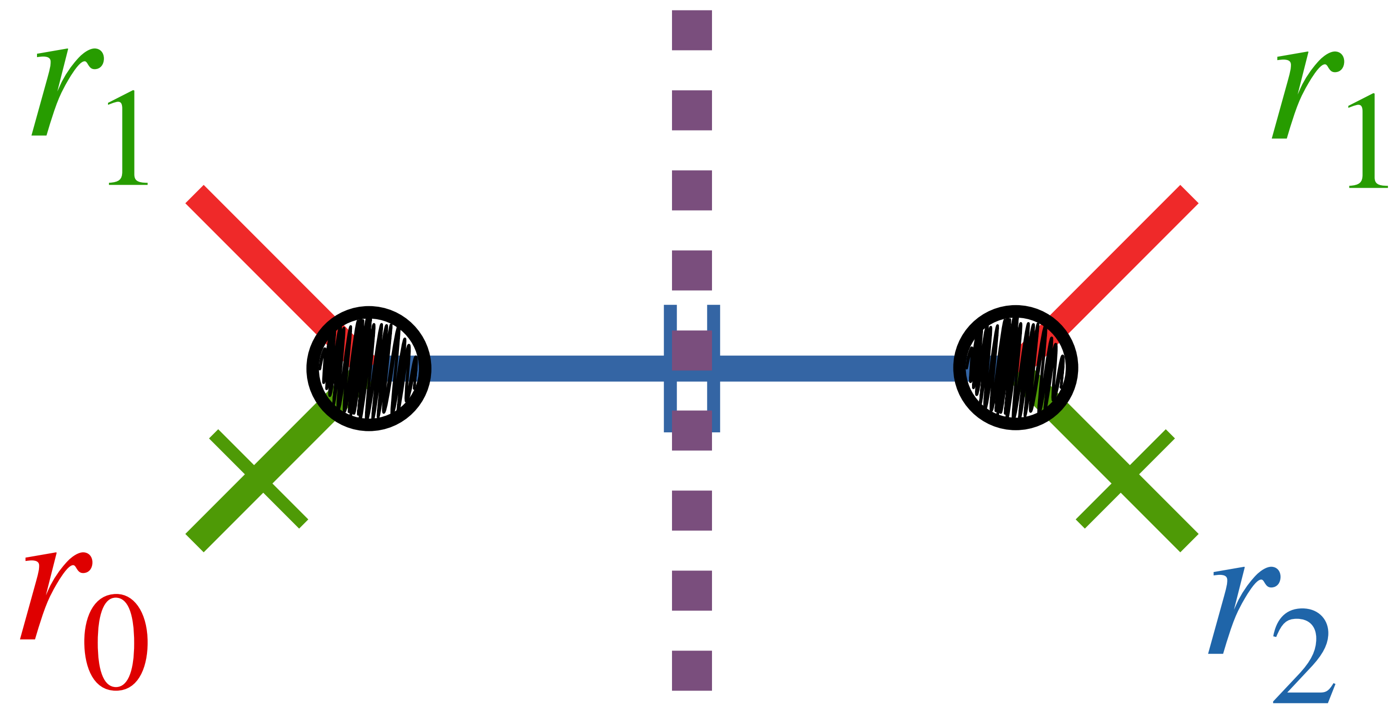
Symmetries of voltage operations



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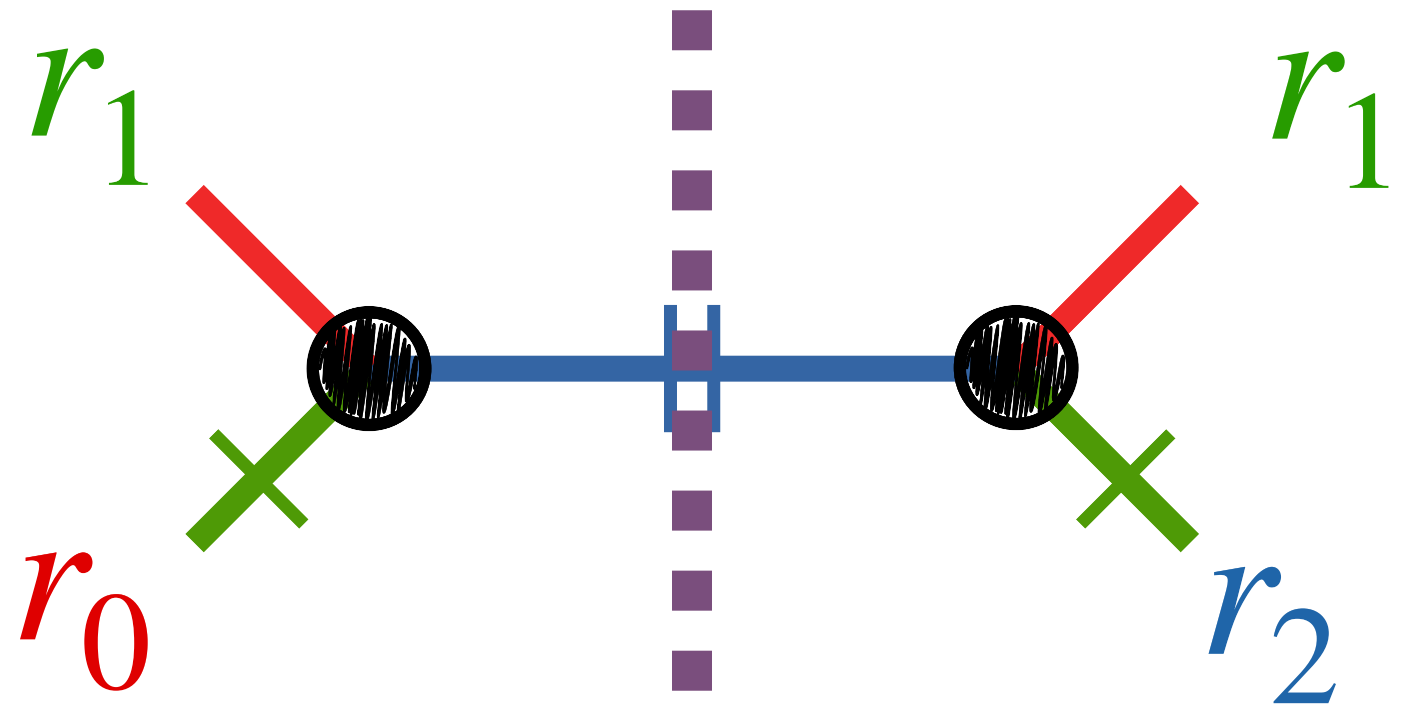
- If $\mathscr{X} \cong \mathscr{X}^{\tau^{\#}}$, then τ lifts an automorphism of $\mathscr{X} \rtimes_\eta \mathscr{Y}$.

Symmetries of voltage operations



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- If $\Gamma \leq \mathrm{Aut}(\mathscr{Y})$ is the group of such automorphisms, then Γ lifts to group $\tilde{\Gamma} \leq \mathrm{Aut}(\mathscr{X} \rtimes_\eta \mathscr{Y})$ that is a extension of $\mathrm{Aut}(\mathscr{X})$ by Γ

Symmetries of voltage operations

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Is there anything else?

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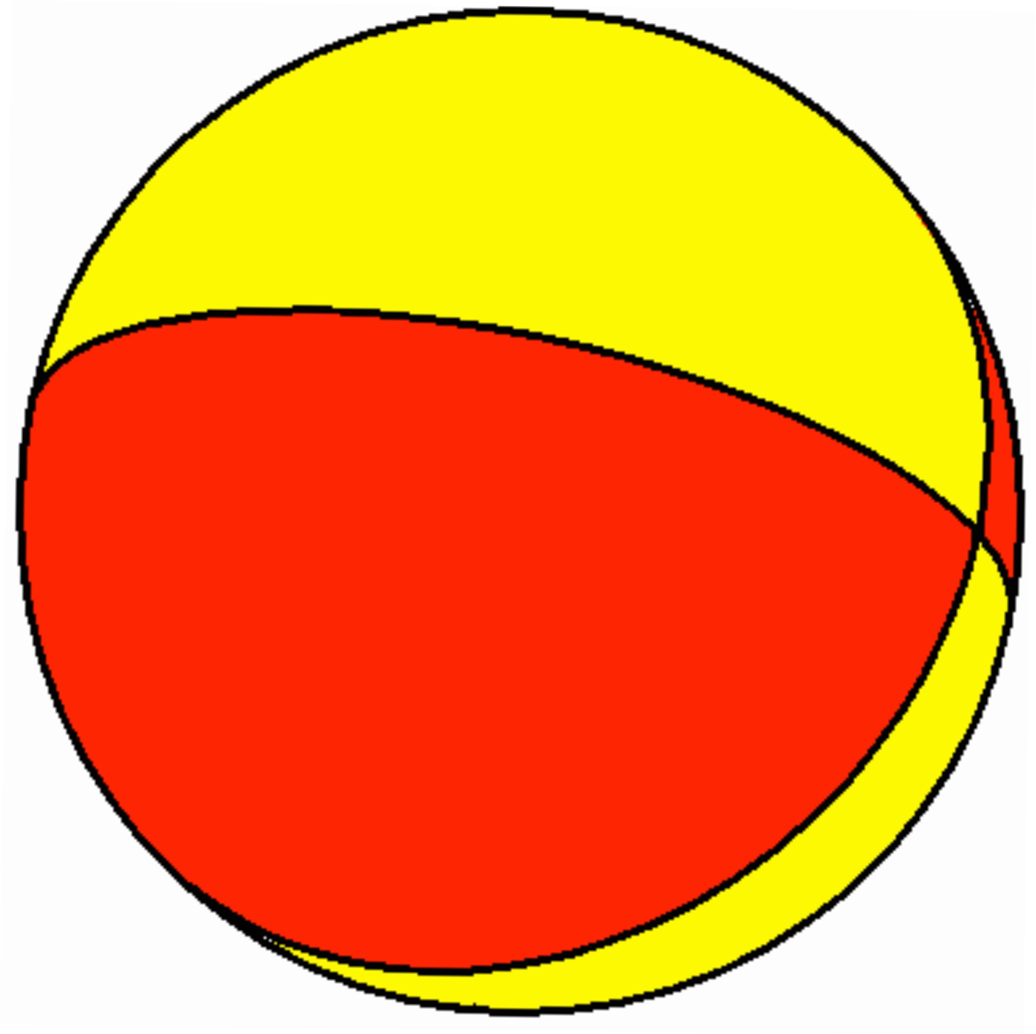

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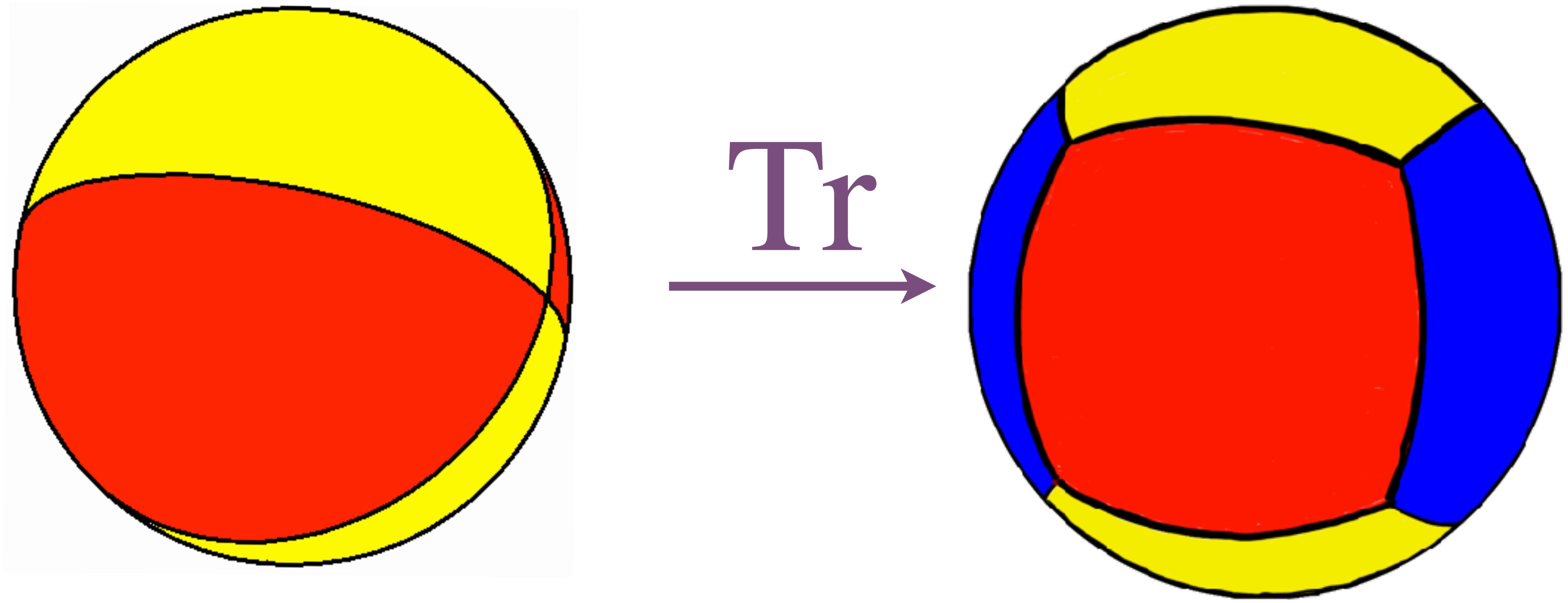

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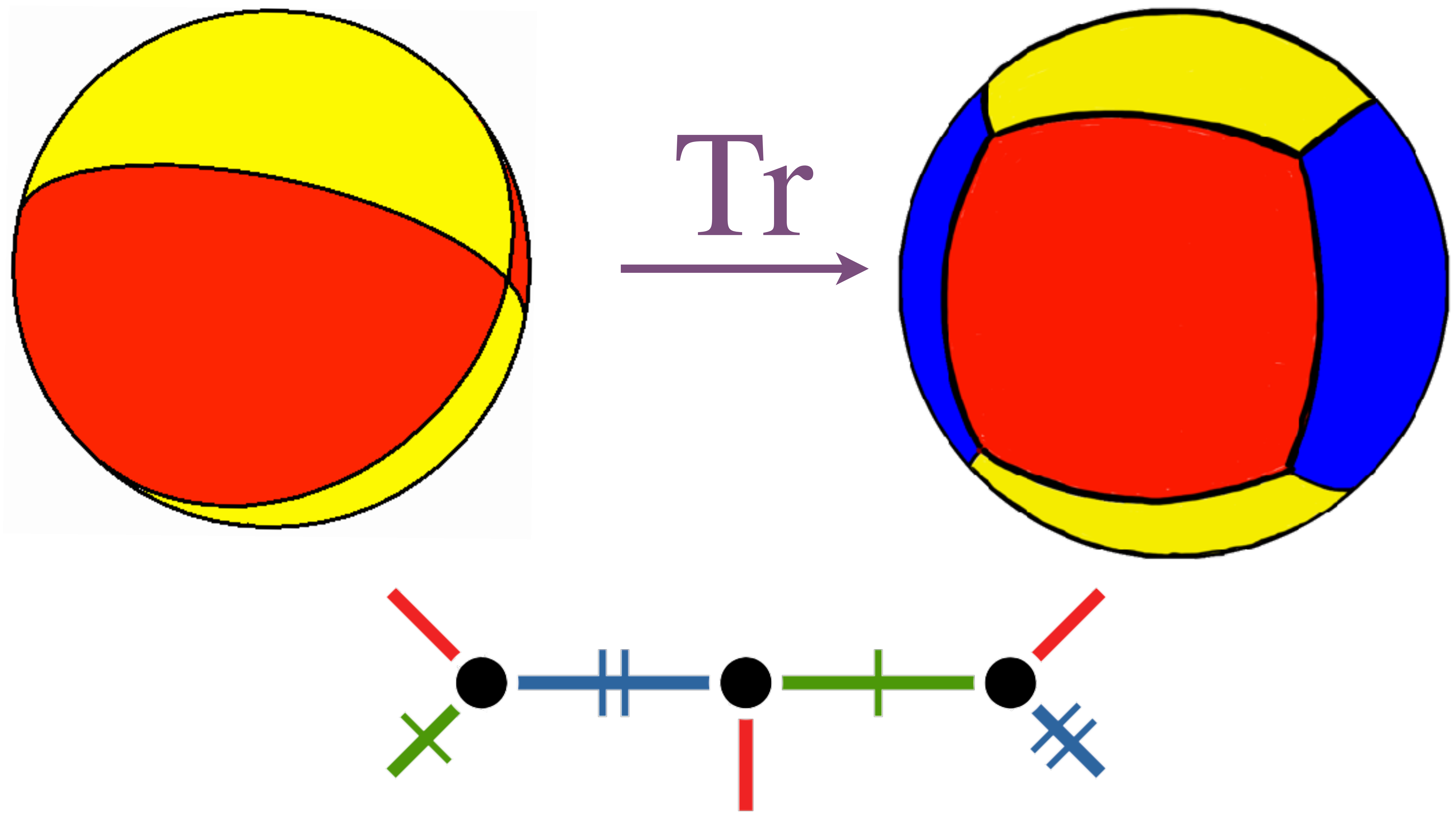

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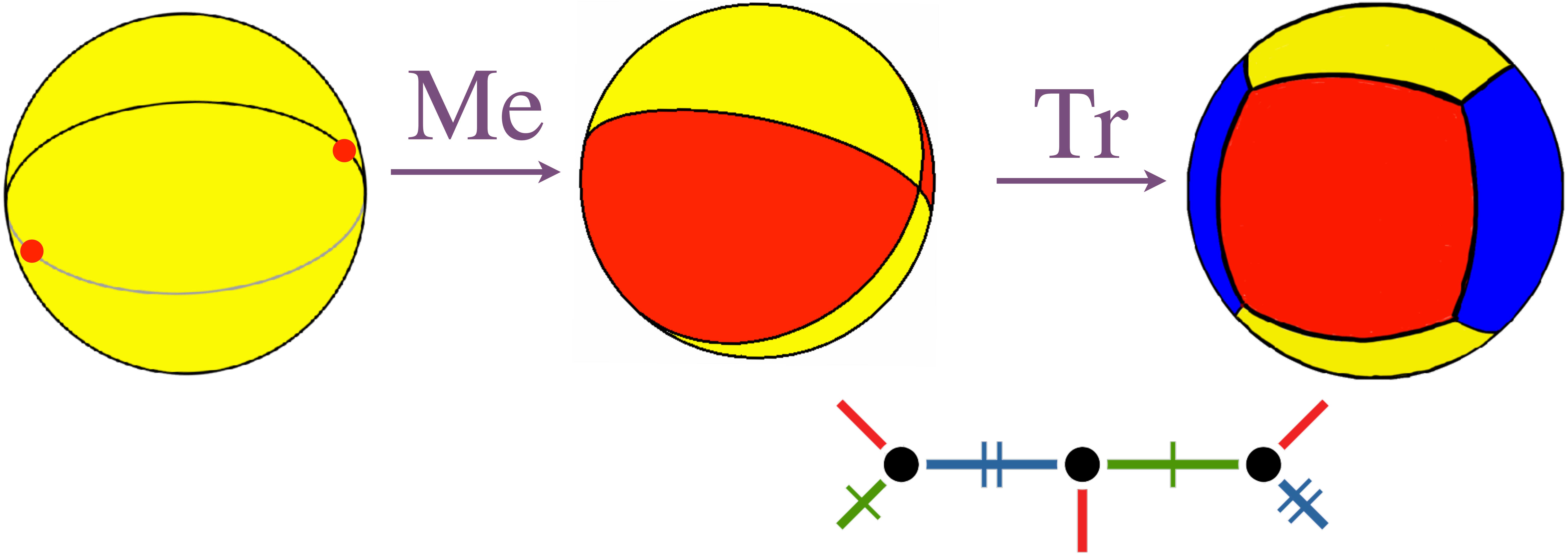

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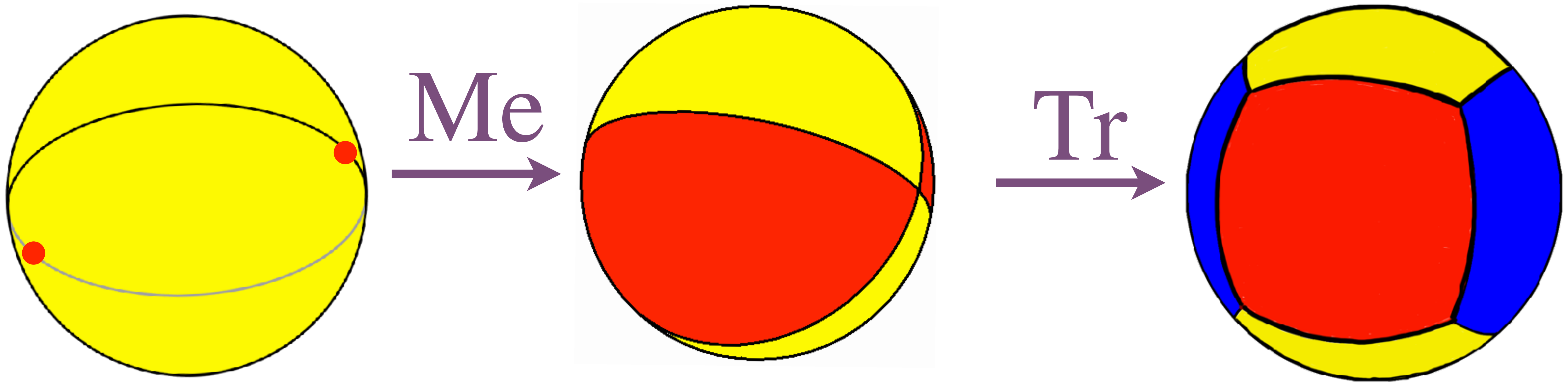

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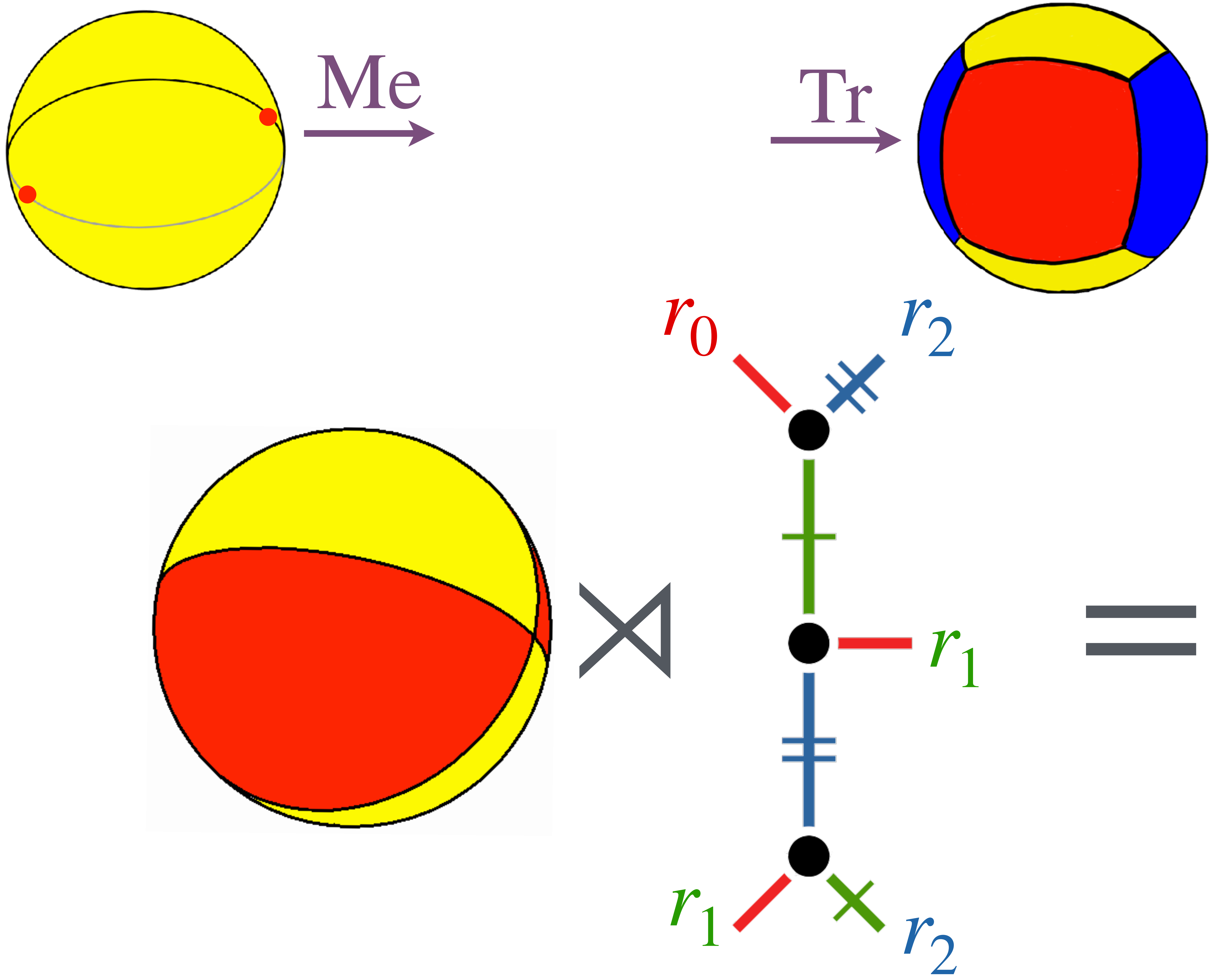

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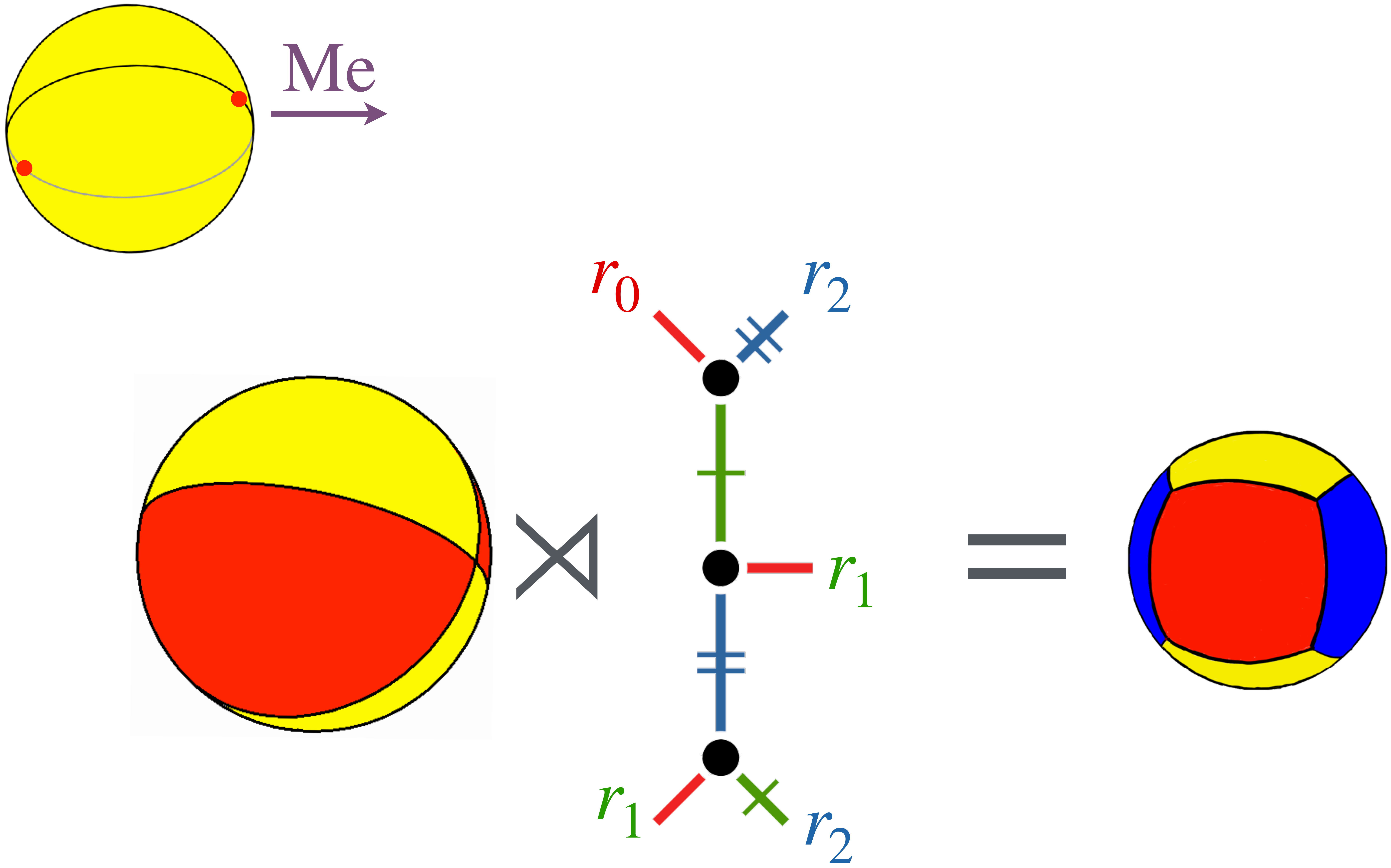

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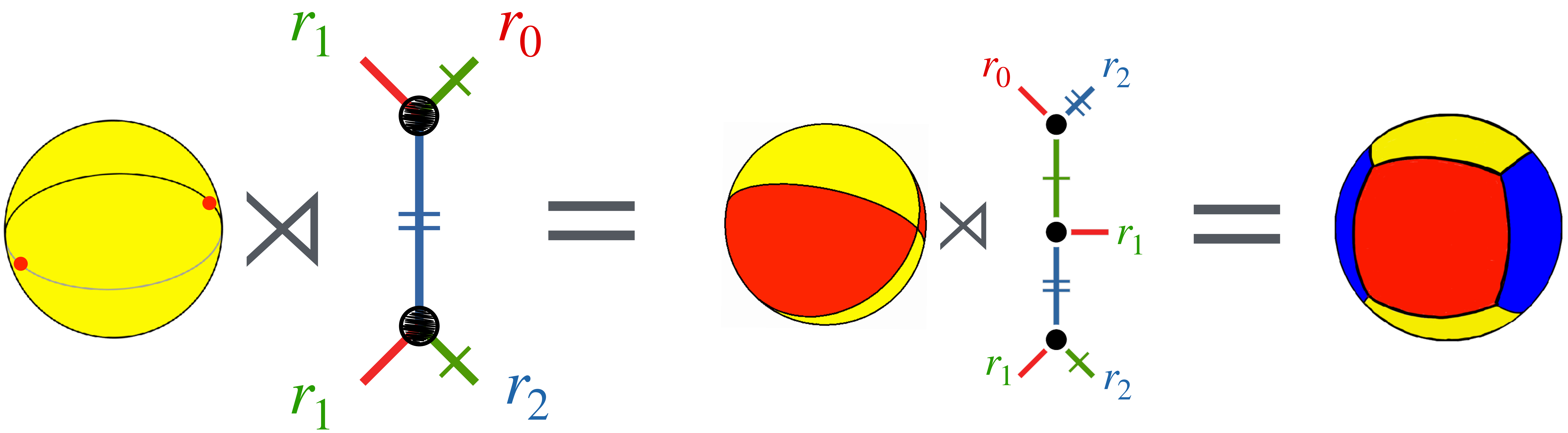

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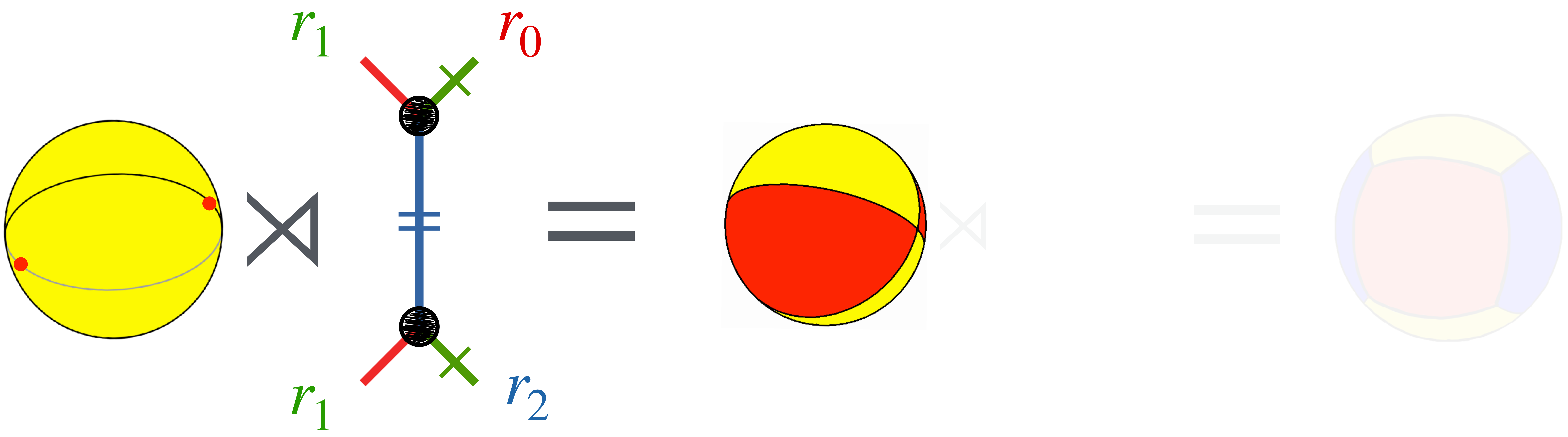
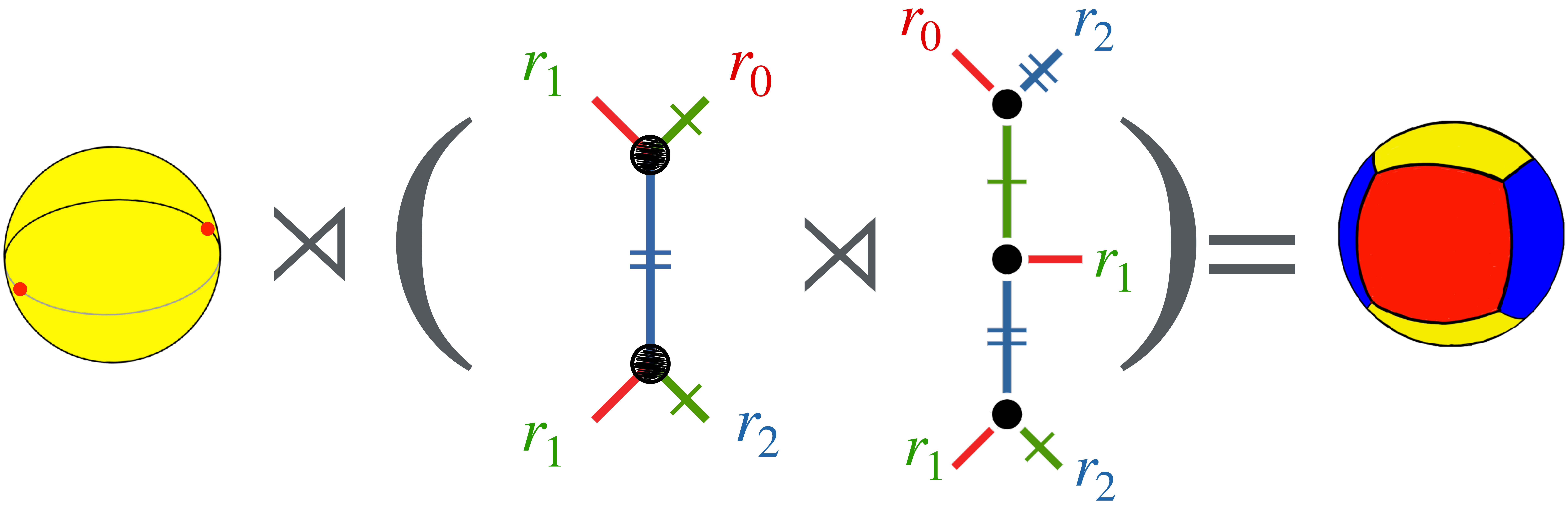


Symmetries of voltage operations



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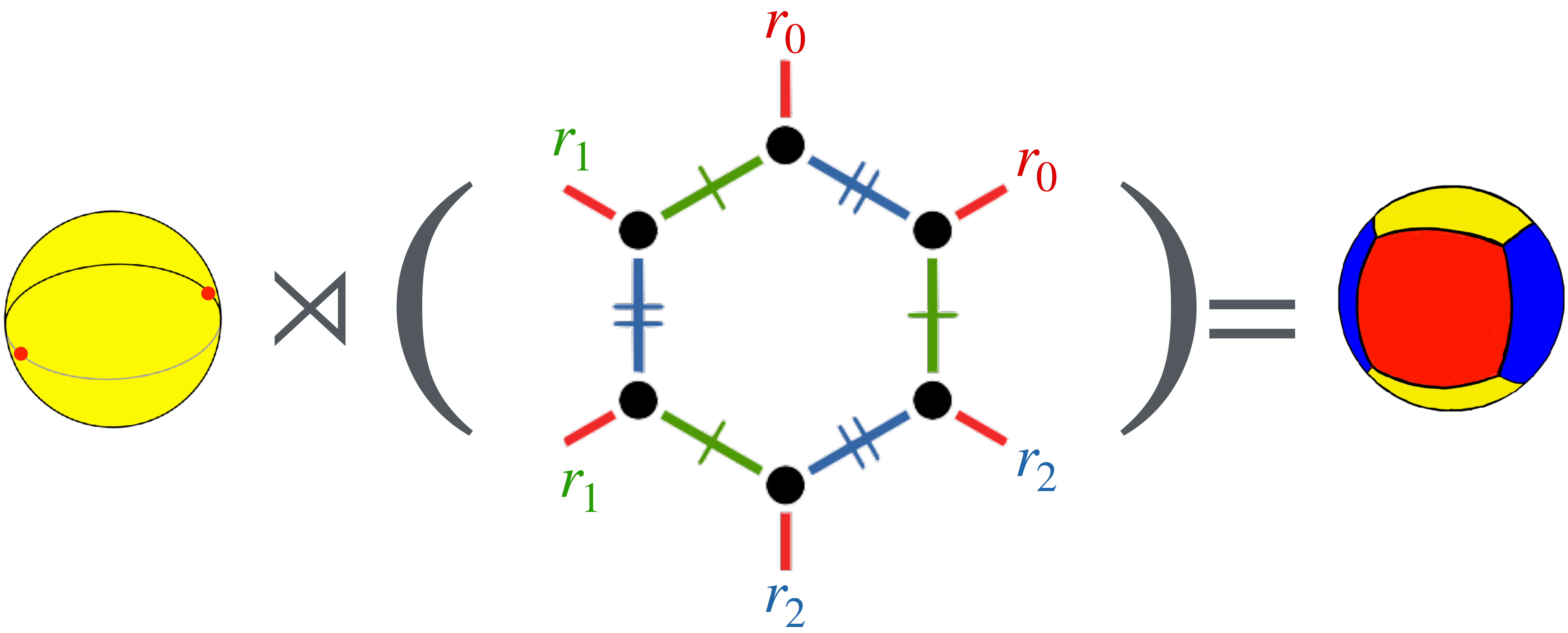
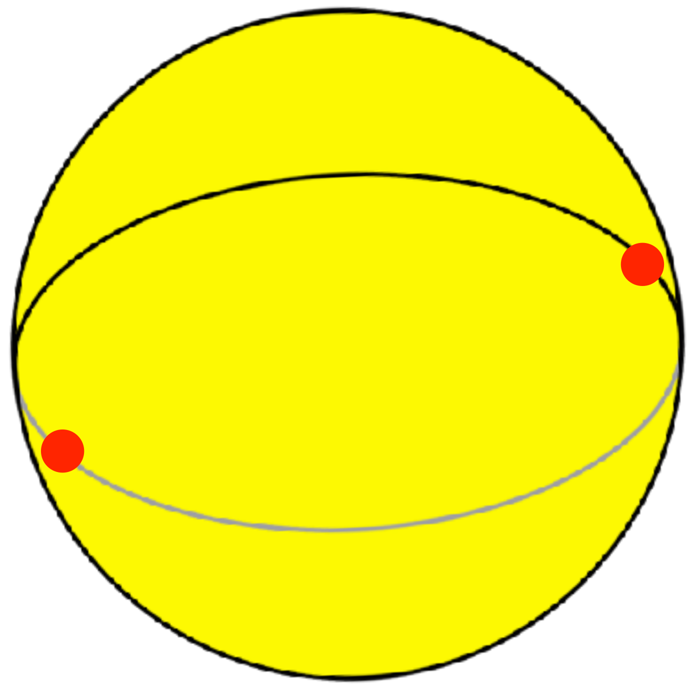


Symmetries of voltage operations



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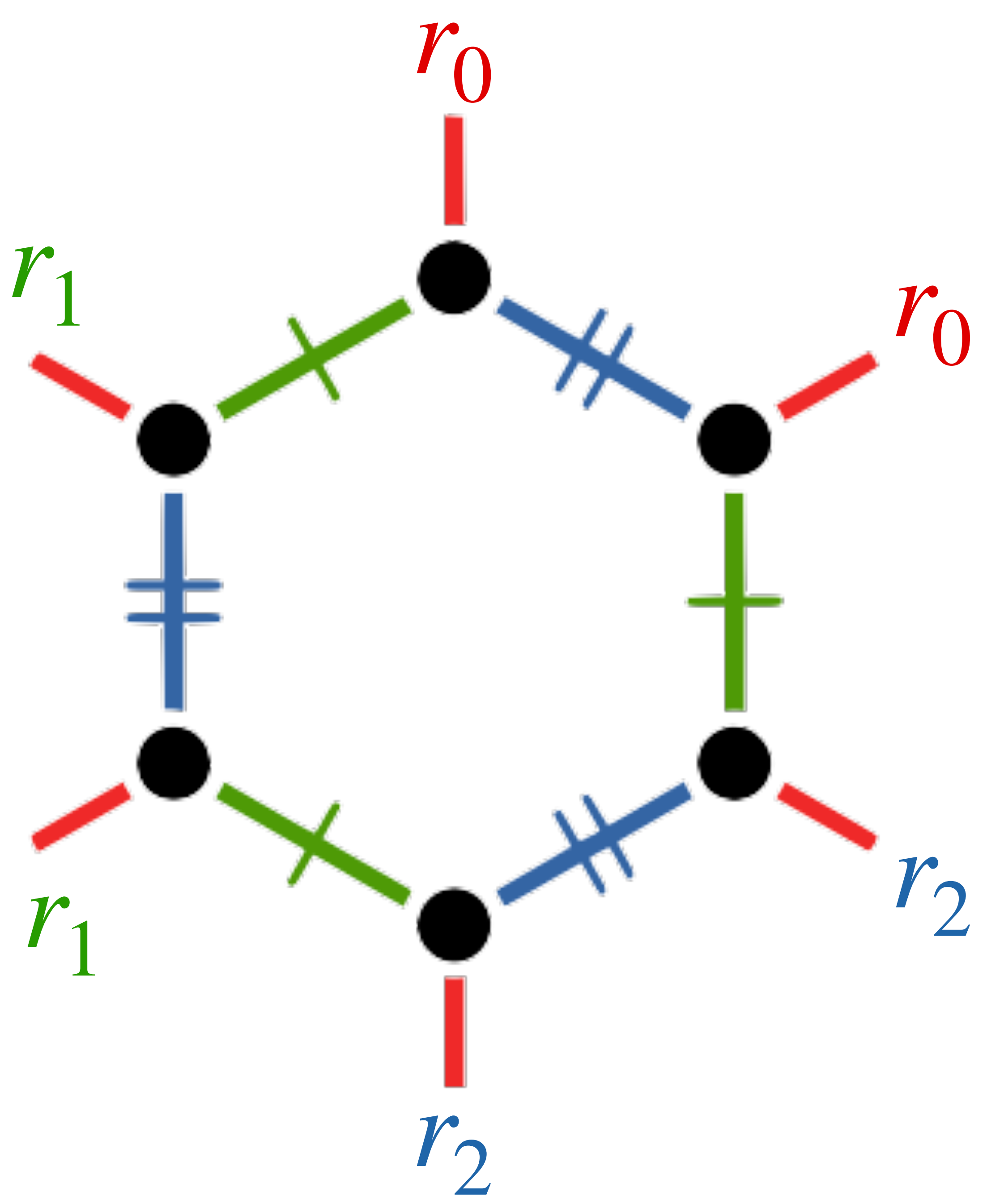

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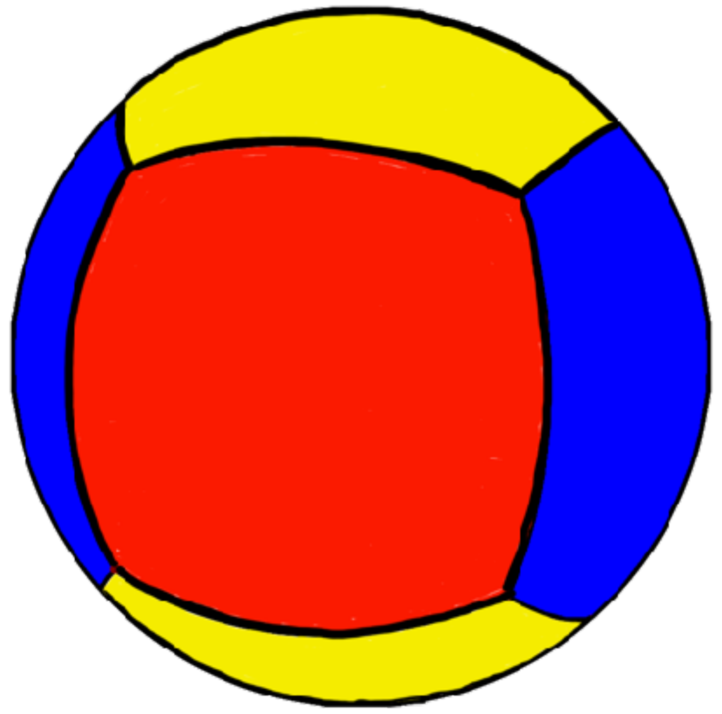
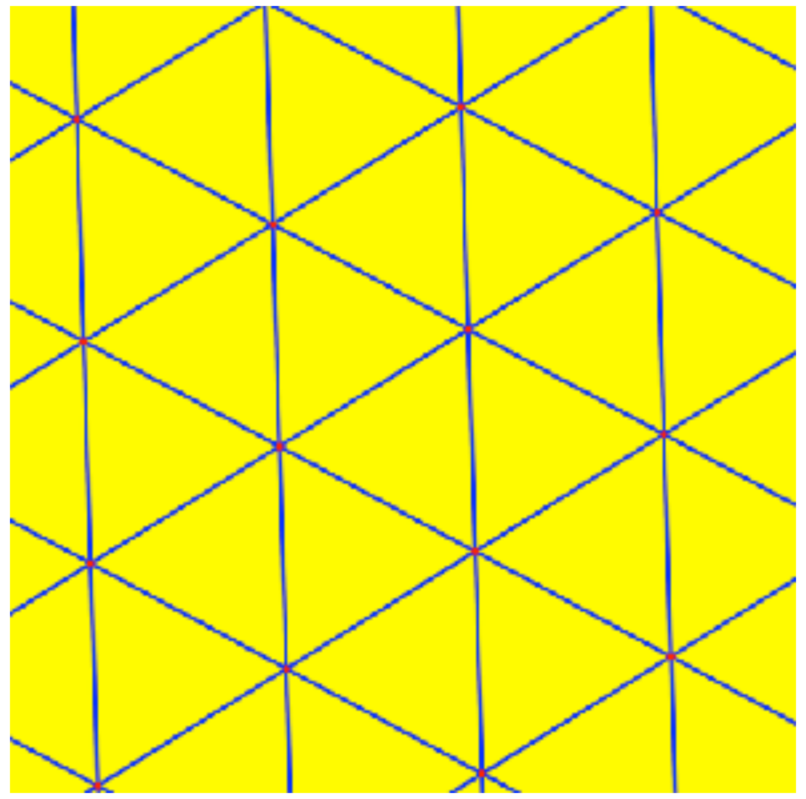


$\mathrm{Aut}(\mathscr{X}) = \mathbb{Z}_2^3$

$\mathrm{Aut}(\mathscr{Y}) = S_3$

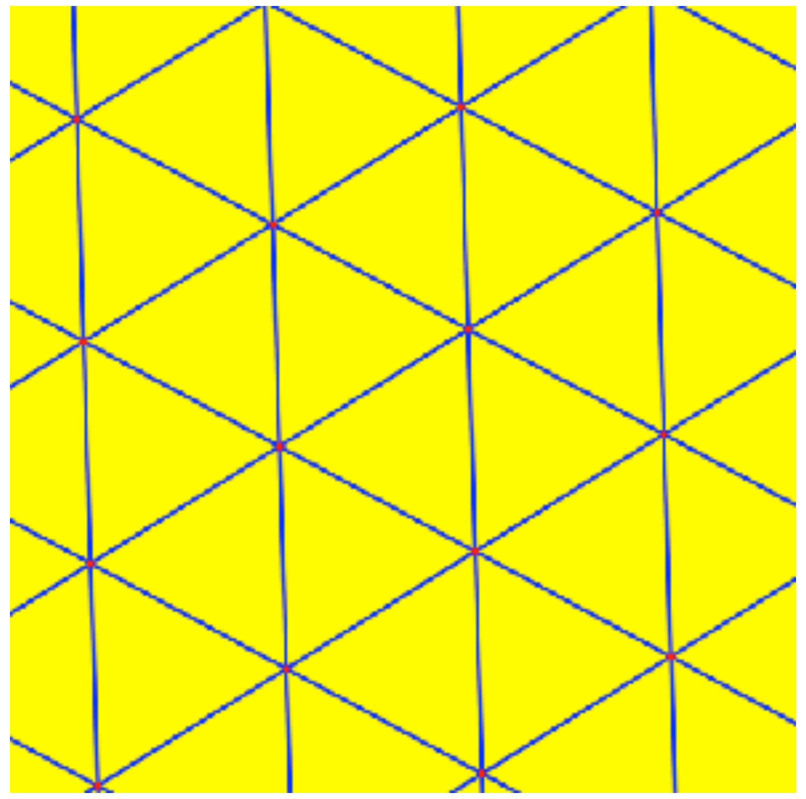
$\mathrm{Aut}(\mathscr{X} \rtimes_\eta \mathscr{Y}) = \mathbb{Z}_2^3 \rtimes S_3$

Symmetries of voltage operations

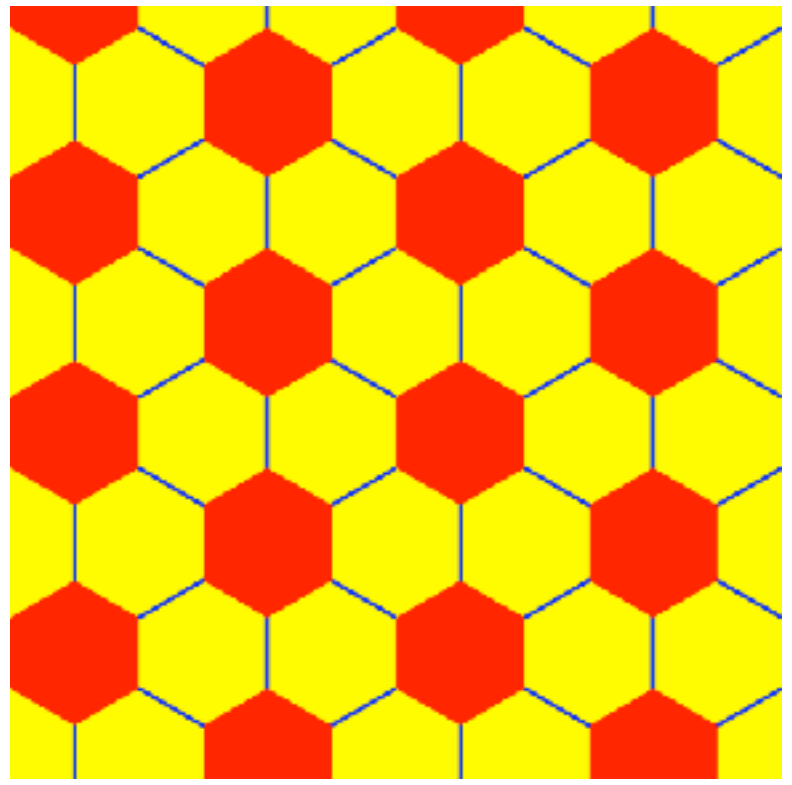


{3,6}

Symmetries of voltage operations



Tr

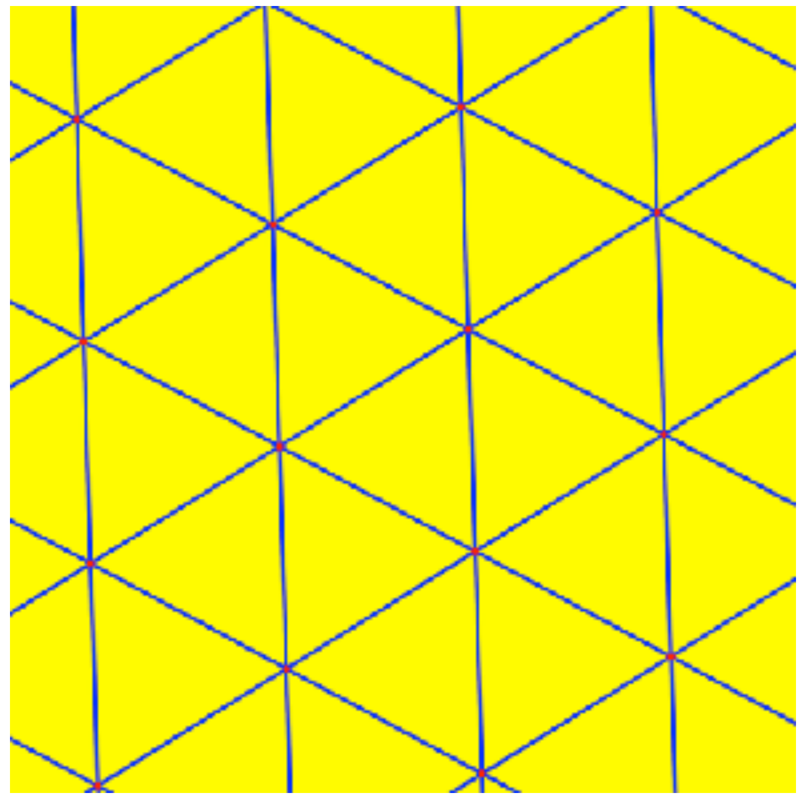


$\{3,6\}$

$\mathrm{Tr}\{3,6\} = \{6,3\}$

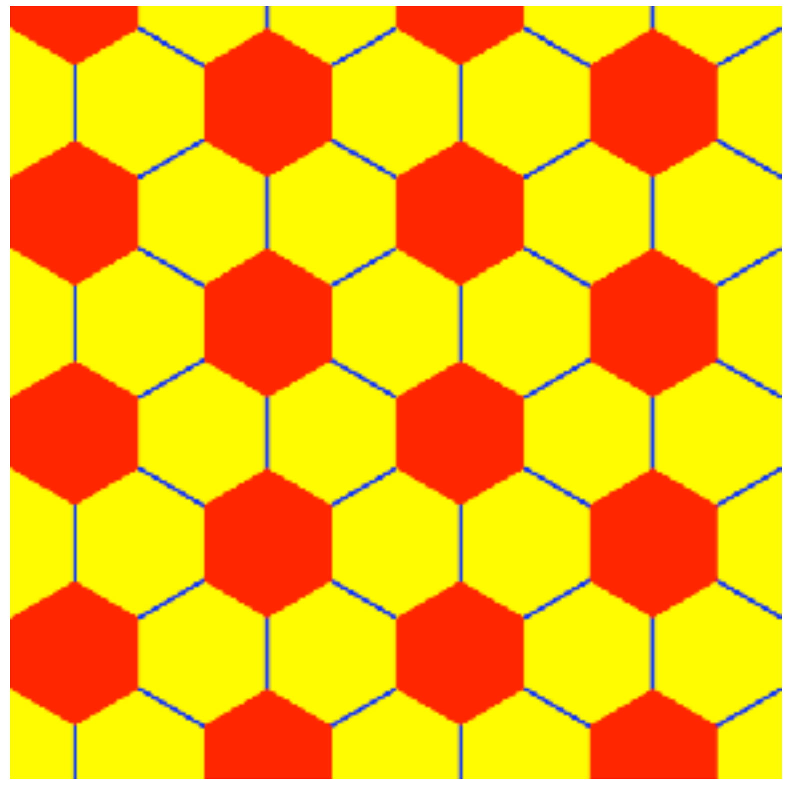
Symmetries of voltage operations

Me $\longrightarrow$



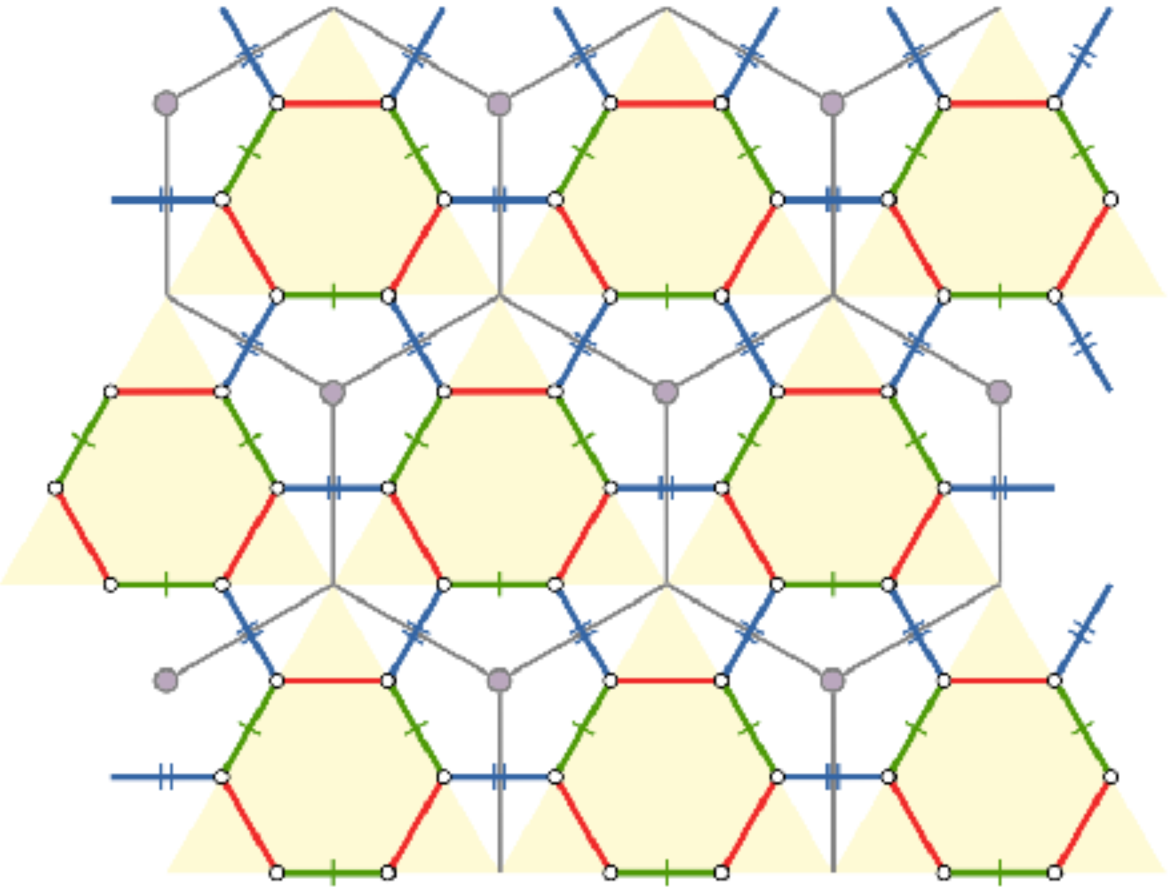
{3,6}

Tr $\longrightarrow$



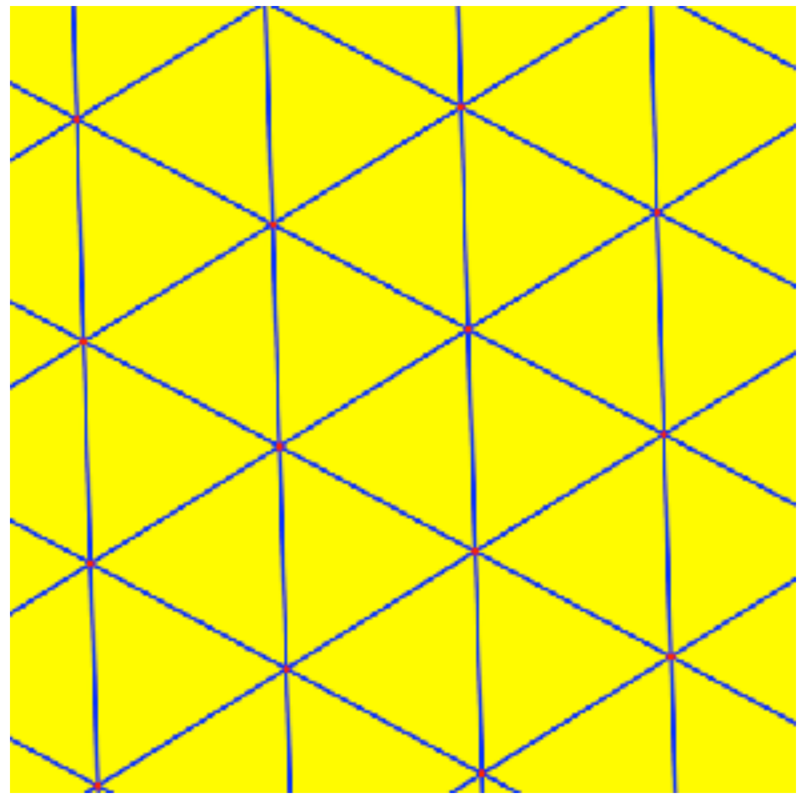
Tr{3,6} = {6,3}

Symmetries of voltage operations



(3,3,3)

Me



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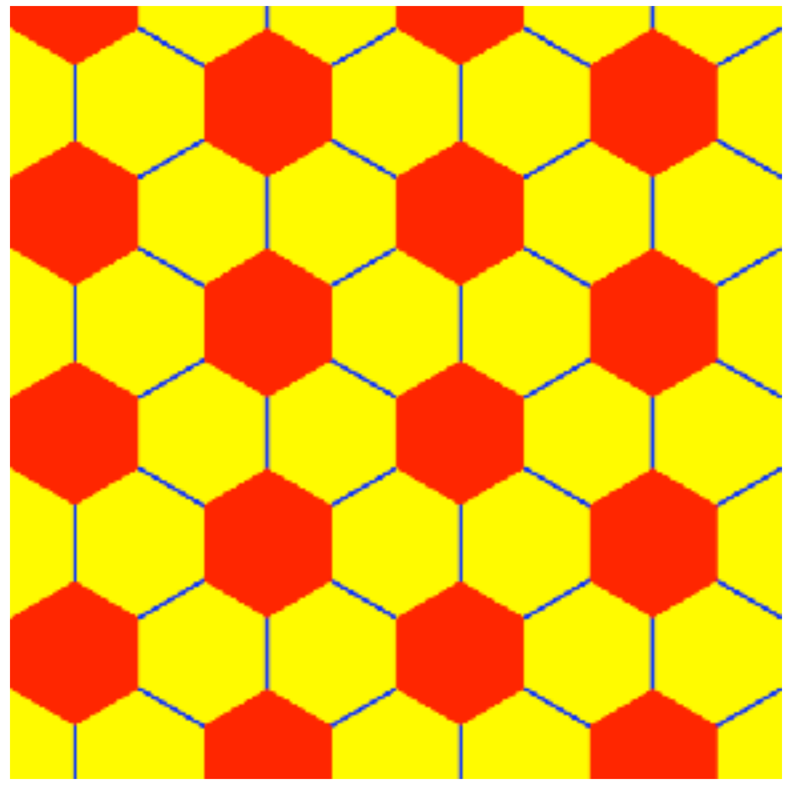


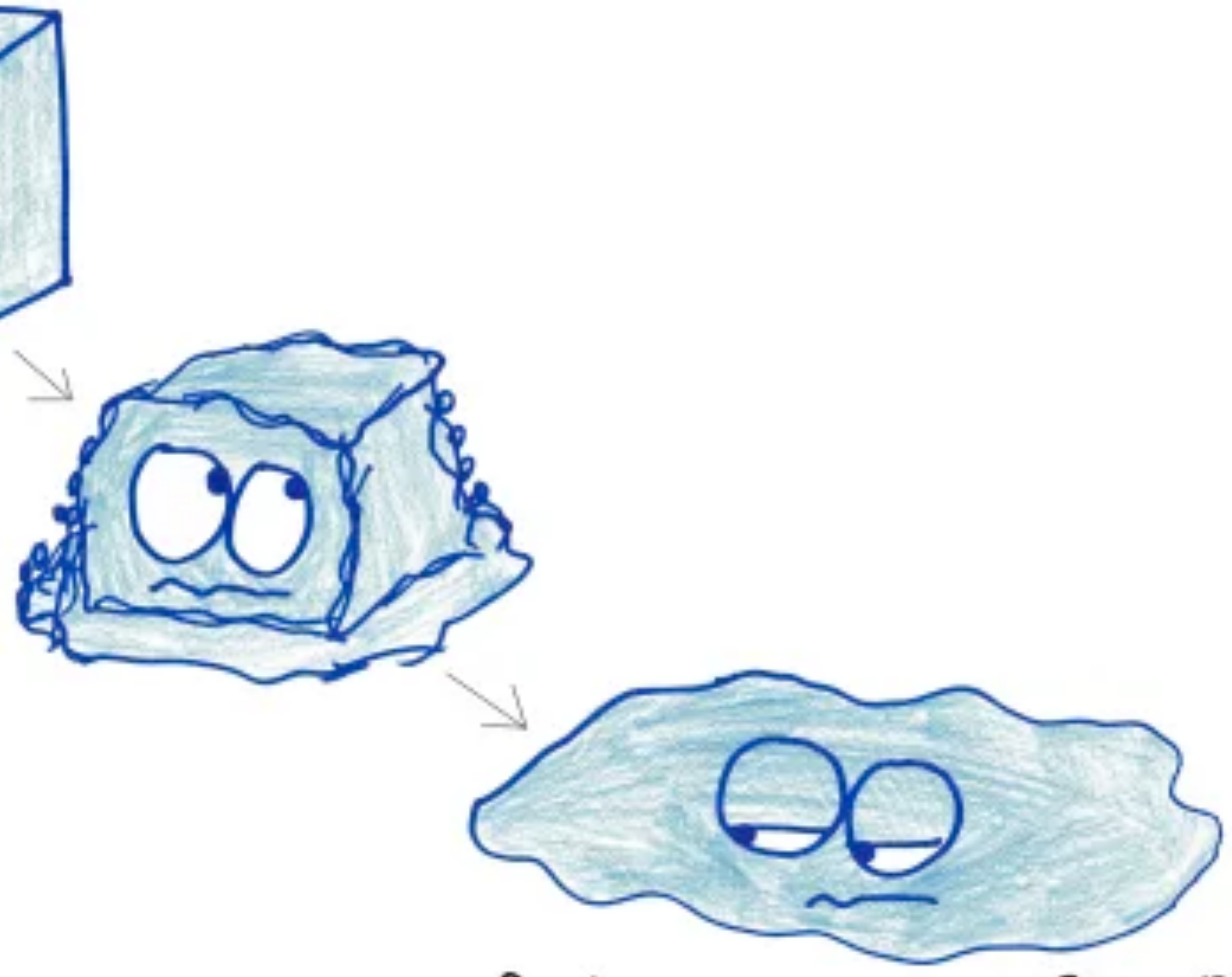
Tr{3,6} = {6,3}

Symmetries of voltage operations

Conjecture

If $\mathscr{X}$ cannot be regarded as $\mathscr{X} \cong \mathscr{W} \rtimes_\theta \mathscr{Z}$ for some suitable choice of $(\mathscr{Z}, \theta)$, then all the automorphisms of $\mathscr{X} \rtimes_\eta \mathscr{Y}$ are lifts of automorphisms of $\mathscr{Y}$.

Not a voltage operation:



Thank you for your attention